2) $\frac{5x-2}{x^2-4} = \frac{5x-2}{(x-2)(x+2)}$

$$\frac{5x-2}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

بضرب المعادلة في $(x-2)(x+2)$

$$5x-2 = A(x+2) + B(x-2)$$

* عندما $x=2 \Leftarrow$

$$8 = 4A$$

$$\boxed{A=2}$$

* عندما $x=-2 \Leftarrow$

$$-12 = -4B$$

$$\boxed{B=3}$$

نعكس الكسر ليصبح

$$\frac{5x-2}{x^2-4} = \frac{2}{x-2} + \frac{3}{x+2}$$

$$\int \frac{5x-2}{x^2-4}\,dx = \int \frac{2}{x-2}\,dx + \int \frac{3}{x+2}\,dx$$

$$= 2\ln|x-2| + 3\ln|x+2| + c$$

4)

$$\frac{3x}{x^2-3x-4} = \frac{3x}{(x-4)(x+1)}$$

$$\frac{3x}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$3x = A(x+1) + B(x-4)$$

$12 = 5A$ ← $x=4$

$A = \frac{12}{5}$

$-3 = -5B$ ← $x=-1$

$B = \frac{3}{5}$

$$\int \frac{3x}{x^2-3x-4}\,dx = \frac{12}{5}\int \frac{1}{x-4}\,dx + \frac{3}{5}\int \frac{1}{x+1}\,dx$$

$$= \frac{12}{5}\ln|x-4| + \frac{3}{5}\ln|x+1| + c$$

6) $\dfrac{3x+8}{x^3+5x^2+6x} = \dfrac{3x+8}{x(x^2+5x+6)}$

$$\frac{3x+8}{x(x+3)(x+2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x+2}$$

$$3x+8 = A(x+3)(x+2) + Bx(x+2) + Cx(x+3)$$

$8 = 6A \Rightarrow A = \frac{8}{6} = \frac{4}{3}$ ← $x=0$

$2 = -2C \Rightarrow C = -1$ ← $x=-2$

← $x=1$

$$11 = \frac{4}{3}(4)(3) + B(3) + 4(-1)$$

$11 = 16 + 3B - 4 \Rightarrow 3B = -1 \quad B = \frac{-1}{3}$

$$\int \text{الدالة} = \int \left(\frac{4/3}{x} - \frac{1/3}{x+3} - \frac{1}{x+2}\right) dx$$

$$= \frac{4}{3}\ln|x| - \frac{1}{3}\ln|x+3| - \ln|x+2| + c$$

10) $$\frac{4x-5}{x^3-3x^2} = \frac{4x-5}{x^2(x-3)}$$

$$\frac{4x-5}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$4x-5 = Ax(x-3) + B(x-3) + Cx^2$$

$$-5 = -3B \Rightarrow B = \frac{5}{3} \quad \Leftarrow \quad x=0$$

$$7 = 9C \Rightarrow C = \frac{7}{9} \quad \Leftarrow \quad x=3$$

$$11 = 4A + \frac{5}{3} + 16\left(\frac{7}{9}\right) \Leftarrow \quad x=4$$

$$A = -7/9$$

$$\int \text{السؤال} \, dx = \frac{-7}{9}\int \frac{1}{x}dx + \frac{5}{3}\int \frac{1}{x^2}dx + \frac{7}{9}\int \frac{1}{x-3}dx$$

$$= \frac{-7}{9}\ln|x| - \frac{5}{3x} + \frac{7}{9}\ln|x-3| + C$$

14) $\frac{x^3+x}{x^2-1}$

$$\begin{array}{r|l} x^2-1 & x^3+x \\ & -(x^2-x) \\ \hline & x \\ & 2x \end{array}$$

$\frac{x^3+x}{x^2-1}$ = المقسوم عليه + $\frac{\text{الباقي}}{\text{المقسوم عليه}}$ + المتبقي

$$= x + \frac{2x}{x^2-1}$$

$$\int \frac{x^3+x}{x^2-1}\,dx = \int x + \frac{2x}{x^2-1}\,dx$$

$$= \frac{x^2}{2} + \ln|x^2-1| + C$$

18) $\frac{4}{x^3-2x^2+4x} = \frac{4}{x(x^2-2x+4)}$

$$= \frac{A}{x} + \frac{Bx+C}{x^2-2x+4}$$

$$4 = A(x^2-2x+4) + x(Bx+C)$$

$$4 = Ax^2 - 2Ax + 4A + Bx^2 + Cx$$

$$4 = x^2(A+B) + x(-2A+C) + 4A$$

نساوي المعاملات في الطرفين حسب الدرجة

$$0 = A+B \quad \text{.....(1}$$

$$0 = -2A+C \quad \text{.....(2}$$

$$4A = 4 \Rightarrow A = 1 \quad \text{.....(3}$$

نعوض قيمة A في معادلة (1)

$$0 = 1+B \rightarrow B = -1$$

نجد C من المعادلة (3)

$$0 = -2(1) + C \Rightarrow C = 2$$

$$\int \text{السؤال} \, dx$$

$$= \int \left(\frac{1}{x} + \frac{-x+2}{x^2-2x+4} \right) dx$$

$$\int \left(\frac{1}{x} + \frac{-x+1+1}{x^2-2x+4} \right) dx$$

$$\int \frac{1}{x} dx + \int \frac{-x+1}{x^2-2x+4} dx + \int \frac{1 \, dx}{x^2-2x+4}$$

$$\ln|x| + \frac{-1}{2} \int \frac{-2(-x+1)}{(x^2-2x+4)} dx$$

$$+ \int \frac{1}{x^2-2x+1+3} dx$$

$$\ln|x| - \frac{1}{2} \ln|x^2-2x+4|$$

$$+ \int \frac{1}{(x-1)^2+3} dx$$

$$\rightarrow \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-1}{\sqrt{3}} \right) + c$$

22)

$$I = \int \frac{x^2+1}{x^2-5x-6}\,dx$$

$$x^2-5x-6 \;\overline{)\; x^2+1 \;\big|\; 1}$$

$$-(x^2-5x-6)$$

$$5x+7$$

$$I = \int \left(1 + \frac{5x+7}{x^2-5x-6}\right) dx$$

$$= \int \left[1 + \frac{5x+7}{(x-6)(x+1)}\right] dx$$

28)

$$\int \frac{3x+7}{x^4-16}\,dx$$

$$\frac{3x+7}{(x^2-4)(x^2+4)} = \frac{3x+7}{(x-2)(x+2)(x^2+4)}$$

$$\frac{3x+7}{(x^2-4)(x^2+4)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$$

$$3x+7 = A(x+2)(x^2+4) + B(x-2)(x^2+4) + (Cx+D)(x-2)(x+2)$$

ابحث C ، B ، A

$$A = \frac{13}{32} \qquad B = \frac{-1}{32}$$

$$C = \frac{-3}{8} \qquad D = \frac{7}{8}$$

36) $\int \frac{2e^x}{e^{3x}+e^x}\,dx$

$$\frac{2e^x}{e^x(e^{2x}+1)} = \frac{2}{e^{2x}+1}$$

$u = e^x$

$du = e^x dx$

$dx = \frac{du}{e^x}$

$$\int \frac{2}{u^2+1} \cdot \frac{du}{e^x}$$

$$\int \frac{2}{(u^2+1)\cdot u}\,du$$

$$\frac{2}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1}$$

$$2 = A(u^2+1) + (Bu+C)u$$

$$2 = Au^2 + A + Bu^2 + cu$$

$0 = A+B$, $\boxed{C=0}$ $\boxed{A=2}$

$\boxed{B=-2}$

$$\int \frac{2}{u} + \frac{-2u}{u^2+1} \, du$$

$$2\ln|u| - \ln|u^2+1| + c$$

$$2\ln|e^x| - \ln|e^{2x}+1| + c$$