



Mathematics

Trimester 3

Grade 12 General

Inferential Statistics

Math Teacher : Moussa Salem

- 1- Find area under normal distribution curves.
- 2- Find probabilities for normal distributions and find data values

The Normal Distribution

KeyConcept Characteristics of the Normal Distribution

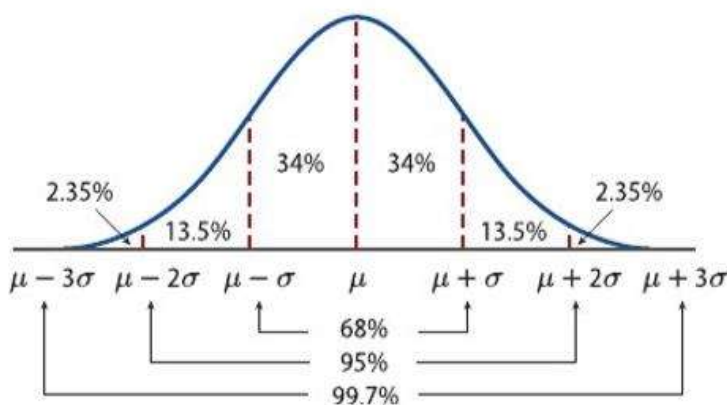
- The graph of the curve is bell-shaped and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve is continuous.
- The curve approaches, but never touches, the x-axis.
- The total area under the curve is equal to 1 or 100%.

New Vocabulary

normal distribution
empirical rule
z-value
standard normal
distribution

KeyConcept The Empirical Rule

In a normal distribution with mean μ and standard deviation σ :



- approximately 68% of the data values fall between $\mu - \sigma$ and $\mu + \sigma$.
- approximately 95% of the data values fall between $\mu - 2\sigma$ and $\mu + 2\sigma$.
- approximately 99.7% of the data values fall between $\mu - 3\sigma$ and $\mu + 3\sigma$.

<https://www.hackmath.net/en/calculator/normal-distribution>

Example 1 Use the Empirical Rule

$$z = \frac{x - \mu}{\sigma}$$

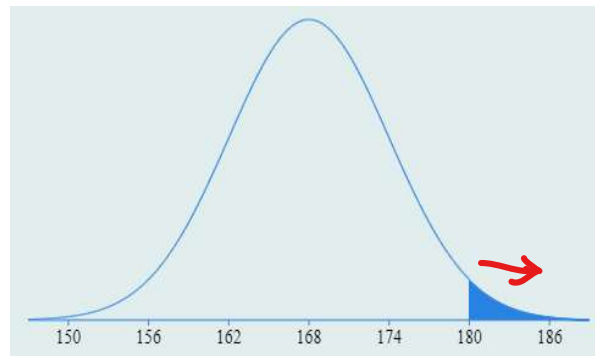
HEMPT The heights of the 880 students at Al-Sharq Secondary School are normally distributed with a mean of 168 cm and a standard deviation of 6 cm.

→ a. Approximately how many students are more than 180 cm tall?

$$\mu = 168, \sigma = 6, n = 880$$

$$\textcircled{1} z = \frac{x - \mu}{\sigma} = \frac{180 - 168}{6} = \textcircled{2}$$

$$\begin{aligned} \textcircled{2} P(X > 180) &= ? \quad 1 - 0.9772 = 0.0228 \\ &= 880 \times 0.0228 = 20 \end{aligned}$$



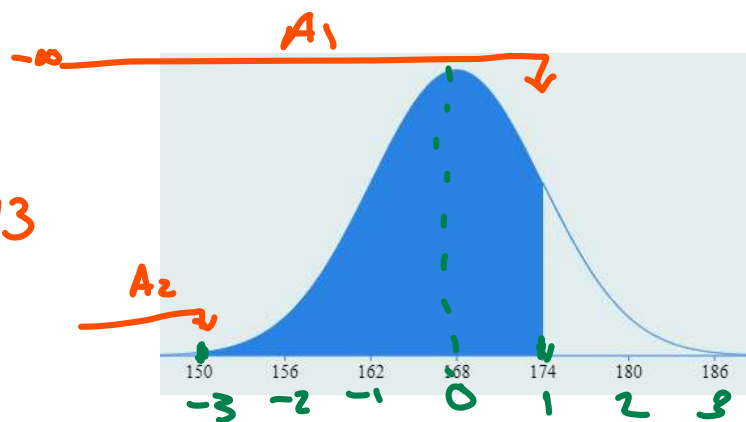
$$\mu = 168, \sigma = 6$$

b. What percent of the students are between x_1 and x_2 150 and 174 cm tall?

$$\textcircled{1} \quad z_1 = \frac{x_1 - \mu}{\sigma} = \frac{150 - 168}{6} = -3$$

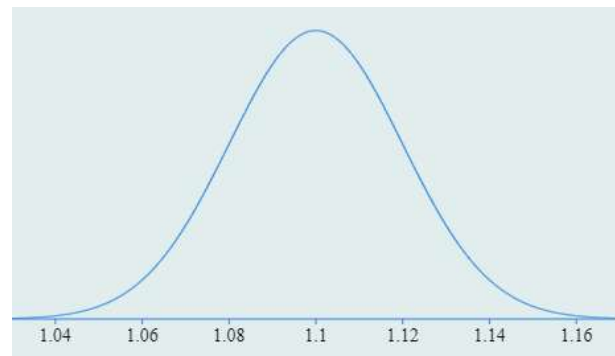
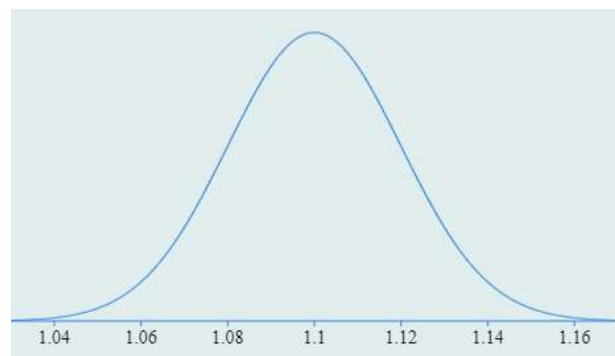
$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{174 - 168}{6} = 1$$

$$\textcircled{2} \quad P(150 < X < 174) = 0.8413 - 0.0044 = 0.8369 \approx 84\%$$



Guided Practice

1. **MANUFACTURING** A machine used to fill water bottles dispenses slightly different amounts into each bottle. Suppose the volume of water in 120 bottles is normally distributed with a mean of 1.1 liters and a standard deviation of 0.02 liter.
 - Approximately how many bottles of water are filled with less than 1.06 liters?
 - What percent of the bottles have between 1.08 and 1.14 liters?



KeyConcept Formula for z-Values

The z-value for a data value in a set of data is given by $z = \frac{X - \mu}{\sigma}$, where X is the data value, μ is the mean, and σ is the standard deviation.

Example 2 Find z-Values

Find each of the following.

2A. z if $X = 32$, $\mu = 28$, and $\sigma = 1.7$

$$z = \frac{x - \mu}{\sigma} = \frac{32 - 28}{1.7}$$

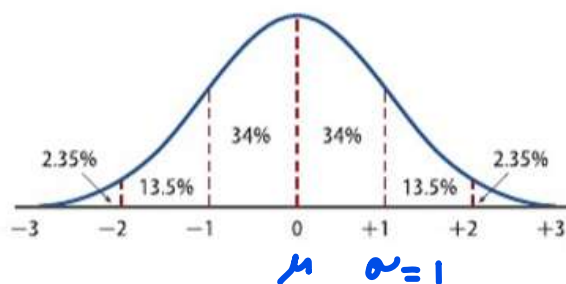
$$z = 2.35$$

2B. X if $z = 2.15$, $\mu = 39$, and $\sigma = 0.4$

$$z = \frac{x - \mu}{\sigma}$$
$$\downarrow$$
$$2.15 = \frac{x - 39}{0.4}$$
$$x - 39 = 0.86$$
$$x = 39.86$$

KeyConcept Characteristics of the Standard Normal Distribution

- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.
- The curve approaches, but never touches, the x-axis.



Using the Normal distribution table to determine the area corresponding to $z = 1.42$

- Locate the table of positive z-values.
- Locate in the first column the value 1.4 and in the first row the value 0.02.
- The area corresponding to the z-value of 1.42 is located at the intersection of the row and column, which is 0.9222.

Example 3 Use the Standard Normal Distribution

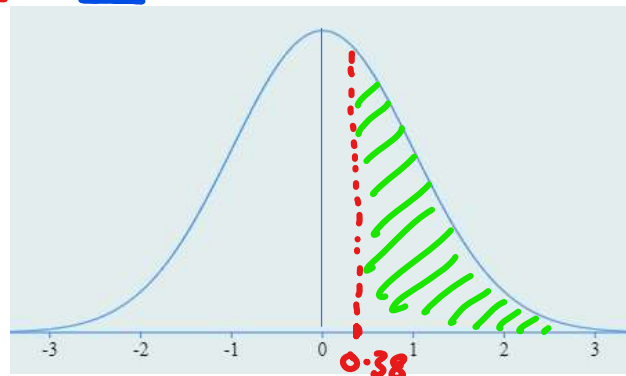
BASKETBALL The average number of points that a basketball team scored during a single season was 63 with a standard deviation of 18. If there were 15 games during the season, → find the percentage of games in which the team scored more than 70 points. Assume that the number of points is normally distributed.

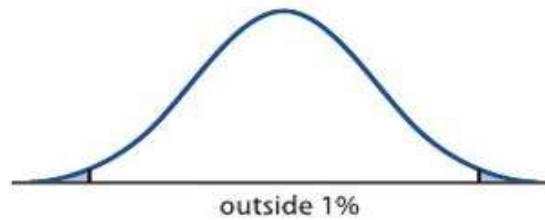
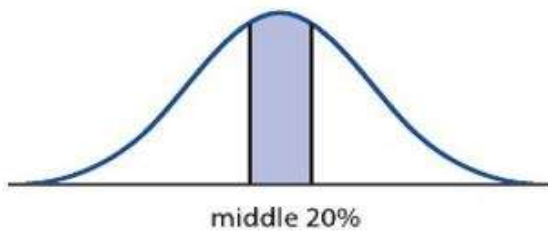
$$\mu = 63, \sigma = 18$$

$$x = 70, n = 15$$

$$\textcircled{1} z = \frac{x - \mu}{\sigma} = \frac{70 - 63}{18} = 0.38$$

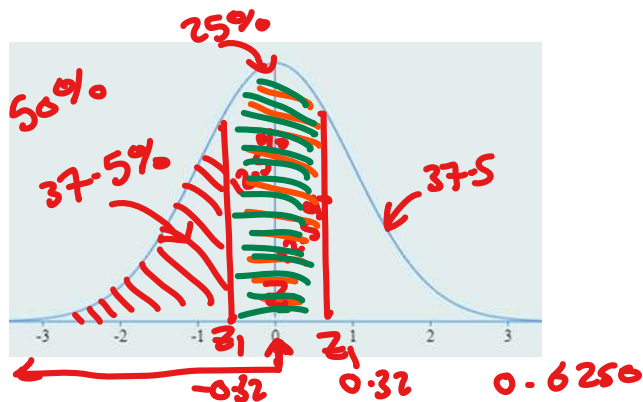
$$\textcircled{2} P(X > 70) = 1 - 0.6480 = 0.3520$$
$$= 35.20\%$$





Example 4 Find z-Values Corresponding to a Given Area

4A. the middle 25% of the data

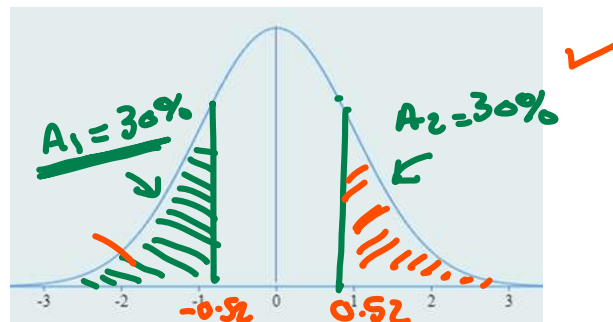


$$A_1 = 37.5\% = 0.3750$$

$$z_1 = -0.32 \Rightarrow z_2 = 0.32$$

$$-0.32 < z < 0.32$$

4B. the outside 60% of the data



$$A_1 = 30\% = 0.3000$$

$$z_1 = -0.52 \rightarrow z_2 = 0.52$$

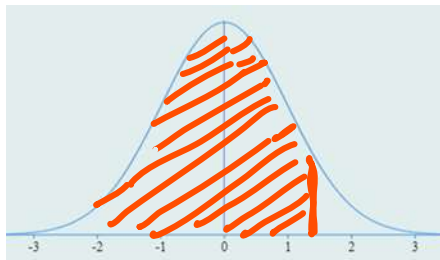
$$z_1 < -0.52 \text{ or } z_2 > 0.52$$

Probability and the Normal Distribution

Example 5 Find Probabilities

TESTING The scores on a standardized test are normally distributed with $\mu = 72$ and $\sigma = 11$. Find each probability and use a graphing calculator to sketch the corresponding area under the curve.

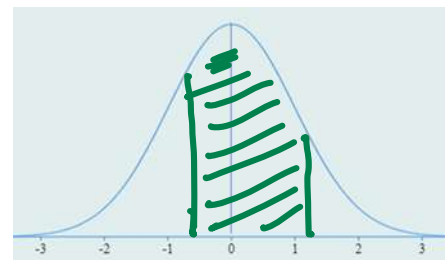
A. $P(X < 89)$



$$\textcircled{1} z = \frac{x - \mu}{\sigma} = \frac{89 - 72}{11} = 1.55$$

$$\textcircled{2} P(X < 89) = 0.9394$$

B. $P(65 < X < 85)$



$$\textcircled{1} z_1 = \frac{65 - 72}{11} = -0.64$$

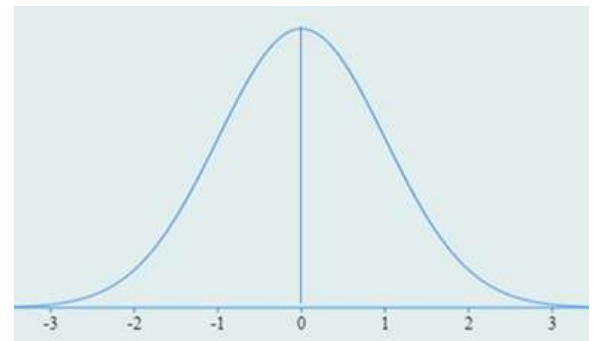
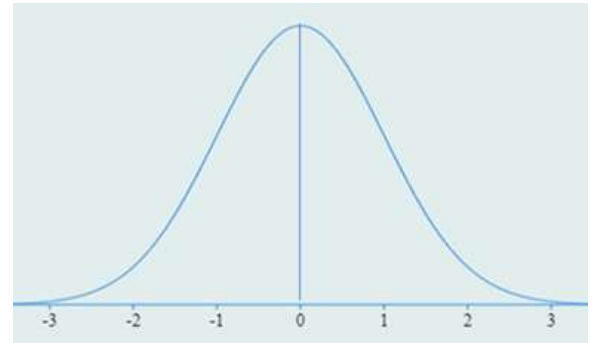
$$z_2 = \frac{85 - 72}{11} = 1.18$$

$$\textcircled{2} P(65 < X < 85) = P(1.18) - P(-0.64) = 0.8810 - 0.2611 = 0.6199$$

Real-World Example 6 Find Intervals of Data

RESEARCH As part of a medical study, a researcher selects a study group with a mean weight of 86 kg and a standard deviation of 5.5 kg. Assume that the weights are normally distributed.

- A. If the study will mainly focus on participants whose weights are in the middle 80% of the data set, what range of weights will this include?
- B. If participants whose weights fall in the outside 5% of the distribution are contacted 2 weeks after the study, people in what weight range will be contacted?



COLLEGE The scores for the entrance exam for a college's mathematics department is normally distributed with $\mu = 65$ and $\sigma = 8$.

- a. If Asma wants to be in the top 20%, what score must she get?



$$A_1 = 20\%$$

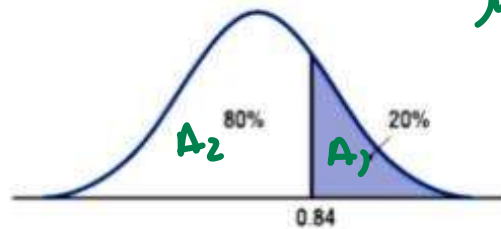
$$A_2 = 80\% = 0.8000$$

$$Z_1 = 0.84$$

$$Z = \frac{x - \mu}{\sigma}$$

$$0.84 = \frac{x - 65}{8} \Rightarrow x - 65 = 6.72 \quad x = 71.72$$

$$x > 71.72$$



- b. Asma expects to earn a grade in the middle 90% of the distribution. What range of scores fall in this category?

$$Z = \frac{x - \mu}{\sigma}$$

$$A_1 = 5\% = 0.0500$$

$$Z_1 = -1.64 \rightarrow Z_2 = 1.64$$

$$-1.64 = \frac{x - 65}{8}$$

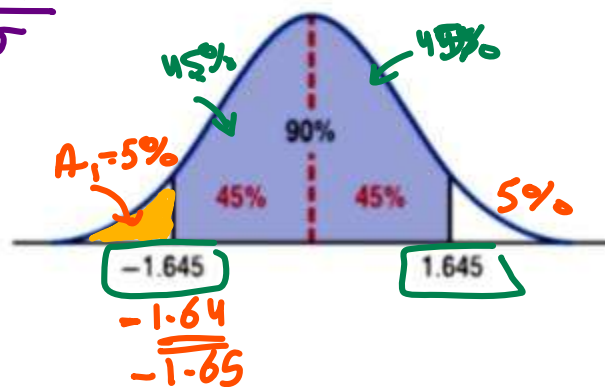
$$x - 65 = -13.12$$

$$x = 51.9$$

$$1.64 = \frac{x - 65}{8}$$

$$x - 65 = 13.12$$

$$x = 78.1$$



$$51.9 < x < 78.1$$

TABLE A The Standard Normal Distribution**Cumulative Standard Normal Distribution**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

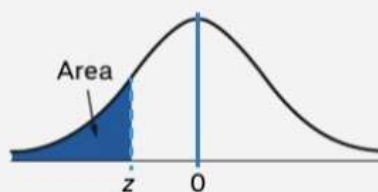
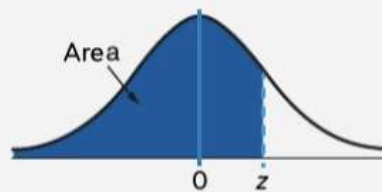
For z values less than -3.49 , use 0.0001 .

TABLE A (continued)

Cumulative Standard Normal Distribution

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For *z* values greater than 3.49, use 0.9999.

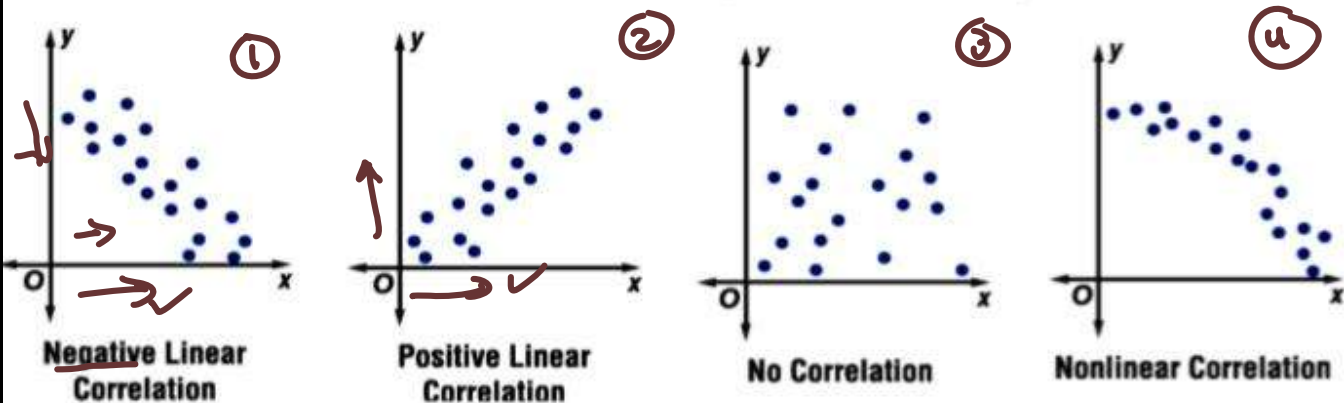


Objectives:

1. Measure the linear correlations. (Direct Learning)
2. Generate least -squares regression lines. (Self-Based) given

Correlation is another area of inferential statistics that involves determining whether a relationship exists between two variables in a set of **bivariate** data.

Bivariate data can be represented as ordered pairs (x, y) , where x is the independent or **explanatory variable** and y is the dependent or **response variable**. To determine whether there may be a linear, a nonlinear, or no correlation between the variables, you can use a scatter plot.



Key Concept Correlation Coefficient

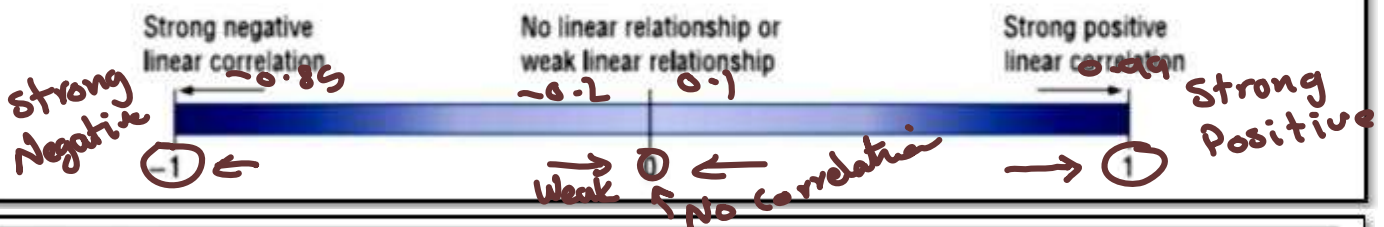
For n pairs of sample data for the variables x and y , the correlation coefficient r between x and y is given by

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

where x_i and y_i represent the values for the i th pair of data, \bar{x} and \bar{y} represent the means of the two variables, and s_x and s_y represent the standard deviations of the two variables.

The correlation coefficient can take on values from -1 to 1 . This value indicates the strength and type of linear correlation between x and y as shown in the diagram below.

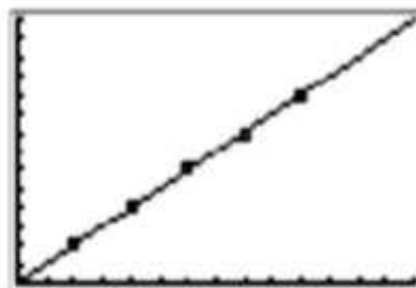
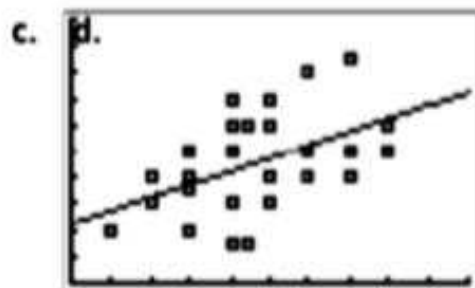
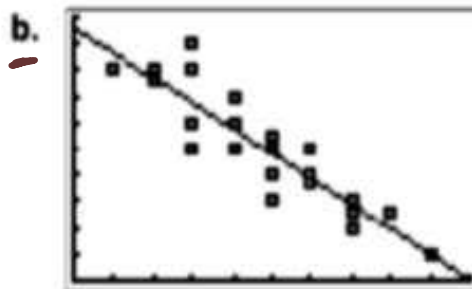
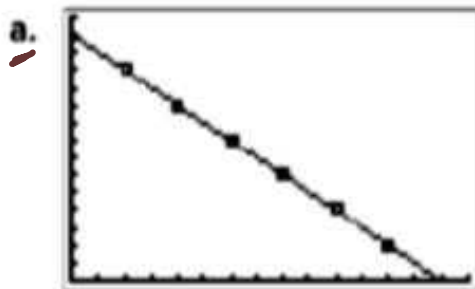
$$-1 \leq r \leq 1$$



Concept Summary Analyzing Bivariate Data

- Step 1** Make a scatter plot, and decide whether the variables appear to be linearly related.
- Step 2** If they appear to be linearly related, calculate the strength of the relationship by calculating the correlation coefficient.
- Step 3** Use a t -test to determine if the correlation is significant.
- Step 4** If significant, find the least-squares regression equation that models the data.

1 Match each graph to the corresponding correlation coefficient.



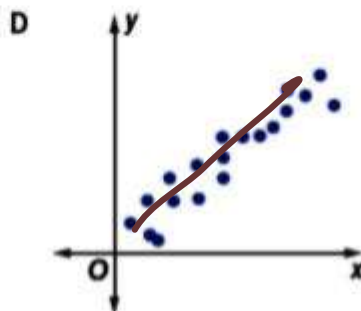
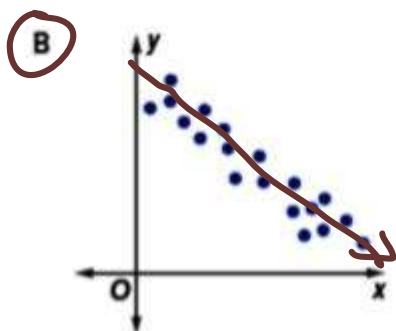
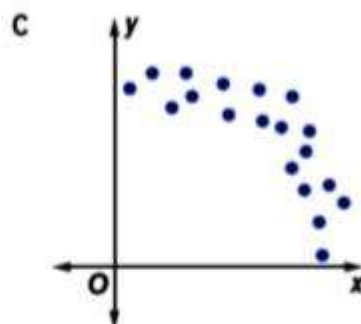
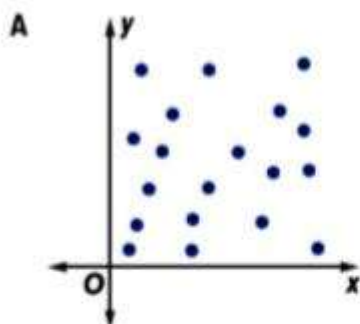
7. $r = -0.90$

8. $r = 0.50$

9. $r = 1.00$

10. $r = -1.00$

2 **MULTIPLE CHOICE** Identify the graph that could have a correlation coefficient of -0.96 in a linear regression.



3

SLEEP/GPA STUDY The regression equation for the average hours of sleep x and GPA y from Example 3 was $\hat{y} = 0.457x - 0.667$. Use this equation to predict the expected GPA (to the nearest tenth) for a student who averages the following hours of sleep and state whether this prediction is reasonable. Explain.

a. 8 hours

$$\hat{y} = 0.457x - 0.667$$

$$\hat{y} = 0.457(8) - 0.667 = 2.7 \approx 3$$

b. 10.5 hours

$$\hat{y} = 0.457(10.5) - 0.667 = \underline{4.13} \quad \text{not reasonable}$$

$4.13 > \underline{\underline{GPA}}$

Probability and Odds

Objectives:

1. Find the probability of an event
2. Find the opportunities for success and failure occurred

PROBABILITY OF SUCCESS AND OF FAILURE

1 If an event can succeed in s ways and fail in f ways, then the probability of success $P(s)$ and the probability of failure $P(f)$ are as follows.

$$P(s) = \frac{s}{s+f} \quad P(f) = \frac{f}{s+f}$$

$$\text{Odds} = \frac{P(s)}{P(f)}$$

1 A bag contains 5 yellow, 6 blue, and 4 white marbles.

- a. What is the probability that a marble selected at random will be yellow? $\frac{5}{15} = \frac{1}{3}$
- b. What is the probability that a marble selected at random will not be white?

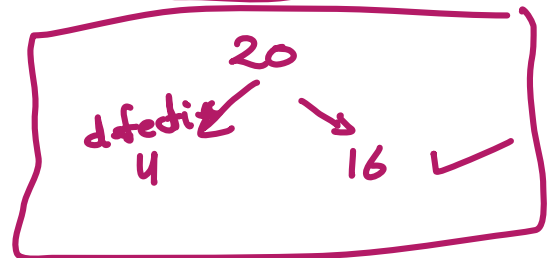
a) $P(\text{yellow}) = \frac{5}{15} = \frac{1}{3}$

b) $P(\text{not a white}) = \frac{11}{15}$

$\frac{5}{15} = \frac{1}{3}$
 yellow 5 + blue 6 = $\frac{11}{15}$

2 A circuit board with 20 computer chips contains 4 chips that are defective. If 3 chips are selected at random, what is the probability that all 3 are defective?

$$P(s) = \frac{{}^4C_3}{{}^{20}C_3} = \frac{1}{285}$$



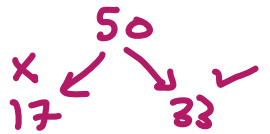
$$P(s) = \frac{{}^{16}C_3}{{}^{20}C_3} = \frac{28}{75}$$

- 3 The CyberToy Company has determined that out of a production run of 50 toys, 17 are defective. If 5 toys are chosen at random, what is the probability that at least 1 is defective?

ok
33

$$P(F) = \frac{{}^{33}C_5}{{}^{50}C_5} = 0.112$$

$$P(S) = 1 - 0.112 = 0.888$$



$$\text{odds} = \frac{P(S)}{P(F)} = \frac{0.888}{0.112} =$$

ODDS

2 The odds of the successful outcome of an event is the ratio of the probability of its success to the probability of its failure.

$$\text{Odds} = \frac{P(S)}{P(F)}$$

- 4 Ayesha must select at random a chip from a box to determine which question she will receive in a mathematics contest. There are 6 blue and 4 red chips in the box. If she selects a blue chip, she will have to solve a trigonometry problem. If the chip is red, she will have to write a geometry proof.

a. What is the probability that Ayesha will draw a red chip?

b. What are the odds that Ayesha will have to write a geometry proof?

$$a) P(S) = \frac{4}{10} = \frac{2}{5} \rightarrow P(F) = \frac{3}{5}$$

$$b) \text{odds} = \frac{P(S)}{P(F)} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

10 = 6 blue 4 red

$$P(F) = 1 - \frac{1}{6} = \frac{5}{6}$$

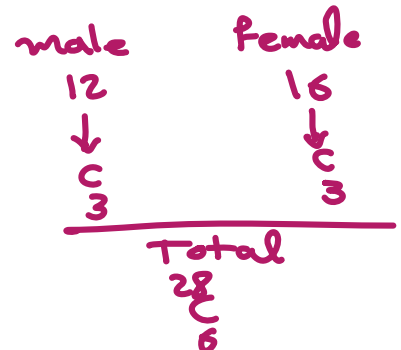
- 5 Twelve male and 16 female students have been selected as equal qualifiers for 6 college scholarships. If the awarded recipients are to be chosen at random, what are the odds that 3 will be male and 3 will be female?

Probability

$$P(S) = \frac{{}^{12}C_3 \times {}^{16}C_3}{{}^{28}C_6} = \frac{880}{2691}$$

$$P(F) = 1 - \frac{880}{2691} = \frac{1811}{2691}$$

$$\text{odds} = \frac{P(S)}{P(F)} = \frac{\frac{880}{2691}}{\frac{1811}{2691}} = \frac{880}{1811}$$



- 6 In an office, there are 7 seniors and 4 juniors. If one person is randomly called on the phone, find the probability the person is a senior.

$$\rightarrow P(S) = \frac{7}{11}$$

$$7S + 4J = 11$$

odds $\rightarrow P(S) = \frac{7}{11}$
 $\rightarrow P(F) = \frac{4}{11}$
 $= \frac{7}{4} = \frac{7}{4}$

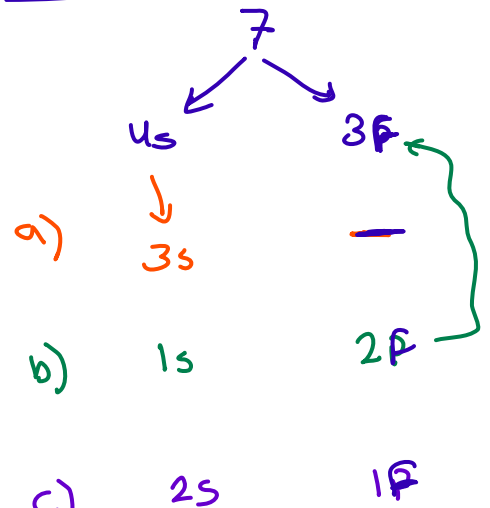
- 7 Of 7 kittens in a litter, 4 have stripes. Three kittens are picked at random. Find the odds of each event.

a. All three have stripes.

b. Only 1 has stripes.

c. One is not striped.

$$\text{odds} = \frac{P(S)}{P(F)}$$



a) $P(S) = \frac{{}^4C_3}{{}^7C_3} = \frac{4}{35} \Rightarrow P(F) = 1 - \frac{4}{35} = \frac{31}{35} \Rightarrow \text{odds} = \frac{4}{31}$

b) $P(S) = \frac{{}^4C_1 \times {}^3C_2}{{}^7C_3} = \frac{12}{35} \Rightarrow P(F) = \frac{23}{35} \Rightarrow \text{odds} = \frac{12}{23}$

c) $P(S) = \frac{{}^4C_2 \times {}^3C_1}{{}^7C_3} = \frac{18}{35} \Rightarrow P(F) = \frac{17}{35} \Rightarrow \text{odds} = \frac{18}{17}$

- 8 **METEOROLOGY** A local weather forecast states that the probability of rain on Saturday is 80%. What are the odds that it will not rain Saturday? (Hint: Rewrite the percent as a fraction.)

rain $P(F) = 80\% = 0.8$

Not $P(S) = 1 - 0.8 = 0.2$

not rain
 $\text{odds} = \frac{P(S)}{P(F)} = \frac{0.2}{0.8} = \frac{1}{4}$

rain
 $\text{odds} = \frac{0.8}{0.2} = \frac{4}{1}$