### **GRADE 7 ASP - CHAPTER 12 REVISION NOTES**

#### Lesson 12-1

## **Measures of Center**

#### What You'll Learn

- · Use the mean, median, and mode as measures of center.
- Choose an appropriate measure of center and recognize measures of statistics.

### **Choose Appropriate Measures**

Different circumstances determine which measures of center are most appropriate.

### Concept Summary Using Mean, Median, and Mode

, , , , , , , , , , , , , , , , , , , ,				
Me asure	Most Useful When			
mean	the data have no extreme values (values that are much greater or much less than the rest of the data)			
median	the data have extreme values     there are no big gaps in the middle of the data			
mode	data have many repeated numbers			

#### Standardized Test Practice

21. The number of books read by the students in each reading class this year is shown in the table. Which measure of center would the school use to show that their students read a lot of books?

104	90
162	134
110	97
145	126
_	

A mode

C mean

**B** median

D cannot be determined

- 22. The high temperatures, in degrees Celsius, for one week are 26°, 27°, 25°, 27°, 28°, 24°, and 25°. If the temperature on the eighth day is 27°, which of the following would be true?
  - F The mode will change.
  - G The mean will increase and the median will remain the same.
  - H The median will increase and the mean will remain the same.
  - J Both the mean and the median will increase.

23. Jamal said that the number that best represented the following set of data is 27. Which measure of center is he referring to?
B

28, 32, 21, 25, 33, 32, 20, 26

A mean

C mode

**B** median

D all of the above

 Short Response Ahmed and Ali have the bowling scores shown.

Game #	Ahmed	Ali
1	124	125
2	135	132
3	109	128
4	116	130
5	141	125

Ahmed's score for his sixth game is 155. What must Ali bowl in the sixth game in order to have the same average as Ahmed? 140

#### Lesson 12-1 Measures of Center

Find the mean, median, and mode for each set of data. Round to the nearest tenth if necessary.

- number of students in each math class: 22, 23, 24,
   22, 21 22.4 students; 22 students; 22 students
- 2. grams of fat per serving: 2, 7, 4, 5, 6, 4, 5, 6, 3, 5
- **4.7 g; 5 g; 5 g 3.** millimeters of rain last week: 1.5, 2, 2.5, 2, 1.5, 2.5, 3 **2.1 mm; 2 mm; 1.5 mm, 2 mm, 2.5 mm**
- 4. At the movie theater, six movies are playing and their lengths are 138, 117, 158, 145, 135, and 120 minutes. Which measure of center best represent the data? Justify your selection and then find the measure of center. See margin.

### Example 1

Find the mean, median, and mode of 2, 3, 2, 4, 4, 6, 4, and 7.

**Mean:** 
$$\frac{2+3+2+4+4+6+4+7}{8} = \frac{32}{8}$$
 or 4

#### Median:

2, 2, 3, 4, 4, 4, 6, 7 Arrange the numbers from least to

greatest.

 $\frac{4+4}{2} = 4$  Find the middle number or the mean of

the two middle numbers.

Mode: 4 Find the data value(s) that occur most

often.

#### Lesson 12-2

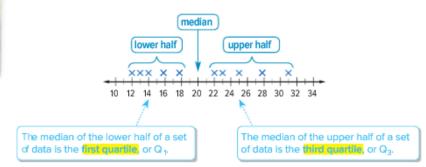
# **Measures of Variability**

#### What You'll Learn

- · Find measures of variability.
- Use measures of variability to interpret and analyze data.

Quartiles

One half of the data lies between the first quartile and the third quartile. In a set of data, the quartiles are the values that divide the data into four equal parts. Recall that the median of a set of data separates the set in half.



#### Key Concept Interquartile Range

Words The interquartile range is the range of the middle half of a set of data. It is the difference between the third quartile and the first quartile.

Symbols Interquartile range =  $Q_3 - Q_1$ 

#### Outliers

A data value that is much larger or much smaller than the median is an outlier. Data that are more than 1.5 times the value of the interquartile range beyond the quartiles are called **outliers**.

first

quartile ....)

median ....)

quartile ....)

third

### Example 3

Find any outliers in the data set.

Step 1 Find the interquartile range.

35 - 15 = 20

Step 2 Multiply the interquartile range, 20, by 1.5.

 $20 \times 1.5 = 30$ 

Step 3 Subtract 30 from the first quartile and add 30 to the third quartile.

15 - 30 = -15 35 + 30 = 65

The only outlier is 70 because it is greater than 65.

Animal Speeds				
Animal	Speed (km/h)			
squirrel	12			
turkey	15			
elephant	25			
cat	30			
reindeer	32			
rabbit	35			
cheetah	70			

#### Get It? Do this problem to find out.

3. Find any outliers in the data set. 155

Movie Running Time (min)	105	120	155	115
	96	100	110	120



### Standardized Test Practice

18. The table shows the total number of wins by the team that won the women's basketball tournament for the past 15 years.

Total Wins						
34	39	29	31	34		
34	34	32	37	39		
33	36	35	33	31		

Which of the following statements is *not* supported by these data? **D** 

- A Less than half of the teams won more than 34 games and half won less than 34 games.
- B The range of the data is 10 games.
- C About one fourth of the teams won 32 or fewer games.
- D An outlier of the data is 29 games.

19. Refer to the table in Exercise 18. What is the interquartile range of the data?

F 4

**H** 8

**G** 5

**J** 10

20. What is the first quartile of the following set of data?

37, 12, 7, 8, 10, 5, 14, 19, 7, 15, 11

**A** 7

C 11

**B** 7.5

**D** 15

Short Response Find the measures of variability and any outliers for the set of data.

30, 62, 35, 80, 12, 24, 30, 39, 53, 38

R: 68; M: 36.5; Q 4: 30; Q3: 53; IR: 23; none

### Lesson 12-2 Measures of Variability

Find the measures of variability and any outliers for each set of data.

5–8. See margin.

- the number of minutes spent reading each night: 31, 33, 32, 34, 35, 33
- the number of fish in each fish tank: 6, 5, 7, 8, 5, 6, 7, 9, 8, 6
- Hessa earned 5, 7, 10, 6, and 8 dirhams doing errands for her neighbors. Find the measures of variability and any outliers for the set of data.
- The scores that Mr. Khalid's students earned on their last test are shown in the table. Use measures of variability to describe the data in the table.

	Test S	Scores	
99	88	81	89
77	58	92	80
83	82	74	84
76	73	99	74
82	87	82	74
86	76	85	92

### Example 2

During a baking contest, bakers sampled 26, 20, 21, 24, 23, 22, 21, 27, 23, 24, and 25 cookies. Find the measures of variability and any outliers for the data.

Range: 27 - 20 or 7 cookies

Median, First Quartile, Third Quartile:

List the data from the least to greatest.

The median is 23, the first quartile is 21, and the third quartile is 25.

Interquartile Range: 25 - 21 or 4.

Outliers:

Multiply the interquartile range by 1.5.

$$4 \times 1.5 = 6$$

Subtract 6 from the lower quartile and add 6 to the upper quartile.

$$21 - 6 = 15$$
  $25 + 6 = 31$ 

Since there are no values less than 15 or greater than 31, there are no outliers.

Lesson 12-3

# **Mean Absolute Deviation**

#### What You'll Learn

- Find the mean absolute deviation of a set of data.
- Compare the mean absolute deviations for two data sets.

### Find Mean Absolute Deviation

You have found measures of center to describe the middle of a set of data, and you have used the interquartile range to describe the spread of a set of data. The **mean absolute deviation** is the average distance between each data value and the mean.

### Compare Variation

You can compare the mean absolute deviations for two data sets. A data set with a greater mean absolute deviation has data values that are more spread out from the mean than a data set with a smaller mean absolute deviation.

. . . .

#### Standardized Test Practice

15. Which describes the mean absolute deviation of the data?
B

Distance (kilometers)						
60	40	15	25	30	35	40

- A The average distance between each data value and the mean absolute deviation is 10 kilometers.
- **B** The average distance between each data value and the mean is 10 kilometers.
- C The average distance between the mean and the mean absolute deviation is 10 kilometers.
- D The average distance between each data value is 10 kilometers.
- Which data set has a mean absolute deviation of 4?

F 1, 6, 11

H 3, 10, 14

G 0, 12, 24

J 4, 5, 9

17. What is the mean absolute deviation of Muna's science scores, shown below? C

70 82 84 94

A 82.5

**B** 80

C 6.5

**D** 6

18. Short Response Salem wants to invest in the fund with the smaller mean absolute deviation of profits. Which fund should he choose? What is the mean absolute deviation for that fund?

Fund A	5%	11%	8%	2%	10%
Fund B	9%	7%	5%	3%	7%

Fund B; 1.76%

### Lesson 12-3 Mean Absolute Deviation

Find the mean absolute deviation of each data set.

9. 25, 70, 75, 100 21.25

10. 0, 20, 175, 190, 175 81.6

**11.** 35, 50, 40, 55, 45 **6** 

12. 250, 240, 300, 295, 280 22.4

**13.** 144, 137, 156, 149 **6** 

14. The table shows the numbers of two colors of candies in several bags. Which data set has a greater mean absolute deviation? Justify your answer.

Brown Candies	18	14	20	13	17
Red Candies	10	8	5	12	8

Brown candies; the mean absolute deviation is 2.32, which is greater than 1.92, the mean absolute deviation of the red candies.

### Example 3

The number of defective light bulbs in each shipment of bulbs is given. Find the mean absolute deviation. Describe what it represents.

The mean is 9. Find the absolute value of the differences between each data value and the mean.

$$|8 - 9| = 1$$

$$|6 - 9| = 3$$

$$|14 - 9| = 5$$

$$|3 - 9| = 6$$

$$|11 - 9| = 2$$

$$|12 - 9| = 3$$

Find the average of these differences.

$$\frac{1+3+5+6+2+3}{6} = 3\overline{3}$$

The average distance between each data value and the mean is  $3.\overline{3}$ .

### Lesson 12-4

# **Compare Populations**

#### What You'll Learn

- Compare two populations using the measures of center and variability.
- Compare two populations when only one is symmetric.

### **Compare Two Populations**

A **box plot** uses a number line to show the distribution of a set of data. It divides a set of data into four parts using the median and quartiles. A *box* is drawn around the quartile values, and *whiskers* extend from each quartile to the minimum and maximum values that are not outliers. A **double box plot** consists of two box plots graphed on the same number line. You can draw inferences about two populations in a double box plot by comparing their centers and variability.

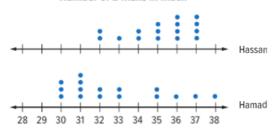
#### Got It? Do this problem to find out.

2. The double dot plot shows the number of new e-mails in each of Hassan's and Hamad's inboxes for sixteen days. Use the dot plots to compare the centers and variations of the two populations. Write an inference you can draw about the two populations. Sample answer: The median of Hassan's data is 35.5 e-mails with an

#### Most Appropriate Measures

To compare the centers and variations when both data sets are symmetric, use the mean and the mean absolute deviation. When neither set or only one is symmetric, use the median and the interquartile range to compare.

#### Number of E-mails in Inbox



interquartile range of 2.5. The median for Hamad's data is 32 e-mails with an interquartile range of 4. There is a greater spread of data around Hamad's e-mails, but Hassan's data centers around a higher number of e-mails. So, you would generally expect Hassan to have more e-mails.

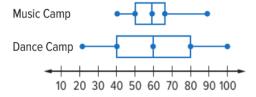
### **Only One Symmetric Population**

You can compare two populations when only one is symmetric. The mean and median of a symmetric distribution are very similar, if not the same. So, you can use the median and interguartile range for both populations.

#### **Need Another Example?**

The double box plot shows the number of weekly participants at two community center camps. Compare the centers and variations of the two populations. Which camp has the greater number of participants?

#### **Number of Weekly Participants**



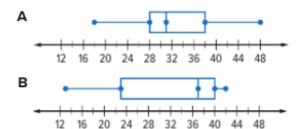
Dance Camp; Sample answer: Both camps have a median of 60 weekly participants, but the dance camp has a greater variation. The music camp data are more consistent.

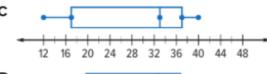
#### Standardized Test Practice

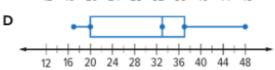
20 24 28

15. A data distribution has a minimum value of 16, a maximum value of 45, a median of 35, a first quartile value of 22, and a third quartile value of 39. Which box plot shows data values that are generally greater than the ones given? B

44 48



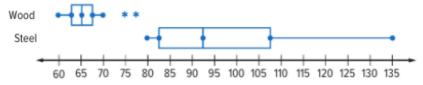




16. Which of the following is not true about the double box plot?

36 40

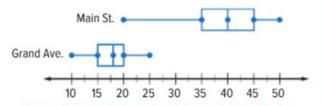
#### Speed of Roller Coasters (km/h)



- **F** The data for the steel roller coasters is symmetric.
- G The data for the steel roller coasters is not symmetric.
- H The fastest steel roller coaster travels 135 kilometers per hour.
- J The slowest wooden roller coaster travels 60 kilometers per hour.

### Lesson 12-4 Compare Populations

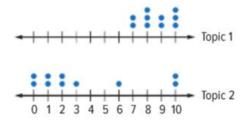
**15.** The double box plot shows the numbers of floors for buildings on two streets. Compare the centers and variations. Write an inference you can draw about the two populations.



The median number of floors on Main St. is 40 with an interquartile range of 10. The median number of floors on Grand Ave. is 18 with an interquartile range of 5. Main St. has more variability and centers around a higher number of floors than Grand Ave.

### Example 4

The double dot plot shows the guiz scores of students on two topics. Compare their centers and variability. Write an inference you can draw about the two populations.



The median score for Topic 1 is 8.5 with an interquartile range of 2. The median score for Topic 2 is 2 with an interquartile range of 5. Topic 1 centers around a higher score than Topic 2, while Topic 2 has a greater variability than Topic 1.

# **Using Sampling to Predict**

#### What You'll Learn

- · Identify various sampling techniques.
- Determine the validity of a sample and predict the actions of a larger group.

### **Identify Sampling Techniques**

Suppose a school conducted a survey about new clubs. They cannot survey every student, so a randomly selected smaller group called a **sample** is chosen from the larger group, or **population**. The best sample is an **unbiased sample**, or one that is

- representative of the larger population,
- selected at random or without preference, and
- large enough to provide accurate data. If a sample is too small, data accurately representing the larger population may not be available.

Concept Summary Unbiased Samples						
Words	Definition	Example				
simple random sample	Each item or person in a population is as likely to be chosen as any other.	Thirty student ID numbers are randomly selected by a computer.				
stratified random sample	The population is divided into similar, nonoverlapping groups. A simple random sample is then selected from each group.	A population of election districts can be separated into urban, suburban, and rural strata.				
systematic random sample	The items or people are selected according to a specific time or item interval.	Every 20 minutes a customer is chosen, or every 10th customer in line is chosen.				

Concept Summary   Biased Samples						
Words	Definition	Example				
convenience sample	Includes members of the population that are easily accessed.	The first 10 students in the cafeteria line.				
voluntary response sample	Involves only those who want to or can participate in the sampling.	The principal sent an e-mail to graduating seniors asking them where to hold commencement. Seniors are asked to vote through an online poll.				

#### **Making Predictions**

To make a valid prediction or inference about a population, the sample would need to be random, and either simple, stratified, or systematic.

### Validating and Predicting Samples

Depending on the sampling method used, you can make predictions about larger populations. When you make a prediction about a population from a sample of data, you are drawing an *inference* about that population.

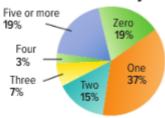


#### Standardized Test Practice

- 21. A real estate agent surveys people about their housing preferences at an open house for a luxury townhouse. Which of the following best explains why the results of this survey might not be valid? B
  - A The survey is biased because the agent should have conducted the survey by telephone.
  - B The survey is biased because the sample consisted of only people who already are interested in townhouses.
  - C The survey is biased because the sample was a voluntary response sample.
  - D The survey is biased because the agent should have conducted the survey at a single-family home.
- 22. Short Response One hundred people in a music store were surveyed about what type of music they prefer. If 35% of them said they prefer rock music, how many people out of 1500 can be expected to prefer rock music?
  525

An online survey of about 38,000 children produced the results shown in the circle graph.

#### How Many Cans of Soda Do You Drink in a Day?



23. About how many of the children surveyed drink two cans of soda or fewer per day?

**F** 5700

H 14,060

**G** 8993

**J** 26,980

24. Based on the results, about how many children in a class of 30 would drink two or more cans of soda per day?

**A** 5

C 13

**B** 8

**D** 21

### Lesson 12-5 Using Sampling to Predict

- 16. To determine the weekly top ten songs, the local radio station asks people to log on to their website and vote for their favorite song. Identify the sample as biased or unbiased and describe its type. Explain.
- 17. Forty-five out of 60 people at a steakhouse said their favorite meal was steak. Is this sample representative of the entire town? If so, how many of the 13,000 residents would say steak was their favorite meal?

### Example 5

Is polling students on the football team about their favorite sports a biased or unbiased sample? Then describe the type of sample.

biased, convenience sample

Students who play football are more likely to choose football as their favorite sport.

### **Additional Answers**

- 16. Biased, voluntary response survey; only those people who are interested in participating in the survey are part of the sample.
- 17. No; the sample is biased. Diners at a steakhouse are more likely to choose steak as their favorite meal than randomly-chosen diners.

#### What You'll Learn

- · Find the probability of simple events.
- · Find the probability of the complement of an event.

### **Probability**

When you attempt a free throw, there are two possible results: making or missing the shot. Each results is an **outcome**. A **simple event** is one outcome or a collection of outcomes The **probability** of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The probability of an event is a number between 0 and 1 that expresses the likelihood of the event occuring. The closer a probability is to 1, the more likely it is to occur.





### **Find Probability of the Complement**

The **complement** of a set is the set of all objects that do not belong to the given set. Two events are complementary if one or the other must happen, but they cannot happen at the same time. For example, when you roll a number cube, the number cube must land on an even number or an odd number, but not both. The sum of the probability of an event and its complement is 1 or 100%.



### Example 3

Find P(not blue) in Example 1. Then describe the likelihood of the event. Write impossible, unlikely, equally likely, likely, or certain.

Selecting a blue marble and not selecting a blue marble are complementary events. So the sum of the probabilities is 1.

$$P(\text{blue}) + P(\text{not blue}) = 1$$

$$\frac{2}{5} + P(\text{not blue}) = 1$$

$$\frac{2}{5} - \frac{2}{5} + P(\text{not blue}) = 1 - \frac{2}{5}$$

$$P(\text{not blue}) = \frac{3}{5}$$
The sum of the probabilities is 1.

Replace  $P(\text{blue})$  with  $\frac{2}{5}$ .

Subtraction Property of Equality
$$P(\text{not blue}) = \frac{3}{5}$$
Simplify.

The probability of selecting a marble that is not blue is  $\frac{3}{5}$  or 60%. Because the probability of the event is greater than 50%, the event is likely to occur.

### **Need Another Example?**

A box contains 8 red, 7 blue, and 5 yellow blocks. A block is selected without looking. Find P(not yellow). Then describe the likelihood of the event. Write impossible, unlikely, equally likely, likely, or certain.  $\frac{3}{4}$  or 75%; likely



#### Standardized Test Practice

- 45. The production records of a toy manufacturing company show that 5 out of every 75 toys have a defect. What is the probability that a randomly selected toy manufactured at the company will not have a defect?
  - A 7%
- C 70%
- B 27%
- D 93.3%
- 46. If Ahmed spins a spinner like the one shown, what is the probability that it lands on the space with the square or the space with the triangle?
  - F 1/4
  - G 1/2
- $H^{\frac{2}{3}}$ 
  - $\frac{3}{4}$



- 47. Short Response Noura has a bag containing 3 white, 8 red, 2 blue, and 2 yellow marbles. She randomly picks a marble from the bag. What is the probability that the marble Noura picks will be white?

  1 or 20%
- 48. A bag contains some tiles. Each tile has the number 1, 2, 3, 4, 5, or 6 written on it. The table shows the frequency of each number in the bag. You choose a tile at random. Which is closest to the probability of choosing a 3?

Number	1	2	3	4	5	6	
Frequency	13	9	20	8	10	15	

- A 13%
- C 20%
- **B** 17%
- D 27%

### Lesson 12-6 Probability of Simple Events

Ahmed rolls a ten-sided solid whose identical faces are numbered with the first ten square numbers. Find each probability. Then describe the likelihood of the event as impossible, unlikely, equally likely, likely, or certain.

- **18.** P(ones digit is 2) **0 or 0%; impossible**
- 19.  $P(\text{two-digit number}) \frac{3}{5} \text{ or } 60\%$ ; likely
- 20.  $P(\text{multiple of 8}) = \frac{1}{5} \text{ or 20\%; unlikely}$
- 21. P(ones digit not 3) 1 or 100%; certain
- 22.  $P(\text{odd}) = \frac{1}{2} \text{ or 50\%; equally likely}$

### Example 6

Rana rolls a seven-sided solid whose identical faces are numbered 4 through 10. Find *P*(odd). Then describe the likelihood of the event as *impossible*, *unlikely*, *equally likely*, *likely*, or *certain*.

sample space: {4, 5, 6, 7, 8, 9, 10}

favorable outcomes: odd numbers, 5, 7, and 9

 $P(\text{odd}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{7}$ 

The probability is about 43%. This is less than 50%, so the event is unlikely.

#### What You'll Learn

- Find and compare experimental and theoretical probabilites.
- · Predict the actions of a larger group.

### Theoretical and Experimental Probability

In a uniform probability model, each outcome has an equal probability of occurring. For example, some carnival games involve spinning a wheel to win a prize. If each prize is equally likely, you can use a uniform probability model to calculate your chances of winning a particular prize.

**Theoretical probability** is based on uniform probability — what *should* occur when conducting a probability experiment. **Experimental probability** is based on relative frequency — what *actually* occurs during such an experiment. If the number of trials is very small, there can be a wide variation in results.



#### Standardized Test Practice

18. Najat has a set of five cards numbered 1 through 5. She shuffles the cards and chooses one card at random. She repeats the process several times and records the results in the table below. Based on her results, what is the experimental probability of choosing a 1? B

Number	1	2	3	4	5
Frequency	6	4	5	3	7

- A 20% C 25%
- **B** 24% **D** 30%
- 19. Short Response Yousif flips a coin several times. The table shows his results. What is the experimental probability that the coin lands tails-up? Express the probability as a fraction and as a percent.

30				
Result	Heads	Tails		
Frequency	31	19		

20. In a survey, 60 students were asked to choose their favorite type of film, and 27 of the students chose action films. Out of a similar group of 480 students, which is the best prediction of the number of students who would choose action films as their favorite type of film?

**F** 45 **H** 216 **G** 130 **J** 288

21. Wafa has a bag containing red, blue, and green marbles. She chooses a marble at random, notes the color, and replaces it. She does this 12 times and finds that she chose a red marble r times. Which of the following is the experimental probability of choosing a red marble?

A  $\frac{r}{12}$  C  $\frac{r}{3}$ 

12 D

### Lesson 12-7 Theoretical and Experimental Probability

In a survey of randomly selected students, students chose their preference from among three meal options for a school event.

Meal	Pizza	Burgers	Vegetarian
Frequency	47	31	22

- 23. What was the experimental probability that a student chose a burger?
  31/100 or 31%
- 24. What was the experimental probability that a student chose the vegetarian option? 11/50 or 22%
- **25.** Out of a similar group of 450 students, predict how many would choose the vegetarian option. **99**

### Example 7

A spinner has five equal-sized sections. Two are red, one is green, one is blue, and one is orange. Alia spins the spinner 60 times. Find the theoretical and experimental probability of red.

Result	Red	Green	Blue	Orange
Frequency	22	15	11	12

theoretical: 
$$P(\text{red}) = \frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{2}{5} \text{ or } 40\%$$

experimental: 
$$P(\text{red}) = \frac{\text{red results}}{\text{all results}} = \frac{22}{60} = \frac{11}{30} \text{ or } 36.\overline{6}\%$$

# **Probability of Compound Events**

#### What You'll Learn

- · Find the number of outcomes for an event.
- · Find the probability of a compound event.

## Outcomes of Compound Events

A **compound event** consists of two or more simple events. The probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. When you work with compound events, you can make a table, draw a **tree diagram**, or make a list to display and count all of the outcomes in the sample space.

## **Probability of Compound Events**

When you know the number of outcomes, you can find the probability that a compound event will occur.



### Example 3

Eissa has 2 counters. Each counter has one side marked with an E, for Eissa, and the other side marked with a J, for Jamal. Both counters are tossed. If one counter lands with E up and the other lands with J up, Eissa wins. Otherwise, Jamal wins. What is the probability that Eissa will win?

First find the number of outcomes. The possible outcomes are EE, EJ, JE, and JJ.

There are four equally likely outcomes, with two favoring Eissa. So, the probability of Eissa winning is  $\frac{1}{2}$  or 50%.

## Key Concept Fundamental Counting Principle

Words If event M can occur in m ways and is followed by event N that can occur

in n ways, then the event M followed by N can occur in  $m \cdot n$  ways.

Example If there are 4 possible sizes for fish tanks and 3 possible shapes, then there are 4 • 3 or 12 possible fish tanks.

#### **Need Another Example?**

A toy robot moves straight ahead until it hits an obstacle. Then it turns, with equal chances of turning left or right. If the robot makes three turns, what is the probability that all three will be left turns?

#### Standardized Test Practice

23. The spinner is spun twice. What is the probability that it will land on 2 after the first spin and on 5 after the second spin?



A 
$$\frac{1}{64}$$

$$c\frac{1}{8}$$

D 5/8

24. A bicycle lock has 4 rotating discs and each contains the digits 0–9. How many different lock combinations are possible?

F 3024

G 5040

H 6561

J 10,000

25. A restaurant offers a combo special with 3 different sandwiches, 2 different salads, and 5 different drinks. From how many different combos could Rana choose? C

A 10

B 11

**C** 30

**D** 50

26. Short Response Tarek can make outfits out of the clothes shown in the table. Draw a tree diagram to show all of the possible outcomes See Answer Appendix.

Shoes	Shirts	Pants
white	blue	tan
black	red	navy
	green	denim
	black	

### Lesson 12-8 Probability of Compound Events

A coin is tossed and a number cube is rolled. Find each probability.

**28.** 
$$P(\text{tails and odd number})$$

**29.** 
$$P(\text{heads and a number greater than 2})  $\frac{1}{3}$$$

### Example 8

A card is drawn from a deck of eight cards numbered from 1 to 8 and a number cube is rolled. Find the probability of drawing a 3 and then rolling a 6.

Use the Fundamental Counting Principle to find the number of possible outcomes.

$$P(3, 6) = P(drawing a 3) \cdot P(rolling a 6)$$

number of cards	the total number of times	sides on number cube	equals	number of outcomes
8	×	6	=	48

There are 48 outcomes.

Find the probability. There is one way draw a 3 and roll a 6. So, the probability is  $\frac{1}{48}$  or 2%.

### Al Foaa C2 School

## Final Exam Revision WorkSheets-Trimester(3)-2017/2018

Student Name: ----/06/2018

**Question 1:** Choose the correct answer.

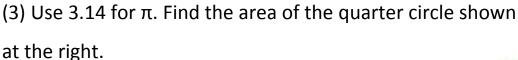
(1) Find the circumference of a circle with a diameter of 14cm.

Use 
$$\frac{22}{7}$$
 for  $\pi$ .

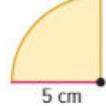
- a. 40 cm
- b. 44 cm
- c. 22 cm
- d. 14 cm
- (2) Use 3.14 for  $\pi$ . Find the area of the semicircle shown

at the right.

- a. 18.84 cm
- b. 37.68 cm
- c.56.52 cm
- d. 226.08 cm

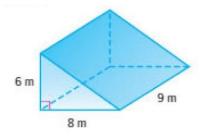


- a. 7.58 cm
- b. 19.625 cm
- c. 39.25 cm
- d. 78.5 cm



12 cm

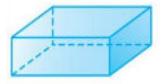
- (4) Find the volume of the triangular prism shown.
  - a. 40 cm
- b. 200 cm
- c. 216 cm
- d. 220 cm



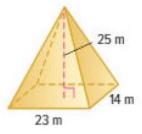
- (5) Find the volume of the prism shown at the right.
  - a. 60cm<sup>3</sup>
- b. 90 cm<sup>3</sup>
- c. 100cm<sup>3</sup>
- d. 120 cm<sup>3</sup>



- (6) If the volume of the box shown is 24 cm<sup>3</sup>.
- Choose the possible dimensions for the box.
  - a. 3cm, 2cm, 2cm
- b. 3cm, 3cm, 2cm
- c. 4cm, 2cm, 2cm
- d. 3cm, 4cm, 2cm



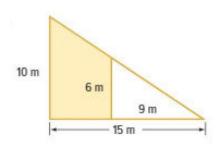
- (7) Find the volume of the pyramid shown.
  - a. 2,683 cm<sup>3</sup>
- b. 4,025 cm<sup>3</sup>
- c. 8,050 cm<sup>3</sup>
- d. 8,100 cm<sup>3</sup>



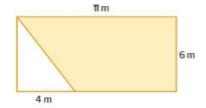
- (8) Write the surface area formula of a cube with side length x.
  - a. S.A =  $x^3$
- b. S.A =  $6x^3$
- c. S.A =  $6x^2$  d. S.A =  $4x^2$



- (9) Find the area of the shaded part.
  - a. 25cm<sup>2</sup>
- b. 48 cm<sup>2</sup>
- c. 10 cm<sup>2</sup>
- $d.80 \text{ cm}^2$



- (10) Find the area of the shaded part.
  - a. 42cm<sup>2</sup>
- b. 54 cm<sup>2</sup>
- c. 62 cm<sup>2</sup>
- d. 78 cm<sup>2</sup>



- (11) Find the probability of rolling a 4 or 6 on the number cube.
  - a.  $\frac{1}{2}$
- b.  $\frac{2}{3}$
- c.  $\frac{1}{6}$
- d.  $\frac{1}{3}$
- (12) A letter tile is chosen randomly. Find p(not I) =
  - a.  $\frac{4}{7}$
- b. 4
- c.  $\frac{5}{7}$
- d.  $\frac{3}{7}$



- (13) A coin is tossed 20 times, and it lands on picture 7 times.
- Find the experimental Probability of the coin landing on picture.
  - a.  $\frac{1}{10}$
- b.  $\frac{7}{20}$
- c.  $\frac{13}{20}$
- d.  $\frac{1}{2}$



- (14) Find the total number of outcomes when rolling a number cube and spinning a spinner with 3 equal sections.
  - a. 9
- b. 15
- c. 16
- d. 18
- (15) In how many ways can a president, vice president, and secretary be randomly selected from 10 candidates?
  - a. 270
- b. 810
- c. 700
- d. 720
- (16) A number cube is rolled and a marble is selected at random from the bag at the right. Find p (3 and red)
  - a.  $\frac{1}{24}$  b.  $\frac{1}{4}$
  - c.  $\frac{3}{14}$  d.  $\frac{1}{6}$



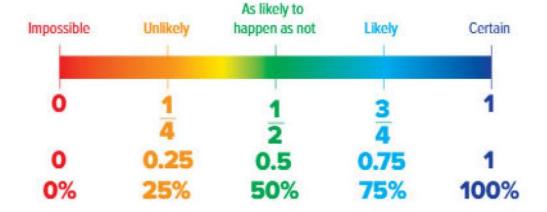


- (17) A number cube is rolled and a letter is selected from the word **EMIRATES**. Find p(even and a constant)
  - **a.** 48

- c.  $\frac{1}{12}$  d.  $\frac{1}{2}$
- (18) Hind is packing for a trip. In her closet, there are 3 red, 4 black, 2 green, and 2 yellow blouses. She randomly select two blouses. Find P( red and red)

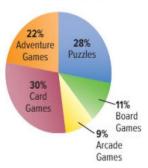
  - a.  $\frac{3}{55}$  b.  $\frac{3}{121}$
  - c.  $\frac{6}{121}$  d.  $\frac{3}{11}$

- (19) Mariam won 25 of the last 30 games she played. Find the probability of Mariam winning the next game she plays.
  - a.  $\frac{1}{3}$
- b.  $\frac{1}{6}$
- c.  $\frac{5}{6}$
- d.  $\frac{2}{3}$
- (20) What number is 20% of 80?
  - a. 20
- b. 16
- c. 80
- d. 60
- (21) What is the percentage of likely probability?
  - a. 75%
- b. 100%
- c. 25%
- d. 50%



- (22) The circle graph shows the results of an online gaming site conducted a survey to determine the types of games people play online. If 1,500 people participated in the study, How many people would play card games?

  Games People Play Online
  - a. 330
- b. 420
- c. 450
- d. 45,000



- (23) A store manager sends an E-mail survey to customers who have registered at the stor's Website.
  - a. simple random sample
- b. systematic random sample
- c. convenience sample
- d. voluntary response sample
- (24) A store sells 3 types of pants: jeans, capris, and cargos. The store workers survey 50 customers at random about their favorite type of pants.
- a. simple random sample
- c. convenience sample
- b. systematic random sample
- d. voluntary response sample

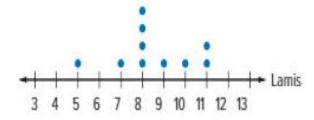
Туре	Number
Jeans	25
Capris	15
Cargos	10

- (25) A magazine asks its readers to complete and return a questionnaire about popular scientists. The majority of those who replied liked one scientist the most, so the magazine decides to write more articles about the scientist.
  - a. simple random sample
- b. systematic random sample
- c. convenience sample
- d. voluntary response sample
- (26) A school district surveys the family of every tenth student to determine if they would vote in favor of the construction of a new school building.
  - a. simple random sample
- b. systematic random sample
- c. convenience sample
- d. voluntary response sample
- (27) The School Council advisor asked every tenth in the lunch line how they prefered to be coducted with school news. The results are shown in the table.

Method	Number
E-mail	16
Newsletter	12
Announcement	5
Telephone	3

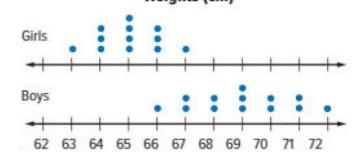
- a. simple random sample
- b. systematic random sample
- c. convenience sample
- d. voluntary response sample

- (28) The dot plot shows Lamis's number of hours worked. Find the median?
  - a. 7
- b. 8
- c. 10
- d. 11

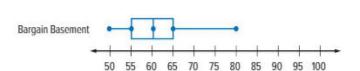


- (29) The double dot plot shows the heights in centimeters for the girls and boys in Ms. Alia's Math Class. Find the boy's heights mean.

  Heights (cm)
  - a. 69
- b. 162.5
- c. 172
- d. 172.5



- (30) The box plot shows the cost of MP3 player at a store. Find the interquartile range.
  - a. 10
- b. 50
- c. 60
- d. 80



Cost of MP3 Players (AED)

- (31) Select an appropriate display for the data.
  - a. circle graph
- b. double box plot
- c. line graph
- d. histogram

(32) Select an appropriate display for the data.

a. circle graph

b. double box plot

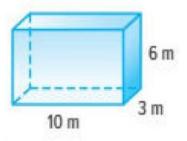
c. line graph

d. histogram

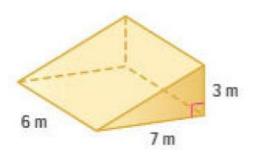
Age Group	Number of Texts per Day
11-15	25
16-20	23
21-25	17
26-30	10

**Question 2:** Answer the following questions. Show all your work.

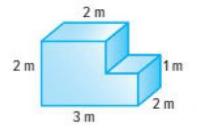
(33) Find the surface area of the prism on the right.



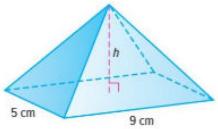
(34) Find the volume of the prism at the right.



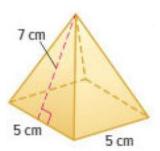
(35) Find the volume of the composite figure shown at the right.



(36) The rectangular pyramid shown has a volume of 90 cubic centimeters and a base area of 45 square centimeters. Find the height of the pyramid.



(37) Find the total surface area of the pyramid shown.



(38) A football team has many different uniforms. The coach can choose from three colors of blouse: green, yellow, and white.

There are two colors of pants: black and blue.

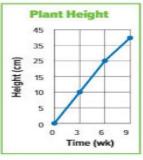
Use a tree diagram to find all blouse and pant combinations.



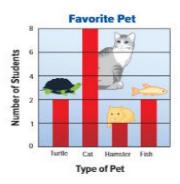
(39) There are 3 oranges, 3 bananas, and 4 apples in a fruit basket. Laila selects a piece of fruit at random and then Aisha selects a piece of fruit at random.

Find p( apple then orange).

(40) The graph shows the height of a plant after 9 weeks of growth. Why the graph is misleading?



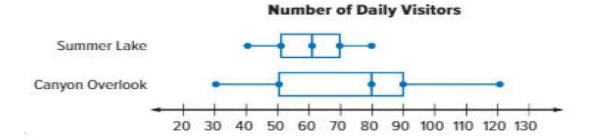
(41) The graph at the right shows the results of a survey to determine students' favorite pets. Why the graph is misleading?



(42) To determine how often her students tardy, Ms. Rana considered the attendance record for her first period class. Why is this graph misleading?



(43) The double box plot shows the number of daily visitors to two different parks. Compare the centers and variations of two populations. In general, which park has more daily visitors?



All Best Wishes Dear Students ...!



Ms. Suha AbdelHadi

## The Circumference.

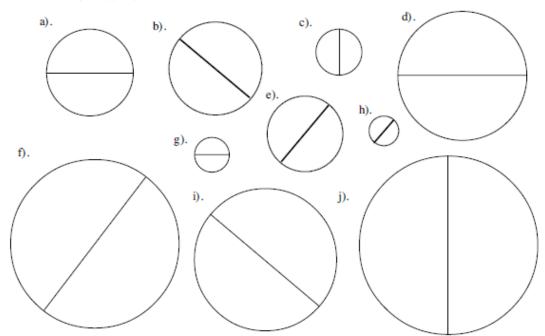
### Circles 1.

Circumference =  $\pi$  x diameter  $C = \pi d$ Circumference =  $2 \times \pi \times \text{radius}$  $C = 2\pi r$ 



- 1). Use  $\pi = 3.14$ . Calculate the circumference of each circle to 2 d.p., if the diameter is :
  - a). 12 cm
- b). 20 cm
- c). 35 cm
- d). 90 cm
- e). 2 cm

- f). 6.5 cm
- g). 10.5 mm h). 0.5 Km
- i).
- 105.1 m j). 6.35 m n). 23.34 Km o). 323 mm
- p). 4502 cm q). 0.125 Km r). 1.625 m
- k). 88.15 mm l). 80.05 cm m). 25.6 Km
- s). 11.65 cm t).
- 2). Measure the diameter of each of these circles and calculate their circumferences to 1 d.p.. Take  $\pi = 3.142$ .



Using the  $\pi$  button on your calculator, calculate the circumference of these circles to 2 d.p. when the radius is:



- a). 10 cm
- b). 55 cm
- c). 12 m
- d). 560 m

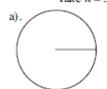
50 in

490 Km e).

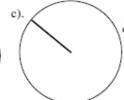
- f). 0.6 mm
- g). 0.125 m h). 35.8 mm
- i). 345 Km
- j). 80.04 Km

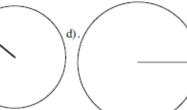
- k). 0.04 m

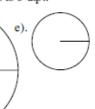
- 1). 0.012 Km m). 23.99 cm n). 1000 Km o). 987.6 m
- 64 ft
- q). 20.25 yds r). 4.75 miles s).
- 6.5 ft.
- Measure the radii of each of these circles and calculate their circumferences to 3 d.p., Take  $\pi = 3.142$ .

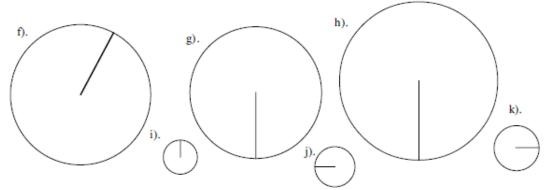






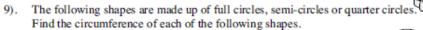






For the following questions use calulator  $\pi$  and leave answers to a sensible degree of accuracy.

- The minute hand on a watch is 1.5 cm long. What distance does the tip of this hand travel a). 1 hour? through in
  - b). 1 day?
- A farmer has a circular field which is 250 metres across. He wishes to put a fence around the field. What length of fencing does he require?
- 7). A bicycle tyre has a 40 cm radius.
  - a). If the wheel travels through 1 complete revolution, how far has the bicycle travelled?
  - The wheel rotates 120 times, how far has the bicycle travelled ?
- 8). A car tyre has a 55 cm radius.
  - If the wheel travels through 1 complete revolution, how far has the car travelled
  - The wheel rotates 2500 times, how far has the car travelled
    - in cm, i).
- ii). in m,
- iii). in Km?

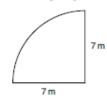




8 cm

17 mm

12 cm

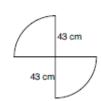




d).

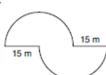


17 mm



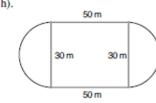
f).

c).

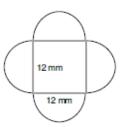


h).

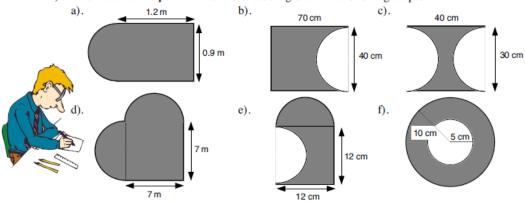
e).



i).



14). Find the area and perimeter of the shaded regions in the following shapes.

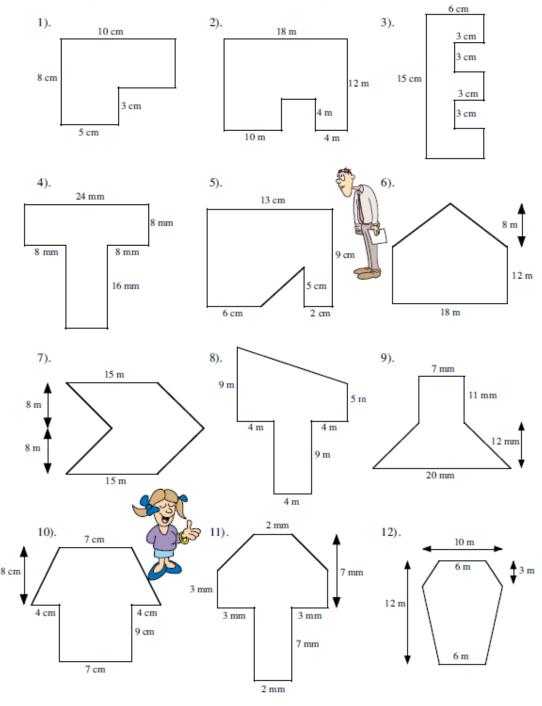




### Compound Areas.



Find the area of each of the shapes below. Where diagrams are shaded, find only the shaded area. Diagrams not to scale.



Take  $\pi = 3.14$ . Leave answers to 1 d.p.. 14 mm 20 cm 15). 13). 14). 8 Km 30 cm 12 mm 8 Km 16). 17). 18). 14 m 18 cm 6 Km 5 m 6 Km 9 m 12 cm 5 m 19). 20). 21). 12 mm 40 cm 32 m 6 mm 6 mm 22). 23). 24). 28 Km 10 cm 4 cm 10 mm 15 cm 6 mm



### **Shape Worded Questions.**

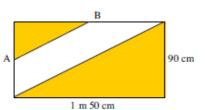
Take  $\pi = 3.14$ . Leave answers to an appropriate degree of accuracy.

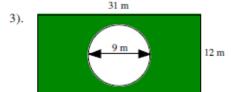


- A circle has a radius of 8 cm. Find its area and circumference.
- c). A circle has an area of 350 cm2. Find its radius then its circumference.
- A rectangular flag has two colours. The measurements are as shown in the diagram.

The line AB is drawn from the midpoints of either side.

- Calculate the area of the flag.
- b). Calculate the shaded area of the flag.
- c). Calculate the white area of the flag.
- d). The material to make the flag costs £8.20 per m². How much will the material for the flag cost?





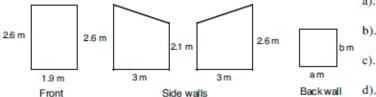
A rectangular garden, has a fruit section in the shape of a circle, diameter 9 metres.

#### Find

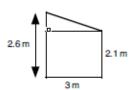
- a). the area of the circular fruit section,
- b). the shaded area.
- 4). In the diagram the radius of the circle is 3 cm.
  - a). Find the area of i). the minor segment,
    - ii). the major segment.
  - b). Find the
- minor arc length,
- ii). major arc length.



- The Millenium Wheel (The London Eye) was the world's tallest observation wheel when it was built. It stands 135 m high. Take the height as its diameter.
  - a). What is its circumference?
  - b). It has 32 pods on its circumference. It takes a pod 30 minutes to complete one revolution. At what speed is the pod travelling in Km/h?
- A shed is made up of the following shapes. The floor is a rectangle



- Work out the area of the front.
  - What is the width, a, of the back wall?
  - What is the height, b, of the back wall?
  - Work out the area of the back wall.
- To find the area of the side wall it can be split into a triangle and a rectangle.
  - i). Find the area of the rectangle.
  - ii). Find the area of the triangle.
  - iii). Find the area of the side wall.
- f). Work out the total area of the four walls.





### Prisms 1.

c).

- 1). Draw each of the following shapes. Underneath each drawing write
  - i). the name of the solid.
  - ii). the number of Faces, Vertices and Edges that belong to the solid.

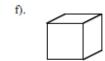








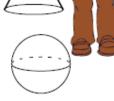












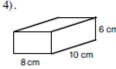
A prism is a solid with a uniform cross section (the same shape and size).

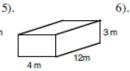
For each of the above solids, state if it is "a prism" or "not a prism".

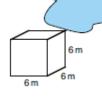
#### The volume of a prism = area of cross section x length

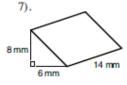
Find the volume of the following prisms.

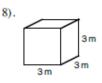


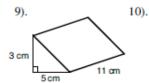


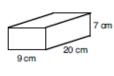


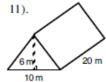


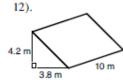


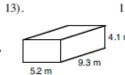


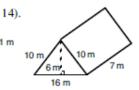


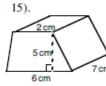


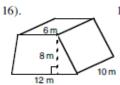


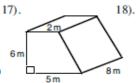


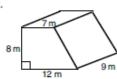


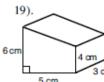


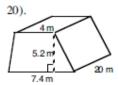


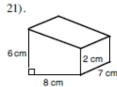


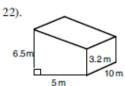










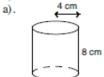




## Prisms 2.

Take  $\pi = 3.14$  where necessary.

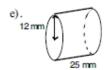
1). Find the curved surface area of each of these cylinders.

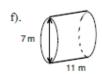










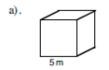


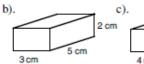


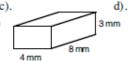
d).



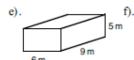
- 2). Find the total surface area of the cylinders above.
- 3). For each of the following solids find
- i). the volume,
- ii). the total surface area.



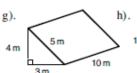


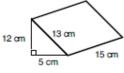


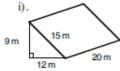


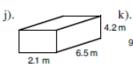


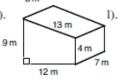


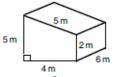




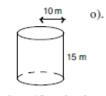












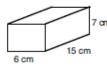




4). A rectangular box has a base 15 cm by 6 cm.

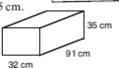
n).

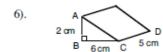
- a). What is the area of the base? The height is 7 cm.
- b). What is the volume of the box ?





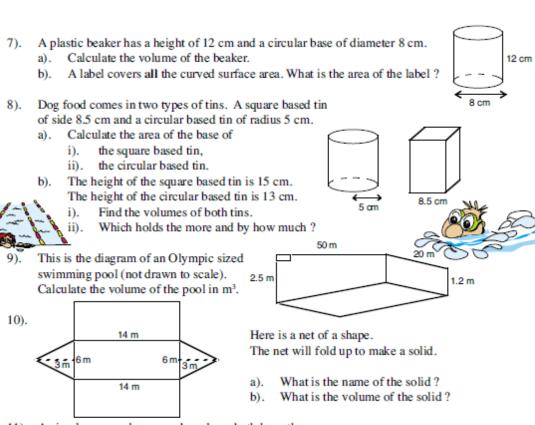
- 5). Billy buys a fish tank. The dimensions are 32 cm by 91 cm by 35 cm.
  - a). Calculate the volume of the fish tank in cm3.
  - b). How many litres of water will it hold when full? (1000 cm³ = 1 litre)



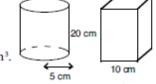


The diagram shows a triangular prism.

- a). Calculate the area of triangle ABC.
- b). Calculate the volume of the prism.



11). A circular can and a square based can both have the same height of 20 cm. The circular based can has a radius of 5 cm and the square based can has a side of length 10 cm. Both are used to hold soup.



- Calculate the volume of each type of can in cm3. i). Change these volumes to capacity in litres.

A School Kitchen pan can hold 32 litres of soup.

- b). i). How many square based cans will it take to fill the pan?
  - How many circular based cans will it take to fill the pan?

12). Smith's Soup is canned at the factory. Each morning they make 500 litres of soup. This is put into cylindrical cans, each of which is 8.4 cm tall and has a diameter of 7 cm. How many cans are filled from the 500 litres of soup?



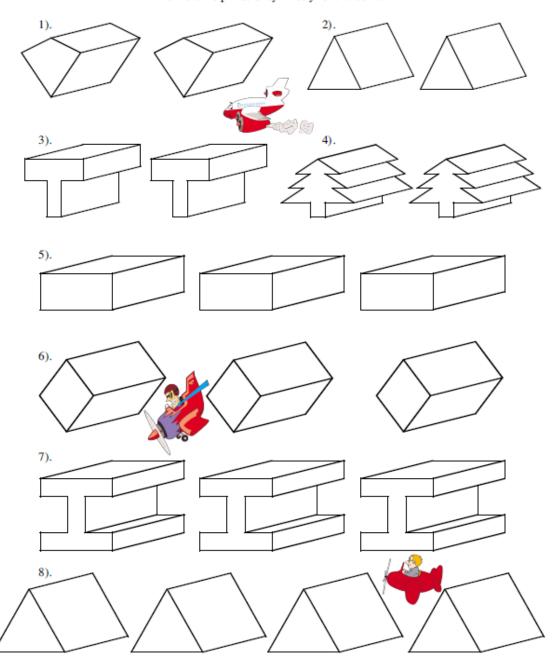
#### Dimensional Units



# Planes of Symmetry.

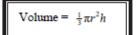
For each of the following solids draw on the diagrams all the planes of symmetry.

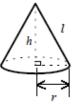
To make it easier there are exactly the same number of drawings
as there are planes of symmetry for that solid.





## Volume of a Cone.

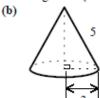


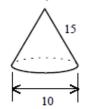


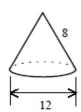


1. Find the volume of the following cones (all lengths are in cm).

(a) 12 l







 Use the formulae to fill in the blanks in the following table. Give missing lengths in cm, areas in cm<sup>2</sup>, and volumes in cm<sup>3</sup>, to three significant figures.

	r (cm)	h (cm)	l (cm)	Volume (cm <sup>3</sup> )
a	3	4		
b	10	24		
c		7	25	
d		6	10	
e		3		12
f		5		125
g	3			9
h	2			28
	ı			

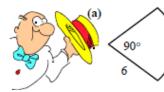


(d)

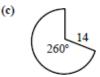
- A cone of height 20 cm, and base radius 5 cm is cut out of a cylindrical block of wood.
  - (a) Calculate the volume of the cone.
  - (b) Calculate the volume of wood that is wasted from the block after the cone has been cut out.

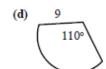


- 4. Find the height of a cone of volume 20 litres and base radius 80 cm.
- 5. Rainwater flows into an inverted cone of base radius 50 cm and height 2 m at a rate of 0.5 litres per minute. How long does it take to fill the cone from empty?
- When each of the following sectors is made into a right circular cone, (all lengths are in cm) find.
  - (i) the base radius of the cone, (ii) the height of the cone, (iii) the volume of the cone.









# The Sphere.

Consider two spheres, of radii 1 unit and r units. One sphere is an enlargement of the other, so if the volume of the sphere of radius 1 (let this be k say) is known, then the volume of a sphere of radius r will be given by,



$$V = bx^3$$
.

It turns out that the volume of a sphere of radius 1 unit is  $\frac{4}{3}\pi$  units<sup>3</sup>, hence the formula for the volume of a sphere of radius r is,



The formula for the surface area of a sphere is

$$S = 4\pi r^2$$

Copy and fill in the missing spaces in the following table, which refer to spheres.

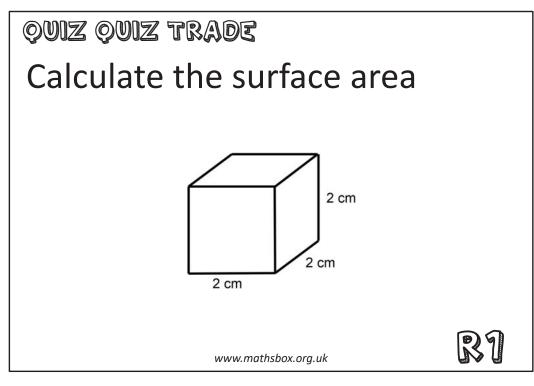
All dimensions are in cm, cm<sup>2</sup>, and cm<sup>3</sup>, respectively. Give missing answers to 3 sig. Figs.

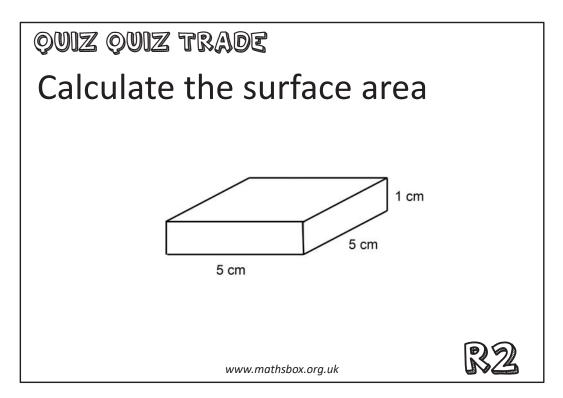
	Radius	Diameter	Volume	Surface Area
1	10			
2	6			
3	1			
4	3.8			
5	5			
6		12		
7		7		
8		6		
9		50		
10		100		
11			250	
12			9	
13			15	
14			3000	
15			1.43 x 10 <sup>27</sup>	
16				60
17				1
18				$4.71 \times 10^{14}$
19				1000
20				9.16 x 10 <sup>-5</sup>

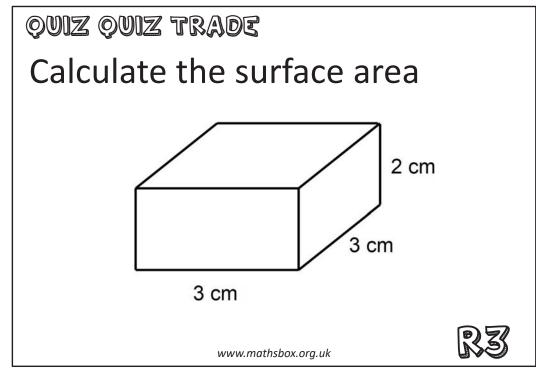
In questions 21 to 27 find the volumes and surface areas in terms of  $\pi$ . All dimensions are in cm, cm<sup>2</sup>, and cm<sup>3</sup>, respectively.

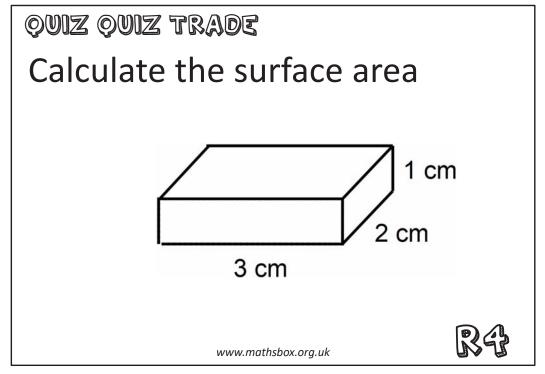
	Radius	Volume	Surface Area
21	4		
22	5		
23		288π	
24		$10^{2}/_{3}\pi$	
25			36π
26			400π
27			π

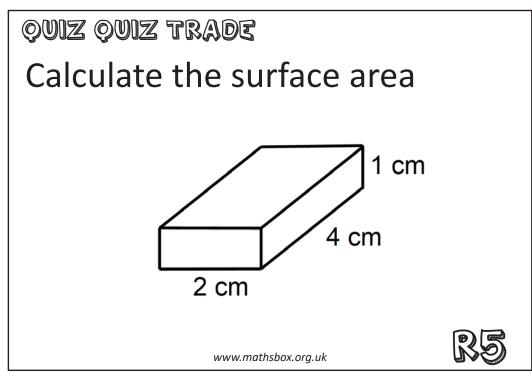


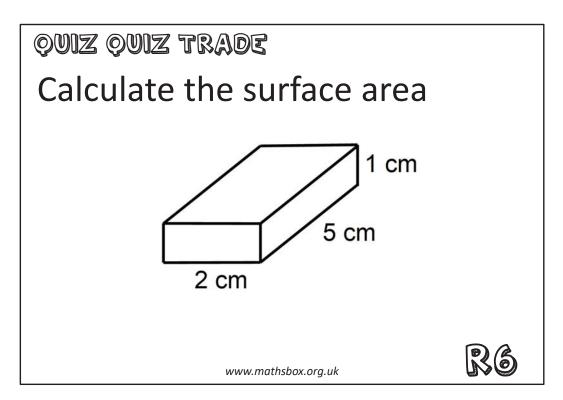


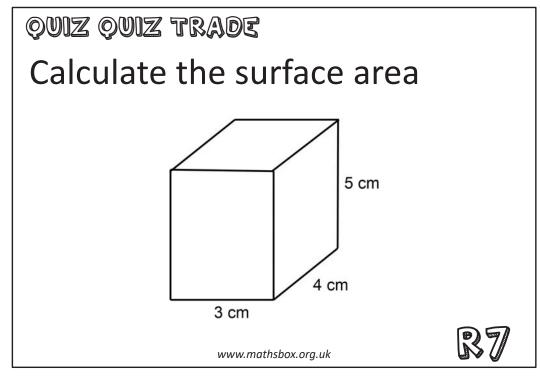


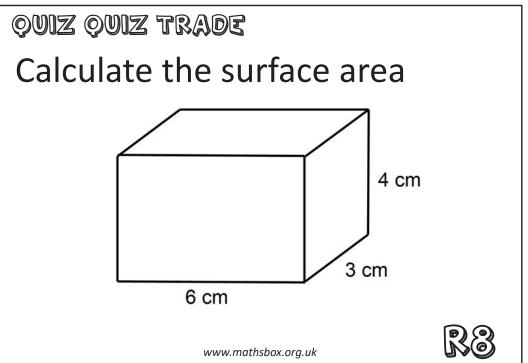


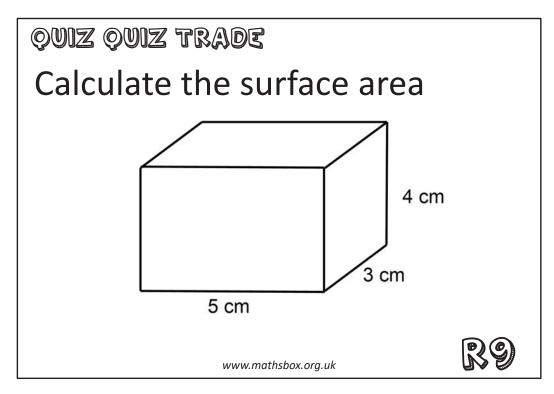


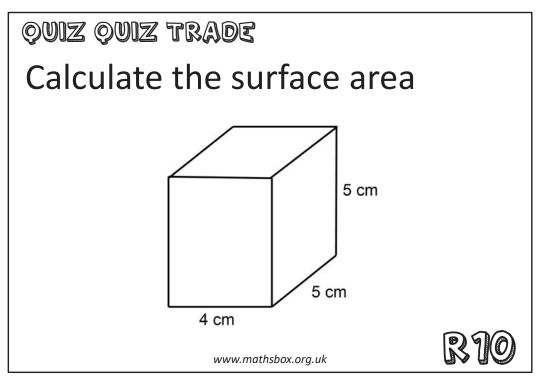


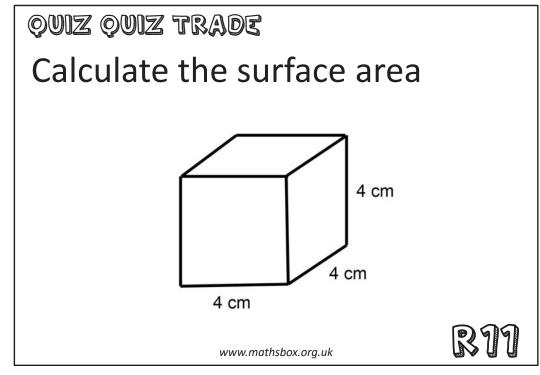


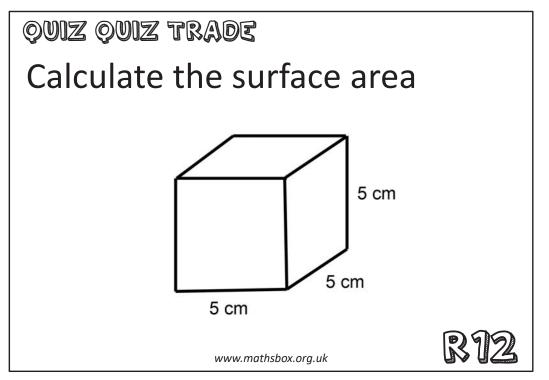


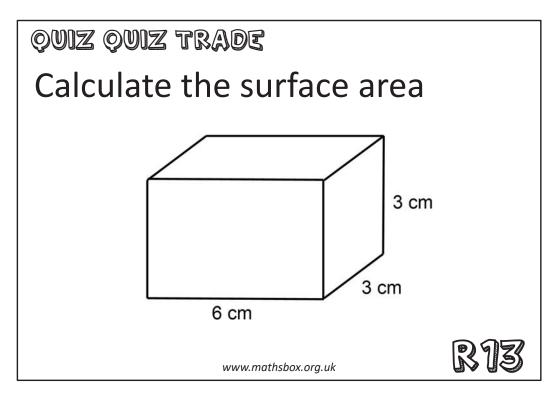


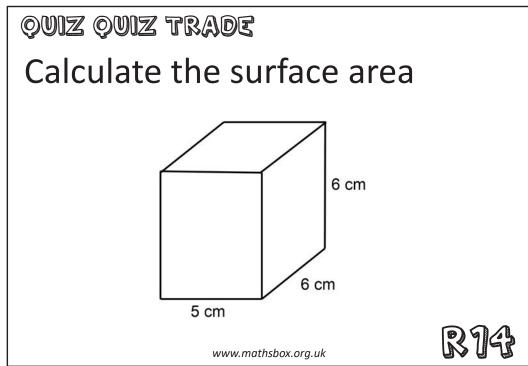


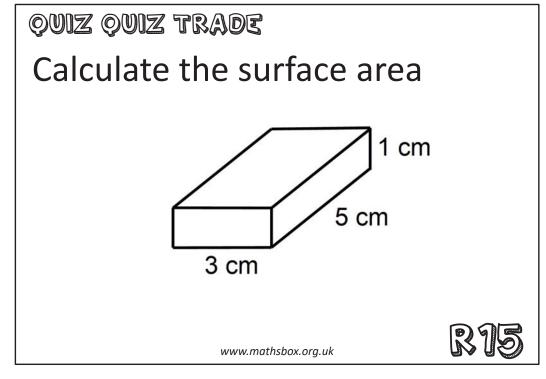


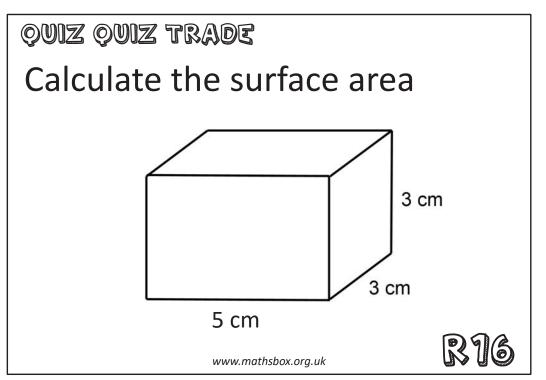


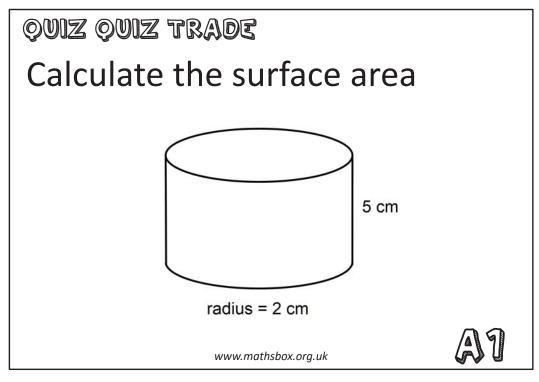


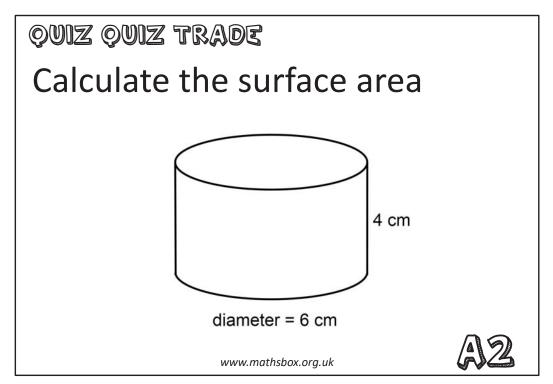


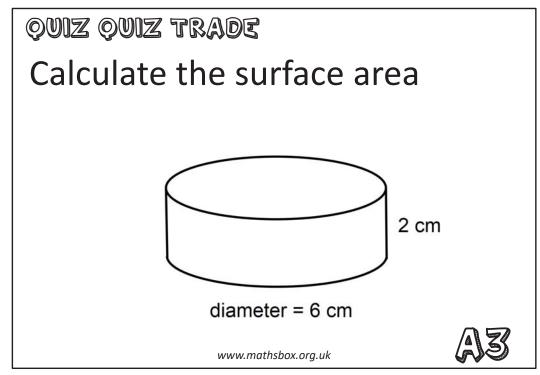


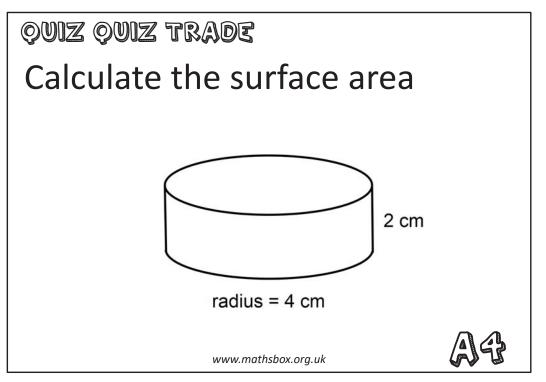


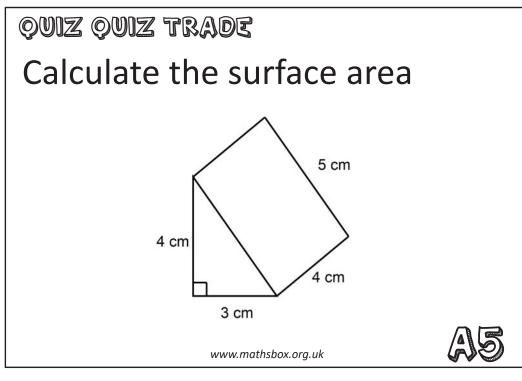


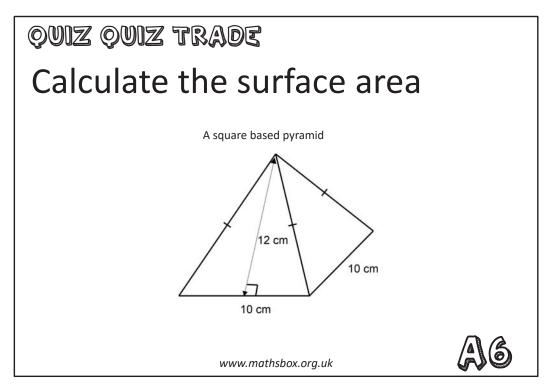


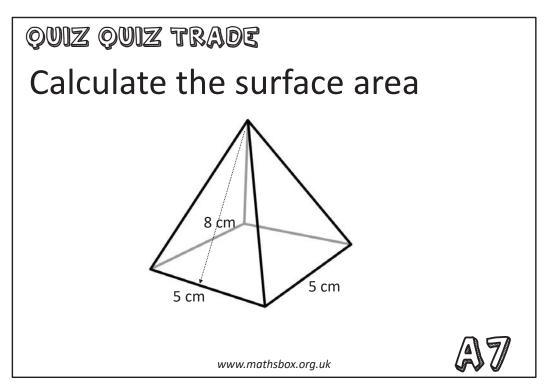


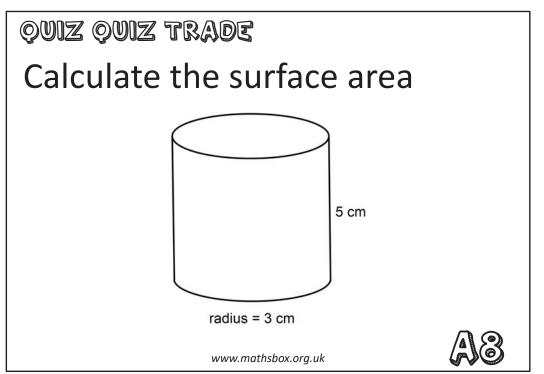




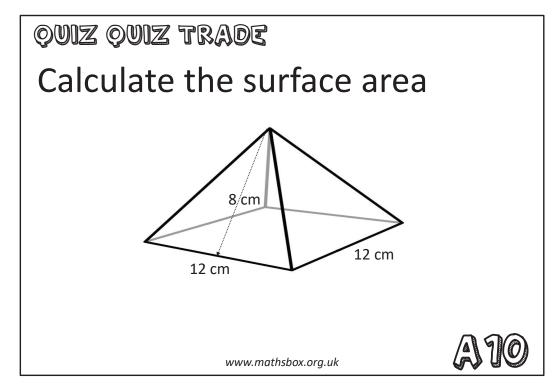


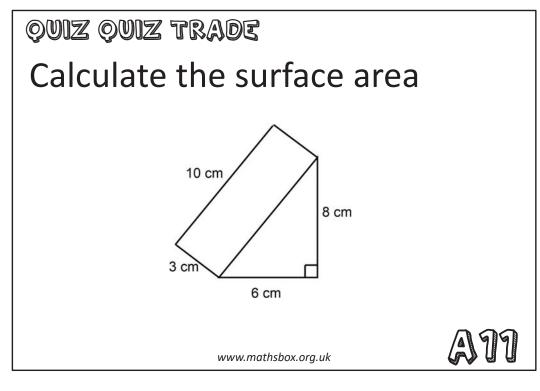


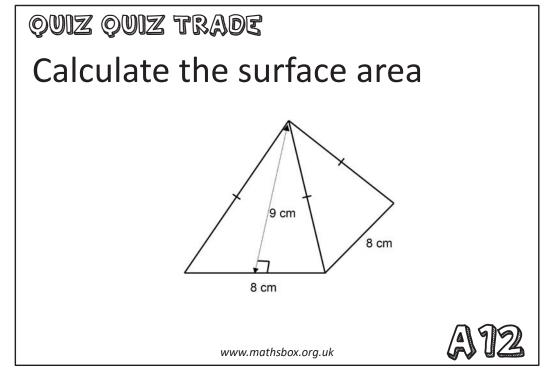


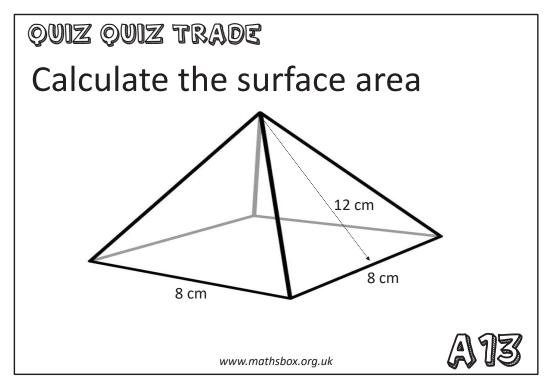


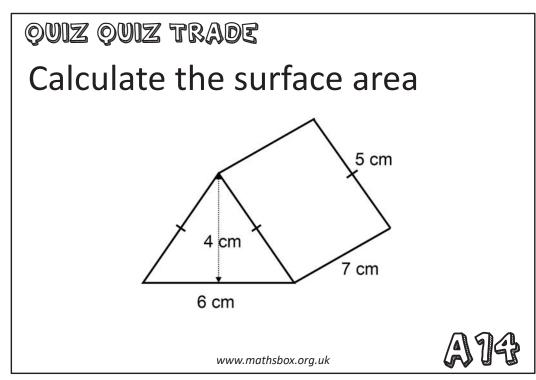
# Calculate the surface area 5 cm 13 cm 12 cm www.mathsbox.org.uk

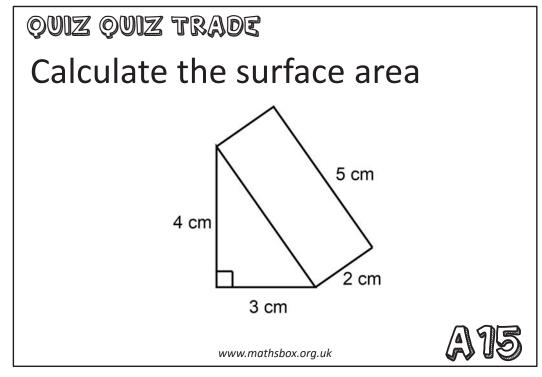


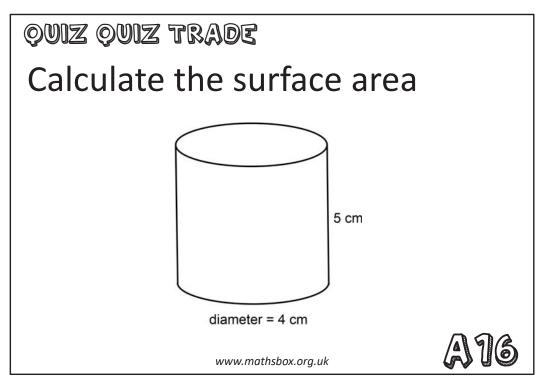


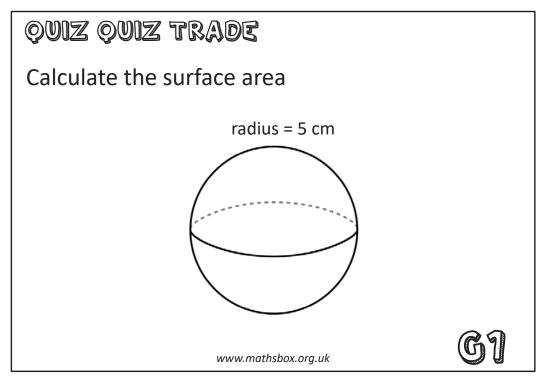


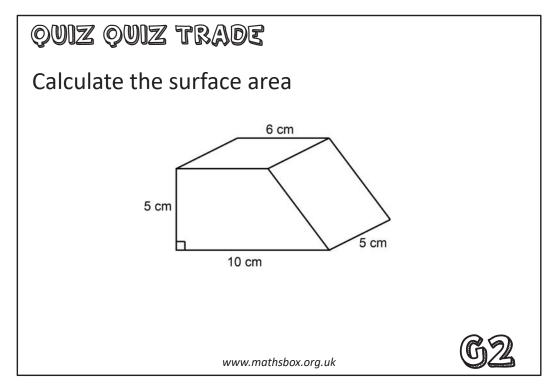


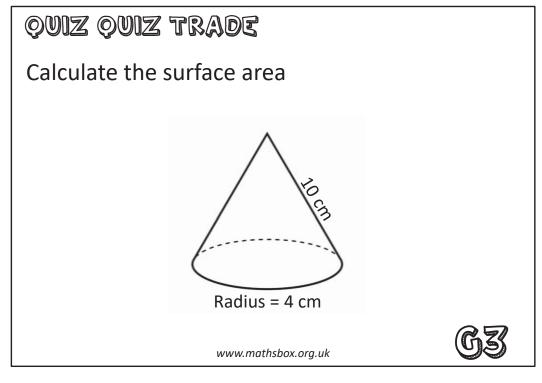


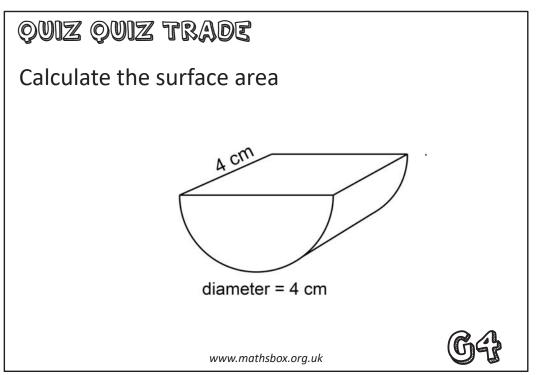


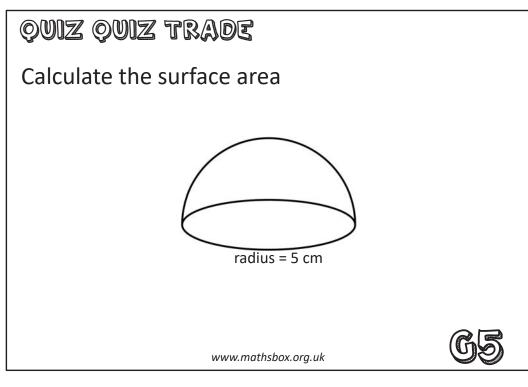


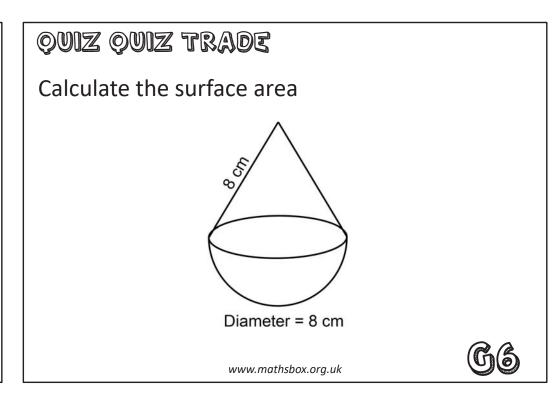


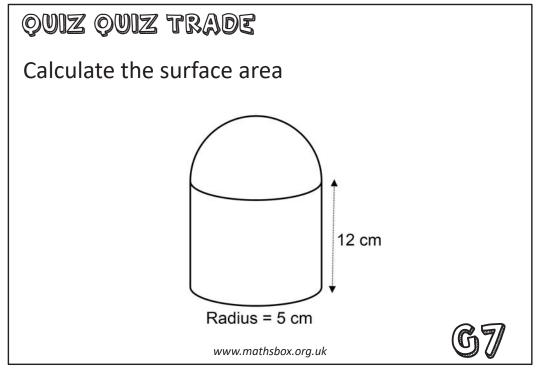


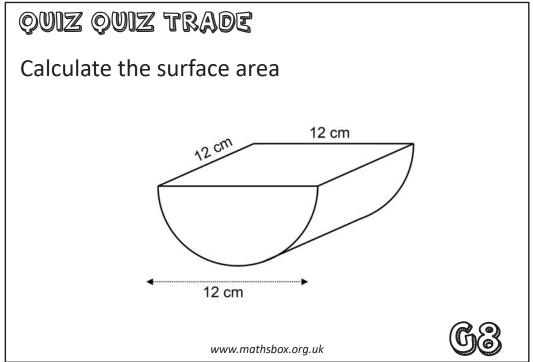


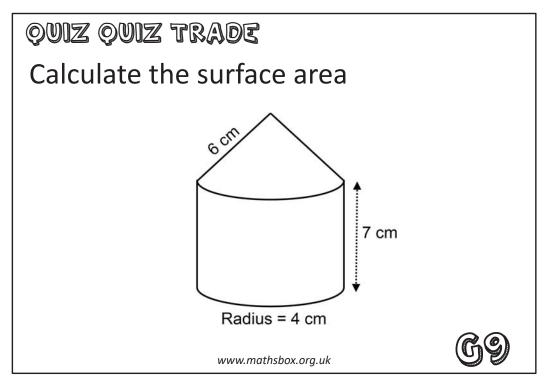


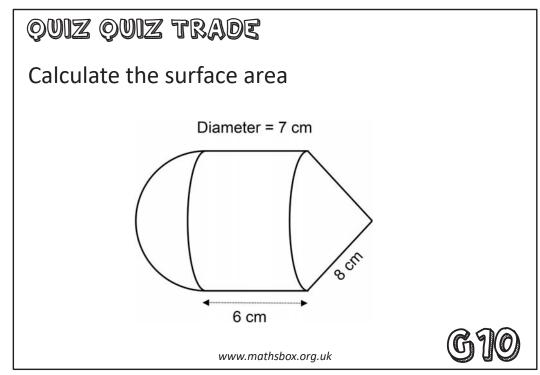


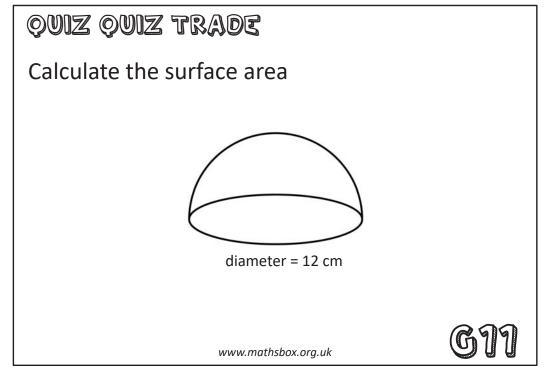


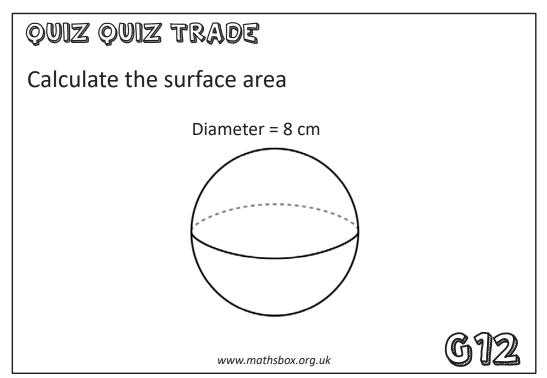


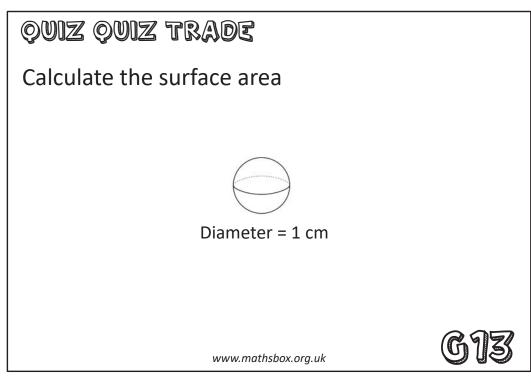


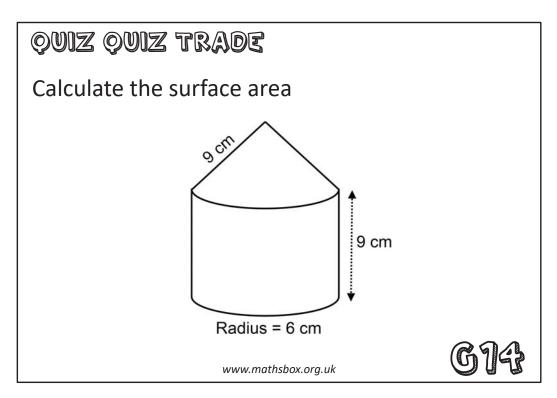


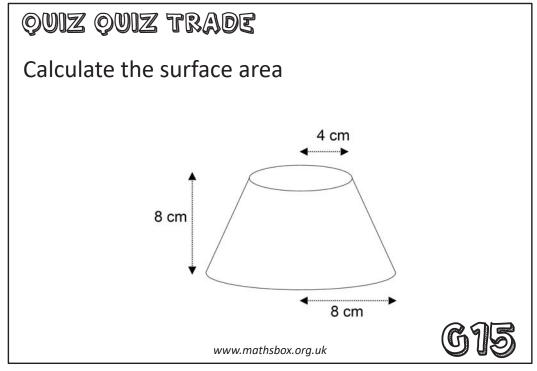


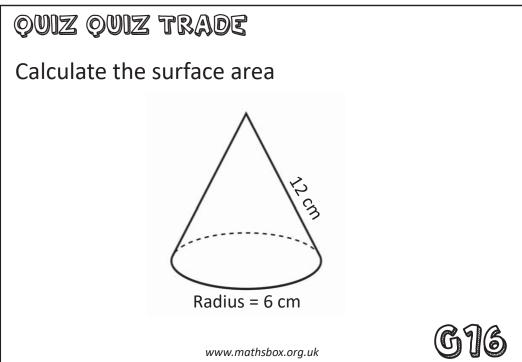






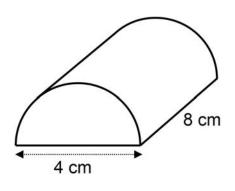






# Quiz Quiz trade

Calculate the surface area

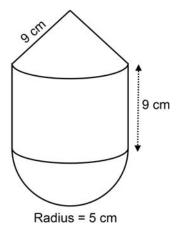


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# Quiz Quiz trade

# Calculate the surface area

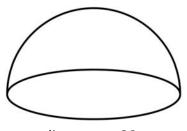


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# Quiz Quiz trade

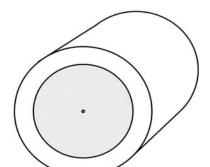
Calculate the surface area



diameter = 20 cm

QUIZ QUIZ TRADE

Calculate the total surface area



A hollow cylinder of length 10 cm

External diameter = 16 cm Internal diameter = 12 cm

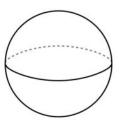
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# QUIZ QUIZ TRADE

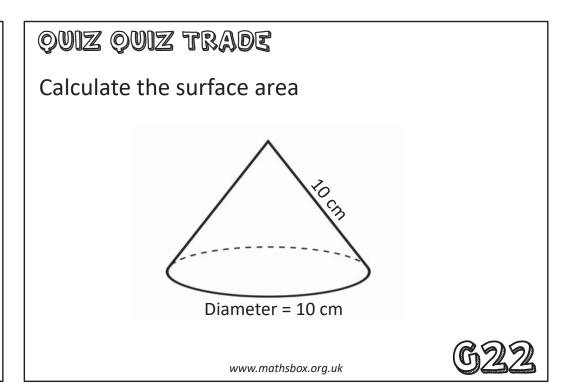
Calculate the surface area

radius = 3 cm



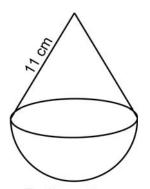
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# QUIZ QUIZ TRADE

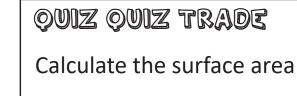
Calculate the surface area

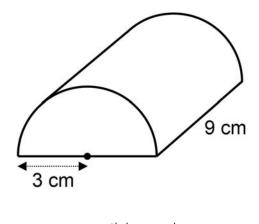


Radius = 5 cm

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**G23** 





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