M5-1

Determine whether each function represents *exponential growth* or *exponential decay*.

17. $f(x) = 7^x$

18. $g(x) = 0.99^x$

19. $h(x) = \left(\frac{2}{3}\right)^x$

20. $j(x) = \left(\frac{5}{4}\right)^x$

21. $k(x) = 0.75^x$

22. $m(x) = 1.02^x$

Graph each function. Find the domain, range, *y*-intercept, asymptote, and end behavior.

23. $f(x) = 0.25^x$

24. $f(x) = 0.8^x$

25. $f(x) = \left(\frac{1}{2}\right)^x$

26. $f(x) = \left(\frac{2}{3}\right)^x$

Graph each function.

27. $f(x) = -4\left(\frac{3}{5}\right)^{x+4} + 3$

28. $f(x) = 3\left(\frac{2}{5}\right)^{x-3} - 6$

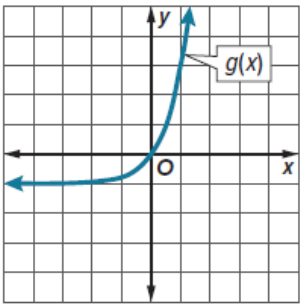
29. $f(x) = \frac{1}{2}\left(\frac{1}{5}\right)^{x+5} + 8$

30. $f(x) = \frac{3}{4}\left(\frac{2}{3}\right)^{x+4} - 2$

31. $f(x) = -\frac{1}{2}\left(\frac{3}{8}\right)^{x+2} + 9$

32. $f(x) = -\frac{5}{4}\left(\frac{4}{5}\right)^{x+4} + 2$

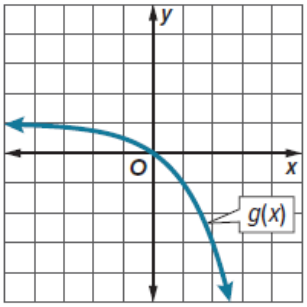
33. Consider $f(x) = \begin{cases} \left(\frac{3}{4}\right)^x \text{ for } x < 0 \\ 3x + 1 \text{ for } x \geq 0 \end{cases}$ and $g(x)$ shown in the graph.



a. Graph $f(x)$.

b. How do the key features, such as domain and range, intercepts, increasing and decreasing, positive and negative, minimum and maximum, symmetry, and end behavior of $f(x)$ and $g(x)$ compare?

34. Consider $f(x) = \begin{cases} -\left(\frac{1}{2}\right)^x + 3 \text{ for } x < -1 \\ -\frac{1}{4}x + \frac{3}{4} \text{ for } x \geq -1 \end{cases}$ and $g(x)$ shown in the graph.



a. Graph $f(x)$.

b. How do the key features, such as domain and range, intercepts, increasing and decreasing, positive and negative, minimum and maximum, symmetry, and end behavior of $f(x)$ and $g(x)$ compare?

M5-2

Solve each equation or inequality.

21. $8^{4x+2} = 64$

22. $5^{x-6} = 125$

23. $625 \geq 5^{a+8}$

24. $256^{b+2} = 4^{2-2b}$

25. $9^{3c+1} = 27^{3c-1}$

26. $\left(\frac{1}{27}\right)^{2d-2} \leq 81^{d+4}$

27. $\left(\frac{1}{2}\right)^{4x+1} = 8^{2x+1}$

28. $\left(\frac{1}{5}\right)^{x-5} = 25^{3x+2}$

29. $\left(\frac{1}{64}\right)^{c-2} < 32^{2c}$

30. $\left(\frac{1}{8}\right)^{3x+4} = \left(\frac{1}{4}\right)^{-2x+4}$

31. $\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{27}{8}\right)^{x-4}$

32. $\left(\frac{1}{9}\right)^{3t+5} \leq \left(\frac{1}{243}\right)^{t-6}$

33. $81^{a+2} = 3^{3a+1}$

34. $10^{5b+2} > 1000$

35. $216 = \left(\frac{1}{6}\right)^{x+3}$

36. $8^{2y+4} = 16^{y+1}$

37. $\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}$

38. $\left(\frac{1}{36}\right)^{w+2} \leq \left(\frac{1}{216}\right)^{4w}$

39. $4^{2x} = 8^{x+4}$

40. $\left(\frac{1}{3}\right)^{m} = 27^{m+2}$

41. $25^{4t+1} \geq 125^{2t}$

42. $6^{2x-1} = 36^{-x}$

Write an exponential function with a graph that passes through the given points.

44. (0, 3) and (3, 375)

45. (0, −1) and (6, −64)

46. (0, 7) and (−2, 28)

47. $\left(0, \frac{1}{2}\right)$ and (2, 40.5)

48. (0, 15) and (1, 12)

49. (0, −6) and (−4, −1536)

50. $\left(0, \frac{1}{3}\right)$ and (3, 9)

51. (0, 1) and (6, 4096)

52. (0, −2) and (−1, −4)

M6-1

Write each equation in exponential form.

1. $\log_{15} 225 = 2$

2. $\log_3 \frac{1}{27} = -3$

3. $\log_5 \frac{1}{25} = 2$

4. $\log_3 243 = 5$

5. $\log_4 64 = 3$

6. $\log_4 32 = \frac{5}{2}$

Write each equation in logarithmic form.

7. $2^7 = 128$

8. $3^{-4} = \frac{1}{81}$

9. $7^{-2} = \frac{1}{49}$

10. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$

11. $2^9 = 512$

12. $64^{\frac{2}{3}} = 16$

Evaluate each expression.

13. $\log_2 64$

14. $\log_{100} 100{,}000$

15. $\log_5 625$

16. $\log_{27} 81$

17. $\log_4 \frac{1}{32}$

18. $\log_{10} 0.00001$

19. $\log_8 512$

20. $\log_9 1$

21. $\log_8 4$

43. **EARTHQUAKES** The intensity of an earthquake can be measured on the Richter scale using the formula $y = 10^{R-1}$, where y is the absolute intensity of the earthquake and R is its Richter scale measurement. In 1906, an earthquake in San Francisco had an absolute intensity of 6,000,000. What is the Richter scale measurement of the earthquake?

Richter Scale Number	Absolute Intensity
1	1
2	10
3	100
4	1000
5	10,000

44. **INVESTING** Maria invests $1000 in a savings account that pays 4% interest compounded annually. The value of the account A after x years can be determined from the equation $A = 1000(1.04^x)$. Solve the equation for x.

45. CONSTRUCT ARGUMENTS Julio and Natalia are competing in a math competition. They each select a logarithmic function and compare their functions to see which has a greater value. Julio selected $f(x) = 10 \log_2 x$, and Natalia selected $g(x) = 2 \log_{10} x$.

a. Which function has a greater value when $x = 7$? Explain your reasoning.

b. Which function has a greater value when $x = 1$? Explain your reasoning.

46. BIOLOGY An amoeba divides into two amoebas every hour. The number N of amoebas after t hours can be represented by $t = \log_2 N$. How many hours will it take for a single amoeba to divide into 2048 amoebas?

47. STRUCTURE Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ on the same coordinate plane. Define the domain, range, intercepts, and asymptotes of each. Compare the two graphs.

48. WRITE What should you consider when using exponential and logarithmic models to make decisions?

49. ANALYZE Let $f(x) = \log_{10}(x)$ and $g(x) = 10^x$. Find $f(g(x))$ and $g(f(x))$ and justify your conclusion.

50. **FIND THE ERROR** Betsy says that the graphs of all logarithmic functions cross the y-axis at (0, 1) because any number to the zero power equals 1. Tyrone disagrees because he says log 0 is undefined. Is either of them correct? Explain your reasoning.

51. **PERSEVERE** Without using a calculator, compare $\log_7 51$, $\log_8 61$, and $\log_9 71$. Which of these is the greatest? Explain your reasoning.

52. **CREATE** Write a logarithmic equation of the form $y = \log_b x$ for each of the following conditions.

a. y is equal to 25.

b. y is negative.

c. y is between 0 and 1.

d. x is 1.

e. x is 0.

M6-2

Solve each equation. Check your solution.

27. $\log_3 56 - \log_3 n = \log_3 7$

28. $\log_2 (4x) + \log_2 5 = \log_2 40$

29. $5 \log_2 x = \log_2 32$

30. $\log_{10} a + \log_{10} (a + 21) = \log_{10} 100$

31. $\log_2 x + \log_2 (x + 2) = \log_2 8$

32. $\log_4 (x^2 + 2x + 1) = \log_4 (11 - x)$

33. $\log_3 \frac{x^2}{4} = \log_3 25$

34. $\log_3 3d = \log_3 9$

35. $\log_{10} (3x^2 - 5x) = \log_{10} 2$

36. $\log_4 (2x^2 - 3x) = \log_4 2$

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to approximate the value of each expression.

37. $\log_5 40$

38. $\log_5 30$

39. $\log_5 \frac{3}{4}$

40. $\log_5 \frac{4}{3}$

41. $\log_5 9$

42. $\log_5 16$

M6-4

Write each exponential equation in logarithmic form.

1. $e^{15} = x$

2. $e^{3x} = 45$

3. $e^{-5x} = 0.2$

4. $e^{8.2} = 10x$

5. $e^{x} = 3$

6. $e^{4} = 8x$

7. $e^{4} = x - 3$

8. $e^{x} = 8$

9. $e^{5} = 10x$

Write each logarithmic equation in exponential form.

10. $\ln 50 = x$

11. $\ln 36 = 2x$

12. $\ln 20 = x$

13. $\ln x = 8$

14. $\ln (4x) = 9.6$

15. $\ln 0.0002 = x$

16. $\ln x \approx 0.3345$

17. $\ln 15 = x$

18. $\ln 5.34 = 2x$

Write each expression as a single logarithm.

19. $3 \ln 3 - \ln 9$

20. $4 \ln 16 - \ln 256$

21. $2 \ln x + 2 \ln 4$

22. $3 \ln 4 + 3 \ln 3$

23. $\ln 125 - 2 \ln 5$

24. $3 \ln 10 + 2 \ln 100$

25. $4 \ln \frac{1}{3} - 6 \ln \frac{1}{9}$

26. $7 \ln \frac{1}{2} + 5 \ln 2$

27. $8 \ln x - 4 \ln 5$

M6-5

6. **RADIOACTIVE DECAY** The half-life of Rubidium-87 is about 48.8 billion years.

a. Determine the value of k and the equation of decay for Rubidium-87.

b. A specimen currently contains 50 milligrams of Rubidium-87. How long will it take the specimen to decay to only 18 milligrams of Rubidium-87?

c. How many milligrams of the 50-milligram sample will be left after 800 million years?

d. How long will it take Rubidium-87 to decay to one-sixteenth its original amount?

7. **USE A MODEL** Consider a certain bacteria which is undergoing continuous exponential growth.

a. If there are 80 cells initially and 675 cells after 30 minutes, determine the value of k.

b. When will the bacteria reach a population of 6000 cells?

c. If a second type of bacteria is growing exponentially according to the model $y = 35e^{0.0978t}$, determine how long before the number of cells of this bacteria exceed the number of cells in the original bacteria.

8. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of Plutonium-239 is 24,360 years, what is the value of k for this element?

9. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for $12,500. The GPS is now worth $8600. If the value of the system depreciates at a rate of 6.2% annually, how many years ago did Abu buy the GPS system?

10. **WHALES** Modern whales appeared between 5 and 10 million years ago. A paleontologist claims to have discovered a whale vertebrae which contains 80% less Carbon-14 than it originally contained. Is it possible for this vertebrae to be from a modern whale? Use the decay constant of 0.00012 for Carbon-14. Justify your response.

11. **REGULARITY** Luisa read that the population of her town has increased exponentially. The current population of her town is 68,735. One year ago, the population was 67,387.

a. Based on this information, write an exponential growth equation. Let y represent the population after t years.

b. Use the equation to estimate the population 100 years ago.

12. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, Cobalt-60, is radioactive and has a half-life of 5.7 years. What is the value of rate of decay for Cobalt-60?

13. **WILDLIFE** The initial population of rabbits in an area is 8000 and the population grows continuously at a rate of 26% each year. Write an equation to represent the rabbit population P in thousands after t years. Then, determine how long it will take for the population to reach 25,000.

M7-1

Simplify each expression, and state when the original expression is undefined.

1. $\dfrac{x(x-3)(x+6)}{x^2+x-12}$

2. $\dfrac{y^2(y^2+3y+2)}{2y(y-4)(y+2)}$

3. $\dfrac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$

4. $\dfrac{(x^2-16x+64)(x+2)}{(x^2-64)(x^2-6x-16)}$

5. $\dfrac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$

6. $\dfrac{3y(y-8)(y^2+2y-24)}{15y^2(y^2-12y+32)}$

Simplify each expression.

7. $\dfrac{x^2 - 5x - 14}{28 + 3x - x^2}$

8. $\dfrac{9x^2 - x^3}{x^2 - 3x - 54}$

9. $\dfrac{(x - 4)(x^2 + 2x - 48)}{(36 - x^2)(x^2 + 4x - 32)}$

10. $\dfrac{16 - c^2}{c^2 + c - 20}$

Simplify each expression.

11. $\dfrac{3ac^3f^3}{8a^2bcf^4} \cdot \dfrac{12ab^2c}{18ab^3c^2f}$

12. $\dfrac{14xy^2z^3}{21w^4x^2z} \cdot \dfrac{7wxyz}{12w^2y^3z}$

13. $\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2}$

14. $\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2}$

15. $\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3}$

16. $\frac{14c^2f^5}{9a^2} \div \frac{35cf^4}{18ab^3}$

Simplify each expression.

20. $\dfrac{\frac{x^2-9}{6x-12}}{\frac{x^2+10x+21}{x^2-x-2}}$

21. $\dfrac{\frac{y-x}{z^3}}{\frac{x-y}{6z^2}}$

22. $\dfrac{\dfrac{a^2 - b^2}{b^3}}{\dfrac{b^2 - ab}{a^2}}$

23. $\dfrac{\dfrac{x - y}{a + b}}{\dfrac{x^2 - y^2}{b^2 - a^2}}$

Simplify each expression.

24. $\dfrac{y^2 + 8y + 15}{y - 6} \cdot \dfrac{y^2 - 9y + 18}{y^2 - 9}$

25. $\dfrac{c^2 - 6c - 16}{c^2 - d^2} \div \dfrac{c^2 - 8c}{c + d}$

26. $\dfrac{x^2 + 9x + 20}{8x + 16} \cdot \dfrac{4x^2 + 16x + 16}{x^2 - 25}$

27. $\dfrac{3a^2 + 6a + 3}{a^2 - 3a - 10} \div \dfrac{12a^2 - 12}{a^2 - 4}$

28. $\dfrac{9-x^2}{x^2-4x-21} \cdot \left(\dfrac{2x^2+7x+3}{2x^2-15x+7}\right)^{-1}$

29. $\left(\dfrac{2x^2+2x-12}{x^2+4x-5}\right)^{-1} \cdot \dfrac{2x^3-8x}{x^2-2x-35}$

30. $\left(\dfrac{3xy^3z}{2a^2bc^2}\right)^3 \cdot \dfrac{16a^4b^3c^5}{15x^7yz^3}$

31. $\dfrac{20x^2y^6z^{-2}}{3a^3c^2} \cdot \left(\dfrac{16x^3y^3}{9acz}\right)^{-1}$

32. $\dfrac{\dfrac{8x^2 - 10x - 3}{10x^2 + 35x - 20}}{\dfrac{2x^2 + x - 6}{4x^2 + 18x + 8}}$

33. $\dfrac{\dfrac{2x^2 + 7x - 30}{-6x^2 + 13x + 5}}{\dfrac{4x^2 + 12x - 72}{3x^2 - 11x - 4}}$

34. $\dfrac{x^2 + 4x - 32}{2x^2 + 9x - 5} \cdot \dfrac{3x^2 - 75}{3x^2 - 11x - 4} \div \dfrac{6x^2 - 18x - 60}{x^3 - 4x}$

35. $\dfrac{8x^2 + 10x - 3}{3x^2 - 12x - 36} \div \dfrac{2x^2 - 5x - 12}{3x^2 - 17x - 6} \cdot \dfrac{4x^2 + 3x - 1}{4x^2 - 40x + 24}$

M7-2

Simplify each expression.

20. $\frac{2}{a+2} - \frac{3}{2a}$

21. $\frac{5}{3b+d} - \frac{2}{3bd}$

22. $\frac{1}{x^2+2x+1} + \frac{x}{x+1}$

23. $\frac{2x+1}{x-5} - \frac{4}{x^2-3x-10}$

24. $\frac{5a}{24cf^4} + \frac{a}{36bc^4f^3}$

25. $\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z}$

26. $\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2}$

27. $\frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy}$

28. $\frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2}$

29. $\frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3}$

30. $\dfrac{8}{x^2 - 6x - 16} + \dfrac{9}{x^2 - 3x - 40}$

31. $\dfrac{6}{y^2 - 2y - 35} + \dfrac{4}{y^2 + 9y + 20}$

32. $\dfrac{12}{3y^2 - 10y - 8} - \dfrac{3}{y^2 - 6y + 8}$

33. $\dfrac{6}{2x^2 + 11x - 6} - \dfrac{8}{x^2 + 3x - 18}$

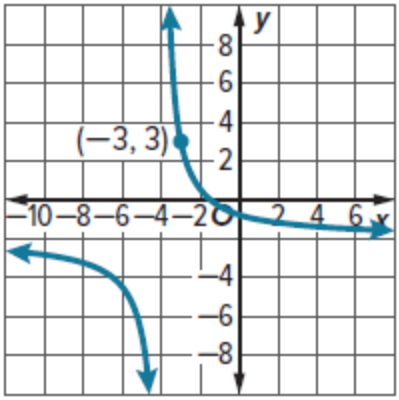
34. $\dfrac{2x}{4x^2 + 9x + 2} + \dfrac{3}{2x^2 - 8x - 24}$

35. $\dfrac{4x}{3x^2 + 3x - 18} - \dfrac{2x}{2x^2 + 11x + 15}$

M7-3

Example 5 Write a Reciprocal Function from a Graph

Identify the values of a, h, and k. Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.



M7-5

If x varies directly as y, find x when $y = 8$.

1. $x = 6$ when $y = 32$

2. $x = 11$ when $y = -3$

3. $x = 14$ when $y = -2$

4. $x = -4$ when $y = 10$

5. If y varies directly as x, and $y = 35$ when $x = 7$, find y when $x = 11$.

6. If y varies directly as x, and $y = 360$ when $x = 180$, find y when $x = 270$.

If a varies jointly as b and c, find a when $b = 4$ and $c = -3$.

7. $a = -96$ when $b = 3$ and $c = -8$

8. $a = -60$ when $b = -5$ and $c = 4$

9. $a = -108$ when $b = 2$ and $c = 9$

10. $a = 24$ when $b = 8$ and $c = 12$

11. If y varies jointly as x and z, and $y = 18$ when $x = 2$ and $z = 3$, find y when x is 5 and z is 6.

12. If y varies jointly as x and z, and $y = -16$ when $x = 4$ and $z = 2$, find y when x is -1 and z is 7.

62. **COMMUNICATIONS** On average, the number of calls c each day between two cities is directly proportional to the product of the populations P_1 and P_2 of the cities and inversely proportional to the square of the distance between them. That is, $c = \frac{kP_1P_2}{d^2}$. Use the function and the population and distance information in the table below.

	Birmingham, AL (pop. 210,710)	Indianapolis, IN (pop. 872,680)	Tallahassee, FL (pop. 191,049)	San Francisco, CA (pop. 884,363)
Birmingham	—	479 mi	301 mi	2327 mi
Indianapolis	479 mi	—	778 mi	2274 mi
Tallahassee	301 mi	778 mi	—	2637 mi
San Francisco	2327 mi	2274 mi	2637 mi	—

a. The average number of daily calls between Indianapolis and Birmingham is about 16,000. Find the value of k. Round to the nearest hundredth.

b. Find the average number of daily calls between Tallahassee and each other city listed.

c. Could you use this formula to find the average number of phone calls made within a city? Explain.

63. INVESTING You decide to invest 10% of your before-tax income in a retirement fund, so your employer deducts this money from your weekly paycheck.

a. Write an equation to represent the amount deducted from your paycheck d for investment in your retirement fund for a week during which you worked h hours and are paid r dollars per hour.

b. Is your equation a *direct*, *joint*, or *inverse* variation? Explain your reasoning.

c. If you earn \$19.50 per hour and worked 36 hours last week, explain how to determine the amount deducted last week for your retirement fund.

64. FIND THE ERROR Jamil and Savannah are setting up a proportion to begin solving the combined variation in which z varies directly as x, and z varies inversely as y. Who has set up the correct proportion? Explain your reasoning.

Jamil

$$z_1 = \frac{kx_1}{y_1} \text{ and } z_2 = \frac{kx_2}{y_2}$$

$$k = \frac{z_1 y_1}{x_1} \text{ and } k = \frac{z_2 y_2}{x_2}$$

$$\frac{z_1 y_1}{x_1} = \frac{z_2 y_2}{x_2}$$

Savannah

$$z_1 = \frac{kx_1}{y_1} \text{ and } z_2 = \frac{kx_2}{y_2}$$

$$k = \frac{z_1 x_1}{y_1} \text{ and } k = \frac{z_2 x_2}{y_2}$$

$$\frac{z_1 x_1}{y_1} = \frac{z_2 x_2}{y_2}$$

65. PERSEVERE If a varies inversely as b, c varies jointly as b and f, and f varies directly as g, how are a and g related?

66. ANALYZE Explain why some mathematicians consider every joint variation a combined variation, but not every combined variation a joint variation.

67. CREATE Describe three real-life quantities that vary jointly with each other.

68. WRITE Determine the type(s) of variation(s) for which 0 cannot be one of the values. Explain your reasoning.

M8-1

Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple random*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning.

1. Berton divides his sports T-shirts by team. Then he randomly selects four T-shirts from each team and records the size.

2. The project manager at a new business inspects every tenth smart phone produced to check that it is operating correctly.

3. A grocery store manager asks its customers to submit suggestions for items on the salad bar during the week.

Identify each sample or question as *biased* or *unbiased*. Explain your reasoning.

4. A random sample of eight people is asked to select their favorite food for a survey about Americans' food preferences.

5. Every tenth student at band camp is asked to name his or her favorite band for a survey about the campers.

6. Every fifth person entering a museum is asked to name his or her favorite type of book to read for a survey about reading interests of people in the city.

7. Do you think that the workout facility needs a new treadmill and racquetball court?

8. Which is your favorite type of music, pop, or country?

9. Are you a member of any after-school clubs?

10. Don't you agree that employees should pack their lunch?

M8-2

1. A student spun a spinner with 4 equal sections 100 times and recorded the results.

   a. Find the theoretical probability of spinning blue. Write your answer as a percentage rounded to the nearest tenth, if necessary.

Spinner Section	Frequency
Red	35
Blue	38
Green	13
Yellow	14

b. Find the experimental probability of spinning blue. Write your answer as a percentage rounded to the nearest tenth, if necessary.

2. A student flipped a coin 125 times and recorded the results.

a. Find the theoretical probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

Coin Result	Frequency
Heads	73
Tails	52

b. Find the experimental probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

3. A fair 6-sided die is rolled 150 times.

a. Find the theoretical probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.

Number on Die	Frequency
1	32
2	18
3	27
4	16
5	33
6	24

b. Find the experimental probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.

4. **INTERNET** Tiana sells handmade earrings online. Last month she sold 60% of her inventory. Design and run a simulation that can be used to estimate the probability of selling inventory.

5. **PROGRAMMING** Lamar designed a soccer computer game. He coded the program such that a player will make a goal on 35% of the attempts. Paola is testing the game and thinks there may be an error in the game's programming. She attempted to make 30 goals and only 4 were successful. Run and evaluate a simulation, and decide whether Paola is correct.

M8-4

6. TRACK The preliminary times for a 110-meter hurdles race are shown. Create a histogram of the set of data. Determine whether the data can be approximated with a normal distribution.

Times (seconds)
14.75, 14.77, 14.31, 14.83, 14.84, 14.35, 14.69, 14.63, 14.74, 14.82, 14.25, 14.93

7. A normal distribution has a mean of 186.4 and a standard deviation of 48.9.

a. What range of values represents the middle 99.7% of the data?

b. What percent of data will be greater than 235.3?

c. What range of values represents the upper 2.5% of the data?

Find the *z*-value for each standard normal distribution.

8. $\sigma = 9.8$, $X = 55.4$, and $\mu = 68.34$

9. $\sigma = 11.6$, $X = 42.80$, and $\mu = 68.2$

10. $\sigma = 11.9$, $X = 119.2$, and $\mu = 112.4$

M9-1

66. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 P.M.

67. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian.

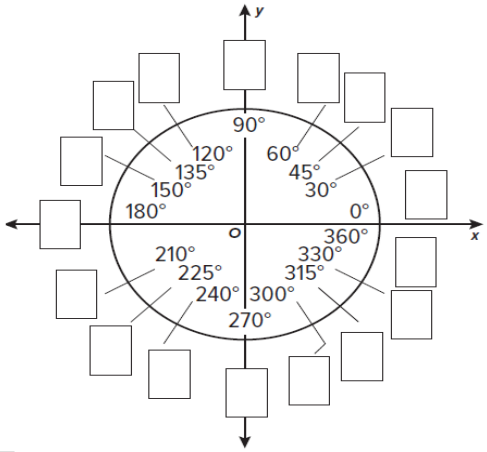
68. PLANETS Earth makes one full rotation on its axis every 24 hours. How long does it take Earth to rotate through 150°? Neptune makes one full rotation on its axis every 16 hours. How long does it take Neptune to rotate through 150°?

69. SURVEYING If a surveyor's wheel with a diameter of 19 inches completes $\frac{5}{6}$ of a rotation, what is the total distance traveled in inches? Round to the nearest hundredth if necessary.

70. STRUCTURE It is convenient to know the measures of some specific angles in both degree and radian measures.

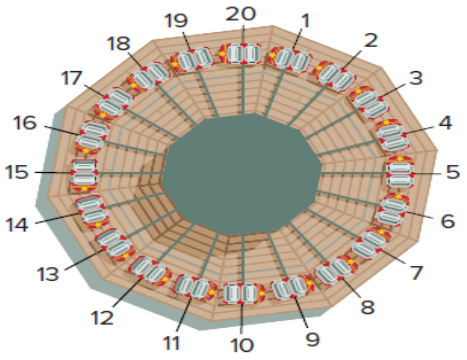
a. Copy and complete the figure at the right by writing the radian measure for each angle.

b. Describe at least one pattern or symmetry that you notice in the completed figure.



71. CLOCKS Through what angle, in degrees and radians, does the hour hand on a clock rotate between 4 P.M. and 7 P.M.? Assuming the length of the hour hand is 6 inches, find the arc length of the circle made by the hour hand during that time.

72. **AMUSEMENT PARKS** The carousel at an amusement park has 20 horses spaced evenly around its circumference. The horses are numbered consecutively from 1 to 20. The carousel completes one rotation about its axis every 40 seconds.



a. What is the central angle, in degrees, formed by horse #1 and horse #8?

b. What is the speed of the carousel in rotations per minute?

c. What is the speed of the carousel in radians per minute?

d. A child rides the carousel for 6 minutes. Through how many radians will the child pass in the course of the carousel ride?

73. **FERRIS WHEELS** The London Eye is one of the world's largest Ferris wheels. The diameter of the wheel is 135 meters, and it makes a complete rotation in 30 minutes. A passenger gets on the ride and travels for 5 minutes before the ride stops. The passenger wants to know how far she traveled during this time.

a. During the 5-minute interval, what is the measure of the central angle of the wheel's rotation in radians? Explain.

b. Explain how to find the distance the passenger traveled to the nearest meter.

c. Explain how you know your answer is reasonable.

M9-2

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

13. (5, 12)

14. (3, 4)

15. (8, −15)

16. (−4, 3)

17. (−9, −40)

18. (1, 2)

19. (8, 4)

20. (4, 4)

21. (6, 2)

22. $(-5, 5\sqrt{2})$

23. (3, −9)

24. (−8, 12)

25. (3, 0)

26. (0, −7)

27. (0, 4)

Sketch each angle. Then find the measure of each reference angle.

28. $\frac{31\pi}{36}$

29. $230°$

30. $205°$

31. $\frac{4\pi}{3}$

32. $-\frac{\pi}{6}$

33. $\frac{7\pi}{4}$

34. $135°$

35. $200°$

36. $\frac{5\pi}{3}$

37. $\frac{13\pi}{8}$

38. $-210°$

39. $-\frac{7\pi}{4}$

PRECISION **Find the exact value of each trigonometric function.**

40. $\tan 330°$

41. $\cos\left(-\frac{11\pi}{4}\right)$

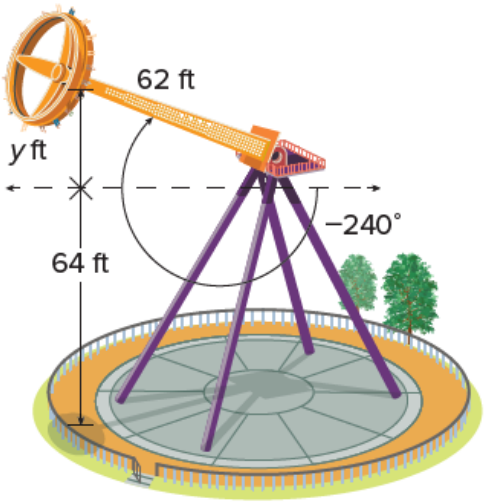
42. $\cot 30°$

43. $\csc \frac{\pi}{4}$

44. $\sin(-150°)$

45. $\tan\left(-\frac{\pi}{4}\right)$

46. **AMUSEMENT RIDES** An amusement park thrill ride swings its riders back and forth on a pendulum that spins. Suppose the swing arm of the ride is 62 feet in length, and the axis from which the arm swings is about 64 feet above the ground. What is the height of the riders above the ground at the peak of the arc? Round to the nearest foot if necessary.



47. **ROOFING** A roofer rests a ladder at a height of 12 feet against a building so that the base of the ladder is x feet from the bottom of the side of the building, forming a 71.6° angle with the ladder and ground. Find the distance from the bottom of the ladder to the side of the building.

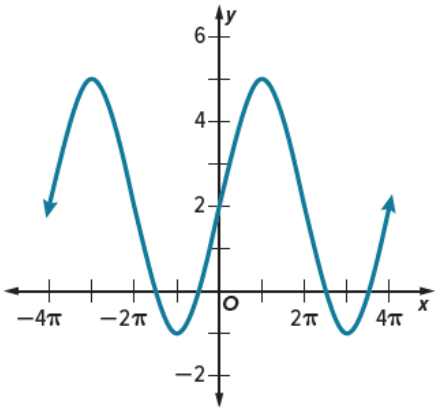
a. Write an equation that can be used to find the distance from the bottom of the ladder to the side of the building.

b. How far is the bottom of the ladder from the side of the building? Round to the nearest tenth if necessary.

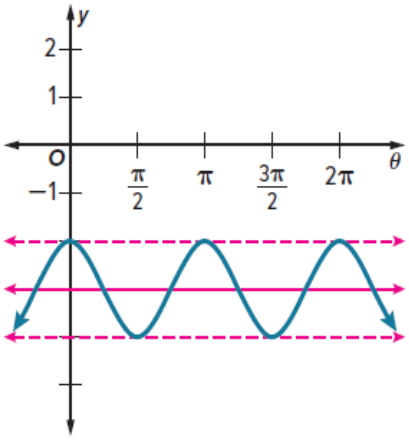
M9-6

Example 5 Write a Trigonometric Function from a Graph

Write a function for the cosine graph shown.



21. Write a function for the cosine graph shown.



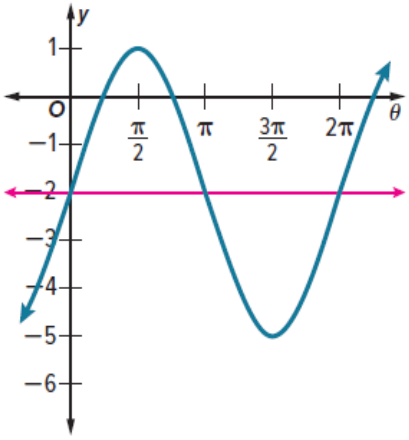
a. Find the vertical shift and amplitude.

b. Find the phase shift, if any.

c. Find the period.

d. Write the function.

22. Write a function for the sine graph shown.



a. Find the vertical shift and amplitude.

b. Find the phase shift.

c. Find the period.

d. Write the function.

State the amplitude, period, and phase shift for each function. Then graph the function and state the domain and range.

1. $y = \sin(\theta + \pi)$

2. $y = \tan\left(\theta - \frac{\pi}{2}\right)$

3. $y = \sin(\theta + 90°)$

4. $y = \cos(\theta - 45°)$

5. $y = \tan\left(\theta + \frac{\pi}{6}\right)$

6. $y = \cos(\theta + 180°)$

State the amplitude, period, and phase shift for each function. Then graph the function and state the domain and range.

7. $y = 3\cos(\theta - 45°)$

8. $y = 2\sin(\theta + 60°)$

9. $y = \frac{1}{2}\sin 3\left(\theta - \frac{\pi}{3}\right)$

10. $y = \frac{1}{2}\cos 3\left(\theta - \frac{\pi}{2}\right)$

11. $y = 2\cos(\theta + 90°)$

12. $y = \frac{1}{4}\sin(\theta - 30°)$

State the amplitude, period, phase shift, vertical shift, and midline equation of each function. Then graph the function and state the domain and range.

13. $y = \cos\theta + 3$

14. $y = \tan\theta - 1$

15. $y = \tan\theta + \frac{1}{2}$

16. $y = 2\cos\theta - 5$

17. $y = 2\sin\theta - 4$

18. $y = \frac{1}{3}\sin\theta + 7$