مدرسة سلامة بنت بطي -Salamah bint buti School

مراجعة أسئلة الهيكل

المادة: رياضيات

الصف الحادي عشر متقدم

الفصل الدراسي الثالث 2023/2022
إعداد المعلمة: فاطمة السعيدي

طالباتي العزيزات، العلم هو الوسيلة الوحيدة التي يرتفع بها شأن الإنسان إلى مراتب الكرامة والشرف.

Academic Year / العام الدراسي	2022/2023
Term / الفصل	3
Subject / المادة	Mathematics/Reveal+Bridge / الرياضيات/ريفيل+ بريدج
Grade / الصف	11
Stream / المسار	Advanced / المتقدم
Number of Main Questions / عدد الأسئلة الأساسية	Part (1) - 10
	Part (2) - 10
	Part (3) - 3
Marks per Main Question / الدرجات لكل سؤال أساسي	Part (1) - 3
	Part (2) - 5
	Part (3) - (6-7)
****Number of Bonus Questions** / عدد الأسئلة الإضافية	2
Marks per Bonus Question / الدرجات لكل سؤال إضافي	5
*** **Type of All Questions** / نوع كافة الأسئلة	Part(1 and 2) MCQ
	Part (3) FRQ
* **Maximum Overall Grade** / *الدرجة القصوى الممكنة	110
Exam Duration - مدة الامتحان	150 minutes
Mode of Implementation- طريقة التطبيق	SwiftAssess & Paper-Based
Calculator / الآلة الحاسبة	Allowed / مسموحة

1	**Graph points with polar coordinates**
	التمثيل البياني للنقاط باستخدام إحداثيات القطب

Find three different pairs of polar coordinates that name the given point if $-360° \leq \theta \leq 360°$ or $-2\pi \leq \theta \leq 2\pi$.

14. $(1, 150°)$

$(1, 150° - 360°) = (1, -210°)$

$(1, 150° + 180°) = (-1, 330°)$

$(1, 150° - 180°) = (-1, -30°)$

15. $(-2, 300°)$

$(2, 300° - 180°) = (2, 120°)$

$(-2, 300° - 360°) = (-2, -60°)$

$(2, -60° - 180°) = (2, -240°)$

16. $\left(4, -\dfrac{7\pi}{6}\right)$

$\left(4, -\dfrac{7\pi}{6} + 2\pi\right) = (4, \dfrac{5\pi}{6})$

$\left(-4, -\dfrac{7\pi}{6} + \pi\right) = (-4, -\dfrac{\pi}{6})$

$\left(-4, -\dfrac{\pi}{6} + 2\pi\right) = (-4, \dfrac{11\pi}{6})$

17. $\left(-3, \dfrac{2\pi}{3}\right)$

$\left(3, \dfrac{2\pi}{3} + \pi\right) = (3, \dfrac{5\pi}{3})$

$\left(3, \dfrac{2\pi}{3} - \pi\right) = (3, -\dfrac{\pi}{3})$

$\left(-3, \dfrac{2\pi}{3} - 2\pi\right) = (-3, -\dfrac{4\pi}{3})$

18. $\left(5, \dfrac{11\pi}{6}\right)$

$\left(5, \dfrac{11\pi}{6} - 2\pi\right) = (5, -\dfrac{\pi}{6})$

$\left(-5, \dfrac{11\pi}{6} - \pi\right) = (-5, \dfrac{5\pi}{6})$

$\left(-5, \dfrac{5\pi}{6} - 2\pi\right) = (-5, -\dfrac{7\pi}{6})$

19. $\left(-5, -\dfrac{4\pi}{3}\right)$

$\left(-5, -\dfrac{4\pi}{3} + 2\pi\right) = (-5, \dfrac{2\pi}{6})$

$\left(5, -\dfrac{4\pi}{3} + \pi\right) = (5, -\dfrac{\pi}{3})$

$\left(5, -\dfrac{\pi}{3} + 2\pi\right) = (5, \dfrac{5\pi}{3})$

20. $(2, -30°)$

$(2, -30° + 360°) = (2, 330°)$

$(-2, -30° + 180°) = (-2, 150°)$

$(-2, -30° - 180°) = (-2, -210°)$

21. $(-1, -240°)$

$(1, -240° - 180°) = (1, 300°)$

$(1, -240° + 180°) = (1, -60°)$

$(-1, -240° - 360°) = (-1, 120°)$

Find a different pair of polar coordinates for each point such that $0 \le \theta \le 180°$ or $0 \le \theta \le \pi$.

44. $(5, 960°)$

We need to subtract $960°$ by $180k$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$(-5, 960° - 5(180°)) = (-5, 60°)$

45. $\left(-2.5, \dfrac{5\pi}{2}\right)$

We need to subtract $\dfrac{5\pi}{3}$ from $k\pi$, such that the result is between 0 and π. Test multiplies of π.

$\left(-2.5, \dfrac{5\pi}{3} - 2\pi\right) = (-2.5, \dfrac{\pi}{2})$

46. $\left(4, \dfrac{11\pi}{4}\right)$

We need to subtract $\dfrac{11\pi}{4}$ from $k\pi$, such that the result is between 0 and π. Test multiplies of π.

$\left(4, \dfrac{11\pi}{4} - 2\pi\right) = (4, \dfrac{3\pi}{4})$

47. $(1.25, -920°)$

We need to add $180k$ to $-920°$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$(1.25, -920° + 6(180°)) = (1.25, 160°)$

48. $\left(-1, -\dfrac{21\pi}{8}\right)$

We need to add $k\pi$ to $-\dfrac{21\pi}{8}$, such that the result is between 0 and π. Test multiplies of π.

$\left(1, -\dfrac{21\pi}{8} + 3\pi\right) = (1, \dfrac{3\pi}{8})$

49. $(-6, -1460°)$

We need to add $180k$ to $-1460°$, such that the result is between $0°$ and $180°$. Test multiples of $180°$.

$(6, -1460° + 9(180°)) = (6, 160°)$

Important Note:

- If k is odd, we need to replace r with $-r$ and $-r$ with r to obtain the correct polar coordinates.
- If k is even, use r obtain the correct polar coordinates.

2	**Graph simple polar equations**
	التمثيل البياني للمعادلات القطبية البسيطة

Write an equation for each polar graph.

57.
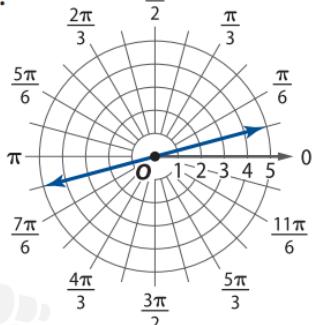
$\theta = \dfrac{\pi}{12}$

58.
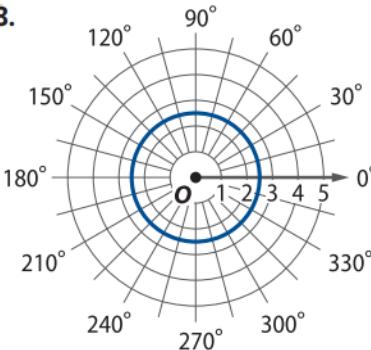
$r = 2.5 \text{ or } r = -2.5$

59.

$r = 4 \text{ or } r = -4$

60.

$\theta = 135°$

3	**Graph polar equations**
	تمثيل المعادلات القطبية بيانيا
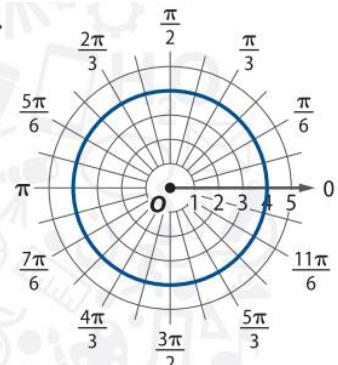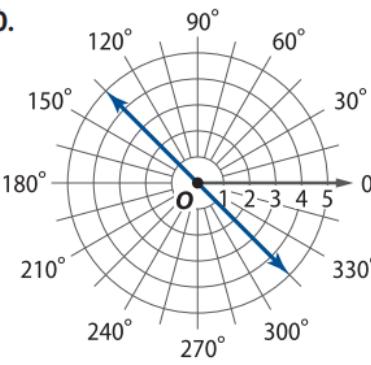
Graph each equation by plotting points.

1. $r = -\cos\theta$

2. $r = \csc\theta$

3. $r = \dfrac{1}{2}\cos\theta$

4
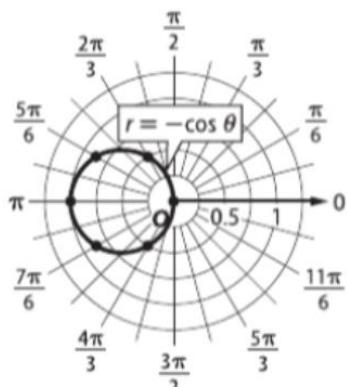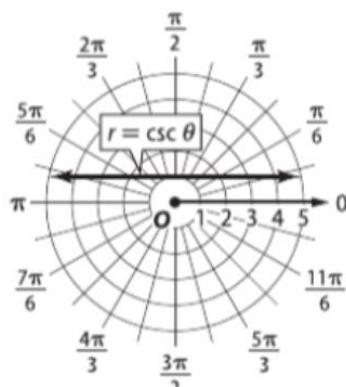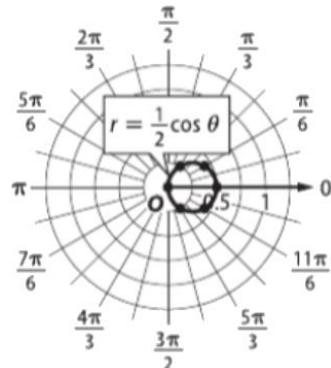
4. $r = 3 \sin \theta$
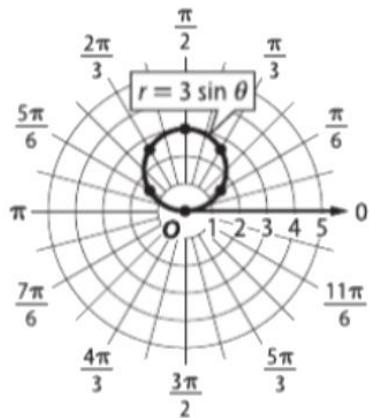
5. $r = -\sec \theta$
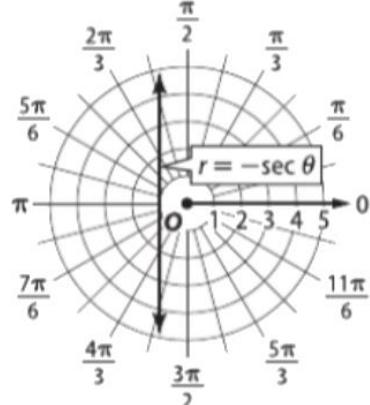
6. $r = \frac{1}{3} \sin \theta$
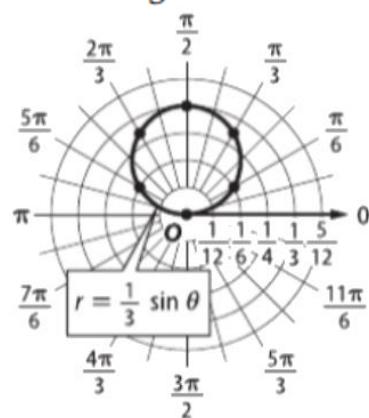
7. $r = -4 \cos \theta$
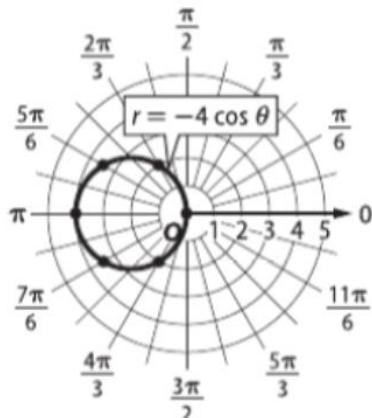
8. $r = -\csc \theta$
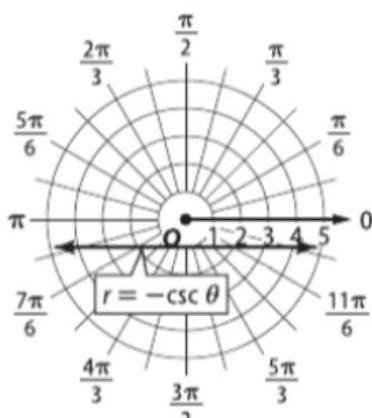
4	Convert between polar and rectangular coordinates
	التحويل بين الإحداثيات القطبية والديكارتية

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest hundredth, if necessary. $x = r \cos \theta$ $y = r \sin \theta$

1. $\left(2, \frac{\pi}{4}\right)$ $r = 2$ and $\theta = \frac{\pi}{4}$

$x = 2 \cos \frac{\pi}{4} = 2\sqrt{2}$

$y = 2 \sin \frac{\pi}{4} = 2\sqrt{2}$

$(2\sqrt{2}, 2\sqrt{2})$

2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$ $r = \frac{1}{4}$ and $\theta = \frac{\pi}{2}$

$x = \frac{1}{4} \cos \frac{\pi}{2} = 0$

$y = \frac{1}{4} \sin \frac{\pi}{2} = \frac{1}{4}$

$\left(0, \frac{1}{4}\right)$

5

3. $(5, 240°)$ $\quad r = 5$ and $\theta = 240°$

$x = 5 \cos 240° = -\dfrac{5}{2}$

$y = 5 \sin 240° = -\dfrac{5\sqrt{3}}{2}$ $\quad \left(-\dfrac{5}{2}, -\dfrac{5\sqrt{3}}{2}\right)$

4. $(2.5, 250°)$ $\quad r = 2$ and $\theta = 250°$

$x = 2.5 \cos 250° = -0.86$

$y = 2.5 \sin 250° = -2.35$

$(-0.86, -2.35)$

5. $\left(-2, \dfrac{4\pi}{3}\right)$ $\quad r = -2$ and $\theta = 240°$

$x = -2 \cos \dfrac{4\pi}{3} = 1$

$y = -2 \sin \dfrac{4\pi}{3} = \sqrt{3}$ $\quad (1, \sqrt{3})$

6. $(-13, -70°)$

$r = -13$ and $\theta = -70°$

$x = -13 \cos(-70°) = -4.45$

$y = -13 \sin(-70°) = 12.22$

$(-4.45, 12.22)$

7. $\left(3, \dfrac{\pi}{2}\right)$ $\quad r = 3$ and $\theta = \dfrac{\pi}{2}$

$x = 3 \cos \dfrac{\pi}{2} = 0$

$y = 3 \sin \dfrac{\pi}{2} = 3$ $\quad (0, 3)$

8. $\left(\dfrac{1}{2}, \dfrac{3\pi}{4}\right)$ $\quad r = \dfrac{1}{2}$ and $\theta = \dfrac{3\pi}{4}$

$x = \dfrac{1}{2} \cos \dfrac{3\pi}{4} = -\dfrac{\sqrt{2}}{4}$

$y = \dfrac{1}{2} \sin \dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{4}$ $\quad \left(-\dfrac{\sqrt{2}}{4}, \dfrac{\sqrt{2}}{4}\right)$

9. $(-2, 270°)$ $\quad r = -2$ and $\theta = 270°$

$x = -2 \cos 270° = 0$

$y = -2 \sin 270° = 2$ $\quad (0, 2)$

10. $(4, 210°)$ $\quad r = 4$ and $\theta = 210°$

$x = 4 \cos 210° = -2\sqrt{3}$

$y = 4 \sin 210° = -2$ $\quad (-2\sqrt{3}, -2)$

11. $\left(-1, -\dfrac{\pi}{6}\right)$ $\quad r = -1$ and $\theta = -\dfrac{\pi}{6}$

$x = -1 \cos -\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{2}$

$y = -1 \sin -\dfrac{\pi}{6} = \dfrac{1}{2}$ $\quad \left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

12. $\left(5, \dfrac{\pi}{3}\right)$ $\quad r = 5$ and $\theta = \dfrac{\pi}{3}$

$x = 5 \cos \dfrac{\pi}{3} = \dfrac{5}{2}$

$y = 5 \sin \dfrac{\pi}{3} = \dfrac{5\sqrt{3}}{2}$ $\quad \left(\dfrac{5}{2}, \dfrac{5\sqrt{3}}{2}\right)$

Determine whether each sequence is arithmetic. Write yes or no. $\boxed{d = a_2 - a_1}$

1. $8, -2, -12, -22,$ *yes*

$d = -2 - 8 = -10$

$d = -12 - (-2) = -10$

$d = -22 - (-12) = -10$

2. $-19, -12, -5, 2, 9$ *yes*

$d = -12 - (-19) = 7$

$d = -5 - (-12) = 7$

$d = 2 - (-5) = 7$

$d = 9 - (2) = 7$

3. $1, 2, 4, 8, 16$ *no*

$d = 2 - 1 = 1$

$d = 4 - 2 = 2$

4. $0.6, 0.9, 1.2, 1.8, \ldots$ *no*

$d = 0.9 - 0.6 = 0.3$

$d = 1.2 - 0.9 = 0.3$

$d = 1.8 - 1.2 = 0.6$

21. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ *no*

$d = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

$d = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$

22. $-9, -3, 0, 3, 9$ *no*

$d = -3 - (-9) = 6$

$d = 0 - (-3) = 3$

23. $14, -5, -19, \ldots$ *no*

$d = -5 - 14 = -19$

$d = -19 - (-5) = -14$

24. $\frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{11}{9}, \ldots$ *yes*

$d = \frac{5}{9} - \frac{2}{9} = \frac{1}{3}$

$d = \frac{8}{9} - \frac{5}{9} = \frac{1}{3}$

$d = \frac{11}{9} - \frac{8}{9} = \frac{1}{3}$

Find the next three terms of each geometric sequence. Then graph the sequence.

$$r = \frac{a_2}{a_1}$$

14. 8, 12, 18, 27, ...

$$r = \frac{12}{8} = 1.5$$

$$a_5 = 27 \times 1.5 = 40.5$$

$$a_6 = 40.5 \times 1.5 = 60.75$$

$$a_7 = 60.75 \times 1.5 = 91.125$$

40.5, 60.75, 91.125
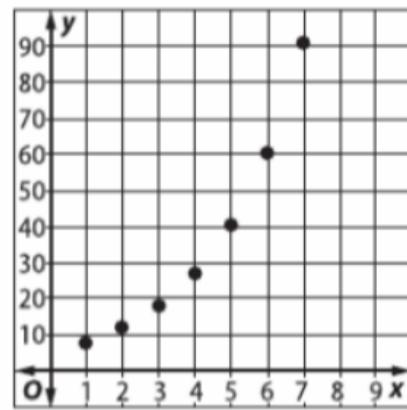
**15.** 8, 16, 32, 64, ...

$$r = \frac{16}{8} = 2$$

$$a_5 = 64 \times 2 = 128$$

$$a_6 = 128 \times 2 = 256$$

$$a_7 = 256 \times 2 = 512$$

128, 256, 512
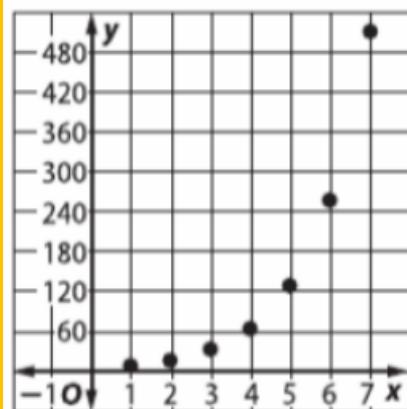

16. 250, 50, 10, 2, ...

$$r = \frac{50}{250} = 0.2$$

$$a_5 = 2 \times 0.2 = \frac{2}{5}$$

$$a_6 = \frac{2}{5} \times 0.2 = \frac{2}{25}$$

$$a_7 = \frac{2}{25} \times 0.2 = \frac{2}{125}$$

$$\frac{2}{5}, \frac{2}{25}, \frac{2}{125}$$

8
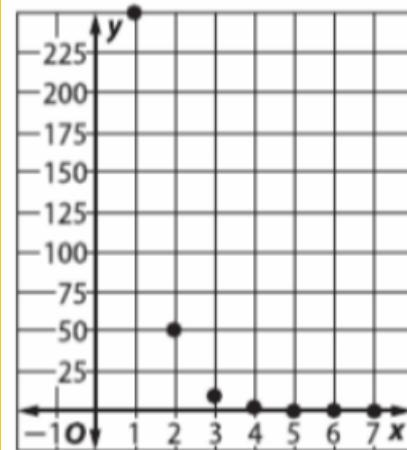
17. $9, -3, 1, -\dfrac{1}{3}, \ldots$

$$r = \dfrac{-3}{9} = -\dfrac{1}{3}$$

$$a_5 = -\dfrac{1}{3} \times -\dfrac{1}{3} = \dfrac{1}{9}$$

$$a_6 = \dfrac{1}{9} \times -\dfrac{1}{3} = -\dfrac{1}{27}$$

$$a_7 = -\dfrac{1}{27} \times -\dfrac{1}{3} = \dfrac{1}{81}$$

$$\dfrac{1}{9}, \dfrac{-1}{27}, \dfrac{1}{81}$$
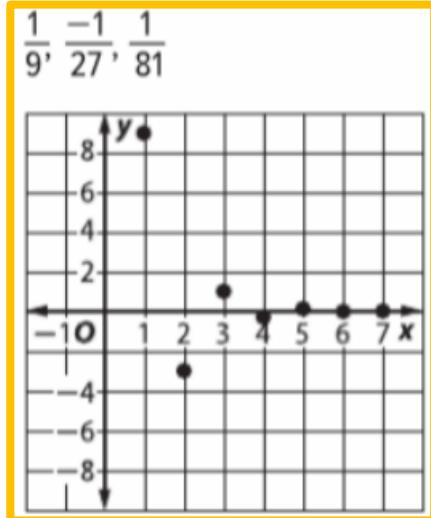
7	Find the nth term and arithmetic means of arithmetic sequences
	إيجاد الحد النوني والوسط الحسابي للمتتاليات الحسابية

Write an equation for the nth term of each arithmetic sequence. $\boxed{a_n = a_1 + (n-1)d}$

20. $24, 35, 46, \ldots$ $a_n = 11n + 13$	**21.** $31, 17, 3, \ldots$ $a_n = -14n + 45$	**22.** $a_9 = 45, d = -3$ $a_n = -3n + 72$
23. $a_7 = 21, d = 5$ $a_n = 5n - 14$	**24.** $a_4 = 12, d = 0.25$ $a_n = 0.25n + 11$	**25.** $a_5 = 1.5, d = 4.5$ $a_n = 4.5n - 21$
26. $9, 2, -5, \ldots$ $a_n = -7n + 16$	**27.** $a_6 = 22, d = 9$ $a_n = 9n - 32$	**28.** $a_8 = -8, d = -2$ $a_n = -2n + 8$
29. $a_{15} = 7, d = \dfrac{2}{3}$ $a_n = \dfrac{2}{3}n - 3$	**30.** $-12, -17, -22, \ldots$ $a_n = -5n - 7$	**31.** $a_3 = -\dfrac{4}{5}, d = \dfrac{1}{2}$ $a_n = \dfrac{1}{2}n - \dfrac{23}{10}$

9

Find a_n for each geometric sequence. $a_n = a_1 r^{n-1}$

18. $a_1 = 2400, r = \frac{1}{4}, n = 7$

$a_n = a_1 r^{n-1}$

$a_n = 2400(\frac{1}{4})^{7-1}$

$a_n = \frac{75}{128}$

19. $a_1 = 800, r = \frac{1}{2}, n = 6$

$a_n = a_1 r^{n-1}$

$a_n = 800(\frac{1}{2})^{6-1}$

$a_n = 25$

20. $a_1 = \frac{2}{9}, r = 3, n = 7$

$a_n = a_1 r^{n-1}$

$a_n = \frac{2}{9}(3)^{7-1}$

$a_n = 162$

21. $a_1 = -4, r = -2, n = 8$

$a_n = a_1 r^{n-1}$

$a_n = -4(-2)^{8-1}$

$a_n = 512$

Write each repeating decimal as a fraction. $$S_n = \frac{a_1}{1-r}$$

14. $0.\overline{35}$

$0.\overline{35} = 0.35 + 0.0035 + \ ...$

$a_1 = 0.35$ $r = \frac{0.0035}{0.35} = 0.01$

$S_n = \frac{0.35}{1 - 0.01} = \frac{35}{99}$

15. $0.\overline{642}$

$0.\overline{642} = 0.642 + 0.000642$

$a_1 = 0.642$ $r = \frac{0.000642}{0.642} = 0.01$

$S_n = \frac{0.642}{1 - 0.01} = \frac{214}{333}$

Write $0.\overline{63}$ as a fraction.

$S_n = \frac{0.63}{1 - 0.01} = \frac{7}{11}$

Write $0.\overline{21}$ as a fraction.

$S_n = \frac{0.21}{1 - 0.01} = \frac{7}{33}$

Find the first three iterates of each function for the given initial value.

8. $f(x) = 5x + 2, \ x_0 = 8$

$x_1 = f(x_0) = 5(8) + 2 = 42$

$x_2 = f(x_1) = 5(42) + 2 = 212$

$x_3 = f(x_2) = 5(212) + 2 = 1062$

9. $f(x) = -4x + 2, \ x_0 = 5$

$x_1 = f(x_0) = -4(5) + 2 = -18$

$x_2 = f(x_1) = -4(-18) + 2 = 74$

$x_3 = f(x_2) = -4(74) + 2 = -294$

10. $f(x) = 6x + 3, \ x_0 = -4$

$x_1 = f(x_0) = 6(-4) + 3 = -21$

$x_2 = f(x_1) = 6(-21) + 3 = -123$

$x_3 = f(x_2) = 6(-123) + 3 = -735$

11. $f(x) = 8x - 4, \ x_0 = -6$

$x_1 = f(x_0) = 8(-6) - 4 = -52$

$x_2 = f(x_1) = 8(-52) - 4 = -420$

$x_3 = f(x_2) = 8(-420) - 4 = -3364$

33. $f(x) = 12x + 8, \ x_0 = 4$

$x_1 = f(x_0) = 12(4) + 8 = 56$

$x_2 = f(x_1) = 12(56) + 8 = 680$

$x_3 = f(x_2) = 12(680) + 8 = 8168$

34. $f(x) = -9x + 1, \ x_0 = -6$

$x_1 = f(x_0) = -9(-6) + 1 = 55$

$x_2 = f(x_1) = -9(55) + 1 = -494$

$x_3 = f(x_2) = -9(-494) + 1 = 4447$

11

35. $f(x) = -6x + 3$, $x_0 = 8$

$x_1 = f(x_0) = -6(8) + 3 = -45$

$x_2 = f(x_1) = -6(-45) + 3 = 273$

$x_3 = f(x_2) = -6(273) + 3 = -1635$

36. $f(x) = 8x + 3$, $x_0 = -4$

$x_1 = f(x_0) = 8(-4) + 3 = -29$

$x_2 = f(x_1) = 8(-29) + 3 = -229$

$x_3 = f(x_2) = 8(-229) + 3 = -1829$

37. $f(x) = -3x^2 + 9$, $x_0 = 2$

$x_1 = f(x_0) = -3(2)^2 + 9 = -3$

$x_2 = f(x_1) = -3(-3)^2 + 9 = -18$

$x_3 = f(x_2) = -3(-18)^2 + 9 = -963$

38. $f(x) = 4x^2 + 5$, $x_0 = -2$

$x_1 = f(x_0) = 4(-2)^2 + 5 = 21$

$x_2 = f(x_1) = 4(21)^2 + 5 = 1769$

$x_3 = f(x_2) = 4(1769)^2 + 5 = 12517449$

39. $f(x) = 2x^2 - 5x + 1$, $x_0 = 6$

$x_1 = f(x_0) = 2(6)^2 - 5(6) + 1 = 43$

$x_2 = f(x_1) = 2(43)^2 - 5(43) + 1 = 3484$

$x_3 = f(x_2) = 2(3484)^2 - 5(3484) + 1 = 24259093$

40. $f(x) = -0.25x^2 + x + 6$, $x_0 = 8$

$x_1 = f(x_0) = -0.25(8)^2 + (8) + 6 = -2$

$x_2 = f(x_1) = -0.25(-2)^2 + (-2) + 6 = 3$

$x_3 = f(x_2) = -0.25(3)^2 + (3) + 6 = 6.75$

41. $f(x) = x^2 + 2x + 3$, $x_0 = \dfrac{1}{2}$

$$x_1 = f(x_0) = (0.5)^2 + 2(0.5) + 3 = \text{4.25}$$

$$x_2 = f(x_1) = (4.25)^2 + 2(4.25) + 3 = \text{29.5625}$$

$$x_3 = f(x_2) = (29.5625)^2 + 2(29.5625) + 3 = \text{936.0664}$$

42. $f(x) = 2x^2 + x + 1$, $x_0 = -\dfrac{1}{2}$

$$x_1 = f(x_0) = 2(-0.5)^2 + (-0.5) + 1 = \text{1}$$

$$x_2 = f(x_1) = (1)^2 + 2(1) + 3 = \text{4}$$

$$x_3 = f(x_2) = (4)^2 + 2(4) + 3 = \text{37}$$

Part (2) - **10**

11	Identify and graph classical curves
	تحديد المنحنيات الكلاسيكية وتمثيلها بيانيا

Identify the type of curve given by each equation. Then use symmetry, zeros, and maximum r-values to graph the function.

**26.** $r = \dfrac{1}{3}\cos\theta$

Circle
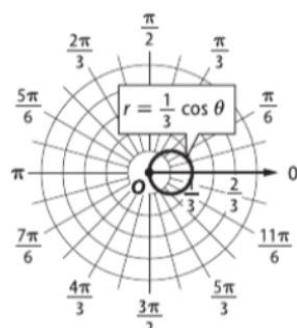
**27.** $r = 4\theta + 1;\ \theta > 0$

Spirals of Archimedes
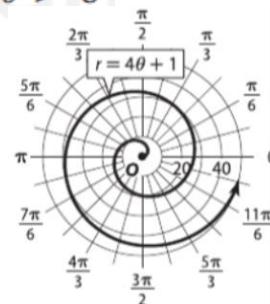
28. $r = 2\sin 4\theta$

Rose
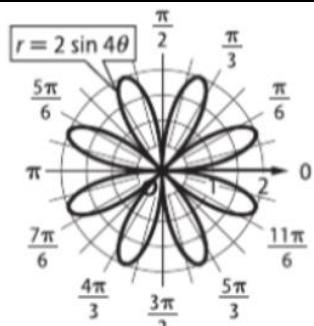


29. $r = 6 + 6\cos\theta$

Cardioid
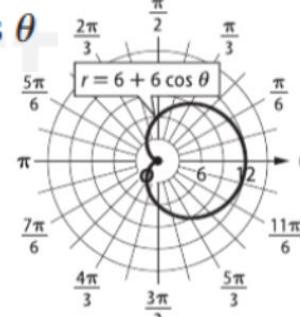
30. $r^2 = 4 \cos 2\theta$

Lemniscate

$r^2 = 4 \cos 2\theta$
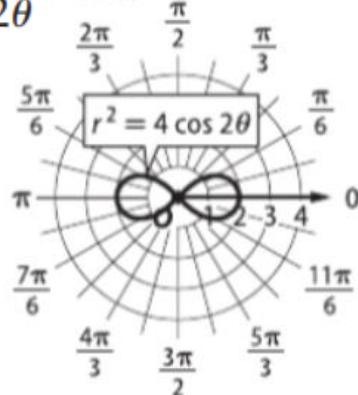
31. $r = 5\theta + 2; \theta > 0$

Spirals of Archimedes

$r = 5\theta + 2$

32. $r = 3 - 2 \sin \theta$

Limaçon

$r = 3 - 2 \sin \theta$
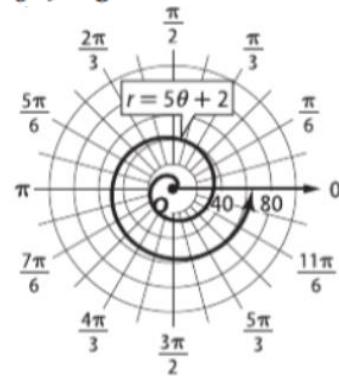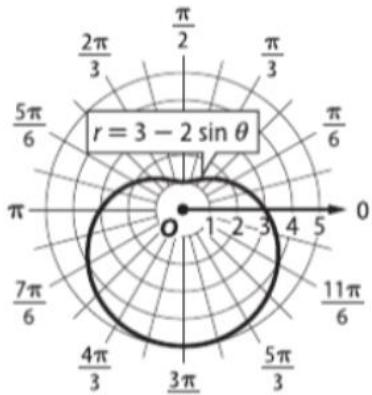
33. $r^2 = 9 \sin 2\theta$

Lemniscate

$r^2 = 9 \sin 2\theta$
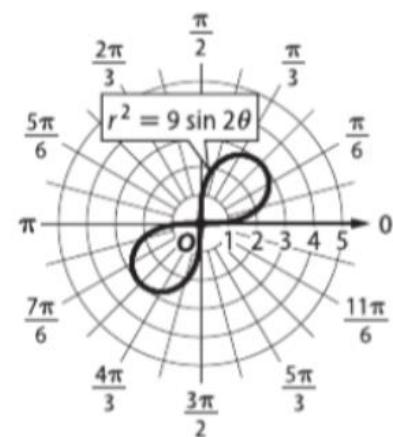
12	Convert between polar and rectangular equations
	التحويل بين المعادلات القطبية والديكارتية

Write each equation in rectangular form, and then identify its graph. Support your answer by graphing the polar form of the equation

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x} \qquad r = x^2 + y^2$$

36. $r = 3 \sin \theta$

$$r = 3 \sin \theta$$

$$r \cdot r = 3 \frac{y}{r} \cdot r$$

$$r^2 = 3y$$

$$x^2 + y^2 = 3y$$

$$x^2 + y^2 - 3y = 0$$

Circle

$r = 3 \sin \theta$

14

37. $\theta = -\dfrac{\pi}{3}$

$\theta = -\dfrac{\pi}{3}$

$\tan\theta = \tan\left(-\dfrac{\pi}{3}\right)$ Line

$\tan\theta = -\sqrt{3}$

$x.\dfrac{y}{x} = -\sqrt{3}.x$

$y = -\sqrt{3}x$
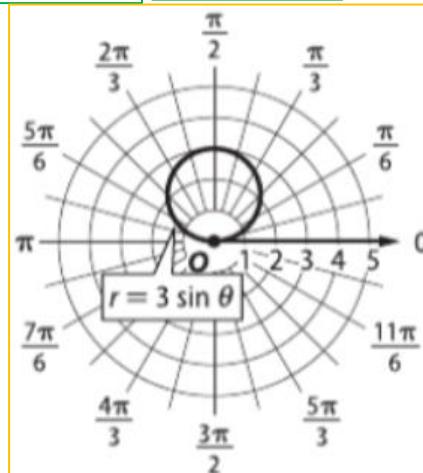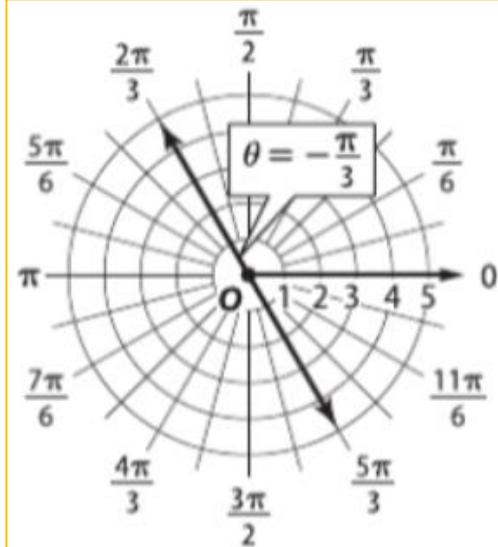
38. $r = 10$

$r = 10$

$r^2 = 100$

$x^2 + y^2 = 100$ Circle

39. $r = 4\cos\theta$

$r = 4\cos\theta$

$r.r = 4\dfrac{x}{r}.r$ Circle

$r^2 = 4x$

$x^2 + y^2 = 4x$

$x^2 + y^2 - 4x = 0$

40. $\tan \theta = 4$

$$\boldsymbol{tan\theta = 4}$$

$$\frac{y}{x} = 4$$

Line

$$x \cdot \frac{y}{x} = 4x$$

$$\boxed{y = 4x}$$

---

41. $r = 8 \csc \theta$

$$\csc(\boldsymbol{\theta}) = \frac{1}{\sin(\boldsymbol{\theta})} = \frac{r}{y}$$

$$\boldsymbol{r = 8\,csc\,\theta}$$

$$\frac{1}{r} \cdot r = 8 \cdot \frac{r}{x} \cdot \frac{1}{r}$$

Line

$$1 = \frac{8}{x}$$

$$x \cdot 1 = \frac{8}{x} \cdot x \qquad \boxed{x = 8}$$

---

42. $r = -4$

$$\boldsymbol{r = -4}$$

$$\boldsymbol{r^2 = 16}$$

Circle

$$\boldsymbol{x^2 + y^2 = 16}$$

43. $\cot\theta = -7$

$$\boldsymbol{cot\theta = -7}$$

$$\frac{x}{y} = -7$$

$$\boxed{\cot(\theta) = \frac{1}{\tan(\theta)}} \quad \boxed{= \frac{x}{y}}$$

==Line==

$$\frac{1}{x}\cdot\frac{x}{y} = -7\cdot\frac{1}{x}$$

$$\frac{1}{y} \bcancel{=} \frac{-7}{x}$$

$$\frac{1}{-7}\cdot x = -7\cdot y\cdot\frac{1}{-7} \qquad \boxed{y = \frac{1}{-7}x}$$

---

44. $\theta = \dfrac{3\pi}{4}$

$$\boldsymbol{tan\,\theta = tan(\frac{3\pi}{4})}$$

$$\boldsymbol{tan\,\theta = -1}$$

==Line==

$$x\cdot\frac{y}{x} = -x$$

$$\boxed{\boldsymbol{y = -x}}$$

---

45. $r = \sec\theta$

$$\boldsymbol{r = \frac{r}{x}}$$

$$\boxed{\sec(\theta) = \frac{1}{\cos(\theta)}} \quad \boxed{= \frac{r}{x}}$$
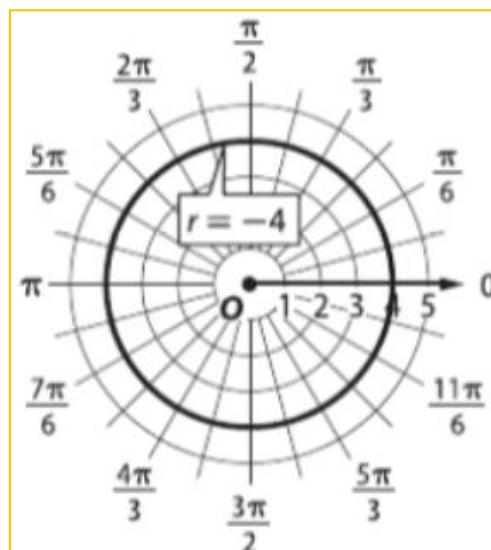
==Line==

$$\frac{1}{r}\cdot r = \frac{r}{x}\cdot\frac{1}{r}$$

$$x\cdot 1 = \frac{1}{x}\cdot x \qquad \boxed{\boldsymbol{x = 1}}$$

Express each complex number in polar form.

$$r = \sqrt{a^2 + b^2}$$

$$\theta = tan^{-1}(\frac{b}{a})$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi \ \ if \ a < 0$$

$$z = r\cos\theta + i\,r\sin\theta$$

10. $4 + 4i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (4)^2} = 4\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{4}{4}\right) = \frac{\pi}{4}$$

$$z = 4\sqrt{2}\cos\frac{\pi}{4} + i\,4\sqrt{2}\sin\frac{\pi}{4} \quad OR \quad z = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

11. $-2 + i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{5}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{1}{-2}\right) + \pi = 2.68$$

$$z = \sqrt{5}\cos2.68 + i\,\sqrt{5}\sin2.68 \quad OR \quad z = \sqrt{5}\left(\cos2.68 + i\sin2.68\right)$$

12. $4 - \sqrt{2}i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (-\sqrt{2})^2} = 3\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{-\sqrt{2}}{4}\right) = -0.34$$

$$z = 3\sqrt{2}\cos(-0.34) + i\,3\sqrt{2}\sin(-0.34) \quad OR \quad z = 3\sqrt{2}\left(\cos - 0.34 + i\sin - 0.34\right)$$

13. $2 - 2i$

Note: you can add $360°$ or 2π if theta was negative

$$r = \sqrt{a^2 + b^2} = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{-2}{2}\right) + 2\pi = \frac{7\pi}{4}$$

$$z = 2\sqrt{2}\cos\frac{7\pi}{4} + i\,2\sqrt{2}\sin\frac{7\pi}{4} \quad OR \quad z = \sqrt{5}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

18

14. $4 + 5i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{41}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{5}{4}\right) = 0.90$$

$z = \sqrt{41}\ cos0.90 + i\ \sqrt{41}sin0.90$ OR $z = \sqrt{41}\ (cos0.90 + i\ sin0.90)$

15. $-2 + 4i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-2)^2 + (4)^2} = 2\sqrt{5}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{4}{-2}\right) + \pi = 2.03$$

$z = 2\sqrt{5}\ cos2.03 + i\ 2\sqrt{5}sin2.03$ OR $z = 2\sqrt{5}\ (cos2.03 + i\ sin2.03)$

16. $-1 - \sqrt{3}i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) + \pi = tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) + \pi = \frac{4\pi}{3}$$

$z = 2cos\dfrac{4\pi}{3} + i\ 2sin\dfrac{4\pi}{3}$ OR $z = 2(cos\dfrac{4\pi}{3} + i\ sin\dfrac{4\pi}{3})$

17. $3 + 3i$

$$r = \sqrt{a^2 + b^2} = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$$

$z = 3\sqrt{2}\ cos\dfrac{\pi}{4} + i\ 3\sqrt{2}sin\dfrac{\pi}{4}$ OR $z = 3\sqrt{2}\ (cos\dfrac{\pi}{4} + i\ sin\dfrac{\pi}{4})$

Determine whether each sequence is convergent or divergent.

- If a sequence has a limit such that the terms approach a unique number, then it is said to converge.

- If not, the sequence is said to diverge.

18. $a_1 = 4, 1.5a_{n-1}, n \geq 2$ Divergent	**20.** $a_n = -n^2 - 8n + 106$ Divergent	**19.** $a_n = \dfrac{5}{10^n}$ Convergent
21. $a_1 = -64, \dfrac{3}{4}a_{n-1}, n \geq 2$ Convergent	**22.** $a_1 = 1, a_n = 4 - a_{n-1},$ $n \geq 2$ Divergent	**23.** $a_n = n^2 - 3n + 1$ Divergent
24. $a_n = \dfrac{n^2 + 4}{3 + n}$ Divergent	**26.** $a_n = \dfrac{5n + 6}{n}$ Convergent	**27.** $a_n = \dfrac{5n}{5^n} + 1$ Convergent

25. $a_1 = 9, a_n = \dfrac{a_{n-1} + 3}{2}, n \geq 2$ Convergent

Find the sum of each arithmetic series. $$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

57. $\displaystyle\sum_{k=1}^{16} (4k - 2)$ $n = 16 - 1 + 1 = 16$ $a_1 = 4(1) - 2 = 2$

$$a_n = 4(16) - 2 = 62$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 16\left(\frac{2 + 62}{2}\right) = 512 \qquad S_{16} = 512$$

58. $\displaystyle\sum_{k=4}^{13} (4k+1)$ $n = 13 - 4 + 1 = 10$ $a_1 = 4(4) + 1 = 17$

$$a_n = 4(13) + 1 = 53$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 10\left(\frac{17 + 53}{2}\right) = 350 \qquad S_{10} = 350$$

59. $\displaystyle\sum_{k=5}^{16} (2k+6)$ $n = 16 - 5 + 1 = 12$ $a_1 = 2(5) + 6 = 16$

$$a_n = 2(16) + 6 = 38$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 12\left(\frac{16 + 38}{2}\right) = 324 \qquad S_{12} = 324$$

60. $\displaystyle\sum_{k=0}^{12} (-3k+2)$ $n = 12 - 0 + 1 = 13$ $a_1 = -3(0) + 2 = 2$

$$a_n = -3(12) + 2 = -34$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = 13\left(\frac{2 + (-34)}{2}\right) = -202 \qquad S_{13} = -202$$

16	Find the nth term and geometric means of geometric sequences
	إيجاد الحد النوني والوسط الحسابي للمتتاليات الهندسية

Find the geometric means of each sequence. $\boxed{a_n = a_1 r^{n-1}}$

35. $810, \underline{\ ?\ }, \underline{\ ?\ }, \underline{\ ?\ }, 10$ $n = 5$ $a_1 = 810$ $a_n = 10$

$a_n = a_1 r^{n-1}$ $\longrightarrow$ $10 = 810 r^{5-1}$ $\longrightarrow$ $r = \pm\dfrac{1}{3}$

$a_2 = 810\left(\dfrac{1}{3}\right)^{2-1} = 270$ $\qquad$ $a_2 = 810\left(-\dfrac{1}{3}\right)^{2-1} = -270$

$a_3 = 810\left(\dfrac{1}{3}\right)^{3-1} = 90$ $\qquad$ $a_3 = 810\left(-\dfrac{1}{3}\right)^{3-1} = 90$

$a_4 = 810\left(\dfrac{1}{3}\right)^{4-1} = 30$ $\qquad$ $a_4 = 810\left(-\dfrac{1}{3}\right)^{4-1} = -30$

36. $640, \underline{?}, \underline{?}, \underline{?}, 2.5$ $n = 5$ $a_1 = 640$ $a_n = 2.5$

$$a_n = a_1 r^{n-1} \longrightarrow 2.5 = 640 r^{5-1} \longrightarrow r = \pm 0.25$$

$a_2 = 640(0.25)^{2-1} = 160$ $a_2 = 640(-0.25)^{2-1} = -160$

$a_3 = 640(0.25)^{3-1} = 40$ $a_3 = 640(-0.25)^{3-1} = 40$

$a_4 = 640(0.25)^{4-1} = 10$ $a_4 = 640(-0.25)^{4-1} = -10$

---

37. $\frac{7}{2}, \underline{?}, \underline{?}, \underline{?}, \frac{56}{81}$ $n = 5$ $a_1 = \frac{7}{2}$ $a_n = \frac{56}{81}$

$$a_n = a_1 r^{n-1} \longrightarrow \frac{56}{81} = \frac{7}{2} r^{5-1} \longrightarrow r = \pm \frac{2}{3}$$

$a_2 = \frac{7}{2}\left(\frac{2}{3}\right)^{2-1} = \frac{7}{3}$ $a_2 = \frac{7}{2}\left(-\frac{2}{3}\right)^{2-1} = -\frac{7}{3}$

$a_3 = \frac{7}{2}\left(\frac{2}{3}\right)^{3-1} = \frac{14}{9}$ $a_3 = \frac{7}{2}\left(-\frac{2}{3}\right)^{3-1} = \frac{14}{9}$

$a_4 = \frac{7}{2}\left(\frac{2}{3}\right)^{4-1} = \frac{28}{27}$ $a_4 = \frac{7}{2}\left(-\frac{2}{3}\right)^{4-1} = -\frac{28}{27}$

---

38. $\frac{729}{64}, \underline{?}, \underline{?}, \underline{?}, \frac{324}{9}$ $n = 5$ $a_1 = \frac{729}{64}$ $a_n = \frac{324}{9}$

$$a_n = a_1 r^{n-1} \longrightarrow \frac{324}{9} = \frac{729}{64} r^{5-1} \longrightarrow r = \pm \frac{4}{3}$$

$a_2 = \frac{729}{64}\left(\frac{4}{3}\right)^{2-1} = \frac{243}{16}$ $a_2 = \frac{729}{64}\left(-\frac{4}{3}\right)^{2-1} = -\frac{243}{16}$

$a_3 = \frac{729}{64}\left(\frac{4}{3}\right)^{3-1} = \frac{81}{4}$ $a_3 = \frac{729}{64}\left(-\frac{4}{3}\right)^{3-1} = \frac{81}{4}$

$a_4 = \frac{729}{64}\left(\frac{4}{3}\right)^{4-1} = 27$ $a_4 = \frac{729}{64}\left(-\frac{4}{3}\right)^{4-1} = -27$

39. Find two geometric means between 3 and 375.

$$n = 4 \qquad a_1 = 3 \qquad a_n = 375$$

$$a_n = a_1 r^{n-1} \longrightarrow 375 = 3r^{4-1} \longrightarrow r = 5$$

$$a_2 = 3(5)^{2-1} = 15$$

$$a_3 = 3(5)^{3-1} = 75$$

40. Find two geometric means between 16 and -2.

$$n = 4 \qquad a_1 = 16 \qquad a_n = -2$$

$$a_n = a_1 r^{n-1} \longrightarrow -2 = 16r^{4-1} \longrightarrow r = -0.5$$

$$a_2 = 16(-0.5)^{2-1} = -8$$

$$a_3 = 16(-0.5)^{3-1} = 4$$

17	Find sums of geometric series
	إيجاد مجاميع المتسلسلات الهندسية

Find a_1 for each geometric series described.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \qquad S_n = \frac{a_1 - a_n r}{1 - r}$$

51. $S_n = -2912,\ r = 3,\ n = 6$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \longrightarrow -2912 = \frac{a_1 - a_1(3)^6}{1 - (3)}$$

$$\longrightarrow -2912 = \frac{a_1(1 - (3)^6)}{1 - (3)} \longrightarrow -2912 = \frac{-728a_1}{-2}$$

$$\longrightarrow -2912 = 364a_1 \longrightarrow a_1 = -8$$

52. $S_n = -10{,}922, r = 4, n = 7$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \longrightarrow -10922 = \frac{a_1 - a_1(4)^7}{1 - (4)}$$

$$\longrightarrow -10922 = \frac{a_1(1 - (4)^7)}{1 - (4)} \longrightarrow -10922 = \frac{-16383a_1}{-3}$$

$$\longrightarrow -10922 = 5461a_1 \longrightarrow \boxed{a_1 = -2}$$

53. $S_n = 1330, a_n = 486, r = \frac{3}{2}$

$$S_n = \frac{a_1 - a_n r}{1 - r} \longrightarrow 1330 = \frac{a_1 - (486)(\frac{3}{2})}{1 - (\frac{3}{2})}$$

$$\longrightarrow 1330 = \frac{a_1 - 729}{-0.5} \longrightarrow 1330(-0.5) = a_1 - 729$$

$$\longrightarrow -665 = a_1 - 729 \longrightarrow -665 + 729 = a_1 \longrightarrow \boxed{a_1 = -2}$$

54. $S_n = 4118, a_n = 128, r = \frac{2}{3}$

$$S_n = \frac{a_1 - a_n r}{1 - r} \longrightarrow 4118 = \frac{a_1 - (128)(\frac{2}{3})}{1 - (\frac{2}{3})}$$

$$\longrightarrow 4118 = \frac{a_1 - \frac{256}{3}}{\frac{1}{3}} \longrightarrow 4118(\frac{1}{3}) = a_1 - \frac{256}{3}$$

$$\longrightarrow \frac{4118}{3} = a_1 - \frac{256}{3} \longrightarrow \frac{4118}{3} + \frac{256}{3} = a_1 \longrightarrow \boxed{a_1 = 1458}$$

55. $a_n = 1024, r = 8, n = 5$

$$a_n = a_1 r^{n\cdot} \longrightarrow 1024 = a_1 8^{5-1} \longrightarrow a_1 = 0.25$$

56. $a_n = 1875, r = 5, n = 7$

$$a_n = a_1 r^{n\cdot} \longrightarrow 1875 = a_1 5^{7-1} \longrightarrow a_1 = \frac{3}{25}$$

18	Find sums of infinite geometric series
	إيجاد مجاميع المتسلسلات الهندسية اللانهائية

Find the sum of each infinite series, if it exists.

$$S = \frac{a_1}{1-r}$$

- If $|r| < 1$, the series has sum
- If $|r| \geq 1$, the series has no sum

22. $18 + 21.6 + 25.92 + ...$

$$r = \frac{21.6}{18} = 1.2 \geq 1$$

No sum exists

23. $-3 - 4.2 - 5.88 - ...$

$$r = \frac{-4.2}{-3} = 1.4 \geq 1$$

No sum exists

24. $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + ...$

$$r = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3} < 1$$

Sum exists

$$a_1 = \frac{1}{2} \qquad S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{1}{2}}{1 - \frac{1}{3}} = \frac{3}{4}$$

25. $\frac{12}{5} + \frac{6}{5} + \frac{3}{5} + ...$

$$r = \frac{6}{5} \div \frac{12}{5} = \frac{1}{2} < 1$$

Sum exists

$$a_1 = \frac{12}{5} \qquad S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{12}{5}}{1 - \frac{1}{2}} = \frac{24}{5}$$

26. $21 + 14 + \dfrac{28}{3} + \ldots$

$$r = \dfrac{14}{21} = \dfrac{2}{3} < 1$$

Sum exists

$a_1 = 21$

$$S = \dfrac{a_1}{1 - r} \qquad S = \dfrac{21}{1 - \dfrac{2}{3}} = 63$$

27. $32 + 40 + 50 + \ldots$

$$r = \dfrac{40}{32} = 1.25 \geq 1$$

No sum exists

19	Recognize and use special sequences
	التعرف على المتتاليات الخاصة واستخدامها

Write a recursive formula for each sequence.

Arithmetic: $a_n = a_{n-1} + d$	**Geometric**: $a_n = r a_{n-1}$

5. $3, 8, 18, 38, 78, \ldots$

Not Arithmetic or Geometric

$2(3) + 2 = 8$

$2(8) + 2 = 18$

$2(18) + 2 = 38$

$a_n = 2a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 2a_{n-1} + 2, \ a_1 = 3$

6. $5, 14, 41, 122, 365, \ldots$

Not Arithmetic or Geometric

$3(5) - 1 = 14$

$3(14) - 1 = 41$

$3(41) - 1 = 122$

$a_n = 3a_{n-1} - 1$

A recursive formula for the sequence is

$a_n = 3a_{n-1} - 1, \ a_1 = 5$

24. $16, 10, 7, 5.5, 4.75, \ldots$

Not Arithmetic or Geometric

$0.5(16) + 2 = 10$

$0.5(10) + 2 = 7$

$0.5(7) + 2 = 5.5$

$a_n = 0.5a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 0{,}5a_{n-1} + 2, \ a_1 = 16$

25. $32, 12, 7, 5.75, \ldots$

Not Arithmetic or Geometric

$0.25(32) + 4 = 12$

$0.25(12) + 4 = 7$

$0.25(7) + 4 = 5.75$

$a_n = 2a_{n-1} + 2$

A recursive formula for the sequence is

$a_n = 0.25a_{n-1} + 4, \ a_1 = 32$

26. 4, 15, 224, 50,175, ...

Not Arithmetic or Geometric

$$(4)^2 - 1 = 15$$

$$(15)^2 - 1 = 224$$

$$(224)^2 - 1 = 50175$$

$$a_n = (a_{n-1})^2 - 1$$

A recursive formula for the sequence is

$$a_n = (a_{n-1})^2 - 1, a_1 = 4$$

27. 1, 2, 9, 730, ...

Not Arithmetic or Geometric

$$(1)^3 + 1 = 2$$

$$(2)^3 + 1 = 9$$

$$(9)^3 + 1 = 730$$

$$a_n = (a_{n-1})^3 + 1$$

A recursive formula for the sequence is

$$a_n = (a_{n-1})^3 + 1, a_1 = 1$$

28. 9, 33, 129, 513, ...

Not Arithmetic or Geometric

$$4(9) - 3 = 33$$

$$4(33) - 3 = 129$$

$$4(129) - 3 = 513$$

$$a_n = 4a_{n-1} - 3$$

A recursive formula for the sequence is

$$a_n = 4a_{n-1} - 3, a_1 = 9$$

29. 480, 128, 40, 18, ...

Not Arithmetic or Geometric

$$0.25(480) + 8 = 128$$

$$0.25(128) + 8 = 40$$

$$0.25(40) + 8 = 18$$

$$a_n = 0.25a_{n-1} + 8$$

A recursive formula for the sequence is

$$a_n = 0.25a_{n-1} + 8, a_1 = 480$$

30. 393, 132, 45, 16, ...

Not Arithmetic or Geometric

$$\frac{1}{3}(393) + 1 = 132$$

$$\frac{1}{3}(132) + 1 = 45$$

$$a_n = \frac{1}{3}a_{n-1} + 1$$

$$\frac{1}{3}(45) + 1 = 16$$

A recursive formula for the sequence is

$$a_n = \frac{1}{3}a_{n-1} + 1, a_1 = 393$$

31. 68, 104, 176, 320, ...

Not Arithmetic or Geometric

$$2(68) - 32 = 104$$

$$2(104) - 32 = 176$$

$$2(176) - 32 = 320$$

$$a_n = 2a_{n-1} - 32$$

A recursive formula for the sequence is

$$a_n = 2a_{n-1} - 32, a_1 = 68$$

Find the indicated term of each expression.

If n is a natural number, then $(a + b)^n =$

$_nC_0 \, a^n b^0 + {}_nC_1 \, a^{n-1}b^1 + {}_nC_2 \, a^{n-2}b^2 + \cdots + {}_nC_n \, a^0 b^n$

```
           ♡ 1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
1  7  21  35  35  21   7   1
1  8  28  56  70  56  28   8   1
1  9  36  84 126 126  84  36   9   1
```

1
2
3
4
5
6
7
8
9
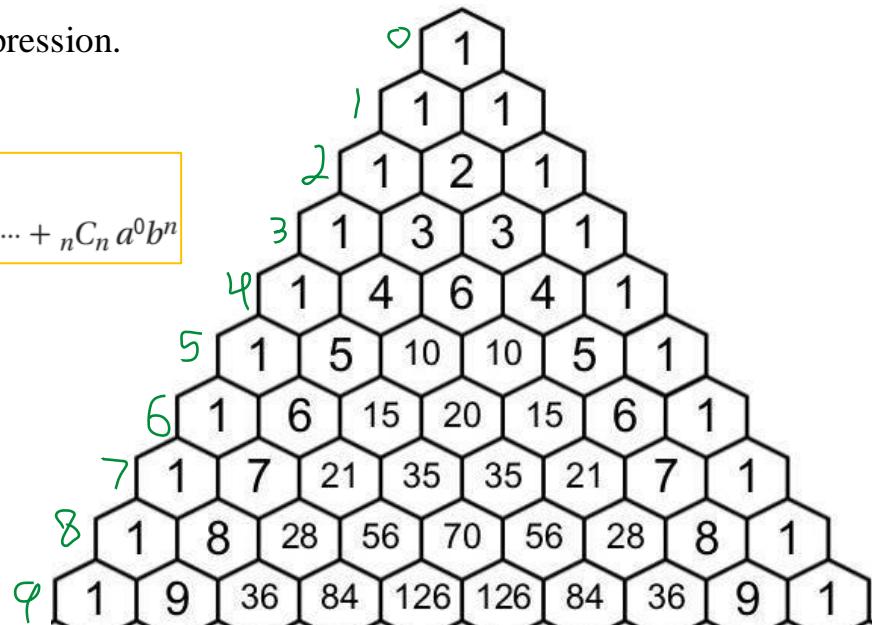
23. third term of $(x + 2z)^7$

$3 - 1 = 2$

$_7C_2 a^5 b^2$

7 6 |5| 4 3 2 1 0

0 1 |2| 3 4 5 6 7

$21(x)^5(2z)^2 \rightarrow 21x^5 4z^2 \rightarrow 84x^5z^2$

24. fourth term of $(y - 3x)^6$

$_6C_3 a^3 b^3$

6 5 4 |3| 2 1 0

0 1 2 |3| 4 5 6

$20(y)^3(-3x)^3 \rightarrow 20y^3(-27)z^3 \rightarrow -540y^5z^2$

25. seventh term of $(2a - 2b)^8$

$_8C_6 a^2 b^6$

8 7 6 5 4 3 |2| 1 0

0 1 2 3 4 5 |6| 7 8

$28(2a)^2(2b)^6 \rightarrow 28(4)a^2(64)b^6 \rightarrow 7168a^2b^6$

26. sixth term of $(4x + 5y)^6$

$_6C_5 a^1 b^5$

6 5 4 3 2 |1| 0

0 1 2 3 4 |5| 6

$6(4x)^1(5y)^5 \rightarrow 6(4x)(3125)y^5 \rightarrow 75000xy^5$

28

27. fifth term of $(x - 4)^9$

$$_9C_4a^5b^4$$

$$126x^5(-4)^4 \rightarrow 126x^5(256) \rightarrow 32256x^5$$

28. fourth term of $(c + 6)^8$

$$_8C_3a^5b^3$$

$$56(c)^5(6)^3 \rightarrow 56c^5(216) \rightarrow 12096c^5$$

Part (3) - 3

21	Find products, quotients, powers, and roots of complex numbers in polar form
	إيجاد ناتج ضرب الأعداد المركبة وناتج قسمتها وأسسها والجذور في الصورة القطبية

Find each product or quotient, and express it in rectangular form.

Product Formula	$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
Quotient Formula	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$, where z_2 and $r_2 \neq 0$

26. $6\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right) \cdot 4\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$ Product

$$(6)(4)\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right]$$

$$24\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$$

$$24\left[-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right]$$

\

Rectangular form: $-12\sqrt{2} + 12\sqrt{2}i$

27. $5(\cos 135° + i \sin 135°) \cdot 2 (\cos 45° + i \sin 45°)$ Product

$$(5)(2)[\cos(135° + 45°) + i\sin(135° + 45°)]$$

$$10[\cos(180°) + i\sin(180°)]$$

$$10[-1 + i0)]$$

Rectangular form: -10

28. $3\left(\cos \dfrac{3\pi}{4} + i \sin \dfrac{3\pi}{4}\right) \div \dfrac{1}{2}(\cos \pi + i \sin \pi)$ Quotien

$$\frac{3}{0.5}\left[\cos\left(\frac{3\pi}{4} - \pi\right) + i\sin(\frac{3\pi}{4} - \pi)\right]$$

$$6\left[\cos\left(-\frac{\pi}{4}\right) + i\sin(-\frac{\pi}{4})\right]$$

$$6\left[\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2})\right]$$

Rectangular form: $3\sqrt{2} - 3\sqrt{2}i$

29. $2(\cos 90° + i \sin 90°) \cdot 2(\cos 270° + i \sin 270°)$ Product

$$(2)(2)[\cos(90° + 270°) + i\sin(90° + 270°)]$$

$$4[\cos(360°) + i\sin(360°)]$$

$$4[1 + i0)]$$

Rectangular form: 4

30. $3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \div 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ Quotien

$$\frac{3}{4}\left[\cos\left(\frac{\pi}{6} - \frac{2\pi}{3}\right) + i\sin(\frac{\pi}{6} - \frac{2\pi}{3})\right]$$

$$\frac{3}{4}\left[\cos\left(-\frac{\pi}{2}\right) + i\sin(-\frac{\pi}{2})\right]$$

$$\frac{3}{4}[0 - i)]$$

Rectangular form: $-\frac{3}{4}i$

31. $4\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) \div 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ Quotien

$$\frac{4}{2}\left[\cos\left(\frac{9\pi}{4} - \frac{3\pi}{2}\right) + i\sin(\frac{9\pi}{4} - \frac{3\pi}{2})\right]$$

$$2\left[\cos\left(\frac{3\pi}{4}\right) + i\sin(\frac{3\pi}{4})\right]$$

$$2\left[-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})\right]$$

Rectangular form: $-\sqrt{2} + \sqrt{2}i$

32. $\frac{1}{2}(\cos 60° + i\sin 60°) \cdot 6(\cos 150° + i\sin 150°)$ Product

$$(0.5)(6)[\cos(60° + 150°) + i\sin(60° + 150°)]$$

$$3[\cos(210°) + i\sin(210°)]$$

$$3\left[-\frac{\sqrt{3}}{2} - i\frac{1}{2})\right]$$

Rectangular form: $-\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

33. $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ Quotien

$$\frac{6}{2}\left[\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) + i\sin(\frac{3\pi}{4}-\frac{\pi}{4})\right]$$

$$3\left[\cos\left(\frac{\pi}{2}\right) + i\sin(\frac{\pi}{2})\right]$$

$$3[0 + i)]$$

Rectangular form: $3i$

---

34. $5(\cos 180° + i\sin 180°) \cdot 2(\cos 135° + i\sin 135°)$ Product

$$(5)(2)[\cos(180° + 135°) + i\sin(180° + 135°)]$$

$$10[\cos(315°) + i\sin(315°)]$$

$$10\left[-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})\right]$$

Rectangular form: $-5\sqrt{2} + 5\sqrt{2}i$

---

35. $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \div 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ Quotien

$$\frac{0.5}{3}\left[\cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right) + i\sin(\frac{\pi}{3}-\frac{\pi}{6})\right]$$

$$\frac{1}{6}\left[\cos\left(\frac{\pi}{6}\right) + i\sin(\frac{\pi}{6})\right]$$

$$\frac{1}{6}\left[\frac{\sqrt{3}}{2} + \frac{1}{2}i)\right]$$

Rectangular form: $\frac{\sqrt{3}}{2} + \frac{1}{12}i$

FINANCIAL LITERACY Nasser had AED 15,000 in credit card debt when he graduated from college. The balance increased by 2% each month due to interest (murabaha), and Nasser could only make payments of AED 400 per month. Write a recursive formula for the balance of his account each month. Then determine the balance after five months.

initial balance + balance times 2% − monthly payment

$$a_2 = \quad a_1 \quad + \quad (a_1 \times 0.02) \quad - \quad 400$$

$a_2 = 1.02a_1 - 400$ The formula is $a_n = 1.02a_{n-1} - 400, \ a_1 = 15,000$

$$a_n = 1.02a_{n-1} - 400$$

$a_1 = 15,000$	$a_2 = (15,000 \times 1.02) - 400$ or **14,900**
$a_2 = 14,900$	$a_3 = (14,900 \times 1.02) - 400$ or **14,798**
$a_3 = 14,798$	$a_4 = (14,798 \times 1.02) - 400$ or **14,693.96**
$a_4 = 14,693.96$	$a_5 = (14,693.96 \times 1.02) - 400$ or **14,587.84**
$a_5 = 14,587.84$	$a_6 = (14,587.84 \times 1.02) - 400$ or 14,479.60

Write a recursive formula for a AED 10,000 debt, at 2.5% interest (murabaha) per month, with a AED 600 monthly payment. Then find the first five balances.

initial balance + balance times 2.5% − monthly payment

$$a_2 = \quad a_1 \quad + \quad (a_1 \times 0.025) \quad - \quad 600$$

$a_2 = 1.025a_1 - 600$ The formula is $a_n = 1.025a_{n-1} - 600, \ a_1 = 10,000$

$$a_n = 1.025a_{n-1} - 600$$

$a_1 = 10,000$	$a_2 = 1.025(10,000) - 600 = 9650$
$a_2 = 9650$	$a_3 = 1.025(9650) - 600 = 9291.25$
$a_3 = 9291.25$	$a_4 = 1.025(9291.25) - 600 = 8923.53$
$a_4 = 8923.53$	$a_5 = 1.025(8923.53) - 600 = 8546.62$

FINANCING Faris financed a *AED* 1,500 rowing machine to help him train for the college rowing team. He could only make a *AED* 100 payment each month, and his bill increased by 1% due to interest (murabaha) at the end of each month.

a. Write a recursive formula for the balance owed at the end of each month.

initial balance + **balance times 1%** − **monthly payment**

$$a_2 = \quad a_1 \quad + \quad (a_1 \times 0.01) \quad - \quad 100$$

$$a_2 = 1.01a_1 - 100 \qquad \text{The formula is} \quad a_n = 1.01a_{n-1} - 100, \; a_1 = 1500$$

b. Find the balance owed after the first four months.

$$a_n = 1.01a_{n-1} - 100$$

$a_1 = 1500$

$a_2 = 1.01(1500) - 100 = 1415$

$a_2 = 1415$

$a_3 = 1.01(1415) - 100 = 1329.15$

$a_3 = 1329.15$

$a_4 = 1.01(1329.15) - 100 = 1242.4415$

$a_4 = 1242.4415$

$a_5 = 1.01(1242.4415) - 100 = 1154.865915$

c. How much interest (murabaha) has accumulated after the first six months?

$a_5 = 1242.44$

$a_6 = 1.01(1154.865915) - 100 = 1066.414574$

Interest

$$= (1500 \times 0.01) + (1415 \times 0.01) + (1329.15 \times 0.01) + (1242.4415 \times 0.01)$$
$$+ (1154.865915 \times 0.01) + (1066.414574 \times 0.01) = 77.1$$

FINANCIAL LITERACY Mr. Adnan and his company deposit AED 20,000 into his retirement account at the end of each year. The account earns 8% interest (murabaha) before each deposit.

a. Write a recursive formula for the balance in the account at the end of each year.

> **initial balance** + **balance times 1%** + **monthly payment**
>
> a_2 = a_1 + $(a_1 \times 0.08)$ + 20000
>
> $a_2 = 1.08a_1 + 20000$ The formula is $a_n = 1.08a_{n-1} + 20000,\ a_1 = 20000$

b. Determine how much is in the account at the end of each of the first 8 years.

> $a_n = 1.08a_{n-1} + 20000$
>
> | | |
> |---|---|
> | $a_1 = 20,000$ | $a_2 = 1.08(20000) + 20000 = 41600$ |
> | $a_2 = 41600$ | $a_3 = 1.08(41600) + 20000 = 64928$ |
> | $a_3 = 64928$ | $a_4 = 1.08(64928) + 20000 = 90122.24$ |
> | $a_4 = 90122.24$ | $a_5 = 1.08(90122.24) + 20000 = 117332.02$ |
> | $a_5 = 90122.24$ | $a_6 = 1.08(117332.02) + 20000 = 146718.58$ |
> | $a_6 = 146718.58$ | $a_7 = 1.08(146718.58) + 20000 = 178456.07$ |
> | $a_7 = 178456.07$ | $a_8 = 1.08(178456.07) + 20000 = 212732.56$ |

23	Use the Binomial Theorem to expand powers of binomials
	استخدام نظرية ذات الحدين لتفكيك أسس ذوات الحدين

Expand each binomial.

1. $(c + d)^5$

$(c + d)^5 = {}_5C_0 c^5 + {}_5C_1 c^4 d + {}_5C_2 c^3 d^2 + {}_5C_3 c^2 d^3 + {}_5C_4 cd^4 + {}_5C_5 d^5$

$= c^5 + 5c^4 d + 10c^3 d^2 + 10c^2 d^3 + 5\,cd^4 + d^5$

2. $(g + h)^7$

$(g + h)^7 =_7 C_0\, g^7 +_7 C_1\, g^6 h +_7 C_2\, g^5 h^2 +_7 C_3\, g^4 h^3 +_7 C_4\, g^3 h^4 +_7 C_5\, g^2 h^5 +_7 C_6\, gh^6 +_7 C_7\, h^7$

$$= g^7 + 7g^6 h + 21\, g^5 h^2 + 35g^4 h^3 + 35\, g^3 h^4 + 21\, g^2 h^5 + 7gh^6 + h^7$$

3. $(x - 4)^6$

$(x - 4)^6 =_6 C_0\, x^6 +_6 C_1\, x^5(-4) +_6 C_2\, x^4(-4)^2 +_6 C_3\, x^3(-4)^3 +_6 C_4\, x^2(-4)^4 +_6 C_5\, x(-4)^5 +_6 C_6\, (-4)^6$

$$= x^6 + 6x^5(-4) + 15x^4(-4)^2 + 20x^3(-4)^3 + 15x^2(-4)^4 + 6x(-4)^5 + (-4)^6$$

$$= x^6 - 24x^5 + 240x^4 - 1280x^3 + 3840x^2 - 6144x + 4096$$

4. $(2y - z)^5$

$(2y - z)^5 =_5 C_0(2y)^5 +_5 C_1(2y)^4(-z) +_5 C_2(2y)^3(-z)^2 +_5 C_3(2y)^2(-z)^3 +_5 C_4(2y)(-z)^4 +_5 C_5(-z)^5$

$$= 32y^5 + 5(16)y^4(-z) + 10(8)y^3 z^2 + 10(4)y^2(-z)^3 + 5(2)yz^4 + (-z)^5$$

$$= 32y^5 - 80y^4 z + 80y^3 z^2 - 40y^2 z^3 + 10yz^4 - z^5$$

5. $(x + 3)^5$

$(x + 3)^5 =_5 C_0\, x^5 +_5 C_1\, x^4(3) +_5 C_2\, x^3(3)^2 +_5 C_3\, x^2(3)^3 +_5 C_4\, x(3)^4 +_5 C_5\, (3)^5$

$$= x^5 + 5x^4(3) + 10x^3(3)^2 + 10x^2(3)^3 + 5x(3)^4 + (3)^5$$

$$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$$

6. $(y - 4z)^4$

$(y - 4z)^4 =_4 C_0\, y^4 +_4 C_1\, y^3(-4z) +_4 C_2\, y^2(-4z)^2 +_4 C_3\, y(-4z)^3 +_4 C_4\, (-4z)^4$

$$= y^4 + 4y^3(-4)z + 6y^2(16)z^2 + 4y(-64)z^3 + (256)z^4$$

$$= y^4 - 16y^3 z + 96y^2 z^2 - 256yz^3 + 256z^4$$

15. $(a - b)^6$

$(a - b)^6 = {}_6C_0\, a^6 + {}_6C_1 a^5(-b) + {}_6C_2\, a^4(-b)^2 + {}_6C_3\, a^3(-b)^3 + {}_6C_4\, a^2(-b)^4 + {}_6C_5\, a(-b)^5 + {}_6C_6\,(-b)^6$

$$= a^6 + 6a^5(-b) + 15a^4(-b)^2 + 20a^3(-b)^3 + 15a^2(-b)^4 + 6a(-b)^5 + (-b)^6$$

$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

16. $(c - d)^7$

$(c - d)^7 = {}_7C_0\, c^7 + {}_7C_1 c^6(-d) + {}_7C_2\, c^5(-d)^2 + {}_7C_3\, c^4(-d)^3 + {}_7C_4\, c^3(-d)^4 + {}_7C_5\, c^2(-d)^5 + {}_7C_6\, c(-d)^6 + {}_7C_7(-d)^7$

$$= c^7 + 7c^6(-d) + 21c^5(-d)^2 + 35c^4(-d)^3 + 35c^3(-d)^4 + 21c^2(-d)^5 + 7c(-d)^6 + (-d)^7$$

$$= c^7 - 7c^6d + 21c^5d^2 - 35c^4d^3 + 35c^3d^4 - 21c^2d^5 + 7cd^6 - d^7$$

17. $(x + 6)^6$

$(x + 6)^6 = {}_6C_0\, x^6 + {}_6C_1\, x^5(6) + {}_6C_2 x^4(6)^2 + {}_6C_3 x^3(6)^3 + {}_6C_4 x^2(6)^4 + {}_6C_5 x(6)^5 + {}_6C_6(6)^6$

$$= x^6 + 6x^5(6) + 15x^4(6)^2 + 20x^3(6)^3 + 15x^2(6)^4 + 6x(6)^5 + (6)^6$$

$$= x^6 + 36x^5 + 540x^4 + 4320x^3 + 19440x^2 + 46656x + 46656$$

18. $(y - 5)^7$

$(y - 5)^7 = {}_7C_0\, y^7 + {}_7C_1 y^6(-5) + {}_7C_2\, y^5(-5)^2 + {}_7C_3\, y^4(-5)^3 + {}_7C_4\, y^3(-5)^4 + {}_7C_5\, y^2(-5)^5 + {}_7C_6\, y(-5)^6 + {}_7C_7(-5)^7$

$$= y^7 + 7y^6(-5) + 21y^5(-5)^2 + 35y^4(-5)^3 + 35y^3(-5)^4 + 21y^2(-5)^5 + 7y(-5)^6 + (-5)^7$$

$$= y^7 - 35y^6 + 525y^5 - 4375y^4 + 21875y^3 - 65625y^2 + 109375y - 78125$$

19. $(2a + 4b)^4$

$(2a + 4b)^4 = {}_4C_0(2a)^4 + {}_4C_1(2a)^3(4b) + {}_4C_2(2a)^2(4b)^2 + {}_4C_3(2a)(4b)^3 + {}_4C_4(4b)^4$

$$= (16)a^4 + 4(8)a^3(4)b + 6(4)a^2(16)b^2 + 4(2)a(64)b^3 + (256)b^4$$

$$= 16a^4 + 128a^3b + 384a^2b^2 + 512ab^3 + 256b^4$$

20. $(3a - 4b)^5$

$(3a - 4b)^5 = {}_5C_0(3a)^5 + {}_5C_1(3a)^4(-4b) + {}_5C_2(3a)^3(-4b)^2 + {}_5C_3(3a)^2(-4b)^3 + {}_5C_4(3a)(-4b)^4 + {}_5C_5(-4b)^5$

$= (243)a^5 + 5(81)a^4(-4)b + 10(27)a^3(16)b^2 + 10(9)a^2(-64)b^3 + 5(3)a(256)b^4 + (-1024)b^5$

$= 243a^5 - 1620a^4b + 4320a^3b^2 - 5760a^2b^3 + 3840ab^4 - 1024b^5$

24	A learning outcome from the SoW
	ناتج من الخطة الفصلية

25	A learning outcome from the SoW
	ناتج من الخطة الفصلية