Lesson 10-4 The Binomial Distribution

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Objectives:

1 Identify and conduct a binomial experiment.

2 Find probabilities using binomial distributions.

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1 Binomial Experiments

KeyConcept Binomial Experiments

- There is a fixed number of independent trials n.
- Each trial has only two possible outcomes, success or failure.
- The probability of success p is the same in every trial. The probability of failure q is $1 - p$.
- The random variable X is the number of successes in n trials.

1 2

H T

$\boxed{q = 1 - p}$

$\boxed{p + q = 1}$

Example 1 Identify a Binomial Experiment

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

a. The spinner at the right is spun $\boxed{20}$ times to see how many times it lands on red.

n

This experiment can be reduced to a binomial experiment with success being that the spinner lands on red and failure being any other outcome. Thus, a trial is a spin, and the random variable X represents the number of reds spun.

$n = 20$ $\quad p = \frac{1}{4}$ $\quad q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

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b. One hundred students are randomly asked their favorite food.

This is not a binomial experiment because there are many possible outcomes.

GuidedPractice

1A. Seventy-five students are randomly asked if they own a car.

This is a binomial experiment.

* Success p is (yes) $\frac{1}{2}$

* failure q (No) $\frac{1}{2}$

$\frac{2}{4}$ $\quad \frac{1}{4}$

$n = 75$

$p = \frac{1}{2} = 0.50$ $\quad q = \frac{1}{2} = 0.50$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

1B. In class that consists of 12 male and 13 female students, a group of 5 students is randomly chosen to see how many males are selected.

This experiment can't be reduced to binomial experiment, because the events are not independent

1C. A school study that 65% of students have seen an episode of specific show. Seven students are randomly asked if they have seen an episode of the show.

This experiment can reduced to binomial experiment

Success: is a (yes) failure is "No"

trial: is asking the student, collected their responsed and find the number of yes

$n=7$ $p=0.65$ $q=1-0.65=0.35$

$\boxed{n=7, \quad P=0.65 \quad q=0.35}$

KeyConcept Conducting Binomial Experiments

Step 1 Describe a trial for the situation, and determine the number of trials to be conducted.

Step 2 Define a success, and calculate the theoretical probabilities of success and failure.

Step 3 Describe the random variable X.

Step 4 Design and conduct a simulation to determine the experimental probability.

1 — 13
1 — 13
1 — 13
1 — 13

Example 2 Design a Binomial Experiment

فردي

Conduct a binomial experiment to determine the probability of drawing an odd numbered flashcard from a set of flashcards, consisting of 52 cards, divided equally between four different colors and each color is numbered 1 to 13. Then compare the experimental and theoretical probabilities of the experiment.

Step 1 A trial is drawing a flashcard from the set. The number of trials conducted can be any number greater than 0. We will use 52.

Step 2 A success is drawing an odd-numbered flashcard. The odd-numbered flashcards in the set are 1, 3, 5, 7, 9, 11 and 13, and they occur once in each of the four colors. Therefore, there are $4 \cdot 7$ or $\boxed{28}$ odd-numbered flashcards in the set. The probability of drawing an odd-numbered flashcard, or the probability of success, is $\frac{28}{52}$ or $\frac{7}{13}$. The probability of failure is $1 - \frac{7}{13}$ or $\frac{6}{13}$.

$P = \frac{7}{13}$ $\boxed{q = \frac{6}{13}}$

Step 3 The random variable X represents the number of odd-numbered flashcards drawn in 52 trials.

Step 4 Use the random number generator on a calculator to create a simulation. Assign the integers 0 – 12 to accurately represent the probability data.

Odd-numbered flashcards 0, 1, 2, 3, 4, 5, 6

Other flashcards 7, 8, 9, 10, 11, 12

$\frac{12}{52}$ 100%

Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Odd-Numbered flashcard	~~||||~~ ~~||||~~ || ←	12
Other flashcards	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~	40

An odd-numbered flashcard was drawn 12 times, so the experimental probability is $\frac{12}{52}$ or about 23.1%. This is less than the theoretical probability of $\frac{28}{52}$ or about 53.8%.

Guided Practice

Conduct a binomial experiment to determine the probability of drawing an even-numbered flashcard from a deck of flashcards. Then compare the experimental and theoretical probabilities of the experiment.

Step 1 A trial: Is drawing a card from the deck, The simulation will consist 20 trials 2 4 6 8 10 12 14 16 18 20

Step 2 success Is drawing even-numbered card

The propability of success is $\frac{10}{20} = \frac{1}{2}$, $q = \frac{1}{2}$

Step 3 The radom variable X: number of even-cards was drawing from a deck in 20 trials

Step 4

Outcomes	Tally	Frequency
Even-Numbered Card	~~||||~~ ||	7
Other Cards	~~||||~~ ~~||||~~ |||	13

The experimental probability is $\frac{7}{20}$ or 35%, the theoretical probability of $\frac{10}{20}$ or 50%

2 Binomial Distribution

A binomial distribution is a frequency distribution of the probability of each value of X, where the random variable X represents the number of successes in n trials.

Because X is a discrete random variable, a binomial distribution is a *discrete probability distribution.*

KeyConcept Binomial Probability Formula

The probability of X successes in n independent trials is

$$P(X) = {}_nC_X\, p^X q^{n-X}$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($q = 1 - p$).

$\boxed{nCx}$ $5C_2$ | 5 $\boxed{\text{shift}}$ $\boxed{\div}$ 2 X

Notice that the Binomial Probability Formula is an adaptation of the Binomial Theorem you have already studied. The expression ${}^nC_X\, p^X q^{n-X}$ represents the $p^X q^{n-X}$ term in the binomial expansion of $(p+q)^n$.

Standardized Test Example 3 Find a Probability

Khamis is selling items from a catalog to raise money for school. He has a 40% chance of making a sale each time he solicits a potential customer. Khamis asks 10 people to purchase an item. Find the probability that 6 people make a purchase.

A 8.6% **B** 11.1% **C** 24% **D** 40%

$X = 6$

$n = 10$

$p = 40\% = 0.4$

$q = 1 - 0.4 = 0.6$

$nCx\ p^x\ q^{n-x}$

$= 10C6\ p^6\ q^{10-6}$

$= 10C6\ p^6\ q^4$

$= 10C6\ (0.4)^6 (0.6)^4 = 0.11147 \times 100$

$= 11.14\%$

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TELEMARKETING At Khawla's telemarketing job, 15% of the calls that she makes to potential customers result in a sale. She makes 20 calls in a given hour. What is the probability that 5 calls result in a sale?

A 6.7% **B** 8.3% **C** 10.3% **D** 11.9%

$p = 15\% = 0.15$

$q = 1 - p = 1 - 0.15$

$= 0.85$

$n = 20 \quad | \quad X = 5$

$\boxed{{}^{20}C_5}\ (0.15)^5\ (0.85)^{15}$

$\boxed{n_{C_X}\ p^X\ q^{n-X}}$

KeyConcept Mean of a Binomial Distribution

Mean

$\boxed{\mu = np}$

The mean μ of a binomial distribution is given by $\boxed{\mu = np}$, where n is the number of trials and p is the probability of success.

Real-World Example 4 Full Probability Distribution

$\boxed{n=5}$

Houriyya forgot to study for her civics quiz. The quiz consists of five multiple choice questions with each question having four answer choices. Houriyya randomly circles an answer for each question. In order to pass, she needs to answer at least four questions correctly.

A B C D

1 X

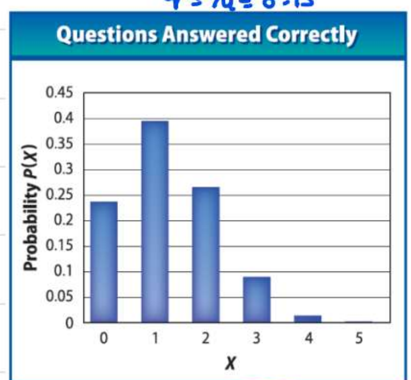
$\boxed{P = \frac{1}{4}}$ $\boxed{q = \frac{3}{4}}$

$\boxed{4}$ from 5

a. Determine the probabilities associated with the number of questions Houriyya answered correctly by calculating the probability distribution.

$n=5$ $p=\frac{1}{4}=0.25$ $q=\frac{3}{4}=0.75$

$\boxed{X}$	0	1	2	3	4	5



at $x=0$ $P(0) = {}^5C_0 (0.25)^0 (0.75)^5 =$

at $x=1$ $P(1) = {}^5C_1 (0.25)^1 (0.75)^4 =$

at $x=2$

at $x=3$

at $x=4$

at $x=5$

$\boxed{P(X) = {}_nC_x \, p^x q^{n-x}}$

at least 4 question 4, 5

b. What is the probability that Houriyya passes the quiz? $\boxed{X=4}$ or $X=5$

$n=5$
$P=0.25$
$q=0.75$

$P(4)+P(5) = {}^5C_4 (0.25)^4 (0.75)^1 + {}^5C_5 (0.25)^5 (0.75)^0$

$\boxed{0.015}$ $\boxed{0.001}$

$= 0.0146 + 0.00097$

0.016

c. How many questions should Houriyya expect to answer correctly?

Mean

$\mu = np = 5(0.25) = 1.25$ will answer 1 Q

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$n =$ $P = \frac{1}{2}$ $q = \frac{1}{2}$

TEST TAKING Suppose Houriyya's civics quiz consisted of five true-or-false questions instead of multiple-choice questions.

A. Determine the probabilities associated with the number of answers Houriyya answered correctly by calculating the probability distribution.

$n = 5$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

$P(X) = {}_nC_x \; p^x \; q^{n-x}$

$x = 1$ $P(0) = {}_5C_0 (0.5)^0 (0.5)^5 =$

$P(1) = {}_5C_1 (0.5)^1 (0.5)^4 = 0.156$

$P(2) = {}_5C_2 (0.5)^2 (0.5)^3 = 0.3125$

$\boxed{P(3) = {}_5C_3 (0.5)^3 (0.5)^2 =}$ 0.3125

$P(4) = {}_5C_4 (0.5)^4 (0.5)^1 =$

$P(5) = {}_5C_5 (0.5)^5 (0.5)^0 =$

B. What is the probability that Houriyya passes the quiz?

to pass she must answered at least 4 question correctly.

$P(4) + P(5) = {}_5C_4 (0.5)^4 (0.5)^1 + {}_5C_5 (0.5)^5 (0.5)^0$

$=$

C. How many questions should Houriyya expect to answer correctly?

$\mu = np = 5\left(\frac{1}{2}\right) = 2.5$

She will answer about 3 quest

GAMES Saeed has earned five spins of the wheel on the right. He will receive a prize each time the spinner lands on WIN. What is the probability that he receives three prizes?

A 4.2% **C** 7.1%

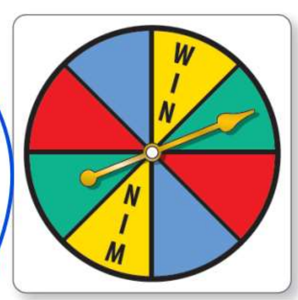
B 5.8% **D** 8.8%

$\boxed{X = 3}$

$\boxed{n = 5}$

$p = \frac{2}{8} = \frac{1}{4} = 0.25$

$q = \frac{6}{8} = \frac{3}{4} = 0.75$



$P(X) = {}_nC_x \; p^x \; q^{n-x}$

$P(3) = {}_5C_3 (0.25)^3 (0.75)^2$

$= \frac{45}{512} = 8.78\%$

p q

SOCCER A certain soccer team has won 75.7% of their games. Find the probability that they win 7 of their next 12 games.

$p = 75.7\% = 0.757$

$q = 1 - p$

$= 1 - 0.757$

$= 0.243$

$n = 12$

$x = 7$

$\boxed{x \leq n}$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$P(7) = {}_{12}C_7\, (0.757)^7 (0.243)^5$

$= 0.0956$

$= 9.56\%$

GARDENING Zayed is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Zayed knows that the probability of having a blue flower is 75%. What is the probability that 20 of the flowers will be blue?

$p = 75\% = 0.75$

$q = 0.25$

$n = 24$

$x = 20$

$P(x) = {}_nC_x\, p^x\, q^{n-x}$

$P(20) = {}_{24}C_{20}\, (0.75)^{20}\, (0.25)^4$

$= 0.1316$

$\simeq 13.16\%$

RUGBY A penalty goal kicker is accurate 75% of the time from within 35 m. What is the probability that he makes exactly 7 of his next 10 kicks from within 35 m?

Range (m)	Accuracy (%)
$\boxed{0–35}$	$\boxed{75}$
35–45	62
45+	20

$p = 0.75$

$q = 0.25$

$n = 10$

$x = 7$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$P(7) = {}^{10}C_7\, (0.75)^7 (0.25)^3 = 0.25028$

$= 25.03\%$

BABIES Mr. and Mrs. Salem are planning to have 3 children. The probability of each child being a boy is 50%. What is the probability that they will have 2 boys?

$p = 0.50$

$q = 0.50$

$x = 2$

$n = 3$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$P(2) = {}_3C_2\, (0.5)^2 (0.5)^1$

$= \frac{3}{8}$

$= 0.375 \xrightarrow{\times 100} = 37.5\%$

A binomial distribution has a 60% rate of success. There are 18 trials.

$P = 0.60$
$q = 0.40$
$n = 18$

a. What is the probability that there will be at least 12 successes?

at least 17

X ~~12~~, ~~13~~, ~~14~~, 15, 16, 17, 18

$^{18}C_{12}(0.6)^{12}(0.4)^{6} + {}^{18}C_{13}(0.6)^{13}(0.4)^{5} + {}^{18}C_{14}(0.6)^{14}(0.4)^{4} + \cdots + {}^{18}C_{18}(0.6)^{18}(0.4)^{0}$

b. What is the probability that there will be 12 failures?

x success   12 failures $\Rightarrow$ success $= 18 - 12 = 6$

$\boxed{X = 6}$ $^{18}C_{6}(0.6)^{6}(0.4)^{12}$

c. What is the expected number of successes? $= 11$

$\mu = np = 18(0.6) = 10.8$

on 11 trials

===

Each binomial distribution has n trials and p probability of success. Determine the most likely number of successes.

a. $n = 8, \ p = 0.6$

$\mu = np$
$= 8 \times 0.6$
$= 4.8$
$\boxed{\simeq 5}$

b. $n = 10, \ p = 0.4$

$\mu = np = 10(0.4)$
$= 4$

===

Each binomial distribution has n trials and p probability of success. Determine the probability of s successes.

a. $n = 8, \quad p = 0.3, \quad s \geq 2$

$\mu = 8 \times (0.3)$
$= 2.4$ $2 \geq$

b. $n = 10, \quad p = 0.2, \quad s \geq 2$

$\mu = 10 \times 0.2 = 2$

c. $n = 6, \quad p = 0.6, \quad s \geq 4$

$\mu = 6(0.6) = 3.6$ $\simeq 4$