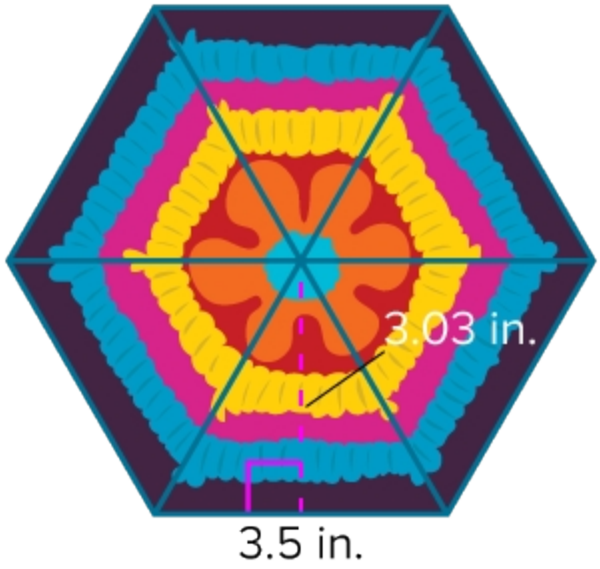
8-4 Area of Regular Polygons

1. Kendra knitted the coaster shown as a present for her grandmother. The coaster is shaped like a regular hexagon. Each side of the hexagon is 3.5 inches. Find the area of the coaster. Round to the nearest hundredth.



SOLUTION:

The coaster decomposes into 6 congruent triangles.
Find the area of each triangle.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(3.5)(3.03)$$

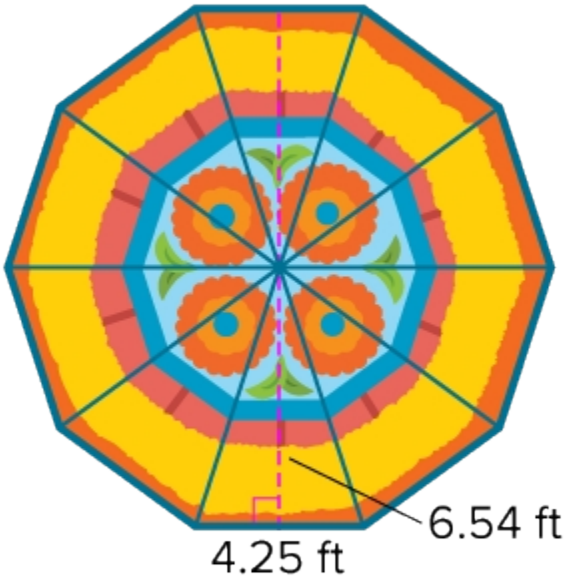
$$= 5.3025$$

The area of each triangle is 5.3 square inches.

The triangles are congruent, so multiply the area of one triangle by 6. Round to the nearest hundredth.
$6(5.3025) = 31.82$

So, the area of the coaster is 31.82 square inches.

2. Paul bought a new rug in the shape of a regular decagon. Each side of the decagon is 4.25 feet. Find the area of the rug. Round to the nearest hundredth.



SOLUTION:

The coaster decomposes into 10 congruent triangles.
Find the area of each triangle.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(4.25)(6.54)$$

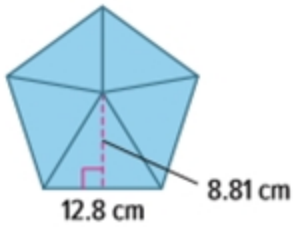
$$= 13.8975$$

The area of each triangle is 13.8975 square feet.

The triangles are congruent, so multiply the area of one triangle by 10. Round to the nearest hundredth.
10(13.8975) = 138.98

So, the area of the rug is 138.98 square feet.

3. **Open Response** A regular pentagon is shown. What is the area of the pentagon?



SOLUTION:

The pentagon decomposes into 5 congruent triangles.
Find the area of each triangle.

$$A = \frac{1}{2}bh$$

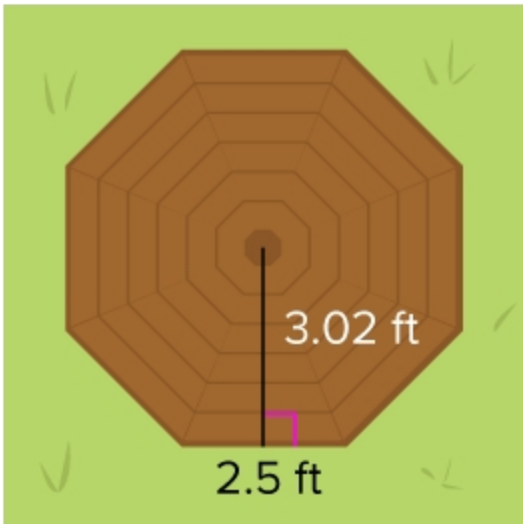
$$= \frac{1}{2}(12.8)(8.81)$$

$$= 56.384$$

The area of each triangle is 56.384 square centimeters .

The triangles are congruent, so multiply the area of one triangle by 5. Round to the nearest hundredth.
5(56.384) = 281.92
So, the area of the rug is 281.92 square centimeters.

4. Julian is going to build a picnic table. The top of the picnic table is shaped like an octagon with sides measuring 2.5 feet. If the wood costs $3.95 per square foot, what is the least he will he spend on the top of the picnic table?



SOLUTION:

The octagon decomposes into 8 congruent triangles.
Find the area of each triangle.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(2.5)(3.02)$$

$$= 3.775$$

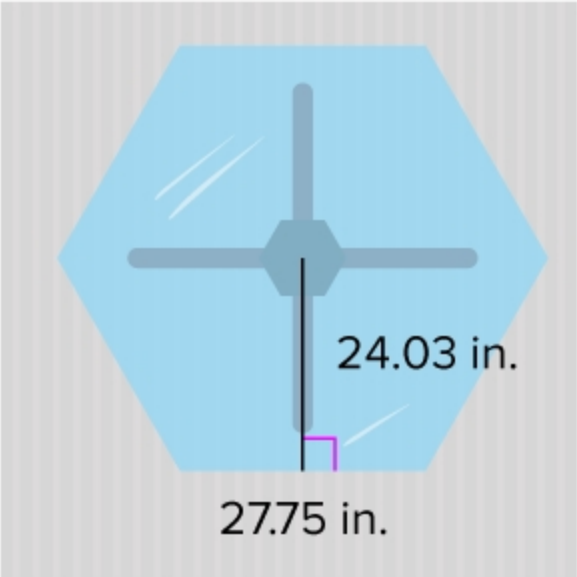
The area of each triangle is 3.775 square feet.

The triangles are congruent, so multiply the area of one triangle by 8.
$8(3.775) = 30.2$

So, the area of the top of the table is 30.2 square feet.

The total cost is $3.95 × 30.2 or $119.29.

5. Williana's mother wants to buy a glass tabletop for their dining room table. The tabletop is shaped like a hexagon with sides measuring 27.75 inches. If the glass costs \$0.06 per square inch, how much will she spend on the glass table top?



SOLUTION:

The hexagon decomposes into 6 congruent triangles.
Find the area of each triangle.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(27.75)(24.03)$$

$$= 333.41625$$

The area of each triangle is 333.41625 square inches.

The triangles are congruent, so multiply the area of one triangle by 6.
$6(333.41625) = 2000.4975$

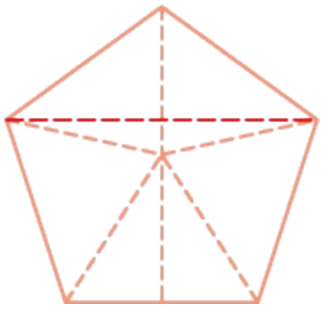
So, the area of the top of the table is 2000.4975 square inches.

The total cost is \$0.06 × 2000.4975 or \$120.03.

6. Draw a regular pentagon and use dashed lines to show the ways it can be decomposed. Describe the shapes in the decomposed figure.

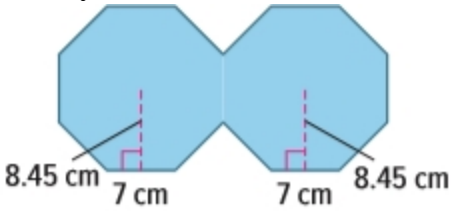
   SOLUTION:

   Sample answer:



   The regular pentagon is decomposed into 5 triangles or 1 triangle and 1 trapezoid.

7. **Identify Structure** What is the area of the figure below?



   SOLUTION:

   The figure is composed of two congruent octagons. The area of one octagon is $8(\frac{1}{2} \times 7 \times 8.45)$ or 236.6 cm^2. So, the area of both figures is 2(236.6) or 473.2 cm^2.

8. **Reason Abstractly** The area of a regular hexagon is about 65 square units. You decompose the figure into 6 triangles. The height of one triangle is about 4.3 units. What is the approximate length of the base of the triangle?

   SOLUTION:

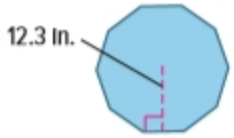
   Sample method: Solve $65 = 6(\frac{1}{2} \times 4.3 \times b)$ for the base b.

   $65 = 6(\frac{1}{2} \times 4.3 \times b)$

   $65 = 12.9b$ Multiply.
   $5 \approx b$ Divide.

   The base is about 5 units.

9. **Reason Inductively** The figure shown is a regular decagon. If the perimeter is 80 inches, what is the area of the decagon? Write an argument that can be used to defend your solution.



SOLUTION:

The base length of each triangle is 80 ÷ 10 or 8 in. So, $10\left(\frac{1}{2} \times 8 \times 12.3\right) = 492$. So, the area is 492 in^2.