Teacher Edition
Volume 1

Reveal MATH™

Integrated I

Mc Graw Hill

Teacher Edition

Volume 1

Reveal MATH®

Integrated I

Mc
Graw
Hill

mheducation.com/prek-12

Cover: (t to b, l to r) whitestone/123RF , Ingram Publishing

Send all inquiries to:
McGraw-Hill Education
8787 Orion Place
Columbus, OH 43240

ISBN: 978-0-07-700693-8
MHID: 0-07-700693-3

Printed in the United States of America.

3 4 5 6 7 8 9 10 11 WEB 27 26 25 24 23 22 21 20

Contents in Brief

Reveal Math Guiding Principles

Academic research and the science of learning provide the foundation for this powerful K–12 math program designed to help reveal the mathematician in every student.

> *Reveal Math* is built on a solid foundation of **RESEARCH** that shaped the **PEDAGOGY** of the program.

Reveal Math Integrated I, Integrated II, and Integrated III (Reveal Math Integrated) used findings from research on teaching and learning mathematics to develop its instructional model. Based on analyses of research findings, these areas form the foundational structure of the program:

- Rigor
- Productive Struggle
- Formative Assessment
- Rich Tasks
- Mathematical Discourse
- Collaborative Learning

Instructional Model

1 Launch

WARM UP

During the **Warm Up,** students complete exercises to activate prior knowledge and review prerequisite concepts and skills.

LAUNCH THE LESSON

In **Launch the Lesson**, students view a real-world scenario and image to pique their interest in the lesson content. They are introduced to questions that they will be able to answer at the end of the lesson.

EXPLORE

During the **Explore** activity, students work in partners or small groups to explore a rich mathematical problem related to the lesson content.

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY

2 Explore and Develop

3 Reflect and Practice

LEARN	EXAMPLES & CHECK	EXIT TICKET	PRACTICE
In the **Learn** section, students gain the foundational knowledge needed to actively work through upcoming Examples.	Students work through **Examples** related to the key concepts and engage in mathematical discourse. Students complete a **Check** after several Examples as a quick formative assessment to help teachers adjust instruction as needed.	The **Exit Ticket** gives students an opportunity to convey their understanding of the lesson concepts.	Students complete **Practice** exercises individually or collaboratively to solidify their understanding of lesson concepts and build proficiency with lesson skills.

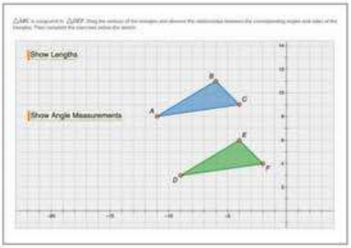
Klaus Vedfelt/DigitalVision/Getty Images

Reveal Math Key Areas of Focus

Reveal Math Integrated I, II, III (Reveal Math Integrated) have a strong focus on rigor—especially the development of conceptual understanding—an emphasis on student mindset, and ongoing formative assessment feedback loops.

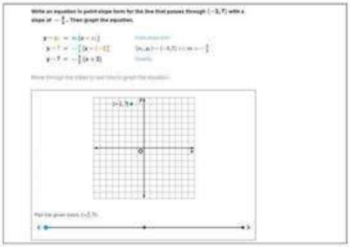
Rigor

Reveal Math Integrated has been thoughtfully designed to incorporate a balance of the three elements of rigor: conceptual understanding, procedural skills and fluency, and application.



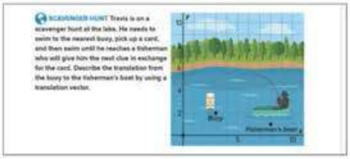
Conceptual Understanding

Explore activities give all students an opportunity to work collaboratively and discuss their thinking as they build conceptual understanding of new concepts. In the **Explore** activity to the left, students use **Web Sketchpad**® to build understanding of the relationships between corresponding sides and angles in congruent triangles.



Procedural Skills and Fluency

Students use different strategies and tools to build procedural fluency. In the **Example** shown, students build proficiency with writing equations in point-slope form.

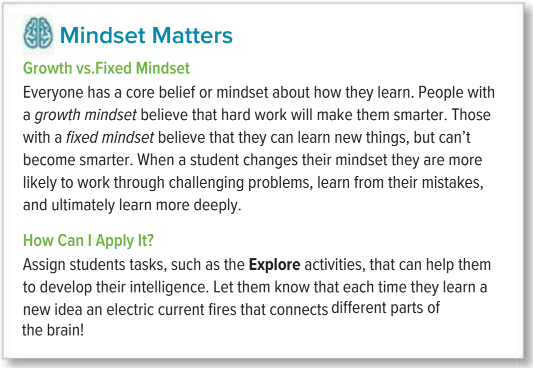


Application

Real-world examples and practice problems are opportunities for students to apply their learning to new situations. In the real-world example shown, students apply their understanding by solving a multi-step problem with translations.

Student Mindset

Mindset Matters tips located in each module provide specific examples of how Reveal Math Integrated content can be used to promote a growth mindset in all students. Another feature focused on promoting a growth mindset is **Ignite! Activities** developed by Dr. Raj Shah to spark student curiosity about why the math works. An **Ignite!** delivers problem sets that are flexible enough so that students with varying background knowledge can engage with the content and motivates them to ask questions, solve complex problems, and develop a can-do attitude toward math.



Mindset Matters

Growth vs.Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that hard work will make them smarter. Those with a *fixed mindset* believe that they can learn new things, but can't become smarter. When a student changes their mindset they are more likely to work through challenging problems, learn from their mistakes, and ultimately learn more deeply.

How Can I Apply It?

Assign students tasks, such as the **Explore** activities, that can help them to develop their intelligence. Let them know that each time they learn a new idea an electric current fires that connects different parts of the brain!

Teacher Edition Mindset Tip

Student Ignite! Activity

Formative Assessment

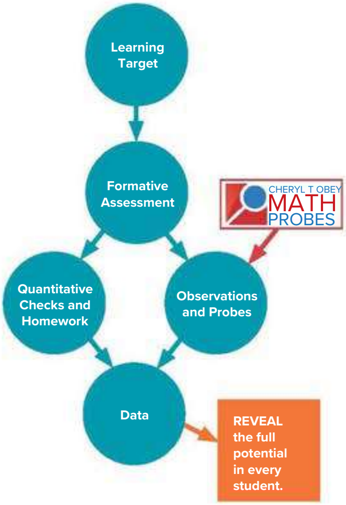
The key to reaching all learners is to adjust instruction based on each student's understanding. Reveal Math Integrated offers powerful formative assessment tools that help teachers to efficiently and effectively differentiate instruction for all students.

Math Probes

Each module includes a **Cheryl Tobey Formative Assessment Math Probe** that is focused on addressing student misconceptions about key math topics. Students can complete these probes at the beginning, middle, or end of a module. The teacher support includes a list of recommended differentiated resources that teachers assign based on students' responses.

Example Checks

After multiple examples, a formative assessment **Check** that students complete on their own allows teachers to gauge students' understanding of the concept or skill presented. When students complete the Check online, the teacher receives resource recommendations which can be assigned to students.


Learning Target
Formative Assessment
CHERYL T OBEY MATH PROBES
Quantitative Checks and Homework
Observations and Probes
Data
REVEAL the full potential in every student.

A Powerful Blended Learning Experience

The *Reveal Math Integrated Course I, Course II, Course III* (Reveal Math Integrated) blended learning experience was designed to include purposeful print and digital components focused on sparking student curiosity and providing teachers with flexible implementation options.

Reveal Math Integrated has been thoughtfully developed to provide a rich learning experience no matter where a district, school, or classroom falls on the digital spectrum. All of the instructional content can be projected or can be accessed via desktop, laptop, or tablet.

Lesson

1 Launch

WARM UP

The **Warm Up** exercise can be projected on an interactive whiteboard.

LAUNCH THE LESSON

Launch the Lesson can be projected or assigned to students to access on their own devices.

EXPLORE

The **Explore** activity can be projected while students record their observations in a notebook or can be assigned for students to complete on individual devices.

Launch the Lesson

Explore

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY
INTERACTIVE PRESENTATION
PRINT STUDENT EDITION

(l)Andrey_Kuzmin/iStock/Getty Images, (r)McGraw-Hill Education

2 Explore and Develop

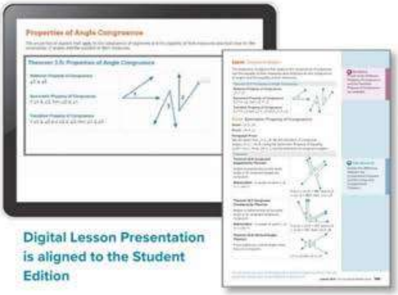
LEARN

As students are introduced to the key lesson concepts, they can progress through the **Learn** by recording notes in a notebook or on their own devices.

EXAMPLES & CHECK

Either in a notebook or on an individual device, students work through one or more **Examples** related to key lesson concepts.

A **Check** follows several Examples in either the Student Edition or on each student device.


Properties of Angle Congruence
Theorem 2.5: Properties of Angle Congruence

Digital Lesson Presentation is aligned to the Student Edition

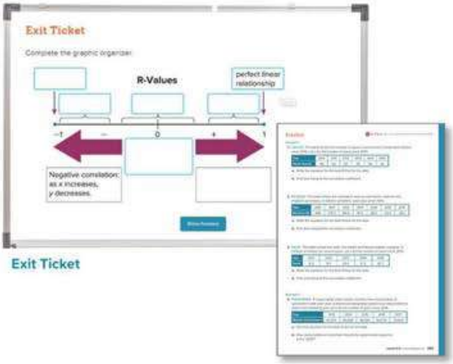
3 Reflect and Practice

EXIT TICKET

The **Exit Ticket** is projected or accessed via student devices to provide students with lesson closure and an opportunity to revisit the lesson concepts.

PRACTICE

Assign students **Practice** problems from their Student Edition or create a digital assignment for them to work on their device in class or at home to solidify lesson concepts.


Exit Ticket

Complete the graphic organizer.

R-Values
perfect linear relationship
−1
0
1
Negative correlation: as x increases, y decreases.

Exit Ticket

Practice

(l)McGraw-Hill Education, (r)Andrey_Kuzmin/iStock/Getty Images

Supporting All Learners

The *Reveal Math Integrated I, II,* and *III* (Reveal Math Integrated) programs were designed so that all students have access to:

- rich tasks that promote productive struggle,
- opportunities to develop proficiency with the habits of mind and thinking strategies of mathematicians, and
- prompts to promote mathematical discourse and build academic language.

Resources for Differentiating Instruction

When needed, resources are available to differentiate math instruction for students who may need to see a concept in a different way, practice prerequisite skills, or are ready to extend their learning.

AL

Approaching Level Resources

- Remediation Activities
- Extra Examples

BL

Beyond Level Resources

- Beyond Level Differentiated Activities
- Extension Activities

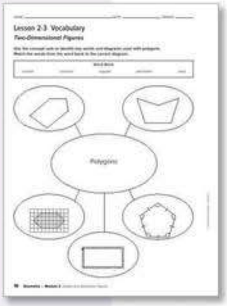
Resources for English Language Learners

Reveal Math Integrated also includes student and teacher resources to support students who are simultaneously learning grade-level math and building their English proficiency. Appropriate, research-based language scaffolds are also provided to support students as they engage in rigorous mathematical tasks and discussions.

ELL

English Language Learners

- Spanish Personal Tutors
- Math Language-Building Activities
- Language Scaffolds
- *Think About It!* and *Talk About It!* Prompts
- Multilingual eGlossary
- Audio
- Graphic Organizers
- Web Sketchpad, Desmos, and eTools



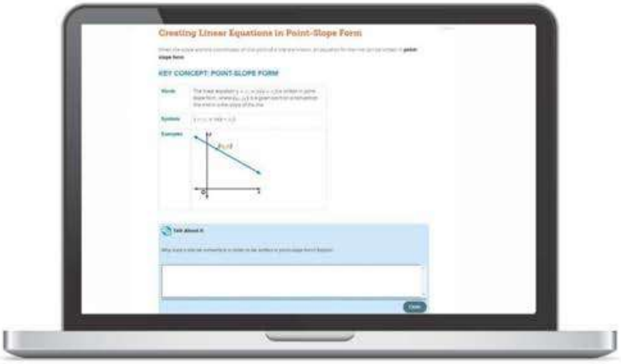
Lesson 2-3 Vocabulary

Two-Dimensional Figures

Polygons

Developing Mathematical Thinking and Strategic Questioning

Reveal Math Integrated I, II, and *III* (Reveal Math Integrated) are comprised of high-quality math content designed to be accessible and relevant to each student. Throughout the program, students are presented with a variety of thoughtfully designed questioning strategies related to the content. Using these questions provides you with an additional, built-in type of formative assessment that can be used to modify instruction. They also strengthen students' ownership of mathematical content knowledge and daily use of the Standards for Mathematical Practice.


Creating Linear Equations in Point-Slope Form

KEY CONCEPT: POINT-SLOPE FORM

Key Concept Introduction followed by a Talk About It question to discuss with a classmate.

You will find these types of questioning strategies throughout Reveal Math Integrated. The related Standard for Mathematical Practice for each is also indicated.

- **Talk About It** questions encourage students to engage in mathematical discourse with classmates (MP3)
- **Alternate Method** shows students another way to solve a problem and asks them to compare and contrast the methods and solutions (MP1)
- **Avoid a Common Error** shows students a problem similar to an example but with a flaw in reasoning, and students have to find and explain the error (MP3)
- **State Your Assumptions** requires that student state the assumptions they made to solve a problem (MP4)
- **Use a Source** asks students to find information using an external source, such as the Internet, and use it to pose or solve a problem (MP5)
- **Think About It** questions help students make sense of mathematical problems (MP1)
- **Concept Checks** prompt students to analyze how the Key Concepts of the lesson apply to various use cases (MP3)

Scott Dunlap/iStockvectors/Getty Images

Reveal Student Readiness with Individualized Learning Tools

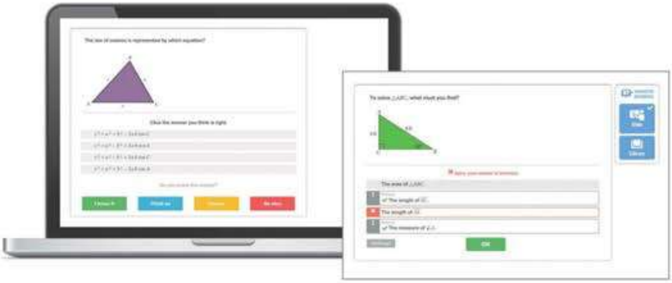
Reveal Math Integrated I, II, and *III* (Reveal Math Integrated) incorporate innovative, technology-based tools that are designed to extend the teacher's reach in the classroom to help address a wide range of knowledge gaps, set and align academic goals, and meet student individualized learning needs.

LEARNSMART®

Topic-Mastery

With embedded **LearnSmart,**® students have a built-in study partner for topic practice and review to prepare for multi-module or mid-year tests.

LearnSmart's revolutionary adaptive technology measures students' awareness of their own learning, time on topic, answer accuracy, and suggests alternative resources to support student learning, confidence, and topic mastery.

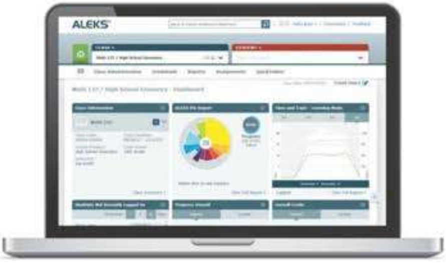


ALEKS®

Individualized Learning Pathways

Learners of all levels benefit from the use of ***ALEKS'*** adaptive, online math technology designed to pinpoint what each student knows, does not know, and most importantly, what each student is ready to learn.

When paired with Reveal Math Integrated, ***ALEKS*** is a powerful tool designed to provide integrated instructionally actionable data enabling teachers to utilize Reveal Math Integrated resources for individual students, groups, or the entire classroom.

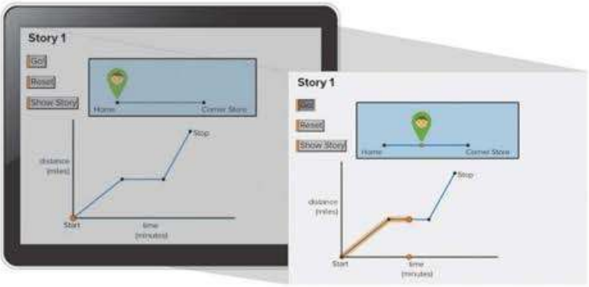


Activity Report

Scott Dunlap/Stockvectors/Getty Images

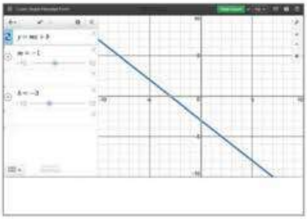
Powerful Tools for Modeling Mathematics

Reveal Math Integrated I, II, and *III* (Reveal Math Integrated) have been designed with purposeful, embedded digital tools to increase student engagement and provide unique modeling opportunities.



Web Sketchpad® Activities

The leading dynamic mathematics visualization software has now been integrated with **Web Sketchpad Activities** at point of use within Reveal Math Integrated. Student exploration (and practice) using **Web Sketchpad** encourages problem solving and visualization of abstract math concepts.



desmos

The powerful **Desmos** graphing calculator is available in Reveal Math Integrated for students to explore, model, and apply math to the real-world.

eTools

By using a wide variety of digital **eTools** embedded within Reveal Math Integrated, students gain additional hands-on experience while they learn and teachers have the option to create problem-based learning opportunities.

Technology-Enhanced Items

Embedded within the digital lesson, technology-enhanced items—such as drag-and-drop, flashcard flips, or diagram completion—are strategically placed to give students the practice with common computer functions needed to master computer-based testing.

TYPE
SWIPE
DRAG & DROP
FLASHCARDS
eTOOLS
MULTI-SELECT
WATCH
EXPAND

McGraw-Hill Education

Assessment Tools to Reveal Student Progress and Success

Reveal Math Integrated I, II, and *III* (Reveal Math Integrated) provide a comprehensive array of assessment tools, with both print and digital administration options, to measure student understanding and progress. The digital assessment tools include next-generation assessment items, such as multiple-response, selected-response, and technology-enhanced items.

Assessment Solutions

Reveal Math Integrated provides embedded, regular formative checkpoints to monitor student learning and provide feedback that can be used to modify instruction and help direct student learning using reports and recommendations based on resulting scores.

Summative assessments built in Reveal Math Integrated evaluate student learning at the module conclusion by comparing it against the state standards covered.

Formative Assessment Resources

- Cheryl Tobey Formative Assessment Math Probes
- Checks
- Exit Tickets
- Put It All Together

Summative Assessment Resources

- Module Tests
- Performance Tasks
- End-of-Course Tests
- LearnSmart

Or **Build Your Own** assessments focused on standards or objectives. Access to banks of questions, including those with tech-enhanced capabilities, enable a wide range of options to mirror high-stakes assessment formats.

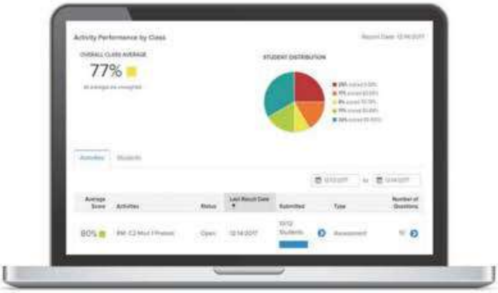
Reporting

Clear, instructionally actionable data is a click away with the Reveal Math Integrated Reporting Dashboard.

Activity Report Real-time class and student reporting of activities completed by the class. Includes average score, submission rate, and skills covered for the class and each student.

- **Item Analysis Report** A detailed analysis of response rates and patterns, answers, and question types in a class snapshot or by student.

Standards Report Performance data by class or individual student are aggregated by standards, skills, or objectives linked to the related activities completed.



Activity Report

Scott Dunlap/Stockvectors/Getty Images

Professional Development Support for Continuous Learning

McGraw-Hill Education supports lifelong learning and demonstrates commitment to teachers with a built-in professional learning environment designed for support during planning or extended learning opportunities.

What You Will Find

- Best-practice resources
- Implementation support
- Teaching Strategies
- Classroom Videos
- Math Misconception Videos
- Content and Pedagogy Videos
- Content Progression Information

Why Professional Development Is so Important

- Research-based understanding of student learning
- Improved student performance
- Evidence-based instructional best practices
- Collaborative content strategy planning
- Extended knowledge of program how-to's

Kevin Dodge/Blend Images/Getty Images

Reveal Math Expert Advisors

Cathy Seeley, Ed.D.

Austin, Texas

Mathematics educator, speaker, and writer, former Senior Fellow at the Charles A. Dana Center at The University of Texas at Austin, past President of NCTM, former Director of K-12 Mathematics for the State of Texas

Areas of expertise:
Mathematics Teaching, Equity, Assessment, STEM Learning, Informal Learning, Upside-Down Teaching, Productive Struggling, Mathematical Practices, Mathematical Habits of Mind, Family and Community Outreach, Mathematics Education Policy, Advocacy

> "We want students to believe deeply that mathematics makes sense–in generating answers to problems, discussing their thinking and other students' thinking, and learning new material."
>
> **—Seeley, 2016, Making Sense of Math**

Cheryl R. Tobey, M.Ed.

Gardiner, Maine

Senior Mathematics Associate at Education Development Center (EDC)

Areas of expertise:
Formative assessment and professional development for mathematics teachers; tools and strategies to uncovering misconceptions

> "Misunderstandings and partial understandings develop as a normal part of learning mathematics. Our job as educators is to minimize the chances of students' harboring misconceptions by knowing the potential difficulties students are likely to encounter using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas.
>
> **— Tobey, et al 2007, 2009, 2010, 2013, 2104, Uncovering Student Thinking Series**

Nevels Nevels, Ph.D.

Saint Louis, Missouri

PK-12 Mathematics Curriculum Coordinator for Hazelwood School District

Areas of expertise:
Mathematics Teacher Education; Student Agency & Identity; Socio-Cultural Perspective in Mathematics Learning

> "A school building is one setting for learning mathematics. It is understood that all children should be expected to learn meaningful mathematics within its walls. Additionally, teachers should be expected to learn within the walls of this same building. More poignantly, I posit that if teachers are not learning mathematics in their school building, then it is not a school."
>
> **—Nevels, 2018**

Raj Shah, Ph.D.

Columbus, Ohio

Founder of Math Plus Academy, a STEM enrichment program and founding member of The Global Math Project

Areas of expertise:
Sparking student curiosity, promoting productive struggle, and creating math experiences that kids love

> "As teachers, it's imperative that we start every lesson by getting students to ask more questions because curiosity is the fuel that drives engagement, deeper learning and perseverance."
>
> **—Shah, 2017**

(l to r, t to b) Photo by Damon Leo, Shannon Larsen, Matthew McClellan, Raj Shah

Walter Secada, Ph.D.

Coral Gables, Florida

Professor of Teaching and Learning
at the University of Miami

Areas of expertise:
Improving education for English language learners, equity in education, mathematics education, bilingual education, school restructuring, professional development of teachers, student engagement, Hispanic dropout and prevention, and reform

> "The best lessons take place when teachers have thought about how their individual English language learners will respond not just to the mathematical content of that lesson, but also to its language demands and mathematical practices."
>
> **—Secada, 2018**

Ryan Baker, Ph.D.

Philadelphia, Pennsylvania

Associate Professor and Director
of Penn Center for Learning Analytics
at the University of Pennsylvania

Areas of expertise:
Interactions between students and educational software; data mining and learning analytics to understand student learning

> "The ultimate goal of the field of Artificial Intelligence in Education is not to promote artificial intelligence, but to promote education... systems that are designed intelligently, and that leverage teachers' intelligence. Modern online learning systems used at scale are leveraging human intelligence to improve their design, and they're bringing human beings into the decision-making loop and trying to inform them."
>
> **—Baker, 2016**

Chris Dede, Ph.D.

Cambridge, Massachusetts

Timothy E. Wirth Professor in
Learning Technologies at Harvard
Graduate School of Education

Areas of expertise:
Provides leadership in educational innovation; educational improvements using technology

> "People are very diverse in how they prefer to learn. Good instruction is like an ecosystem that has many niches for alternative types of learning: lectures, games, engaging video-based animations, readings, etc. Learners then can navigate to the niche that best fulfills their current needs."
>
> **—Dede, 2017**

Dinah Zike, M.Ed.

Comfort, Texas

President of Dinah.com
in San Antonio, Texas and
Dinah Zike Academy

Areas of expertise:
Developing educational materials that include three-dimensional graphic organizers; interactive notebook activities for differentiation; and kinesthetic, cross-curricular manipulatives

> "It is education's responsibility to meet the unique needs of students, and not the students' responsibility to meet education's need for uniformity."
>
> **—Zike, 2017, InRIGORating Math Notebooks**

(l to r, t to b) Walter Secada, Elisabeth Reinkordt, Chris Dede, Sergio Rodriguez

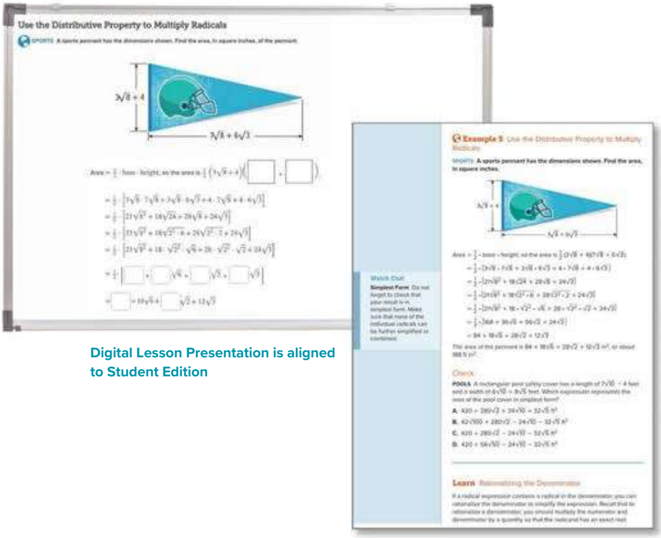
Reveal Everything Needed for Effective Instruction

Reveal Math Integrated I, II, and *III* (Reveal Math Integrated) provide both print and innovative, technology-based tools designed to address a wide range of classrooms. No matter whether you're in a 1:1 district, or have a classroom projector, Reveal Math Integrated provides you with the resources you need for a rich learning experience.

Reveal
INTEGRATED I
Language Development Handbook
Reveal
INTEGRATED II
Language Development Handbook
Reveal
INTEGRATED III
Language Development Handbook

Blended Classrooms

Focused on projection of the **Interactive Presentation,** students follow along, taking notes and working through problems in a notebook during class time. Also included in the Interactive Student Edition is a glossary, selected answers, and a reference sheet.



Use the Distributive Property to Multiply Radicals

Example 5 Use the Distributive Property to Multiply Radicals

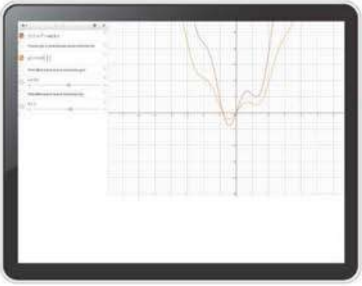
Digital Lesson Presentation is aligned to Student Edition

Andrey_Kuzmin/iStock/Getty Images

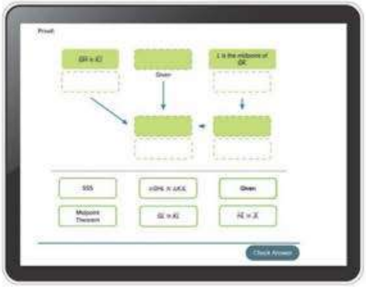
Digital Classrooms

Projection is a focal point for key areas of the course with students interacting with the lesson using their own devices. Each student can access teacher-assigned sections of the lessons for **Explore** activities, **Learn** sections, and **Examples**. Point of use videos, animations, as well as interactive content enable students to experience math in interesting and impactful ways.

Web Sketchpad

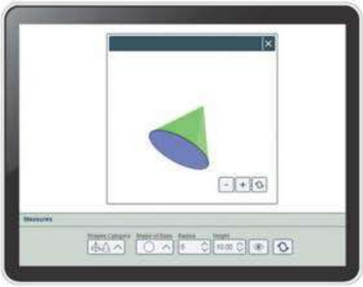


Desmos



Drag-and-Drop

Video



eTools

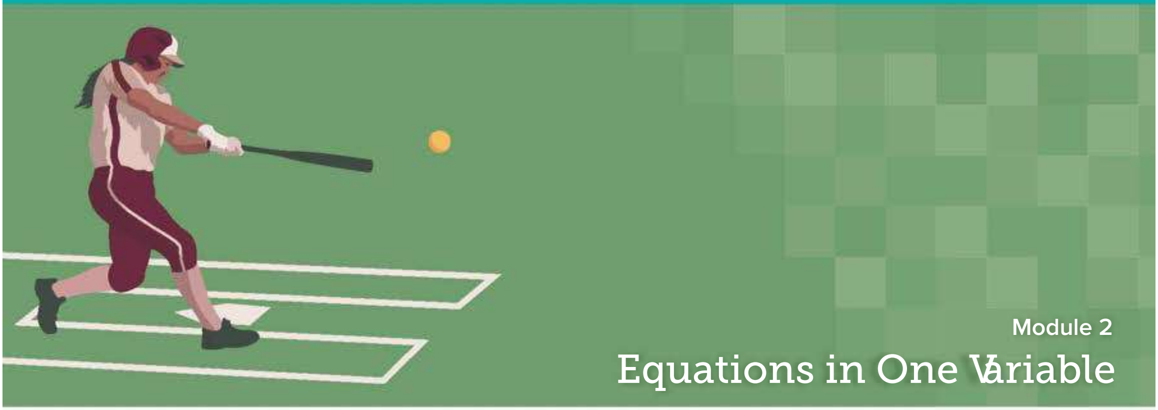
In Reveal Math Integrated, R is for—

- Research
- Rigor
- Relevant Connections

**Are you…
READY to start?**

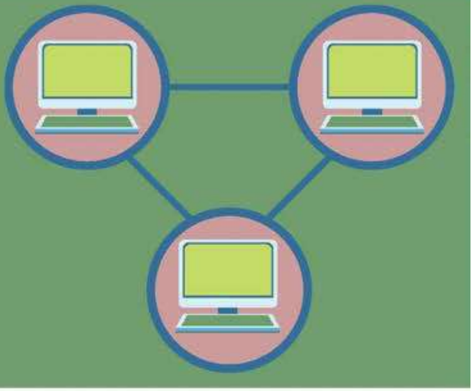
Module 1

Expressions



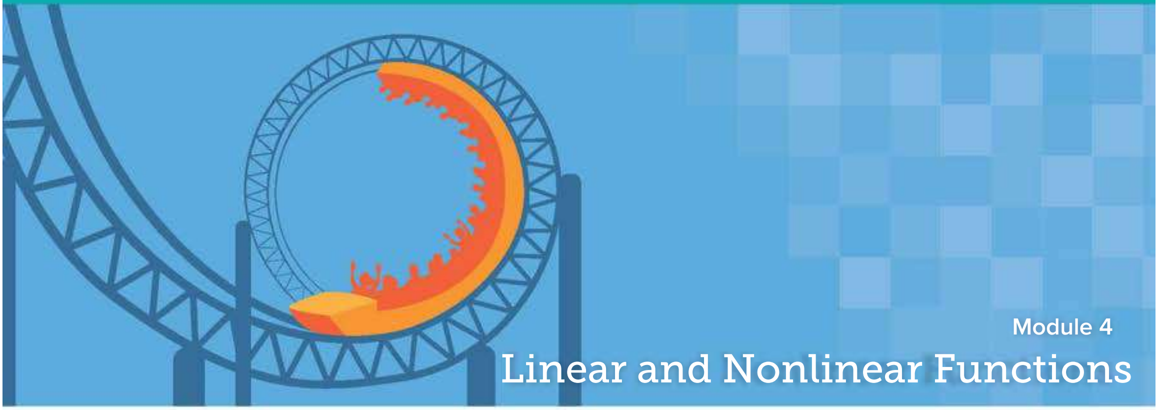
Module 2

Equations in One Variable



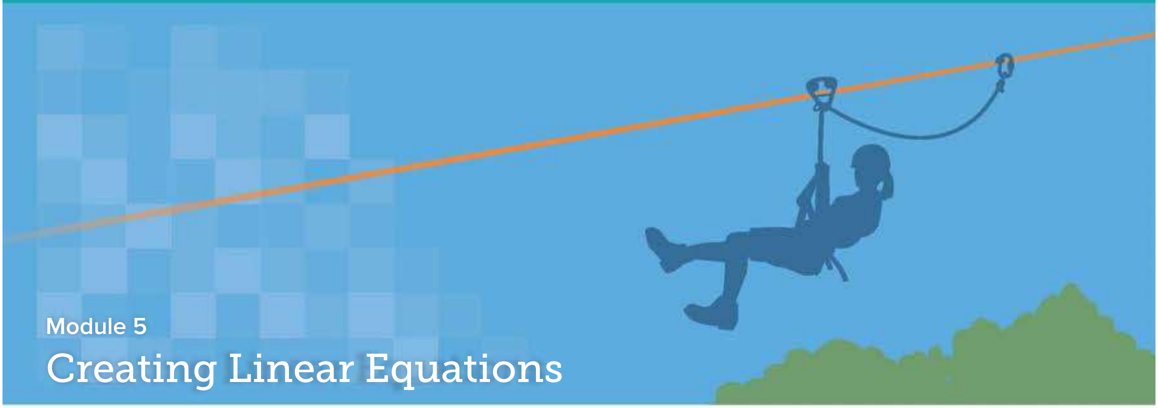
Module 3

Relations and Functions



Module 4

Linear and Nonlinear Functions



Module 5

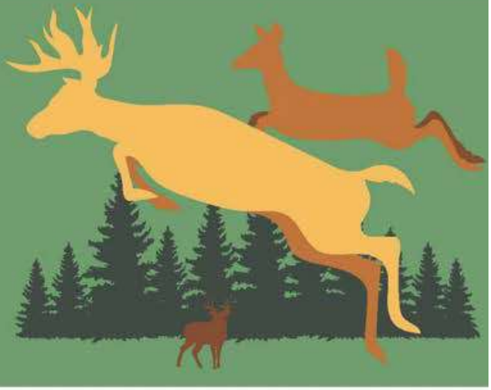
Creating Linear Equations

Module 6

Linear Inequalities

Module 7

Systems of Linear Equations and Inequalities



Module 8

Exponential Functions

Module 9

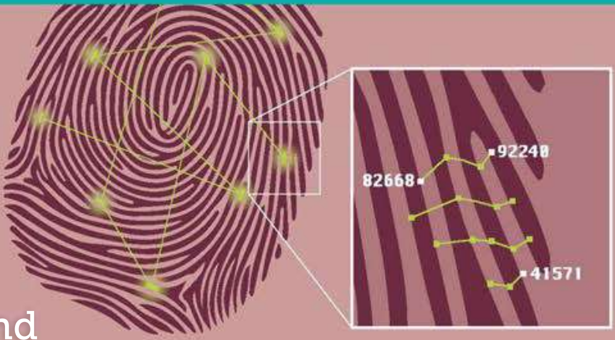
Statistics

Module 10

Tools of Geometry

Module 11

Angles and Geometric Figures

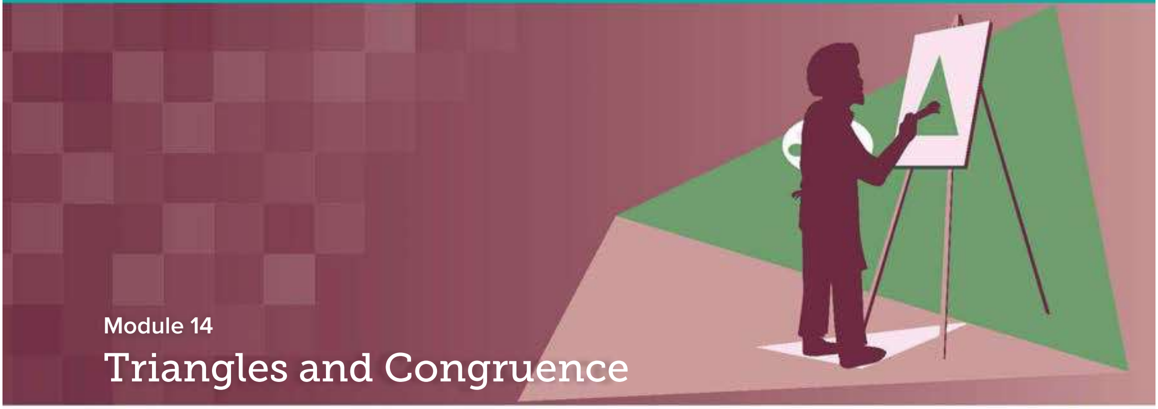


Module 12

Logical Arguments and Line Relationships

Module 13

Transformations and Symmetry



Module 14

Triangles and Congruence

Standards for Mathematical Content, Reveal Math Integrated I

This correlation shows the alignment of *Reveal Math Integrated I* to the Standards for Mathematical Content from the Common Core State Standards for Mathematics.

Standard		Lesson(s)
Number and Quantity		
Quantities ★ N.Q		
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**2-7, 3-1, 9-2, 9-4**
N.Q.2	Define appropriate quantities for the purpose of descriptive modeling.	**1-6**
N.Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**1-6**
Algebra		
Seeing Structure in Expressions A.SSE		
A.SSE.1	Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients.	**1-1, 1-2, 1-4, 4-6, 4-7**
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*	
Creating Equations ★ A.CED		
A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear quadratic functions, and simple rational and exponential functions.	**2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 6-1, 6-2, 6-3, 6-4**
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**4-1, 4-3, 4-4, 4-5, 4-6, 4-7, 5-1, 5-2, 5-3, 5-5, 5-6, 8-1, 8-2, 8-3, 8-5, 8-6**
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	**2-1, 2-7, 3-6, 5-1, 5-2, 6-1, 6-2, 6-3, 6-4, 6-5, 7-1, 7-2, 7-3, 7-4, 7-5**
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	**2-7**
Reasoning with Equations and Inequalities A.REI		
A.REI.1	Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**2-2, 2-3, 2-4, 2-5, 2-6, 4-3, 5-1, 5-2**
A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 6-1, 6-2**
A.REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**7-4**
A.REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**7-1, 7-2, 7-3, 7-4**
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**3-4, 4-1, 4-3, 8-1**

★ Mathematical Modeling Standards

Standard		Lesson(s)
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	**7-1**
A.REI.12	Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**6-5, 7-5**
Functions		
Interpreting Functions F.IF		
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	**3-1, 3-2**
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**3-2**
F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	**4-5, 8-5, 8-6**
F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	**3-3, 3-4, 3-5, 3-6, 4-4, 4-6, 4-7, 8-1**
F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	**3-2, 3-3, 3-6, 8-1**
F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	**4-2, 5-1**
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ **a.** Graph linear and quadratic functions and show intercepts, maxima, and minima. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**4-1, 4-3, 4-4, 8-1, 8-2**
F.IF.9	Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	**3-6, 4-3, 4-6, 8-1**
Building Linear or Exponential Functions F.BF		
F.BF.1	Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations.	**4-5**
F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	**4-5, 8-5, 8-6**

★ Mathematical Modeling Standards

Standard		Lesson(s)
F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	**4-4, 4-7, 8-2**
Linear and Exponential ★ F.LE		
F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. **a.** Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.	**Expand 4-3, 8-1, Expand 8-5**
	b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
	c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**4-5, 8-3, 8-5**
F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.	**Standard F.LE.3 is taught in Integrated Math Course II, 12-8 Modeling and Curve Fitting**
F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context.	**4-1, 4-2, 4-3, 8-1, 8-3, 8-5**
Geometry		
Congruence G.CO		
G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**10-2, 10-3, 10-4, 11-1, 11-2, 12-7**
G.CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	**11-4**
G.CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	**13-6**
G.CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**13-1, 13-2, 13-3, 13-5, 13-6**
G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**13-1, 13-2, 13-3, 13-4, 13-5, 13-6**
G.CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide whether they are congruent.	**13-1, 13-2, 13-3, 13-4, 13-6**
G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**14-2**

★ Mathematical Modeling Standards

Standard		Lesson(s)
G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	**14-3, 14-4**
G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	**10-3, 10-7, 11-1, 11-2, 12-5, 12-9, 12-10, 13-1, 14-3, 14-4, 14-6**
G.CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects.	**Standard G.CO.13 is taught in Integrated Math Course II, 5-5 Tangents**
Expressing Geometric Properties With Equations G.GPE		
G.GPE.4	Use coordinates to prove simple geometric theorems algebraically.	**14-7**
G.GPE.5	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**Expand 4-2, 12-8**
G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	**11-3**
Statistics and Probability ★		
Interpreting Categorical and Quantitative Data S.ID		
S.ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	**9-2, 9-4**
S.ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**9-4, 9-6**
S.ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**9-5, 9-6**
S.ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**9-7**
S.ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models.	**5-3, 5-5**
	b. Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate.	
	c. Fit a linear function for scatter plots that suggest a linear association.	
S.ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**5-1, 5-3**
S.ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	**5-5**
S.ID.9	Distinguish between correlation and causation.	**5-4**

★ Mathematical Modeling Standards

Standards for Mathematical Practice

This correlation shows the alignment of *Reveal Math Integrated I* to the Standards for Mathematical Practice, from the Common Core State Standards.

Standard	Lesson(s)
1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	*Reveal Math Integrated I* requires students to make sense of problems and persevere in solving them in Examples and Practice throughout the program. Some specific lessons for review are: Lessons 1-1, 1-4, 2-5, 3-1, 3-3, 3-4, 3-5, 4-1, 4-3, 4-4, 4-7, 5-1, 5-4, 5-5, 5-6, 6-1, 6-2, 6-4, 7-2, 8-2, 9-2, 9-7, 10-4, 10-7, 11-3, 12-1, 12-7, 12-8, 12-10, 13-2, 14-3, 14-5, 14-7
2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	*Reveal Math Integrated I* requires students to reason abstractly and quantitatively in Think About It features and Higher Order Thinking Skills throughout the program. Some specific lessons for review are: Lessons 1-2, 1-6, 2-1, 2-2, 2-3, 2-4, 2-6, 2-7 3-3, 3-4, 3-5, 4-2, 5-1, 5-2, 6-1, 6-2, 7-3, 7-4, 8-4, 8-5, 9-4, 10-3, 10-4, 11-3, 11-6, 12-2, 12-9, 13-4, 14-3, 14-7
3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	*Reveal Math Integrated I* requires students to construct viable arguments and critique the reasoning of others in Talk About It features and Practice throughout the program. Some specific lessons for review are: Lessons 1-3, 2-4, 3-2, 3-3, 4-5, 5-4, 6-4, 7-5, 8-1, 8-5, 9-1, 9-3, 10-1, 10-2, 10-5, 11-2, 11-8, 12-1, 12-5, 12-6, 12-8, 12-9, 12-10, 13-1, 13-4, 13-5, 14-1, 14-3, 14-5, 14-7
4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	*Reveal Math Integrated I* requires students to model with mathematics, collaborate, and discuss mathematics in Examples and Practice throughout the program. Some specific lessons for review are: Lessons 1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 2-1, 2-5, 2-6, 3-2, 3-5, 4-2, 4-3, 4-6, 5-1, 5-2, 5-6, 6-3, 6-5, 7-3, 7-4, 8-1, 8-4, 8-6, 9-1, 9-2, 9-4, 9-5, 9-6, 9-7, 10-2, 10-6, 10-7, 11-1, 11-4, 11-5, 11-6, 12-3, 12-4, 12-6, 12-9, 12-10, 13-1, 13-4, 14-1, 14-4, 14-5

Standard	Lesson(s)
5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	*Reveal Math Integrated I* requires students to use appropriate tools strategically in Explore activities throughout the program. Some specific lessons for review are: Lessons 1-4, 2-2, 2-3, 3-4, 4-1, 4-3, 4-4, 4-7, 5-3, 5-4, 5-5, 5-6, 6-1, 6-5, 7-1, 8-2, 8-5, 9-5, 9-6, 10-2, 10-6, 11-4, 11-8, 12-1, 12-7, 12-8, 13-1, 13-3, 13-5, 13-6, 14-2, 14-4, 14-6
6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	*Reveal Math Integrated I* requires students to attend to precision in Examples and Practice throughout the program. Some specific lessons for review are: Lessons 1-4, 1-6, 2-7, 3-1, 3-6, 4-6, 5-3, 5-5, 6-4, 7-2, 7-5, 8-3, 8-4, 9-3, 10-1, 11-1, 11-6, 11-7, 11-8, 12-2, 12-3, 12-4, 12-5, 12-6, 12-7, 13-2, 13-3, 13-6, 14-2, 14-4, 14-6
7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	*Reveal Math Integrated I* requires students to look for and make use of structure in Explore activities and Higher Order Thinking Skills throughout the program. Some specific lessons for review are: Lessons 1-2, 1-3, 1-5, 2-2, 2-3, 2-4, 2-5, 3-6, 4-4, 4-7, 5-3, 6-3, 7-5, 8-1, 8-2, 8-6, 9-1, 9-7, 10-5, 11-5, 12-2, 13-3, 13-6, 14-2, 14-6
8 Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	*Reveal Math Integrated I* requires students to look for and express regularity in repeated reasoning in Concept Check and Think About It features and Higher Order Thinking Skills throughout the program. Some specific lessons for review are: Lessons 1-5, 2-7, 3-1, 4-5, 5-2, 6-3, 7-1, 8-3, 8-6, 9-6, 10-3, 11-2, 12-3, 12-4, 13-2, 14-1

Module 1

Expressions

Module Goals

- Students write and evaluate numerical and algebraic expressions.
- Students simplify expressions using the Distributive Property.
- Students evaluate absolute value expressions.

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Also addresses N.RN.3, N.Q.2 and N.Q.3

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Be Sure to Cover

To completely cover N.RN.3, go online to assign the following activity:

- Operations with Rational Numbers (Expand 1-3)

Coherence

Vertical Alignment

Previous

Students performed operations on rational numbers.

7.NS.1

Now

Students take what they have learned about whole numbers and apply that to algebraic expressions.

A.SSE.1, A.SSE.2

Next

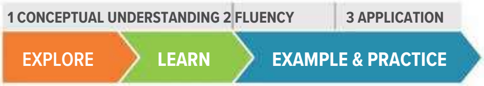
Students will create equations to solve problems.

A.CED.1

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.



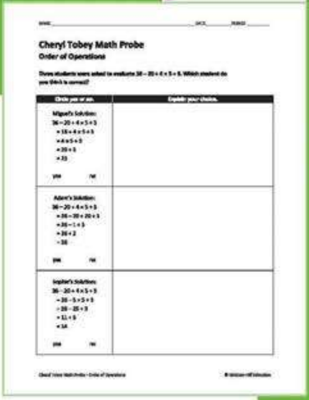
Suggested Pacing

Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
1-1 Numerical Expressions	A.SSE.1b, A.SSE.2	2	1
1-2 Algebraic Expressions	A.SSE.1, A.SSE.2	1	0.5
1-3 Properties of Real Numbers	A.SSE.2	2	1
1-3 Expand Operations with Rational Numbers	N.RN.3	1	0.5
Put It All Together: Lessons 1-1 through 1-3		1	0.5
1-4 Distributive Property	A.SSE.1a, A.SSE.2	2	1
1-5 Expressions Involving Absolute Value	A.SSE.2	1	0.5
1-6 Descriptive Modeling and Accuracy	N.Q.2, N.Q.3	1	0.5
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	**14**	**7**

Formative Assessment Math Probe

Order of Operations

Module Resource



Correct Answers:
Miguel's Solution: no;
Adam's Solution: no;
Sophie's Solution: yes

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which expression has been simplified correctly by using the order of operations and explain their choices.

Targeted Concepts Understand the order in which operations are performed when evaluating a mathematical expression.

Targeted Misconceptions

- Although a concept introduced in earlier grades, high school students often incorrectly work from left to right without using the rules for the order of operations when evaluating expressions.
- Students may use the order of operations rules, but not work from left to right when evaluating expressions (that is, they may always compute multiplication before division and addition before subtraction).

Use the Probe at the beginning of the year as any misunderstandings or misconceptions can affect a student's success in Algebra 1.

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
Miguel's Solution: yes	worked from left to right without using the rules for the order of operations to evaluate the expression. **Example:** Subtracted 20 from 36 first as it was the first operation in the expression.
Adam's Solution: yes	used the rules for the order of operations without working from left to right in the expression. **Example:** Multiplied 4 by 5 instead of dividing 20 by 4.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Operations with Signed Numbers
- Lesson 1-1, Learn, Examples 5 and 7

Revisit the probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

How can mathematical expressions be represented and evaluated? Sample answer: You can represent mathematical expressions verbally, numerically, and algebraically. They can be evaluated by applying properties and rules. For example, you can translate a sentence to a numerical or algebraic expression and use the order of operations to simplify or evaluate the expression.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

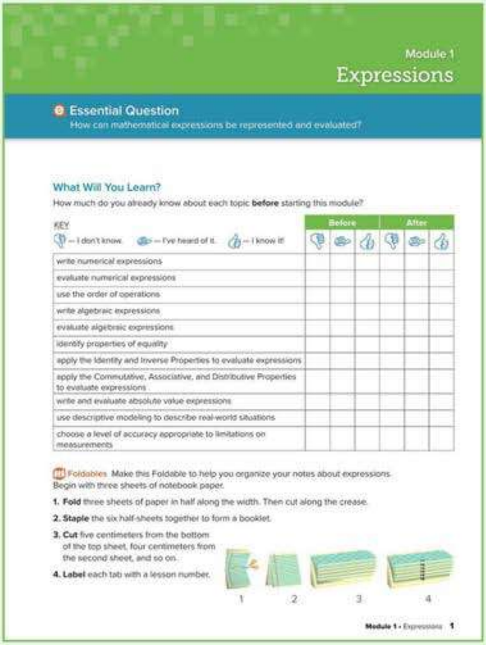
Focus Students organize their notes about expressions, properties of real numbers, and absolute value.

Teach Throughout the module, have students take notes under the tabs of their Foldables while working through each lesson. They should include definitions and key concepts. Encourage students to record examples from each lesson.

When to Use It Use the appropriate tabs as students cover each lesson in this module. Students should add to the vocabulary tab during each lesson.

Launch the Module

For this module, the Launch the Module video uses online shopping to show how algebraic expressions are used in the real world. Students learn how algebraic expressions are used in the trucking industry, and the importance of order of operations.



Module 1

Expressions

Essential Question

How can mathematical expressions be represented and evaluated?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. ✋ — I've heard of it. 👍 — I know it!

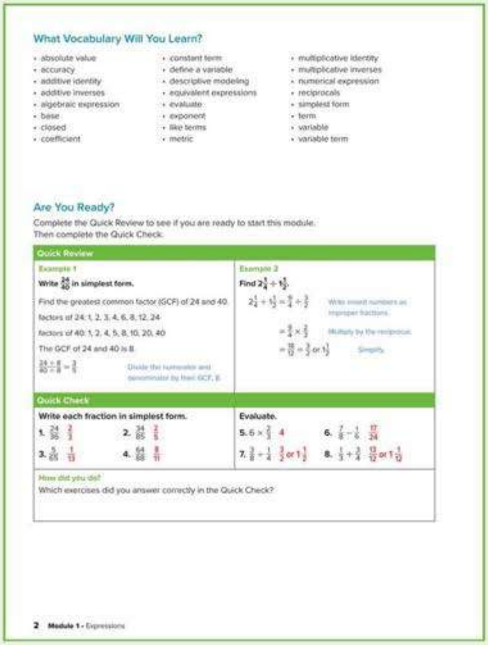
	Before			After		
	👎	✋	👍	👎	✋	👍
write numerical expressions						
evaluate numerical expressions						
use the order of operations						
write algebraic expressions						
evaluate algebraic expressions						
identify properties of equality						
apply the Identity and Inverse Properties to evaluate expressions						
apply the Commutative, Associative, and Distributive Properties to evaluate expressions						
write and evaluate absolute value expressions						
use descriptive modeling to describe real-world situations						
choose a level of accuracy appropriate to limitations on measurements						

Foldables Make this Foldable to help you organize your notes about expressions. Begin with three sheets of notebook paper.

1. **Fold** three sheets of paper in half along the width. Then cut along the crease.
2. **Staple** the six half-sheets together to form a booklet.
3. **Cut** five centimeters from the bottom of the top sheet, four centimeters from the second sheet, and so on.
4. **Label** each tab with a lesson number.

Module 1 • Expressions 1

Interactive Presentation



What Vocabulary Will You Learn?

- absolute value
- accuracy
- additive identity
- additive inverses
- algebraic expression
- base
- closed
- coefficient
- constant term
- define a variable
- descriptive modeling
- equivalent expressions
- evaluate
- exponent
- like terms
- metric
- multiplicative identity
- multiplicative inverses
- numerical expression
- reciprocals
- simplest form
- term
- variable
- variable term

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Write $\frac{24}{40}$ in simplest form.

Find the greatest common factor (GCF) of 24 and 40.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

The GCF of 24 and 40 is 8.

$\frac{24 \div 8}{40 \div 8} = \frac{3}{5}$ Divide the numerator and denominator by their GCF, 8.

Example 2

Find $2\frac{1}{4} \div 1\frac{1}{2}$.

$2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2}$ Write mixed numbers as improper fractions.

$= \frac{9}{4} \times \frac{2}{3}$ Multiply by the reciprocal.

$= \frac{18}{12} = \frac{3}{2}$ or $1\frac{1}{2}$ Simplify.

Quick Check

Write each fraction in simplest form.

1. $\frac{24}{36}$ $\frac{2}{3}$

2. $\frac{34}{85}$ $\frac{2}{5}$

3. $\frac{5}{65}$ $\frac{1}{13}$

4. $\frac{64}{88}$ $\frac{8}{11}$

Evaluate.

5. $6 \times \frac{2}{3}$ 4

6. $\frac{7}{8} - \frac{1}{6}$ $\frac{17}{24}$

7. $\frac{3}{8} \div \frac{1}{4}$ $\frac{3}{2}$ or $1\frac{1}{2}$

8. $\frac{1}{3} + \frac{3}{4}$ $\frac{13}{12}$ or $1\frac{1}{12}$

How did you do?

Which exercises did you answer correctly in the Quick Check?

2 **Module 1** • Expressions

What Vocabulary Will You Learn?

ELL Introduce the key vocabulary by using the following routine.

Define An algebraic expression is a mathematical expression that contains at least one variable.

Example $9y - 7x$

Ask Is there at least one variable? Y es, both $9y$ and $7x$ have variables.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- adding and subtracting integers
- multiplying and dividing integers
- adding and subtracting rational numbers
- using order of operations
- naming points on number lines

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels

You may want to use the **Real Numbers** section to ensure student success in this module.

Mindset Matters

Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a growth mindset believe that hard work will make them smarter. Those with a fixed mindset believe they can learn new things, but cannot become smarter. When a student changes their mindset, they are more likely to work through challenging problems, learn from their mistakes, and ultimately learn more deeply.

How Can I Apply It?

Assign students tasks, celebrate mistakes, and provide opportunities for critique, revision, and reflection. The **Explore** activities and discussion prompts are a great tool to begin this journey.

Lesson 1-1

Numerical Expressions

A.SSE.1b, A.SSE.2

LESSON GOAL

Students write and evaluate numerical expressions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Order of Operations

Develop:

Writing Numerical Expressions
- Translate a Verbal Expression
- Translate a Verbal Expression with Grouping Symbols
- Write a Numerical Expression

Evaluating Numerical Expressions
- Evaluate Expressions
- Order of Operations
- Write and Evaluate a Numerical Expression
- Expressions with Grouping Symbols

Plan for Problem Solving
- Write and Evaluate Algebraic Expressions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Subtract Integers	●	●		●
Extension: The Four Digits Problem		●	●	●

Language Development Handbook

Assign page 1 of the *Language Development Handbook* to help your students build mathematical language related to numerical expressions.

ELL You can use the tips and suggestions on page T1 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students performed operations on rational numbers.
7.NS.1

Now
Students write and evaluate numerical expressions.
A.SSE.1b, A.SSE.2

Next
Students will write and evaluate algebraic expressions.
A.SSE.1, A.SSE.2

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

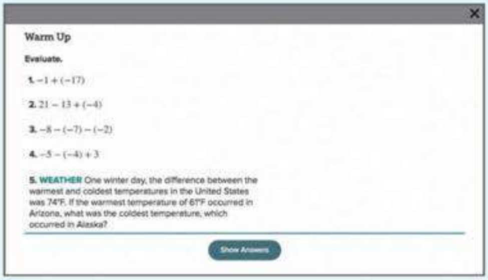
Conceptual Bridge In this lesson, students develop understanding of numerical expressions and use it to build fluency with evaluating numerical expressions. They apply their understanding of numerical expressions by solving real-world problems.

Mathematical Background

When evaluating an expression, students should recall the set of rules that specifies which operation to do first, the order of operations.

- First, perform operations inside grouping symbols. Grouping symbols include parentheses, brackets, braces, and fraction bars. Perform operations inside the innermost grouping symbol first,
- Then, evaluate all powers.
- Next, perform all multiplications and/or divisions from left to right.
- Finally, perform all additions and/or subtractions from left to right.

Interactive Presentation

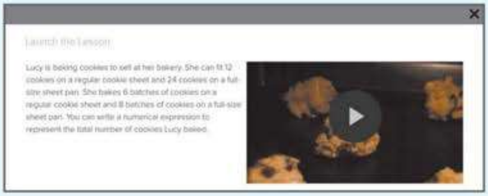


Warm Up

Evaluate.

1. $-1 + (-17)$
2. $21 - 13 + (-4)$
3. $-8 - (-7) - (-2)$
4. $-5 - (-4) + 3$
5. **WEATHER** One winter day, the difference between the warmest and coldest temperatures in the United States was 74°F. If the warmest temperature of 61°F occurred in Arizona, what was the coldest temperature, which occurred in Alaska?

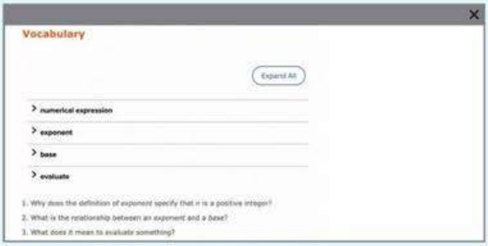
Warm Up



Launch the Lesson

Lucy is baking cookies to sell at her bakery. She can fit 12 cookies on a regular cookie sheet and 24 cookies on a full-size sheet pan. She bakes 6 batches of cookies on a regular cookie sheet and 8 batches of cookies on a full-size sheet pan. You can write a numerical expression to represent the total number of cookies Lucy baked.

Launch



Vocabulary

- numerical expression
- exponent
- base
- evaluate

1. Why does the definition of *exponent* specify that n is a positive integer?
2. What is the relationship between an *exponent* and a *base*?
3. What does it mean to evaluate something?

Today’s Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- adding and subtracting integers

Answers:

1. −18
2. 4
3. 1
4. 2
5. −13°F

Launch the Lesson

Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to analyze the relationships between two different sizes of cookie sheets to write a numerical expression.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today’s Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today’s Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Order of Operations

Objective

Students explore how to use the order of operations to evaluate numerical expressions.

Teaching the Mathematical Practices

4 Apply Mathematics In this Explore, students apply what they know about the order of operations to solving a real-world problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

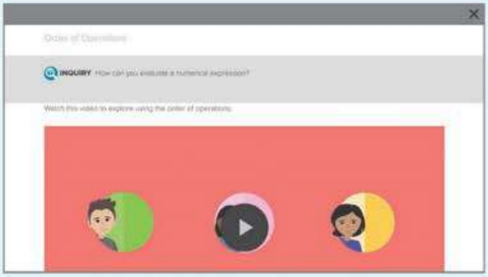
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students watch a video setting up the numerical expression for which the order of operations will be used to calculate. Then students will analyze the incorrect work. The guiding exercises will lead students to determine why the work was incorrect and what is the correct solution path. Then, students will answer the Inquiry Question.

(continued on next page)

Interactive Presentation

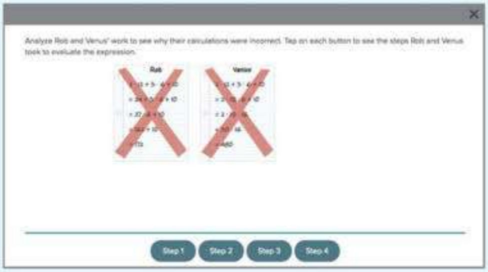


Order of Operations

INQUIRY How can you evaluate a numerical expression?

Watch this video to explore using the order of operations.

Explore



Analyze Rob and Venus' work to see why their calculations were incorrect. Tap on each button to see the steps Rob and Venus took to evaluate the expression.

Explore

TAP

Students click on each step to see the steps each student took to evaluate the given expression.

TYPE

Students answer questions analyzing the presented solutions, and then fill in the correct solution pathway.

Interactive Presentation

INQUIRY How can you evaluate a numerical expression?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Order of Operations (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How could you use a model to help you evaluate the numerical expression? Whose method does this show? Sample answer: The video shows a model with groups of drinks. Add two groups of 12 to get 24, then three groups of 6 to get 18, and one box of 10. $24 + 18 + 10 = 52$. This is similar to Ana's method.
- How would the expression change if Ana brought 2 packages of juice? How many drinks would they have in total? Sample answer: The new expression would be $2 \cdot 12 + 3 \cdot 6 + 2 \cdot 10$. There would be 62 drinks total.

Inquiry

How can you evaluate a numerical expression? Sample answer: Use the order of operations to determine the order in which the operations should be completed.

Go Online to find additional teaching notes and answers for the Explore activity.

Learn Writing Numerical Expressions

Objective

Students write numerical expressions by interpreting words as mathematical symbols.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

Students many times do not use parentheses because the verbal sentence does not mention grouping symbols. Remind students that words like *sum* or *difference* often need parentheses and to always carefully read verbal sentences to identify relationships that may need grouping symbols.

Example 1 Translate a Verbal Expression

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal and numerical expressions in this example.

Questions for Mathematical Discourse

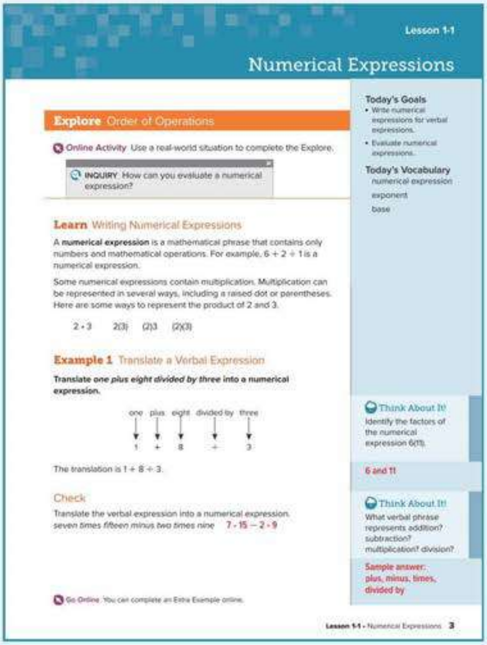
AL Can you tell by the verbal expression what operation is intended to be done first? no

OL How could you adjust the verbal expression to indicate addition should be done first? Sample answer: Add one and eight, and then divide by three.

BL How could you adjust the verbal expression to indicate division should be done first? Sample answer: Add one to the quotient of eight and three.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 1-1

Numerical Expressions

Explore Order of Operations

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you evaluate a numerical expression?

Learn Writing Numerical Expressions

A **numerical expression** is a mathematical phrase that contains only numbers and mathematical operations. For example, $6 + 2 \div 1$ is a numerical expression.

Some numerical expressions contain multiplication. Multiplication can be represented in several ways, including a raised dot or parentheses. Here are some ways to represent the product of 2 and 3.

$2 \cdot 3 \quad 2(3) \quad (2)3 \quad (2)(3)$

Example 1 Translate a Verbal Expression

Translate *one plus eight divided by three* into a numerical expression.

one plus eight divided by three

$1 \quad + \quad 8 \quad \div \quad 3$

The translation is $1 + 8 \div 3$.

Check

Translate the verbal expression into a numerical expression.

seven times fifteen minus two times nine $7 \cdot 15 - 2 \cdot 9$

Go Online You can complete an Extra Example online.

Today's Goals
- Write numerical expressions for verbal expressions.
- Evaluate numerical expressions.

Today's Vocabulary
numerical expression
exponent
base

Think About It! Identify the factors of the numerical expression 6(11).

6 and 11

Think About It! What verbal phrase represents addition? subtraction? multiplication? division?

Sample answer: plus, minus, times, divided by

Lesson 1-1 • Numerical Expressions 3

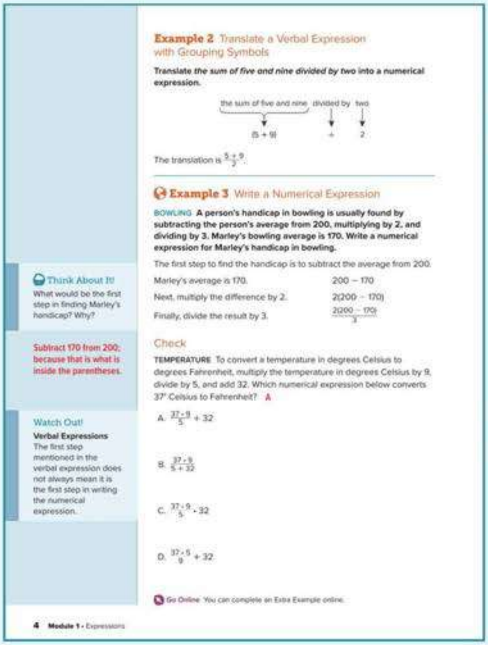
Interactive Presentation

Learn

TYPE

a|

Students identify the factors in a given multiplication expression.



Example 2 Translate a Verbal Expression with Grouping Symbols

Translate *the sum of five and nine divided by two* into a numerical expression.

$\underbrace{\text{the sum of five and nine}}_{(5 + 9)}\ \underset{\div}{\text{divided by}}\ \underset{2}{\text{two}}$

The translation is $\frac{5 + 9}{2}$.

Example 3 Write a Numerical Expression

BOWLING **A person's handicap in bowling is usually found by subtracting the person's average from 200, multiplying by 2, and dividing by 3. Marley's bowling average is 170. Write a numerical expression for Marley's handicap in bowling.**

The first step to find the handicap is to subtract the average from 200.

Marley's average is 170. $200 - 170$

Next, multiply the difference by 2. $2(200 - 170)$

Finally, divide the result by 3. $\frac{2(200 - 170)}{3}$

Think About It!

What would be the first step in finding Marley's handicap? Why?

Subtract 170 from 200, because that is what is inside the parentheses.

Check

TEMPERATURE To convert a temperature in degrees Celsius to degrees Fahrenheit, multiply the temperature in degrees Celsius by 9, divide by 5, and add 32. Which numerical expression below converts 37° Celsius to Fahrenheit? A

A. $\frac{37 \cdot 9}{5} + 32$

B. $\frac{37 \cdot 9}{5 + 32}$

C. $\frac{37 \cdot 9}{5} - 32$

D. $\frac{37 \cdot 5}{9} + 32$

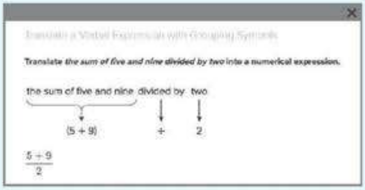
Watch Out!

Verbal Expressions The first step mentioned in the verbal expression does not always mean it is the first step in writing the numerical expression.

Go Online You can complete an Extra Example online.

4 **Module 1** • Expressions

Interactive Presentation



Translate a Verbal Expression with Grouping Symbols

Translate *the sum of five and nine divided by two* into a numerical expression.

$\underbrace{\text{the sum of five and nine}}_{(5 + 9)}\ \underset{\div}{\text{divided by}}\ \underset{2}{\text{two}}$

$\frac{5 + 9}{2}$

Example 2

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Translate a Verbal Expression with Grouping Symbols

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal and numerical expressions in this example.

Questions for Mathematical Discourse

AL What word or phrase represents division? divided by

OL What word or phrase indicates the need for grouping symbols? sum of

BL Could a verbal expression that does not use *divided by* or *quotient of* still result in the same numerical expression? If so, give an example. Yes; sample answer: half the sum of five and nine

Common Error

Students often treat the words *sum* and *difference* like regular addition or subtraction words, even though they require the use of parentheses. Encourage students to underline or highlight sum or difference as a reminder to treat them differently when translating.

Example 3 Write a Numerical Expression

MP Teaching the Mathematical Practices

2 Create Representations Guide students to write a numerical expression that models the situation in this example.

Questions for Mathematical Discourse

AL What three operations are present in the verbal expression? subtraction, multiplication, and division

OL What grouping symbols should be used in the numerical expression? parentheses and a fraction bar

BL Why is the subtraction piece $200 - 170$ instead of $170 - 200$? Sample answer: The average is subtracted from 200, so 200 goes first and the average goes next.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty translating from verbal expressions to numerical expressions,

THEN have them make flash cards with words on one side and the corresponding operation on the other side.

Enrichment Activity BL

Pablo and Martin are both writing a numerical expression for the phrase *three times the sum of four squared and five.* Pablo writes $3(4^2 + 5)$ while Martin writes $3(4^2) + 5$. Who is correct, Pablo or Martin? Explain your reasoning. Pablo; sample answer: Pablo included parentheses around the sum; Martin did not.

Learn Evaluating Numerical Expressions

Objective

Students evaluate numerical expressions by applying the order of operations.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 4 Evaluate Expressions

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What are the bases in each part? 2 and 4 What are the exponents in each part? 4 and 5

OL What does 2^4 indicate? Sample answer: 2 is multiplied by itself 4 times.

BL Write a real-world example where you might use 2^4. Sample answer: There are two cookies on a baking sheet. The baker would like to double the amount of cookies on the sheet. She decides this still isn't enough so she doubles the amount on the sheet again two more times.

Example 5 Order of Operations

Teaching the Mathematical Practices

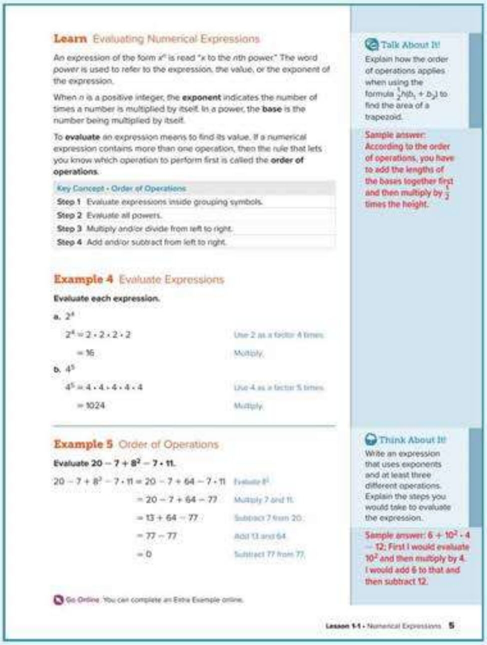
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What step should be performed first when evaluating a numerical expression? grouping symbols

OL Because there are no grouping symbols, which operation should be evaluated first? exponents: $8^2 = 64$

BL How would the value of the expression change if there were parentheses around $8^2 - 7$? Sample answer: Y ou would need to subtract 7 from 64, then multiply this quantity by 11. The addition and subtraction step would become $20 - 7 + 627$, or 640.



Learn Evaluating Numerical Expressions

An expression of the form x^n is read "x to the nth power." The word *power* is used to refer to the expression, the value, or the exponent of the expression.

When *n* is a positive integer, the **exponent** indicates the number of times a number is multiplied by itself. In a power, the **base** is the number being multiplied by itself.

To **evaluate** an expression means to find its value. If a numerical expression contains more than one operation, then the rule that lets you know which operation to perform first is called the **order of operations**.

Key Concept • Order of Operations	
Step 1	Evaluate expressions inside grouping symbols.
Step 2	Evaluate all powers.
Step 3	Multiply and/or divide from left to right.
Step 4	Add and/or subtract from left to right.

Talk About It!

Explain how the order of operations applies when using the formula $\frac{1}{2}h(b_1 + b_2)$ to find the area of a trapezoid.

Sample answer: According to the order of operations, you have to add the lengths of the bases together first and then multiply by $\frac{1}{2}$ times the height.

Example 4 Evaluate Expressions

Evaluate each expression.

a. 2^4

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ Use 2 as a factor 4 times.

$= 16$ Multiply.

b. 4^5

$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ Use 4 as a factor 5 times.

$= 1024$ Multiply.

Example 5 Order of Operations

Evaluate $20 - 7 + 8^2 - 7 \cdot 11$.

$20 - 7 + 8^2 - 7 \cdot 11 = 20 - 7 + 64 - 7 \cdot 11$ Evaluate 8^2.

$= 20 - 7 + 64 - 77$ Multiply 7 and 11.

$= 13 + 64 - 77$ Subtract 7 from 20.

$= 77 - 77$ Add 13 and 64.

$= 0$ Subtract 77 from 77.

Think About It!

Write an expression that uses exponents and at least three different operations. Explain the steps you would take to evaluate the expression.

Sample answer: $6 + 10^2 \cdot 4 - 12$; First I would evaluate 10^2 and then multiply by 4. I would add 6 to that and then subtract 12.

Go Online You can complete an Extra Example online.

Lesson 1-1 • Numerical Expressions 5

Interactive Presentation

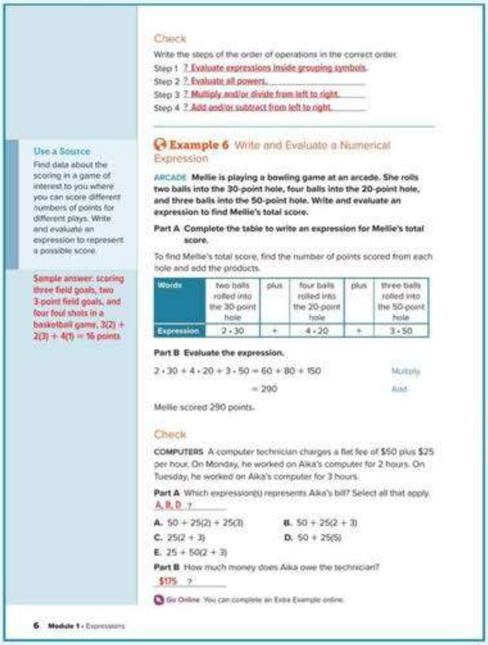
Learn

TAP

Students move through the steps in the order of operations.

TYPE

Students explain how the order of operations applies to finding the area of a trapezoid.



Check

Write the steps of the order of operations in the correct order.

Step 1 ? Evaluate expressions inside grouping symbols.
Step 2 ? Evaluate all powers.
Step 3 ? Multiply and/or divide from left to right.
Step 4 ? Add and/or subtract from left to right.

Use a Source

Find data about the scoring in a game of interest to you where you can score different numbers of points for different plays. Write and evaluate an expression to represent a possible score.

Sample answer: scoring three field goals, two 3-point field goals, and four foul shots in a basketball game. 3(2) + 2(3) + 4(1) = 16 points

Example 6 Write and Evaluate a Numerical Expression

ARCADE **Mellie is playing a bowling game at an arcade. She rolls two balls into the 30-point hole, four balls into the 20-point hole, and three balls into the 50-point hole. Write and evaluate an expression to find Mellie's total score.**

Part A Complete the table to write an expression for Mellie's total score.

To find Mellie's total score, find the number of points scored from each hole and add the products.

Words	two balls rolled into the 30-point hole	plus	four balls rolled into the 20-point hole	plus	three balls rolled into the 50-point hole
Expression	$2 \cdot 30$	+	$4 \cdot 20$	+	$3 \cdot 50$

Part B Evaluate the expression.

$$\begin{aligned} 2 \cdot 30 + 4 \cdot 20 + 3 \cdot 50 &= 60 + 80 + 150 && \text{Multiply.} \\ &= 290 && \text{Add.} \end{aligned}$$

Mellie scored 290 points.

Check

COMPUTERS A computer technician charges a flat fee of \$50 plus \$25 per hour. On Monday, he worked on Aika's computer for 2 hours. On Tuesday, he worked on Aika's computer for 3 hours.

Part A Which expression(s) represents Aika's bill? Select all that apply.
A, B, D ?

A. $50 + 25(2) + 25(3)$
B. $50 + 25(2 + 3)$
C. $25(2 + 3)$
D. $50 + 25(5)$
E. $25 + 50(2 + 3)$

Part B How much money does Aika owe the technician?
\$175 ?

Go Online You can complete an Extra Example online.

6 **Module 1** • Expressions

Interactive Presentation

Example 6

TYPE

Students enter the correct numbers to write a numerical expression.

Example 6 Write and Evaluate a Numerical Expression

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL What mathematical operation will be used when combining the points earned from the three different point levels? addition

OL The problem states the point value for each ball Mellie rolled. What mathematical operation represents the point values when translating to a numerical operation? multiplication

BL How would the expression change if Mellie also rolled one ball into the 100-point hole? The expression would be $2(30) + 4(20) + 3(50) + 1(100)$.

Common Error

Students may not know what understood math operations are attached to the words *triple* and *double*, and may try to add. Remind students that these words represent multiplication.

Essential Question Follow-Up

Students have begun applying the order of operations when evaluating numerical expressions.

Ask:

Why is order important when evaluating numerical expressions?
Sample answer: Order matters because otherwise multiple answers could be found for one problem, when only one answer is actually correct.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

IF students struggle remembering the order of operations,
THEN have students write PEMDAS in their foldable with Parentheses, Exponent, Multiplication, Division, Addition, and Subtraction written by each letter. Have students draw an arrow above MD and AS to remind them to work from left to right.

Enrichment Activity **BL**

Using the numbers 2, 3, 4, and 8 only once and any operation or grouping symbols, write a numerical expression evaluating to 8.
Sample answer: $(4 + 8) \div 3 \times 2$ or $8 \div 4 \times 3 + 2$

Example 7 Expressions with Grouping Symbols

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL In part **a**, which operation should be evaluated first? Explain your reasoning. Either 4 + 5 **or** 7 — 4 could be evaluated first because they are both in parentheses.

OL In part **b**, which operation should be evaluated first? Explain your reasoning. 3 — 2 should be evaluated first because this is in the innermost grouping symbols.

BL Would the value of the expression be the same if the expression in part **b** was $15 - 10 + (3 - 2)^2 + 6$? Explain. No; sample answer: This expression would subtract only 10 from 15, not the entire expression after the subtraction sign.

Common Error

Expressions with multiple grouping symbols can confuse students. Encourage students to start at the innermost part of the grouping symbol and work their way out.

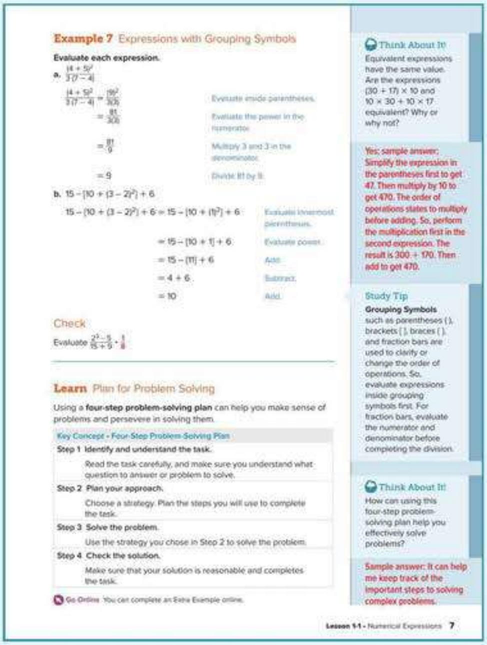
Learn Plan for Problem Solving

Objective

Students use the four-step problem solving plan to solve problems.

Teaching the Mathematical Practices

1 Analyze Givens and Constraints In this Learn, guide students through the use of the four-step plan in order to be able to identify the meaning of problems and look for entry points to their solutions.



Example 7 Expressions with Grouping Symbols

Evaluate each expression.

a. $\frac{(4+5)^2}{3(7-4)}$

$\frac{(4+5)^2}{3(7-4)} = \frac{(9)^2}{3(3)}$ Evaluate inside parentheses.

$= \frac{81}{3(3)}$ Evaluate the power in the numerator.

$= \frac{81}{9}$ Multiply 3 and 3 in the denominator.

$= 9$ Divide 81 by 9.

b. $15 - [10 + (3 - 2)^2] + 6$

$15 - [10 + (3 - 2)^2] + 6 = 15 - [10 + (1)^2] + 6$ Evaluate innermost parentheses.

$= 15 - [10 + 1] + 6$ Evaluate power.

$= 15 - [11] + 6$ Add.

$= 4 + 6$ Subtract.

$= 10$ Add.

Check

Evaluate $\frac{2^3 - 5}{15 + 9} \cdot$ [illegible]

Learn Plan for Problem Solving

Using a **four-step problem-solving plan** can help you make sense of problems and persevere in solving them.

Key Concept • Four-Step Problem-Solving Plan

Step 1 Identify and understand the task.
Read the task carefully, and make sure you understand what question to answer or problem to solve.

Step 2 Plan your approach.
Choose a strategy. Plan the steps you will use to complete the task.

Step 3 Solve the problem.
Use the strategy you chose in Step 2 to solve the problem.

Step 4 Check the solution.
Make sure that your solution is reasonable and completes the task.

Go Online You can complete an Extra Example online.

Think About It!
Equivalent expressions have the same value. Are the expressions $(30 + 17) \times 10$ and $10 \times 30 + 10 \times 17$ equivalent? Why or why not?

Yes; sample answer: Simplify the expression in the parentheses first to get 47. Then multiply by 10 to get 470. The order of operations states to multiply before adding. So, perform the multiplication first in the second expression. The result is 300 + 170. Then add to get 470.

Study Tip
Grouping Symbols such as parentheses ( ), brackets [ ], braces { }, and fraction bars are used to clarify or change the order of operations. So, evaluate expressions inside grouping symbols first. For fraction bars, evaluate the numerator and denominator before completing the division.

Think About It!
How can using this four-step problem-solving plan help you effectively solve problems?

Sample answer: It can help me keep track of the important steps to solving complex problems.

Lesson 1-1 • Numerical Expressions 7

Interactive Presentation

Learn

TAP

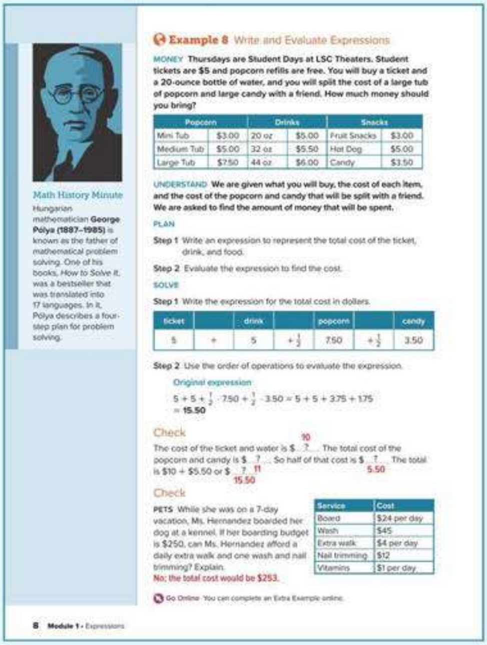
Students move through an explanation for each step of the four-step problem-solving plan.

TYPE

Students explain how the four-step plan helps effectively solve problems.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 8 Write and Evaluate Expressions

MONEY Thursdays are Student Days at LSC Theaters. Student tickets are $5 and popcorn refills are free. You will buy a ticket and a 20-ounce bottle of water, and you will split the cost of a large tub of popcorn and large candy with a friend. How much money should you bring?

Popcorn		Drinks		Snacks	
Mini Tub	$3.00	20 oz	$5.00	Fruit Snacks	$3.00
Medium Tub	$5.00	32 oz	$5.50	Hot Dog	$5.00
Large Tub	$7.50	44 oz	$6.00	Candy	$3.50

UNDERSTAND **We are given what you will buy, the cost of each item, and the cost of the popcorn and candy that will be split with a friend. We are asked to find the amount of money that will be spent.**

PLAN

Step 1 Write an expression to represent the total cost of the ticket, drink, and food.

Step 2 Evaluate the expression to find the cost.

SOLVE

Step 1 Write the expression for the total cost in dollars.

ticket		drink		popcorn		candy
5	+	5	$+\frac{1}{2}$	7.50	$+\frac{1}{2}$	3.50

Step 2 Use the order of operations to evaluate the expression.

Original expression

$$5 + 5 + \frac{1}{2} \cdot 7.50 + \frac{1}{2} \cdot 3.50 = 5 + 5 + 3.75 + 1.75$$
$$= \mathbf{15.50}$$

Check

The cost of the ticket and water is $ $\overset{10}{?}$. The total cost of the popcorn and candy is $ $\overset{11}{?}$. So half of that cost is $ $\underset{5.50}{?}$. The total is $10 + $5.50 or $ $\underset{15.50}{?}$.

Check

PETS While she was on a 7-day vacation, Ms. Hernandez boarded her dog at a kennel. If her boarding budget is $250, can Ms. Hernandez afford a daily extra walk and one wash and nail trimming? Explain.
No; the total cost would be $253.

Service	Cost
Board	$24 per day
Wash	$45
Extra walk	$4 per day
Nail trimming	$12
Vitamins	$1 per day

Go Online You can complete an Extra Example online.

Math History Minute
Hungarian mathematician **George Pólya (1887–1985)** is known as the father of mathematical problem solving. One of his books, *How to Solve It*, was a bestseller that was translated into 17 languages. In it, Pólya describes a four-step plan for problem solving.

8 Module 1 • Expressions

Interactive Presentation

Apply Example 8

TYPE

Students enter the correct values to create a numerical expression.

CHECK

Students complete the Check online to determine whether they are ready to move on.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Example 8 Write and Evaluate Expressions

Teaching the Mathematical Practices

1 Analyze Givens and Constraints Guide students through the use of the four-step plan to identify the meaning of Example 8 and look for entry points to its solution.

Questions for Mathematical Discourse

AL How much does a student ticket cost? $5 How much does a 20-ounce water cost? $5

OL What are the items you are splitting with your friends? a large tub of popcorn and a large candy How do you find your cost based on the total cost? Multiply by $\frac{1}{2}$.

BL Suppose you and your friend also decide to also split a pack of fruit snacks. What effect will this have on your individual cost? Sample answer: The total cost will increase by $1.50 because this is half of $3.

Common Error

Problems that give extra information can lead students to include irrelevant numbers. Encourage students to highlight or circle values that apply to the given scenario and cross off values or information that is not important.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–52
2	exercises that use a variety of skills from this lesson	53–58
3	exercises that emphasize higher-order and critical-thinking skills	59–62

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–57 odd, 59–62
- Extension: The Four Digits Problem
- **ALEKS** Operations with Signed Numbers

OL

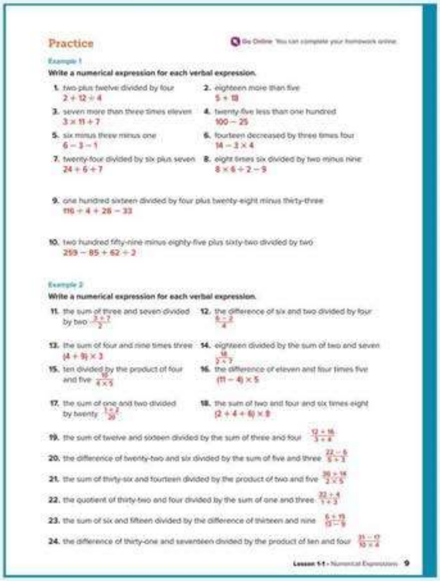
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–61 odd
- Remediation, Review Resources: Subtract Integers
- Personal Tutors
- Extra Examples 1–8
- **ALEKS** Addition and Subtraction with Integers

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–51 odd
- Remediation, Review Resources: Subtract Integers
- *Quick Review Math Handbook:* Variables and Expressions
- Arrive**MATH** Take Another Look
- **ALEKS** Addition and Subtraction with Integers



Practice

Go Online You can complete your homework online.

Example 1

Write a numerical expression for each verbal expression.

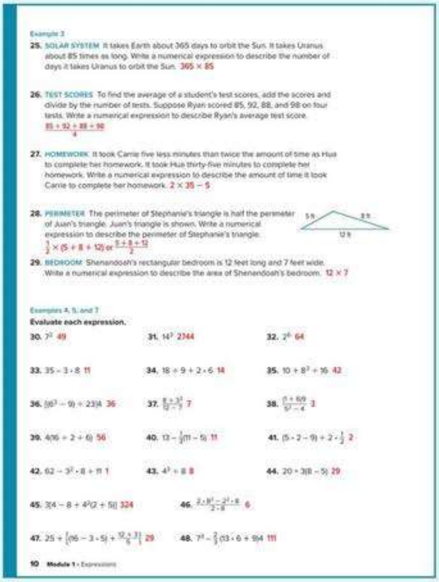
1. two plus twelve divided by four $2 + 12 \div 4$
2. eighteen more than five $5 + 18$
3. seven more than three times eleven $3 \times 11 + 7$
4. twenty-five less than one hundred $100 - 25$
5. six minus three minus one $6 - 3 - 1$
6. fourteen decreased by three times four $14 - 3 \times 4$
7. twenty-four divided by six plus seven $24 \div 6 + 7$
8. eight times six divided by two minus nine $8 \times 6 \div 2 - 9$
9. one hundred sixteen divided by four plus twenty-eight minus thirty-three $116 \div 4 + 28 - 33$
10. two hundred fifty-nine minus eighty-five plus sixty-two divided by two $259 - 85 + 62 \div 2$

Example 2

Write a numerical expression for each verbal expression.

11. the sum of three and seven divided by two $\frac{3+7}{2}$
12. the difference of six and two divided by four $\frac{6-2}{4}$
13. the sum of four and nine times three $(4 + 9) \times 3$
14. eighteen divided by the sum of two and seven $\frac{18}{2+7}$
15. ten divided by the product of four and five $\frac{10}{4 \times 5}$
16. the difference of eleven and four times five $(11 - 4) \times 5$
17. the sum of one and two divided by twenty $\frac{1+2}{20}$
18. the sum of two and four and six times eight $(2 + 4 + 6) \times 8$
19. the sum of twelve and sixteen divided by the sum of three and four $\frac{12+16}{3+4}$
20. the difference of twenty-two and six divided by the sum of five and three $\frac{22-6}{5+3}$
21. the sum of thirty-six and fourteen divided by the product of two and five $\frac{36+14}{2 \times 5}$
22. the quotient of thirty-two and four divided by the sum of one and three $\frac{32 \div 4}{1+3}$
23. the sum of six and fifteen divided by the difference of thirteen and nine $\frac{6+15}{13-9}$
24. the difference of thirty-one and seventeen divided by the product of ten and four $\frac{31-17}{10 \times 4}$

Lesson 1-1 • Numerical Expressions 9



Example 3

25. **SOLAR SYSTEM** It takes Earth about 365 days to orbit the Sun. It takes Uranus about 85 times as long. Write a numerical expression to describe the number of days it takes Uranus to orbit the Sun. 365×85
26. **TEST SCORES** To find the average of a student's test scores, add the scores and divide by the number of tests. Suppose Ryan scored 85, 92, 88, and 98 on four tests. Write a numerical expression to describe Ryan's average test score. $\frac{85 + 92 + 88 + 98}{4}$
27. **HOMEWORK** It took Carrie five less minutes than twice the amount of time as Hua to complete her homework. It took Hua thirty-five minutes to complete her homework. Write a numerical expression to describe the amount of time it took Carrie to complete her homework. $2 \times 35 - 5$
28. **PERIMETER** The perimeter of Stephanie's triangle is half the perimeter of Juan's triangle. Juan's triangle is shown. Write a numerical expression to describe the perimeter of Stephanie's triangle. $\frac{1}{2} \times (5 + 8 + 12)$ or $\frac{5+8+12}{2}$

29. **BEDROOM** Shenandoah's rectangular bedroom is 12 feet long and 7 feet wide. Write a numerical expression to describe the area of Shenandoah's bedroom. 12×7

Examples 4, 5, and 7

Evaluate each expression.

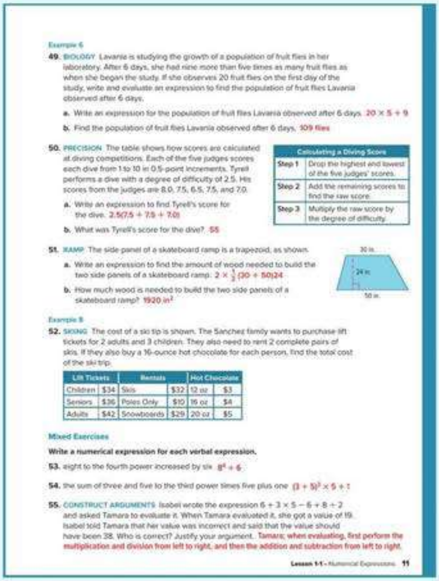
30. 7^2 49
31. 14^3 2744
32. 2^6 64
33. $35 - 3 \cdot 8$ 11
34. $18 \div 9 + 2 \cdot 6$ 14
35. $10 + 8^3 \div 16$ 42
36. $[(6^3 - 9) \div 23]4$ 36
37. [illegible] 7
38. [illegible] 3
39. $4(16 \div 2 + 6)$ 56
40. $13 - \frac{1}{3}(11 - 5)$ 11
41. $(5 \cdot 2 - 9) + 2 \cdot \frac{1}{2}$ 2
42. $62 - 3^2 \cdot 8 + 11$ 1
43. $4^3 \div 8$ 8
44. $20 + 3(8 - 5)$ 29
45. $3[4 - 8 + 4^2(2 + 5)]$ 324
46. [illegible] 6
47. $25 + \left[(16 - 3 \cdot 5) + \frac{12+3}{5}\right]$ 29
48. $7^3 - \frac{2}{3}(13 \cdot 6 + 9)4$ 111

10 Module 1 • Expressions

3 REFLECT AND PRACTICE

A.SSE.1b, A.SSE.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 6

49. BIOLOGY Lavania is studying the growth of a population of fruit flies in her laboratory. After 6 days, she had nine more than five times as many fruit flies as when she began the study. If she observes 20 fruit flies on the first day of the study, write and evaluate an expression to find the population of fruit flies Lavania observed after 6 days.

a. Write an expression for the population of fruit flies Lavania observed after 6 days. $20 \times 5 + 9$

b. Find the population of fruit flies Lavania observed after 6 days. 109 flies

50. PRECISION The table shows how scores are calculated at diving competitions. Each of the five judges scores each dive from 1 to 10 in 0.5-point increments. Tyrell performs a dive with a degree of difficulty of 2.5. His scores from the judges are 8.0, 7.5, 6.5, 7.5, and 7.0.

Calculating a Diving Score	
Step 1	Drop the highest and lowest of the five judges' scores.
Step 2	Add the remaining scores to find the raw score.
Step 3	Multiply the raw score by the degree of difficulty.

a. Write an expression to find Tyrell's score for the dive. $2.5(7.5 + 7.5 + 7.0)$

b. What was Tyrell's score for the dive? 55

51. RAMP The side panel of a skateboard ramp is a trapezoid, as shown.

a. Write an expression to find the amount of wood needed to build the two side panels of a skateboard ramp. $2 \times \frac{1}{2}(30 + 50)24$

b. How much wood is needed to build the two side panels of a skateboard ramp? 1920 in^2

Example 8

52. SKIING The cost of a ski trip is shown. The Sanchez family wants to purchase lift tickets for 2 adults and 3 children. They also need to rent 2 complete pairs of skis. If they also buy a 16-ounce hot chocolate for each person, find the total cost of the ski trip.

Lift Tickets		Rentals		Hot Chocolate	
Children	\$34	Skis	\$32	12 oz	\$3
Seniors	\$36	Poles Only	\$10	16 oz	\$4
Adults	\$42	Snowboards	\$29	20 oz	\$5

Mixed Exercises

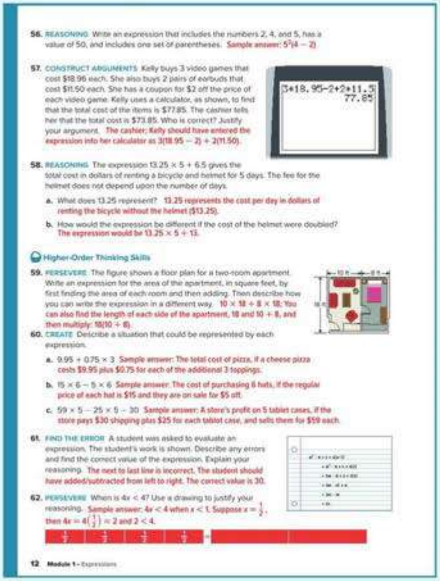
Write a numerical expression for each verbal expression.

53. eight to the fourth power increased by six $8^4 + 6$

54. the sum of three and five to the third power times five plus one $(3 + 5)^3 \times 5 + 1$

55. CONSTRUCT ARGUMENTS Isabel wrote the expression $6 + 3 \times 5 - 6 + 8 \div 2$ and asked Tamara to evaluate it. When Tamara evaluated it, she got a value of 19. Isabel told Tamara that her value was incorrect and said that the value should have been 38. Who is correct? Justify your argument. Tamara; when evaluating, first perform the multiplication and division from left to right, and then the addition and subtraction from left to right.

Lesson 1-1 • Numerical Expressions 11



56. REASONING Write an expression that includes the numbers 2, 4, and 5, has a value of 50, and includes one set of parentheses. Sample answer: $5^2(4 - 2)$

57. CONSTRUCT ARGUMENTS Kelly buys 3 video games that cost \$18.95 each. She also buys 2 pairs of earbuds that cost \$11.50 each. She has a coupon for \$2 off the price of each video game. Kelly uses a calculator, as shown, to find that the total cost of the items is \$77.85. The cashier tells her that the total cost is \$73.85. Who is correct? Justify your argument. The cashier; Kelly should have entered the expression into her calculator as $3(18.95 - 2) + 2(11.50)$.

```
3*18.95-2+2*11.5
                77.85
```

58. REASONING The expression $13.25 \times 5 + 6.5$ gives the total cost in dollars of renting a bicycle and helmet for 5 days. The fee for the helmet does not depend upon the number of days.

a. What does 13.25 represent? 13.25 represents the cost per day in dollars of renting the bicycle without the helmet (\$13.25).

b. How would the expression be different if the cost of the helmet were doubled? The expression would be $13.25 \times 5 + 13$.

Higher-Order Thinking Skills

59. PERSEVERE The figure shows a floor plan for a two-room apartment. Write an expression for the area of the apartment, in square feet, by first finding the area of each room and then adding. Then describe how you can write the expression in a different way. $10 \times 18 + 8 \times 18$; You can also find the length of each side of the apartment, 18 and 10 + 8, and then multiply: $18(10 + 8)$.

60. CREATE Describe a situation that could be represented by each expression.

a. $9.95 + 0.75 \times 3$ Sample answer: The total cost of pizza, if a cheese pizza costs \$9.95 plus \$0.75 for each of the additional 3 toppings.

b. $15 \times 6 - 5 \times 6$ Sample answer: The cost of purchasing 6 hats, if the regular price of each hat is \$15 and they are on sale for \$5 off.

c. $59 \times 5 - 25 \times 5 - 30$ Sample answer: A store's profit on 5 tablet cases, if the store pays \$30 shipping plus \$25 for each tablet case, and sells them for \$59 each.

61. FIND THE ERROR A student was asked to evaluate an expression. The student's work is shown. Describe any errors and find the correct value of the expression. Explain your reasoning. The next to last line is incorrect. The student should have added/subtracted from left to right. The correct value is 30.

62. PERSEVERE When is $4x < 4$? Use a drawing to justify your reasoning. Sample answer: $4x < 4$ when $x < 1$. Suppose $x = \frac{1}{2}$, then $4x = 4\left(\frac{1}{2}\right) = 2$ and $2 < 4$.

12 **Module 1** • Expressions

Lesson 1-2

Algebraic Expressions

A.SSE.1, A.SSE.2

LESSON GOAL

Students will write and evaluate algebraic expressions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Writing Algebraic Expressions

- Write a Verbal Expression
- Write a Verbal Expression with Grouping Symbols
- Write an Algebraic Expression
- Write an Expression

Explore: Using Algebraic Expressions in the Real World

Develop:

Evaluating Algebraic Expressions

- Evaluate an Algebraic Expression
- Write and Evaluate an Algebraic Expression

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Divide Integers	●	●		●
Extension: Toothpick Triangles		●	●	●

Language Development Handbook

Assign page 2 of the *Language Development Handbook* to help your students build mathematical language related to algebraic expressions.

ELL You can use the tips and suggestions on page T2 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students wrote and evaluated numerical expressions.
6.EE.1, A.SSE.1b, A.SSE.2

Now
Students write and evaluate algebraic expressions.
A.SSE.1, A.SSE.2

Next
Students will apply the properties of real numbers to simplify expressions.
A.SSE.2

Rigor

The Three Pillars of Rigor

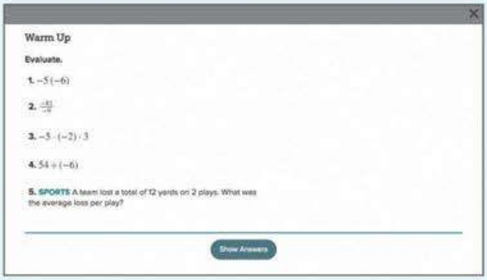
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop an understanding of algebraic expressions and use it to build fluency with evaluating algebraic expressions. They apply their understanding of algebraic expressions by solving real-world problems.

Mathematical Background

Mathematical expressions that contain at least one variable are called algebraic expressions. They can be written as mathematical expressions or verbal expressions, but do not contain an equal sign. A variable is a symbol used to represent an unspecified number or value. Algebraic expressions may contain powers. When evaluating a power, the exponent tells how many times the base is used as a factor.

Interactive Presentation

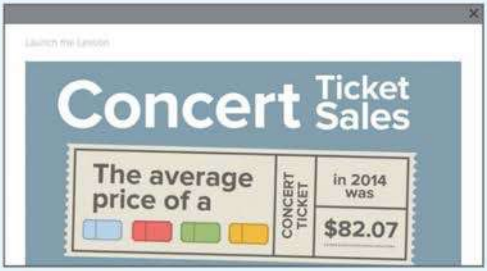


Warm Up

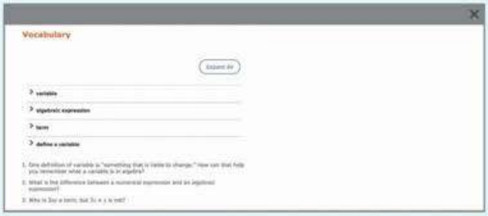
Evaluate.

1. $-5(-6)$
2. $\frac{-81}{-9}$
3. $-5 \cdot (-2) \cdot 3$
4. $54 \div (-6)$
5. **SPORTS** A team lost a total of 12 yards on 2 plays. What was the average loss per play?

Warm Up



Launch



Vocabulary

- variable
- algebraic expression
- term
- define a variable

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- multiplying and dividing integers

Answers:

1. 30
2. 9
3. 30
4. −9
5. 6 yd

Launch the Lesson

Teaching the Mathematical Practices

4 Model with Mathematics Students will analyze the information about concert ticket sales to write an algebraic expression.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Using Algebraic Expressions in the Real World

Objective

Students explore how to use substitution and the order of operations to evaluate algebraic expressions.

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation that models the situation in this Explore. Then use the equation to solve the problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

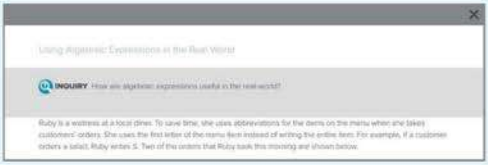
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students will use cost information from a diner to write different orders using variables. The guiding exercises will lead students to write algebraic expressions. Then, students will answer the Inquiry Question.

(continued on next page)

Interactive Presentation

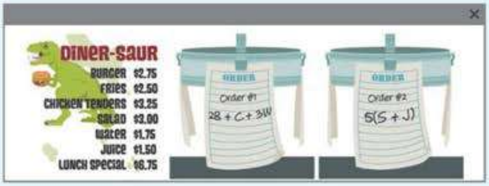


Using Algebraic Expressions in the Real World

INQUIRY How are algebraic expressions useful in the real world?

Ruby is a waitress at a local diner. To save time, she uses abbreviations for the items on the menu when she takes customers' orders. She uses the first letter of the menu item instead of writing the entire item. For example, if a customer orders a salad, Ruby writes S. Two of the orders that Ruby took this morning are shown below.

Explore



Explore

TYPE

Students answer questions analyzing different cost scenarios.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Algebraic Expressions in the Real World (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What did the first table order? How do you know? Two burgers, chicken tenders, and three waters; *B* stands for burger, *C* stands for chicken tenders, and *W* stands for water.
- A third table ordered two lunch specials, three fries, and five juices. What would Ruby write for their order? Describe how you would calculate the cost of the order using multiplication and addition. $2L + 3F + 5J$. Sample answer: Multiply \$6.75 by 2, \$2.50 by 3, and \$1.50 by 5. Then add the three results together.

Inquiry

How are algebraic expressions useful in the real world? Sample answer: Algebraic expressions can be written and evaluated to represent situations in the real world.

Go Online to find additional teaching notes and answers for the Explore activity.

Learn Writing Algebraic Expressions

Objective

Students write algebraic expressions by interpreting words as mathematical symbols.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Essential Question Follow-Up

Students have begun writing verbal and algebraic expressions.

Ask:

Why is it important to use a variable in a real-world situation? Sample answer: It is important to use a variable because many times we do not know a quantity in a real-world situation, so we can use a variable to represent this unknown. Then we can still talk about the situation even with the missing piece.

Example 1 Write a Verbal Expression

MP Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in this example.

Questions for Mathematical Discourse

AL What operation is between the 5 and the x_3? multiplication

OL What word or phrase represents the exponent? cubed or the third power

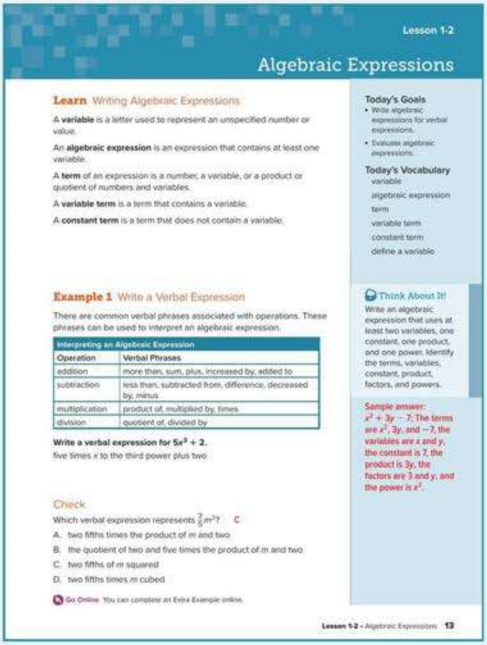
BL Why is the term 2 called the *constant*? Sample answer: The number 2 is not multiplied by a variable, so it will remain 2 no matter what value is substituted into the expression.

Common Error

Students often translate incorrectly when the variable term has a coefficient. Since there is no obvious operation present between the coefficient and variable, they make one up. Remind students that when a variable has a coefficient, there is an understood multiplication sign between them.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 1-2

Algebraic Expressions

Learn Writing Algebraic Expressions

A **variable** is a letter used to represent an unspecified number or value.

An **algebraic expression** is an expression that contains at least one variable.

A **term** of an expression is a number, a variable, or a product or quotient of numbers and variables.

A **variable term** is a term that contains a variable.

A **constant term** is a term that does not contain a variable.

Example 1 Write a Verbal Expression

There are common verbal phrases associated with operations. These phrases can be used to interpret an algebraic expression.

Interpreting an Algebraic Expression

Operation	Verbal Phrases
addition	more than, sum, plus, increased by, added to
subtraction	less than, subtracted from, difference, decreased by, minus
multiplication	product of, multiplied by, times
division	quotient of, divided by

Write a verbal expression for $5x^3 + 2$.

five times x to the third power plus two

Check

Which verbal expression represents $\frac{2}{5}m^2$? C

A. two fifths times the product of m and two

B. the quotient of two and five times the product of m and two

C. two fifths of m squared

D. two fifths times m cubed

Go Online You can complete an Extra Example online.

Today's Goals

- Write algebraic expressions for verbal expressions.
- Evaluate algebraic expressions.

Today's Vocabulary

variable
algebraic expression
term
variable term
constant term
define a variable

Think About It!

Write an algebraic expression that uses at least two variables, one constant, one product, and one power. Identify the terms, variables, constant, product, factors, and powers.

Sample answer: $x^2 + 3y - 7$; The terms are x^2, $3y$, and -7, the variables are x and y, the constant is 7, the product is $3y$, the factors are 3 and y, and the power is x^2.

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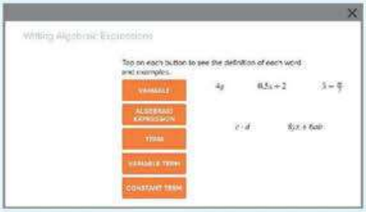
Interactive Presentation



Tap on each button to see the definition of each word and examples.

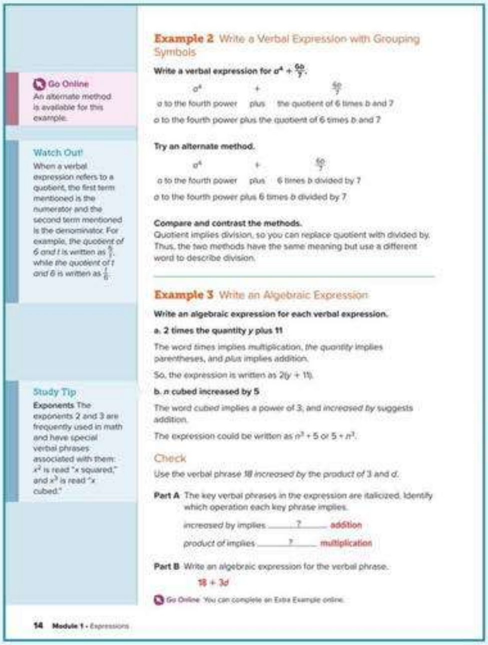
Learn

TAP

Students tap on the words or phrases to see a definition and examples.

TYPE

Students create their own algebraic expression.



Go Online An alternate method is available for this example.

Watch Out! When a verbal expression refers to a quotient, the first term mentioned is the numerator and the second term mentioned is the denominator. For example, *the quotient of 6 and t* is written as $\frac{6}{t}$, while *the quotient of t and 6* is written as $\frac{t}{6}$.

Study Tip **Exponents** The exponents 2 and 3 are frequently used in math and have special verbal phrases associated with them: x^2 is read "*x* squared," and x^3 is read "*x* cubed."

Example 2 Write a Verbal Expression with Grouping Symbols

Write a verbal expression for $a^4 + \frac{6b}{7}$.

a^4	+	$\frac{6b}{7}$
a to the fourth power	plus	the quotient of 6 times *b* and 7

a to the fourth power plus the quotient of 6 times *b* and 7

Try an alternate method.

a^4	+	$\frac{6b}{7}$
a to the fourth power	plus	6 times *b* divided by 7

a to the fourth power plus 6 times *b* divided by 7

Compare and contrast the methods.
Quotient implies division, so you can replace quotient with divided by. Thus, the two methods have the same meaning but use a different word to describe division.

Example 3 Write an Algebraic Expression

Write an algebraic expression for each verbal expression.

a. 2 times the quantity *y* plus 11

The word *times* implies multiplication, *the quantity* implies parentheses, and *plus* implies addition.

So, the expression is written as $2(y + 11)$.

b. *n* cubed increased by 5

The word *cubed* implies a power of 3, and *increased by* suggests addition.

The expression could be written as $n^3 + 5$ or $5 + n^3$.

Check

Use the verbal phrase *18 increased by the product of 3 and d*.

Part A The key verbal phrases in the expression are italicized. Identify which operation each key phrase implies.

increased by implies ___?___ addition

product of implies ___?___ multiplication

Part B Write an algebraic expression for the verbal phrase.

$18 + 3d$

Go Online You can complete an Extra Example online.

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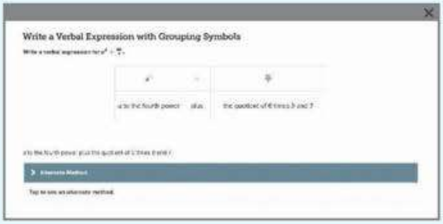
Interactive Presentation



Write a Verbal Expression with Grouping Symbols

Example 2

Tap

Students tap to see an alternate method of writing a verbal expression with grouping symbols.

Type

Students compare and contrast methods of equivalent verbal expressions with grouping symbols.

Example 2 Write a Verbal Expression with Grouping Symbols

Teaching the Mathematical Practices

1 Understand the Approaches of Others Work with students to look at the Alternate Method. Ask students to compare and contrast the original method and the alternate method.

Questions for Mathematical Discourse

AL What words represent addition? Sample answers: plus, added to, increased by, more than

OL What word or phrase represents the exponent? fourth power

BL If the expression were instead $(a^4 + 6b) \div 7$, how would the verbal expression change? Sample answer: The division is now applied to a quantity, so the expression would be the quantity *a* raised to the fourth power plus 6 times *b* all divided by 7.

Example 3 Write an Algebraic Expression

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How do you know the word *quantity* implies that parentheses are used? Sample answer: A quantity usually means more than one thing or a group of things.

OL In part **a**, how would the algebraic expression change if the verbal expression were *2 times the quotient of y and 11*? Sample answer: If *the quantity* is changed to *the quotient of,* then you would need to divide *y* and 11. So the expression would be $2\left(\frac{y}{11}\right)$ or $2 \times \frac{y}{11}$.

BL Write the verbal expression in a different way for the same algebraic expression $2(y + 11)$. Sample answer: The sum of *y* and 11 multiplied by 2.

Common Error

When there are multiple possible solutions to a verbal expression, students may have trouble writing an answer. They get bogged down in which translation is correct, not realizing there are multiple possibilities. Reinforce to students that there are many ways to write verbal expressions.

Example 4 Write an Expression

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about writing algebraic expressions to solving a real-world problem.

Questions for Mathematical Discourse

AL What variables are used to represent a win, tie, and loss? *w, t, z*

OL Why is the expression for the number of points written $3w + t$ instead of $3w + 1t$? Sample answer: The 1 is understood and does not need to be written because anything multiplied by 1 is itself.

BL Why are the losses, *z*, not represented in the expression? Sample answer: $0z$ is simplified to 0 so there is no need to include it in the expression because 0 does not change the expression.

Common Error

When translating a real-world situation, students must define variables before beginning. Without defining variables, the expression is meaningless and students are more likely to make mistakes if they have not decided what the variables represent.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty writing a verbal or algebraic expression, **THEN** pair them with other students as mentors for practicing these skills. The transition from verbal expressions to algebraic expressions is easier for some students than others.

Enrichment Activity BL

Write a verbal expression for $\frac{(x+5)^2}{y-5}$. Sample answer: The square of the sum of *x* and 5 divided by the difference between *y* and 5.

DIFFERENTIATE

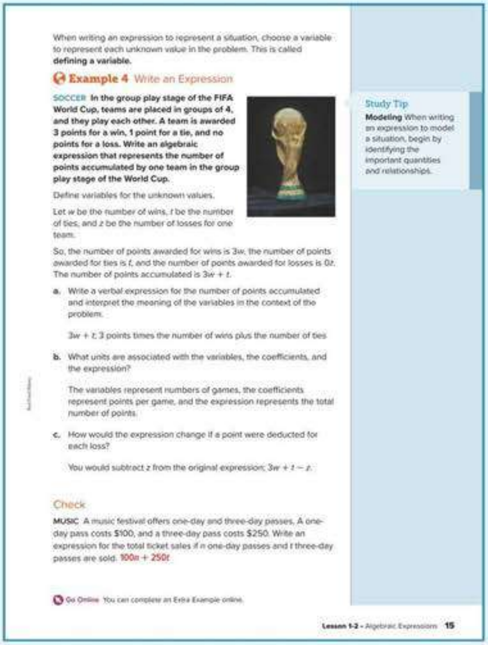
Language Development Activity ELL

Explore the similarities with *quotient* and *difference* compared to *product* and *sum*. Review instances when the order of operations in a written expression must be the same as the order in the numerical or algebraic expression. Also review instances where that order does not have to be maintained, and when the order may be the opposite.

a) The quotient of 8 and 4 means $8 \div 4$; $\frac{8}{4}$ does not mean $4 \div 8$ or $\frac{4}{8}$. Write a sentence or phrase with the same meaning using a phrase like *goes into*. How many times does 4 go into 8?

b) Subtract 8 from 12 means $12 - 8$. How would you say the same thing using *difference*? The difference of 12 and 8 means $12 - 8$.

Expand the discussion to include synonyms for these operations, and how the overall order of words does not always translate to the same order of operations.



When writing an expression to represent a situation, choose a variable to represent each unknown value in the problem. This is called **defining a variable.**

Example 4 Write an Expression

SOCCER **In the group play stage of the FIFA World Cup, teams are placed in groups of 4, and they play each other. A team is awarded 3 points for a win, 1 point for a tie, and no points for a loss. Write an algebraic expression that represents the number of points accumulated by one team in the group play stage of the World Cup.**

Define variables for the unknown values.

Let *w* be the number of wins, *t* be the number of ties, and *z* be the number of losses for one team.

So, the number of points awarded for wins is $3w$, the number of points awarded for ties is *t*, and the number of points awarded for losses is $0z$. The number of points accumulated is $3w + t$.

a. Write a verbal expression for the number of points accumulated and interpret the meaning of the variables in the context of the problem.

$3w + t$; 3 points times the number of wins plus the number of ties

b. What units are associated with the variables, the coefficients, and the expression?

The variables represent numbers of games, the coefficients represent points per game, and the expression represents the total number of points.

c. How would the expression change if a point were deducted for each loss?

You would subtract *z* from the original expression; $3w + t - z$.

Check

MUSIC A music festival offers one-day and three-day passes. A one-day pass costs $100, and a three-day pass costs $250. Write an expression for the total ticket sales if *n* one-day passes and *t* three-day passes are sold. $100n + 250t$

Go Online You can complete an Extra Example online.

Study Tip

Modeling When writing an expression to model a situation, begin by identifying the important quantities and relationships.

Lesson 1-2 • Algebraic Expressions 15

Interactive Presentation

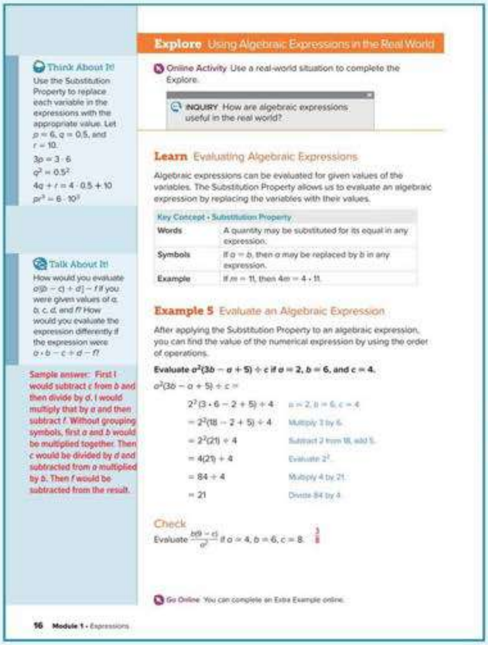
Example 4

TYPE

Students answer questions to analyze the given scenario and write a verbal expression.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Explore Using Algebraic Expressions in the Real World

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How are algebraic expressions useful in the real world?

Think About It!
Use the Substitution Property to replace each variable in the expressions with the appropriate value. Let $p = 6$, $q = 0.5$, and $r = 10$.

$3p = 3 \cdot 6$

$q^2 = 0.5^2$

$4q + r = 4 \cdot 0.5 + 10$

$pr^3 = 6 \cdot 10^3$

Learn Evaluating Algebraic Expressions

Algebraic expressions can be evaluated for given values of the variables. The Substitution Property allows us to evaluate an algebraic expression by replacing the variables with their values.

Key Concept • Substitution Property	
Words	A quantity may be substituted for its equal in any expression.
Symbols	If $a = b$, then a may be replaced by b in any expression.
Example	If $m = 11$, then $4m = 4 \cdot 11$.

Talk About It!
How would you evaluate $a[(b - c) \div d] - f$ if you were given values of a, b, c, d, and f? How would you evaluate the expression differently if the expression were $a \cdot b - c \div d - f$?

Sample answer: First I would subtract c from b and then divide by d. I would multiply that by a and then subtract f. Without grouping symbols, first a and b would be multiplied together. Then c would be divided by d and subtracted from a multiplied by b. Then f would be subtracted from the result.

Example 5 Evaluate an Algebraic Expression

After applying the Substitution Property to an algebraic expression, you can find the value of the numerical expression by using the order of operations.

Evaluate $a^2(3b - a + 5) \div c$ if $a = 2$, $b = 6$, and $c = 4$.

$$
\begin{aligned}
a^2(3b - a + 5) \div c &= 2^2(3 \cdot 6 - 2 + 5) \div 4 && a = 2, b = 6, c = 4\\
&= 2^2(18 - 2 + 5) \div 4 && \text{Multiply 3 by 6.}\\
&= 2^2(21) \div 4 && \text{Subtract 2 from 18, add 5.}\\
&= 4(21) \div 4 && \text{Evaluate } 2^2.\\
&= 84 \div 4 && \text{Multiply 4 by 21.}\\
&= 21 && \text{Divide 84 by 4.}
\end{aligned}
$$

Check
Evaluate $\frac{b(9 - c)}{a^2}$ if $a = 4$, $b = 6$, $c = 8$. $\frac{3}{8}$

Go Online You can complete an Extra Example online.

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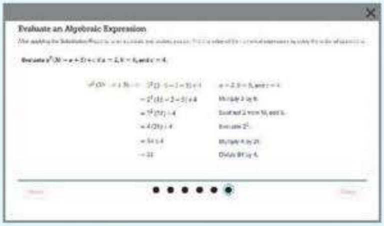
Interactive Presentation



Evaluate an Algebraic Expression

Example 5

Tap

Students tap through the steps of the order of operations to determine the value of an algebraic expression

TYPE

Students evaluate an expression using the order of operations.

Learn Evaluating Algebraic Expressions

Objective

Students evaluate algebraic expressions by substituting values into and simplifying given expressions.

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this Learn.

Common Misconception

Students often replace the variables with their corresponding numerical values, but then fail to finish evaluating the expression. Evaluating an algebraic expression requires two steps. First, use substitution to replace the variable with its value. Then, evaluate the expression by using the order of operations. State that *evaluate* means to find the value.

Example 5 Evaluate an Algebraic Expression

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear mathematical language when answering the question in the Talk About It! feature.

Questions for Mathematical Discourse

AL What is the resulting numerical expression after the Substitution Property is applied? $2^2(3 \times 6 - 2 + 5) \div 4$

OL In what order is the expression inside the parentheses evaluated? multiplication, then subtraction, then addition

BL Why do you multiply before dividing? because multiplication and division are done from left to right and the multiplication came before division in the expression

Common Error

When an expression contains addition and subtraction or multiplication and division, students perform the addition or multiplication first, rather than the operation that appears first when working from left to right. Remind students that addition and subtraction or multiplication and division are performed in one step in the order in which they appear.

Example 6 Write and Evaluate an Algebraic Expression

Teaching the Mathematical Practices

4 Make Assumptions In the Study Tip, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL What operation does the word *times* mean? multiplication

OL Why is the exponent only applied to the radius? Sample answer: The problem states that the radius is squared, not that the quantity four times π multiplied by the radius is squared.

BL Suppose you could blow a bubble with a radius of 10 cm. What is the surface area of your bubble?

$A = 4\pi(10)^2$

$A = 4\pi(100)$

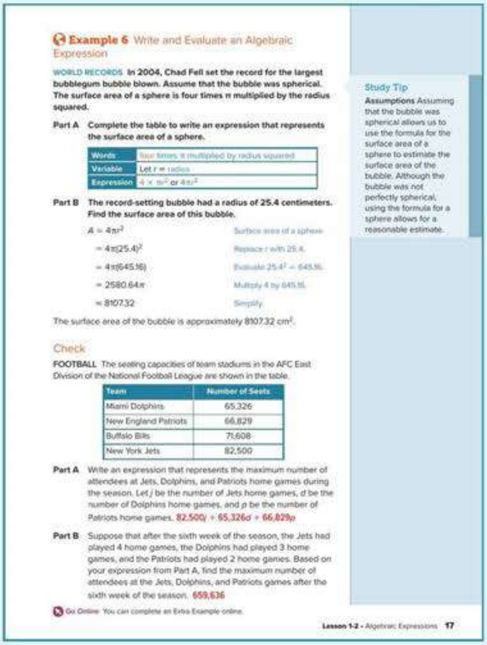
$A = 400\pi \text{ cm}^2$

Essential Question Follow-Up

Students have begun evaluating algebraic expressions.

Ask:

Why may we need to evaluate algebraic expressions? Sample answer: In the real world, algebraic expressions represent unknown values, which we may need to determine. We will need to substitute the known values into an expression and evaluate to determine the value.



Example 6 Write and Evaluate an Algebraic Expression

WORLD RECORDS In 2004, Chad Fell set the record for the largest bubblegum bubble blown. Assume that the bubble was spherical. The surface area of a sphere is four times π multiplied by the radius squared.

Part A Complete the table to write an expression that represents the surface area of a sphere.

Words	four times π multiplied by radius squared
Variable	Let r = radius
Expression	$4 \times \pi r^2$ or $4\pi r^2$

Part B The record-setting bubble had a radius of 25.4 centimeters. Find the surface area of this bubble.

$$\begin{aligned} A &= 4\pi r^2 && \text{Surface area of a sphere} \\ &= 4\pi(25.4)^2 && \text{Replace } r \text{ with 25.4.} \\ &= 4\pi(645.16) && \text{Evaluate } 25.4^2 = 645.16. \\ &= 2580.64\pi && \text{Multiply 4 by 645.16.} \\ &\approx 8107.32 && \text{Simplify.} \end{aligned}$$

The surface area of the bubble is approximately 8107.32 cm².

Study Tip

Assumptions Assuming that the bubble was spherical allows us to use the formula for the surface area of a sphere to estimate the surface area of the bubble. Although the bubble was *not* perfectly spherical, using the formula for a sphere allows for a reasonable estimate.

Check

FOOTBALL The seating capacities of team stadiums in the AFC East Division of the National Football League are shown in the table.

Team	Number of Seats
Miami Dolphins	65,326
New England Patriots	66,829
Buffalo Bills	71,608
New York Jets	82,500

Part A Write an expression that represents the maximum number of attendees at Jets, Dolphins, and Patriots home games during the season. Let j be the number of Jets home games, d be the number of Dolphins home games, and p be the number of Patriots home games. $82{,}500j + 65{,}326d + 66{,}829p$

Part B Suppose that after the sixth week of the season, the Jets had played 4 home games, the Dolphins had played 3 home games, and the Patriots had played 2 home games. Based on your expression from Part A, find the maximum number of attendees at the Jets, Dolphins, and Patriots games after the sixth week of the season. 659,636

Go Online You can complete an Extra Example online.

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Interactive Presentation

Write and Evaluate an Algebraic Expression

WORLD RECORDS In 2004, Chad Fell set the record for the largest bubblegum bubble blown. Assume that the bubble was spherical. The surface area of a sphere is four times π multiplied by the radius squared.

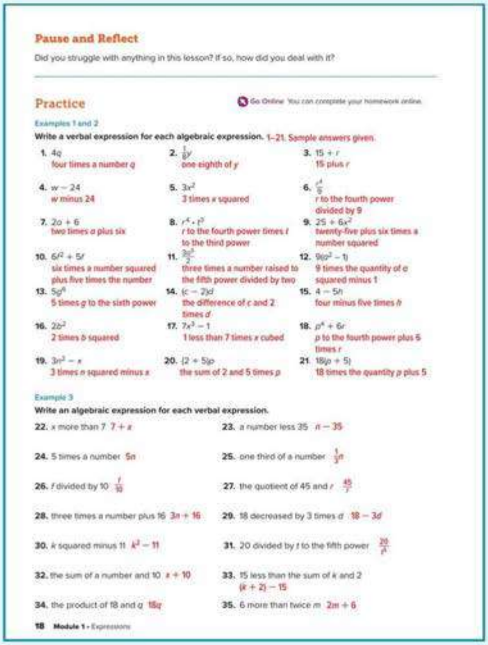
Example 6

Select

Students select the correct definition of the variable.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

Practice

Go Online You can complete your homework online.

Examples 1 and 2

Write a verbal expression for each algebraic expression. 1–21. Sample answers given.

1. $4q$
four times a number q

2. $\frac{1}{8}y$
one eighth of y

3. $15 + r$
15 plus r

4. $w - 24$
w minus 24

5. $3x^2$
3 times x squared

6. $\frac{r^4}{9}$
r to the fourth power divided by 9

7. $2a + 6$
two times a plus six

8. $r^4 \cdot t^3$
r to the fourth power times t to the third power

9. $25 + 6x^2$
twenty-five plus six times a number squared

10. $6f^2 + 5f$
six times a number squared plus five times the number

11. $\frac{3n^5}{2}$
three times a number raised to the fifth power divided by two

12. $9(e^2 - 1)$
9 times the quantity of e squared minus 1

13. $5g^6$
5 times g to the sixth power

14. $(c - 2)d$
the difference of c and 2 times d

15. $4 - 5h$
four minus five times h

16. $2b^2$
2 times b squared

17. $7x^3 - 1$
1 less than 7 times x cubed

18. $p^4 + 6r$
p to the fourth power plus 5 times r

19. $3n^2 - x$
3 times n squared minus x

20. $(2 + 5)p$
the sum of 2 and 5 times p

21. $18(p + 5)$
18 times the quantity p plus 5

Example 3

Write an algebraic expression for each verbal expression.

22. x more than 7 $7 + x$

23. a number less 35 $n - 35$

24. 5 times a number $5n$

25. one third of a number $\frac{1}{3}n$

26. f divided by 10 $\frac{f}{10}$

27. the quotient of 45 and r $\frac{45}{r}$

28. three times a number plus 16 $3n + 16$

29. 18 decreased by 3 times d $18 - 3d$

30. k squared minus 11 $k^2 - 11$

31. 20 divided by t to the fifth power $\frac{20}{t^5}$

32. the sum of a number and 10 $x + 10$

33. 15 less than the sum of k and 2 $(k + 2) - 15$

34. the product of 18 and q $18q$

35. 6 more than twice m $2m + 6$

18 Module 1 • Expressions

DIFFERENTIATE

Reteaching Activity AL ELL

IF students struggle to evaluate algebraic expressions,
THEN use tiles with numbers and operations on them to create the numerical expression after the Substitution Property is applied. Then students remove the operation performed in each step of order of operations and replace with the number tiles that are equivalent.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–66
2	exercises that extend concepts learned in this lesson to new contexts	67–75
3	exercises that emphasize higher-order and critical-thinking skills	76–79

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–75 odd, 76–79
- Extension: Toothpick Triangles
- ALEKS Writing Expressions and Equations

OL

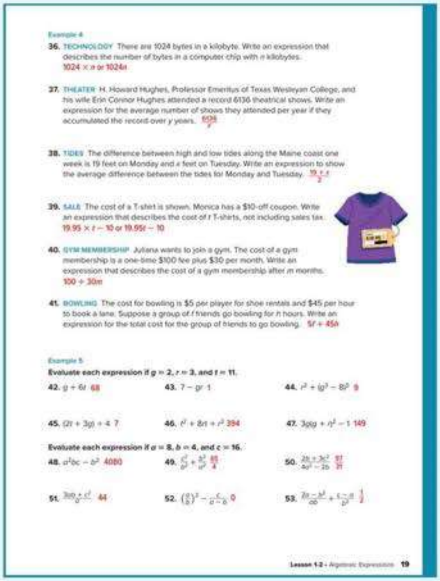
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–79 odd
- Remediation, Review Resources: Divide Integers
- Personal Tutors
- Extra Examples 1–6
- ALEKS Multiplication and Division with Integers

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–65 odd
- Remediation, Review Resources: Divide Integers
- *Quick Review Math Handbook*: Variables and Expressions
- ArriveMATH Take Another Look
- ALEKS Multiplication and Division with Integers



Example 4

36. TECHNOLOGY There are 1024 bytes in a kilobyte. Write an expression that describes the number of bytes in a computer chip with n kilobytes. $1024 \times n$ or $1024n$

37. THEATER H. Howard Hughes, Professor Emeritus of Texas Wesleyan College, and his wife Erin Connor Hughes attended a record 6136 theatrical shows. Write an expression for the average number of shows they attended per year if they accumulated the record over y years. $\frac{6136}{y}$

38. TIDES The difference between high and low tides along the Maine coast one week is 19 feet on Monday and x feet on Tuesday. Write an expression to show the average difference between the tides for Monday and Tuesday. $\frac{19+x}{2}$

39. SALE The cost of a T-shirt is shown. Monica has a $10-off coupon. Write an expression that describes the cost of t T-shirts, not including sales tax. $19.95 \times t - 10$ or $19.95t - 10$

40. GYM MEMBERSHIP Juliana wants to join a gym. The cost of a gym membership is a one-time $100 fee plus $30 per month. Write an expression that describes the cost of a gym membership after m months. $100 + 30m$

41. BOWLING The cost for bowling is $5 per player for shoe rentals and $45 per hour to book a lane. Suppose a group of f friends go bowling for h hours. Write an expression for the total cost for the group of friends to go bowling. $5f + 45h$

Example 5

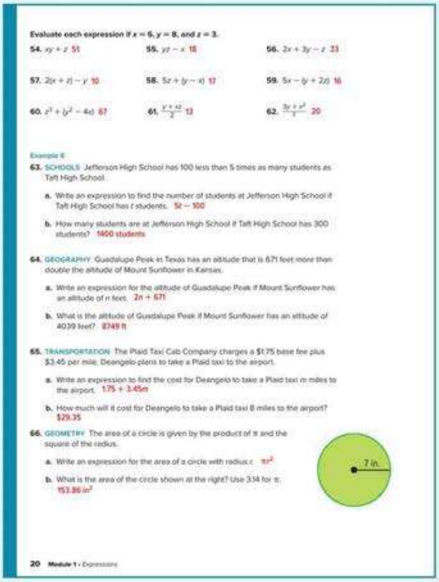
Evaluate each expression if $g = 2$, $r = 3$, and $t = 11$.

42. $g + 6t$ 68
43. $7 - gr$ 1
44. $r^2 + (g^3 - 8)^5$ 9
45. $(2t + 3g) \div 4$ 7
46. $t^2 + 8rt + r^2$ 394
47. $3g(g + r)^2 - 1$ 149

Evaluate each expression if $a = 8$, $b = 4$, and $c = 16$.

48. $a^2bc - b^2$ 4080
49. $\frac{c^2}{b^2} + \frac{b^2}{a^2}$ $\frac{65}{4}$
50. $\frac{2b + 3c^2}{4a^2 - 2b}$ $\frac{97}{31}$
51. $\frac{3ab + c^2}{a}$ 44
52. $\left(\frac{a}{b}\right)^2 - \frac{c}{a - b}$ 0
53. $\frac{2a - b^2}{ab} + \frac{c - a}{b^2}$ $\frac{1}{2}$

Lesson 1-2 • Algebraic Expressions 19



Evaluate each expression if $x = 6$, $y = 8$, and $z = 3$.

54. $xy + z$ 51
55. $yz - x$ 18
56. $2x + 3y - z$ 33
57. $2(x + z) - y$ 10
58. $5z + (y - x)$ 17
59. $5x - (y + 2z)$ 16
60. $z^3 + (y^2 - 4x)$ 67
61. $\frac{y + xz}{2}$ 13
62. $\frac{3y + x^2}{z}$ 20

Example 6

63. SCHOOLS Jefferson High School has 100 less than 5 times as many students as Taft High School.

a. Write an expression to find the number of students at Jefferson High School if Taft High School has t students. $5t - 100$

b. How many students are at Jefferson High School if Taft High School has 300 students? 1400 students

64. GEOGRAPHY Guadalupe Peak in Texas has an altitude that is 671 feet more than double the altitude of Mount Sunflower in Kansas.

a. Write an expression for the altitude of Guadalupe Peak if Mount Sunflower has an altitude of n feet. $2n + 671$

b. What is the altitude of Guadalupe Peak if Mount Sunflower has an altitude of 4039 feet? 8749 ft

65. TRANSPORTATION The Plaid Taxi Cab Company charges a $1.75 base fee plus $3.45 per mile. Deangelo plans to take a Plaid taxi to the airport.

a. Write an expression to find the cost for Deangelo to take a Plaid taxi m miles to the airport. $1.75 + 3.45m$

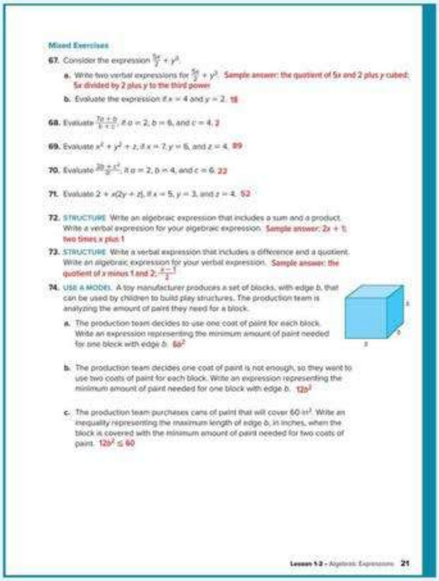
b. How much will it cost for Deangelo to take a Plaid taxi 8 miles to the airport? $29.35

66. GEOMETRY The area of a circle is given by the product of π and the square of the radius.

a. Write an expression for the area of a circle with radius r. πr^2

b. What is the area of the circle shown at the right? Use 3.14 for π. 153.86 in^2

20 Module 1 • Expressions



Mixed Exercises

67. Consider the expression $\frac{5x}{2} + y^3$.
    a. Write two verbal expressions for $\frac{5x}{2} + y^3$. Sample answer: the quotient of 5x and 2 plus y cubed; 5x divided by 2 plus y to the third power
    b. Evaluate the expression if $x = 4$ and $y = 2$. 18

68. Evaluate $\frac{7a + b}{b + c}$, if $a = 2$, $b = 6$, and $c = 4$. 2

69. Evaluate $x^2 + y^2 + z$, if $x = 7$, $y = 6$, and $z = 4$. 89

70. Evaluate $\frac{2b + c^2}{a}$, if $a = 2$, $b = 4$, and $c = 6$. 22

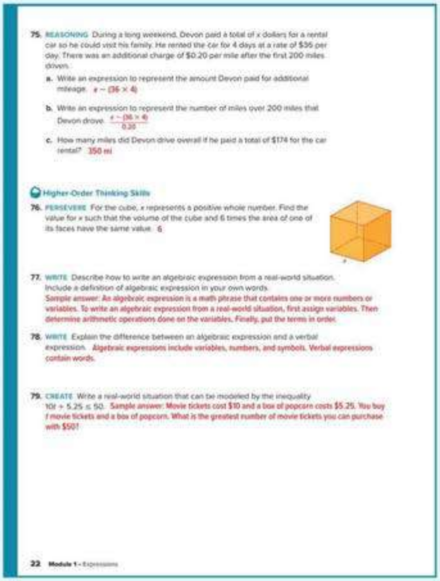
71. Evaluate $2 + x(2y + z)$, if $x = 5$, $y = 3$, and $z = 4$. 52

72. STRUCTURE Write an algebraic expression that includes a sum and a product. Write a verbal expression for your algebraic expression. Sample answer: $2x + 1$; two times x plus 1

73. STRUCTURE Write a verbal expression that includes a difference and a quotient. Write an algebraic expression for your verbal expression. Sample answer: the quotient of x minus 1 and 2; $\frac{x-1}{2}$

74. USE A MODEL A toy manufacturer produces a set of blocks, with edge b, that can be used by children to build play structures. The production team is analyzing the amount of paint they need for a block.
    a. The production team decides to use one coat of paint for each block. Write an expression representing the minimum amount of paint needed for one block with edge b. $6b^2$
    b. The production team decides one coat of paint is not enough, so they want to use two coats of paint for each block. Write an expression representing the minimum amount of paint needed for one block with edge b. $12b^2$
    c. The production team purchases cans of paint that will cover 60 in². Write an inequality representing the maximum length of edge b, in inches, when the block is covered with the minimum amount of paint needed for two coats of paint. $12b^2 \leq 60$

Lesson 1-2 • Algebraic Expressions 21



75. REASONING During a long weekend, Devon paid a total of x dollars for a rental car so he could visit his family. He rented the car for 4 days at a rate of \$36 per day. There was an additional charge of \$0.20 per mile after the first 200 miles driven.
    a. Write an expression to represent the amount Devon paid for additional mileage. $x - (36 \times 4)$
    b. Write an expression to represent the number of miles over 200 miles that Devon drove. $\frac{x - (36 \times 4)}{0.20}$
    c. How many miles did Devon drive overall if he paid a total of \$174 for the car rental? 350 mi

Higher-Order Thinking Skills

76. PERSEVERE For the cube, x represents a positive whole number. Find the value for x such that the volume of the cube and 6 times the area of one of its faces have the same value. 6

77. WRITE Describe how to write an algebraic expression from a real-world situation. Include a definition of algebraic expression in your own words. Sample answer: An algebraic expression is a math phrase that contains one or more numbers or variables. To write an algebraic expression from a real-world situation, first assign variables. Then determine arithmetic operations done on the variables. Finally, put the terms in order.

78. WRITE Explain the difference between an algebraic expression and a verbal expression. Algebraic expressions include variables, numbers, and symbols. Verbal expressions contain words.

79. CREATE Write a real-world situation that can be modeled by the inequality $10t + 5.25 \leq 50$. Sample answer: Movie tickets cost \$10 and a box of popcorn costs \$5.25. You buy t movie tickets and a box of popcorn. What is the greatest number of movie tickets you can purchase with \$50?

22 Module 1 • Expressions

Lesson 1-3

Properties of Real Numbers

A.SSE.2

LESSON GOAL

Students apply the properties of real numbers to simplify expressions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Properties of Equality
- Identify Properties of Equality
- Interpret Properties of Equality
- Use Properties of Equality

Identitites and Inverses
- Evaluate Using the Addition Properties
- Evaluate Using the Multiplicative Identity and Multiplicative Inverse
- Evaluate Using the Multiplicative Property of Zero

Explore: Testing the Associative Property

Develop:

Commutative and Associative Properties
- Evaluate Using the Associative Property
- Evaluate Using the Commutative Property
- Evaluate Using the Associative and Commutative Properties

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Subtract Rational Numbers	●	●		●
Extension: Properties of Operations		●	●	●

Language Development Handbook

Assign page 3 of the *Language Development Handbook* to help your students build mathematical language related to simplifying expressions.

ELL You can use the tips and suggestions on page T3 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice:

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students wrote and evaluated algebraic expressions.

6.EE.2, A.SSE.1b, A.SSE.2

Now

Students apply the properties of real numbers to simplify expressions.

A.SSE.2

Next

Students will use the Distributive Property to simplify expressions.

A.SSE.1a, A.SSE.2

Rigor

The Three Pillars of Rigor

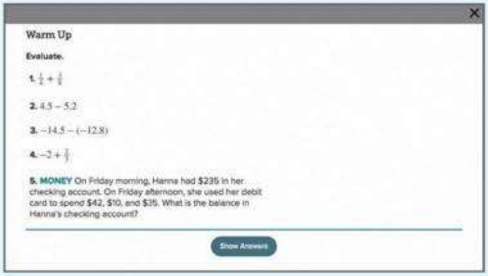
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their understanding of expressions to build fluency with using the properties of real numbers to simplify expressions. They apply their understanding of the properties of real numbers by solving real-world problems.

Mathematical Background

Properties of equality, addition, and multiplication can be used to justify steps when evaluating expressions and solving equations. Using these properties can often help make mental calculations easier.

Interactive Presentation



Warm Up

Evaluate.

1. [illegible]

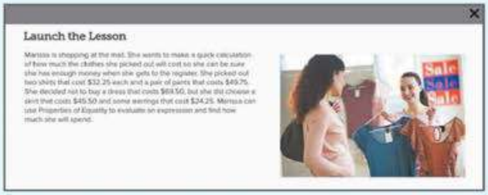
2. $4.5 - 5.2$

3. $-14.5 - (-12.8)$

4. $-2 +$ [illegible]

5. MONEY On Friday morning, Hanna had $235 in her checking account. On Friday afternoon, she used her debit card to spend $42, $10, and $35. What is the balance in Hanna's checking account?

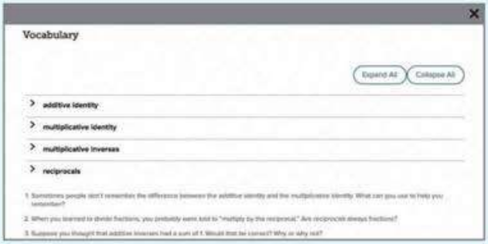
Warm Up



Launch the Lesson

Marissa is shopping at the mall. She wants to make a quick calculation of how much the clothes she picked out will cost so she can be sure she has enough money when she gets to the register. She picked out two shirts that cost $32.25 each and a pair of pants that costs $49.75. She decided not to buy a dress that costs $69.50, but she did choose a skirt that costs $45.50 and some earrings that cost $24.25. Marissa can use Properties of Equality to evaluate an expression and find how much she will spend.

Launch



Vocabulary

- additive identity
- multiplicative identity
- multiplicative inverses
- reciprocals

1. Sometimes people don't remember the difference between the additive identity and the multiplicative identity. What can you use to help you remember?
2. When you learned to divide fractions, you probably were told to "multiply by the reciprocal." Are reciprocals always fractions?
3. Suppose you thought that additive inverses had a sum of 1. Would that be correct? Why or why not?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- adding and subtracting rational numbers

Answers:

1. $\frac{5}{8}$
2. -0.7
3. -1.7
4. $-1\frac{1}{3}$
5. $148

Launch the Lesson

Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to analyze the given information about a shopping trip to write and evaluate an expression.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Testing the Associative Property

Objective

Students compare values of expressions to explore the Associative Property.

MP Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the Associative Property in this Explore to complete the tables.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

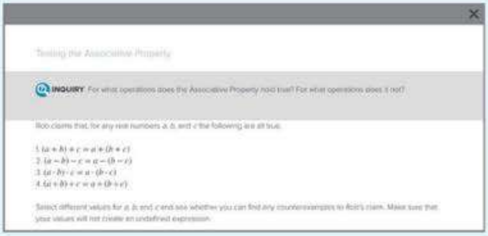
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students select different values for *a*, *b*, and *c* to test eight different claims. They will complete a chart involving addition and subtraction and then another chart involving multiplication and division. The guiding exercises will lead students to discover for which operations the Associative Property holds true. Then, students will answer the Inquiry Question.

(*continued on next page*)

Interactive Presentation



Testing the Associative Property

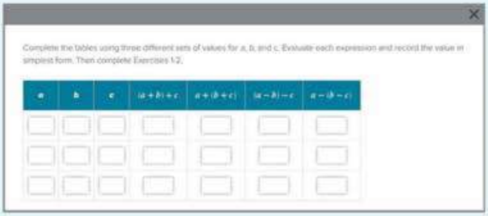
INQUIRY For what operations does the Associative Property hold true? For what operations does it not?

Rob claims that, for any real numbers a, b, and c the following are all true.

1. $(a+b)+c=a+(b+c)$
2. $(a-b)-c=a-(b-c)$
3. $(a\cdot b)\cdot c=a\cdot(b\cdot c)$
4. $(a\div b)\div c=a\div(b\div c)$

Select different values for a, b, and c and see whether you can find any counterexamples to Rob's claim. Make sure that your values will not create an undefined expression.

Explore



Complete the tables using three different sets of values for a, b, and c. Evaluate each expression and record the value in simplest form. Then complete Exercises 1-2.

a	b	c	$(a+b)+c$	$a+(b+c)$	$(a-b)-c$	$a-(b-c)$

Explore

TYPE

Students complete a table by entering possible values for *a*, *b*, and *c* and then the value of three different expressions.

Interactive Presentation

INQUIRY For what operations does the Associative Property hold true? For what operations does it not?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Testing the Associative Property (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is it helpful to reorder or regroup terms when evaluating an expression? Sample answer: Reordering and regrouping terms makes it easier to put terms together that can easily be added or multiplied.

Inquiry

For what operations does the Associative Property hold true? For what operations does it not? addition and multiplication; subtraction and division

Go Online to find additional teaching notes and answers for the Explore activity.

Learn Properties of Equality

Objective

Students recognize the properties of equality by identifying the properties used to justify given statements.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Identify Properties of Equality

MP Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to justify their conclusions.

Questions for Mathematical Discourse

AL In part **a,** how many equations are involved? 2

OL Because part **b** only involves 1 equation, which properties of equality can you eliminate? Symmetric and Transitive Properties

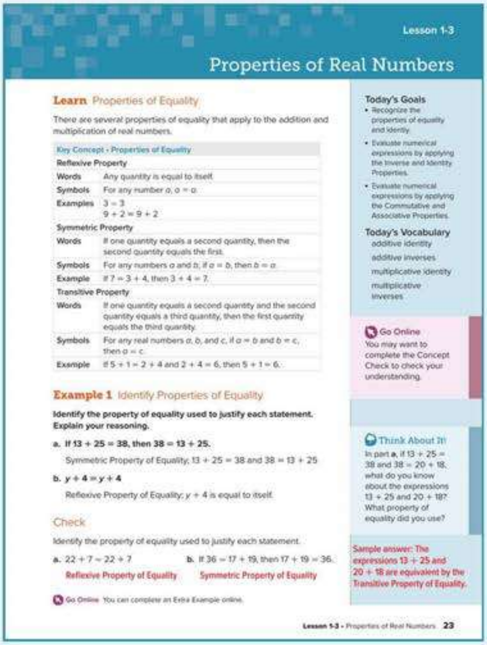
BL Write a statement that illustrates the Transitive Property of Equality. Sample answer: If $10 + 12 = 15 + 7$ and $15 + 7 = 22$, then $10 + 12 = 22$.

Common Error

Students often use the wrong property because they have not fully memorized the names of the properties. They may know the property but not know the name. Encourage students to make flash cards with the property and the definition.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 1-3

Properties of Real Numbers

Learn Properties of Equality

There are several properties of equality that apply to the addition and multiplication of real numbers.

Key Concept • Properties of Equality

Reflexive Property	
Words	Any quantity is equal to itself.
Symbols	For any number a, $a = a$.
Examples	$3 = 3$ $9 + 2 = 9 + 2$
Symmetric Property	
Words	If one quantity equals a second quantity, then the second quantity equals the first.
Symbols	For any numbers a and b, if $a = b$, then $b = a$.
Example	If $7 = 3 + 4$, then $3 + 4 = 7$.
Transitive Property	
Words	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.
Symbols	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Example	If $5 + 1 = 2 + 4$ and $2 + 4 = 6$, then $5 + 1 = 6$.

Example 1 Identify Properties of Equality

Identify the property of equality used to justify each statement. Explain your reasoning.

a. If $13 + 25 = 38$, then $38 = 13 + 25$.

Symmetric Property of Equality; $13 + 25 = 38$ and $38 = 13 + 25$

b. $y + 4 = y + 4$

Reflexive Property of Equality; $y + 4$ is equal to itself.

Check

Identify the property of equality used to justify each statement.

a. $22 + 7 = 22 + 7$
Reflexive Property of Equality

b. If $36 = 17 + 19$, then $17 + 19 = 36$.
Symmetric Property of Equality

Go Online You can complete an Extra Example online.

Today's Goals
- Recognize the properties of equality and identity.
- Evaluate numerical expressions by applying the Inverse and Identity Properties.
- Evaluate numerical expressions by applying the Commutative and Associative Properties.

Today's Vocabulary
additive identity
additive inverses
multiplicative identity
multiplicative inverses

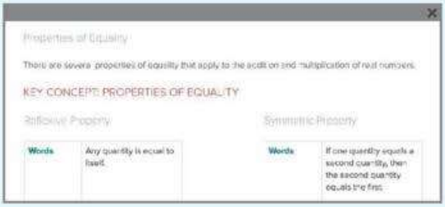
Go Online
You may want to complete the Concept Check to check your understanding.

Think About It!
In part **a**, if $13 + 25 = 38$ and $38 = 20 + 18$, what do you know about the expressions $13 + 25$ and $20 + 18$? What property of equality did you use?

Sample answer: The expressions $13 + 25$ and $20 + 18$ are equivalent by the Transitive Property of Equality.

Lesson 1-3 • Properties of Real Numbers 23

Interactive Presentation



Properties of Equality

There are several properties of equality that apply to the addition and multiplication of real numbers.

KEY CONCEPT: PROPERTIES OF EQUALITY

Reflexive Property

Words	Any quantity is equal to itself.

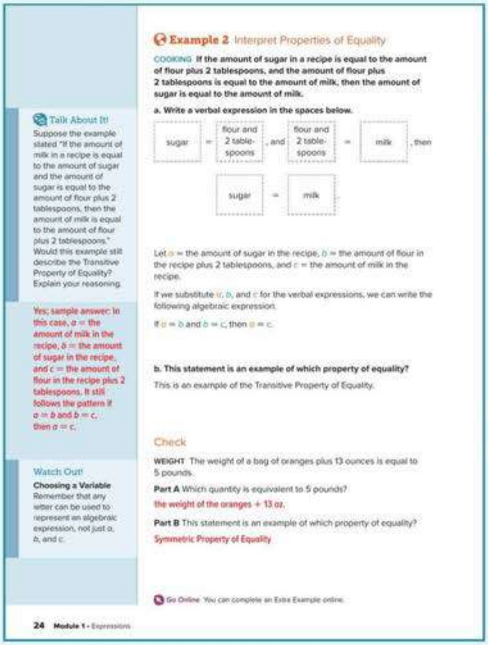
Symmetric Property

Words	If one quantity equals a second quantity, then the second quantity equals the first.

Learn

TYPE

Students use a diagram to write two equations illustrating the Symmetric Property of Equality.



Example 2 Interpret Properties of Equality

COOKING **If the amount of sugar in a recipe is equal to the amount of flour plus 2 tablespoons, and the amount of flour plus 2 tablespoons is equal to the amount of milk, then the amount of sugar is equal to the amount of milk.**

a. Write a verbal expression in the spaces below.

sugar = flour and 2 tablespoons, and flour and 2 tablespoons = milk, then

sugar = milk.

Let a = the amount of sugar in the recipe, b = the amount of flour in the recipe plus 2 tablespoons, and c = the amount of milk in the recipe.

If we substitute a, b, and c for the verbal expressions, we can write the following algebraic expression.

If $a = b$ and $b = c$, then $a = c$.

b. This statement is an example of which property of equality?

This is an example of the Transitive Property of Equality.

Talk About It!

Suppose the example stated "If the amount of milk in a recipe is equal to the amount of sugar and the amount of sugar is equal to the amount of flour plus 2 tablespoons, then the amount of milk is equal to the amount of flour plus 2 tablespoons." Would this example still describe the Transitive Property of Equality? Explain your reasoning.

Yes; sample answer: In this case, a = the amount of milk in the recipe, b = the amount of sugar in the recipe, and c = the amount of flour in the recipe plus 2 tablespoons. It still follows the pattern if $a = b$ and $b = c$, then $a = c$.

Watch Out!

Choosing a Variable Remember that any letter can be used to represent an algebraic expression, not just a, b, and c.

Check

WEIGHT The weight of a bag of oranges plus 13 ounces is equal to 5 pounds.

Part A Which quantity is equivalent to 5 pounds?

the weight of the oranges + 13 oz.

Part B This statement is an example of which property of equality?

Symmetric Property of Equality

Go Online You can complete an Extra Example online.

24 **Module 1** • Expressions

Interactive Presentation

KEY CONCEPT: PROPERTIES OF EQUALITY

Words	Any quantity is equal to itself.

Words	If one quantity equals a second quantity, then the second quantity equals the first.

Example 2

DRAG & DROP

Students drag and drop the images to represent the verbal expression.

TYPE

Students determine if a verbal expression represents the Transitive Property of Equality.

Example 2 Interpret Properties of Equality

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. The Talk About It! feature asks students to justify their conclusions.

Questions for Mathematical Discourse

AL What are the three quantities given in the problem? the amount of sugar, amount of flour plus 2 tablespoons, and the amount of milk

OL Which ingredients belong in the first equation of the Transitive Property? the amount of sugar and the amount of flour plus 2 tablespoons Which ingredients belong in the second equation of the Transitive Property? the amount of flour plus 2 tablespoons and the amount of milk

BL At the end, the amount of sugar and the amount of milk are the same. Why is this an example of the Transitive Property of Equality and not the Reflexive Property of Equality? Sample answer: Sugar and milk are not the same ingredient, and the resulting statement does not set either the amount of sugar or the amount of milk equal to itself.

Common Error

Students may not equate a real-world problem to the properties of equality because there are no given equations. Students will have to define variables first in order to see the property of equality.

Example 3 Use Properties of Equality

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What does the Reflexive Property of Equality state? Sample answer: that any quantity equals itself

OL Which properties of equality can you rule out in part **b** and why? Symmetric and Reflexive because there are three equations

BL Would "If $x + 5 = y - 5$ and $y - 5 = 10$, then $x + 5 = 10$" be an example of the Transitive Property? Explain. Yes; sample answer: When one quantity $(x + 5)$ equals a second $(y - 5)$ and the second quantity $(y - 5)$ equals a third (10), then the first quantity $(x - 5)$ equals the third (10).

Common Error

Students are familiar with numerical statements that they can check for correctness, but when asked to identify properties for algebraic statements, they may try to solve rather than identify the correct property. Remind students that variables are simply unknown numbers so the properties of equality work the same.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to identify or use the correct property of equality,

THEN tell them a good way to remember is:

- The Reflexive Property involves one equation.
- The Symmetric Property involves two equations.
- The Transitive Property involves three equations.

Learn Identities and Inverses

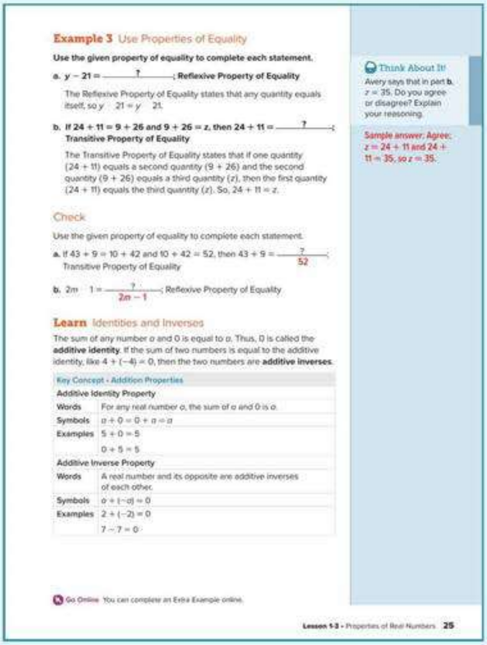
Objective

Students evaluate numerical expressions by applying the Inverse and Identity Properties.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of addition and multiplication in this Learn.

(continued on the next page)



Example 3 Use Properties of Equality

Use the given property of equality to complete each statement.

a. $y - 21 =$ ___?___; **Reflexive Property of Equality**

The Reflexive Property of Equality states that any quantity equals itself, so $y - 21 = y - 21$.

b. If $24 + 11 = 9 + 26$ and $9 + 26 = z$, then $24 + 11 =$ ___?___; **Transitive Property of Equality**

The Transitive Property of Equality states that if one quantity $(24 + 11)$ equals a second quantity $(9 + 26)$ and the second quantity $(9 + 26)$ equals a third quantity (z), then the first quantity $(24 + 11)$ equals the third quantity (z). So, $24 + 11 = z$.

Think About It!

Avery says that in part **b**, $z = 35$. Do you agree or disagree? Explain your reasoning.

Sample answer: Agree; $z = 24 + 11$ and $24 + 11 = 35$, so $z = 35$.

Check

Use the given property of equality to complete each statement.

a. If $43 + 9 = 10 + 42$ and $10 + 42 = 52$, then $43 + 9 =$ ___?___ 52; Transitive Property of Equality

b. $2m - 1 =$ ___?___ $2m - 1$; Reflexive Property of Equality

Learn Identities and Inverses

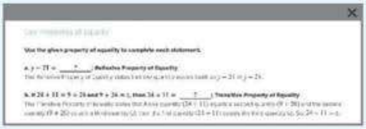
The sum of any number a and 0 is equal to a. Thus, 0 is called the **additive identity**. If the sum of two numbers is equal to the additive identity, like $4 + (-4) = 0$, then the two numbers are **additive inverses**.

Key Concept • Addition Properties	
Additive Identity Property	
Words	For any real number a, the sum of a and 0 is a.
Symbols	$a + 0 = 0 + a = a$
Examples	$5 + 0 = 5$ $0 + 5 = 5$
Additive Inverse Property	
Words	A real number and its opposite are additive inverses of each other.
Symbols	$a + (-a) = 0$
Examples	$2 + (-2) = 0$ $7 - 7 = 0$

Go Online You can complete an Extra Example online.

Lesson 1-3 • Properties of Real Numbers 25

Interactive Presentation



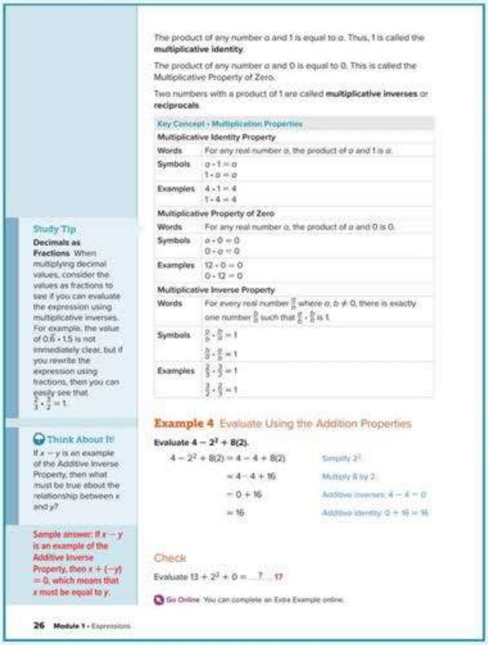
Example 3

TYPE

Students determine if a given claim is correct.

CHECK

Students complete the Check online to determine whether they are ready to move on.



The product of any number a and 1 is equal to a. Thus, 1 is called the **multiplicative identity**.

The product of any number a and 0 is equal to 0. This is called the Multiplicative Property of Zero.

Two numbers with a product of 1 are called **multiplicative inverses** or **reciprocals**.

Key Concept • Multiplication Properties

Multiplicative Identity Property	
Words	For any real number a, the product of a and 1 is a.
Symbols	$a \cdot 1 = a$ $1 \cdot a = a$
Examples	$4 \cdot 1 = 4$ $1 \cdot 4 = 4$
Multiplicative Property of Zero	
Words	For any real number a, the product of a and 0 is 0.
Symbols	$a \cdot 0 = 0$ $0 \cdot a = 0$
Examples	$12 \cdot 0 = 0$ $0 \cdot 12 = 0$
Multiplicative Inverse Property	
Words	For every real number $\frac{a}{b}$ where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a}$ is 1.
Symbols	$\frac{a}{b} \cdot \frac{b}{a} = 1$ $\frac{b}{a} \cdot \frac{a}{b} = 1$
Examples	$\frac{2}{3} \cdot \frac{3}{2} = 1$ $\frac{3}{2} \cdot \frac{2}{3} = 1$

Study Tip

Decimals as Fractions When multiplying decimal values, consider the values as fractions to see if you can evaluate the expression using multiplicative inverses. For example, the value of $0.\overline{6} \cdot 1.5$ is not immediately clear, but if you rewrite the expression using fractions, then you can easily see that $\frac{2}{3} \cdot \frac{3}{2} = 1$.

Think About It!

If $x - y$ is an example of the Additive Inverse Property, then what must be true about the relationship between x and y?

Sample answer: If $x - y$ is an example of the Additive Inverse Property, then $x + (-y) = 0$, which means that x must be equal to y.

Example 4 Evaluate Using the Addition Properties

Evaluate $4 - 2^2 + 8(2)$.

$$\begin{aligned} 4 - 2^2 + 8(2) &= 4 - 4 + 8(2) && \text{Simplify } 2^2. \\ &= 4 - 4 + 16 && \text{Multiply 8 by 2.} \\ &= 0 + 16 && \text{Additive inverses: } 4 - 4 = 0 \\ &= 16 && \text{Additive identity: } 0 + 16 = 16 \end{aligned}$$

Check

Evaluate $13 + 2^2 + 0 =$? 17

Go Online You can complete an Extra Example online.

26 Module 1 • Expressions

Interactive Presentation

Learn

TYPE

a|

Students use the Additive Inverse Property to analyze an expression.

About the Key Concept

Students learn about the additive and multiplicative identities and inverses, along with the Multiplicative Property of Zero. The Identity Properties do not change the value of the expression, while the Inverse Properties either add to zero or multiply to 1. Finally, the Multiplicative Property of Zero states that if the product of two numbers is zero, at least one of the numbers must be zero.

Common Misconception

Students believe the Inverse Properties are illustrated when any expression equals zero or one, regardless of the operation. For example, $-2 + 3 = 1$ and $5(0) = 0$ are often confused as Inverse Properties because they equal zero or one. Encourage students to consider the operation before concluding it is an Inverse Property.

Example 4 Evaluate Using the Addition Properties

MP Teaching the Mathematical Practices

3 Reason Inductively In this example, students will use inductive reasoning to make plausible arguments.

Questions for Mathematical Discourse

AL When using the order of operations, which operation in the expression should be simplified first? the power

OL Why is 8(2) not simplified first even though there are parentheses? Sample answer: Parentheses are done first when they are used as grouping symbols. Those parentheses represent multiplication, which comes after exponents in the order of operations.

BL How can $4 - 4$ be rewritten to more clearly illustrate the use of the Additive Inverse Property? $4 + (-4)$

Example 5 Evaluate Using the Multiplicative Identity and Multiplicative Inverse

Teaching the Mathematical Practices

2 Make Sense of Quantities Mathematically proficient students need to be able to make sense of quantities and their relationships. In this example, notice the relationship between the terms and the Multiplicative Identity and Inverse Properties.

Questions for Mathematical Discourse

AL What is the result of multiplying $\frac{3}{2}$ by $\frac{2}{3}$? $\frac{3}{2}\cdot\frac{2}{3}=\frac{6}{6}=1$

OL How do you know $\frac{3}{2}$ and $\frac{2}{3}$ are multiplicative inverses without multiplying? They are reciprocals.

BL What factor would need to be included in the expression in order for the expression to simplify to 1? Explain. $\frac{1}{7}$ sample answer: In order for the expression to simplify to 1, the multiplicative inverse of 7 would need to be a factor.

Example 6 Evaluate Using the Multiplicative Property of Zero

Teaching the Mathematical Practices

7 Look For a Pattern Students should see the pattern of grouping numbers according to the Multiplicative Property of Zero so the expression can be evaluated more easily.

Questions for Mathematical Discourse

AL According to the order of operations, what should be simplified first? everything inside the parentheses

OL How much of the expression is inside the parentheses? Everything except zero

BL Would the value of the expression change if the expression were instead $7 + [4 - 3(2)] \times 0$? Explain. Yes; sample answer: The zero is no longer multiplied by everything, so only the part in the bracket becomes zero. $7 + 0 = 7$, not zero.

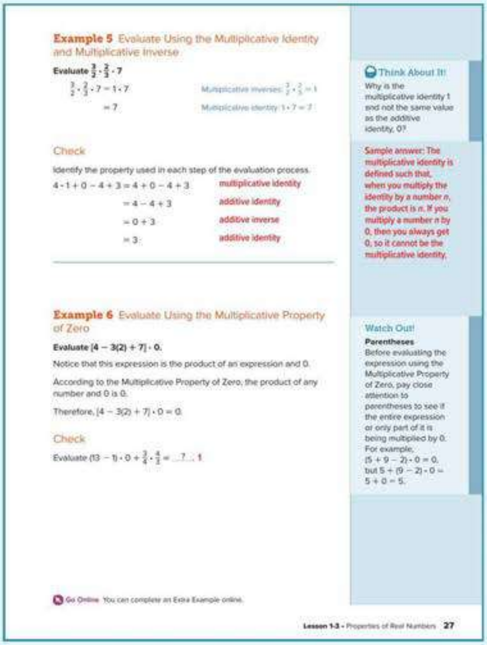
DIFFERENTIATE

Language Development Activity AL ELL

IF students are struggling to identify or use the correct Inverse or Identity Property,

THEN tell them a good way to remember is:

- Identity Properties do not change the starting value. When you add zero or multiply by one, the answer is identical to the starting value, so adding zero or multiplying by one are Identity Properties.
- Inverse Properties involve "undoing," so adding the opposite or multiplying by the reciprocal is the way to get rid of or undo the addition or multiplication that was previously there.



Example 5 Evaluate Using the Multiplicative Identity and Multiplicative Inverse

Evaluate $\frac{3}{2}\cdot\frac{2}{3}\cdot 7$

$\frac{3}{2}\cdot\frac{2}{3}\cdot 7 = 1\cdot 7$ Multiplicative inverses: $\frac{3}{2}\cdot\frac{2}{3}=1$

$= 7$ Multiplicative identity: $1\cdot 7 = 7$

Check

Identify the property used in each step of the evaluation process.

$4\cdot 1 + 0 - 4 + 3 = 4 + 0 - 4 + 3$ multiplicative identity

$= 4 - 4 + 3$ additive identity

$= 0 + 3$ additive inverse

$= 3$ additive identity

Example 6 Evaluate Using the Multiplicative Property of Zero

Evaluate $[4 - 3(2) + 7]\cdot 0$.

Notice that this expression is the product of an expression and 0.

According to the Multiplicative Property of Zero, the product of any number and 0 is 0.

Therefore, $[4 - 3(2) + 7]\cdot 0 = 0$.

Check

Evaluate $(13 - 1)\cdot 0 + \frac{3}{4}\cdot\frac{4}{3} =$? 1

Go Online You can complete an Extra Example online.

Think About It!

Why is the multiplicative identity 1 and not the same value as the additive identity, 0?

Sample answer: The multiplicative identity is defined such that, when you multiply the identity by a number n, the product is n. If you multiply a number n by 0, then you always get 0, so it cannot be the multiplicative identity.

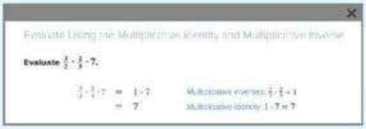
Watch Out!

Parentheses

Before evaluating the expression using the Multiplicative Property of Zero, pay close attention to parentheses to see if the entire expression or only part of it is being multiplied by 0. For example, $(5 + 9 - 2)\cdot 0 = 0$, but $5 + (9 - 2)\cdot 0 = 5 + 0 = 5$.

Lesson 1-3 • Properties of Real Numbers 27

Interactive Presentation



Evaluate Using the Multiplicative Identity and Multiplicative Inverse

Evaluate $\frac{3}{2}\cdot\frac{2}{3}\cdot 7$.

$\frac{3}{2}\cdot\frac{2}{3}\cdot 7 = 1\cdot 7$ Multiplicative inverses: $\frac{3}{2}\cdot\frac{2}{3}=1$

$= 7$ Multiplicative identity: $1\cdot 7 = 7$

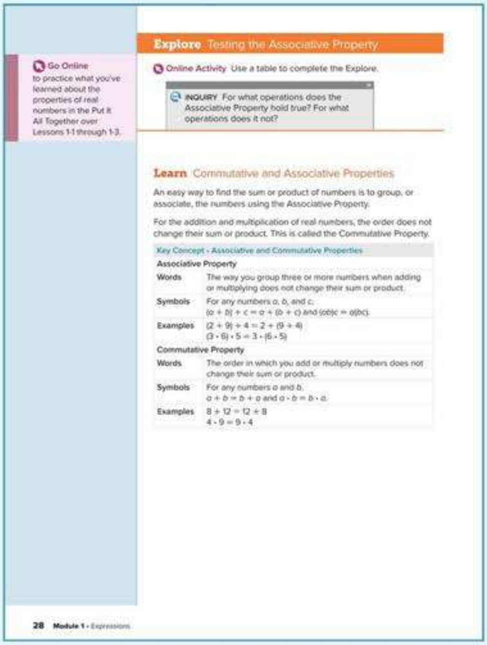
Example 5

TYPE

Students state why the multiplicative identity is 1 and not 0.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Explore Testing the Associative Property

Go Online to practice what you've learned about the properties of real numbers in the Put It All Together over Lessons 1-1 through 1-3.

Online Activity Use a table to complete the Explore.

INQUIRY For what operations does the Associative Property hold true? For what operations does it not?

Learn Commutative and Associative Properties

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the Associative Property.

For the addition and multiplication of real numbers, the order does not change their sum or product. This is called the Commutative Property.

Key Concept • Associative and Commutative Properties	
Associative Property	
Words	The way you group three or more numbers when adding or multiplying does not change their sum or product.
Symbols	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.
Examples	$(2 + 9) + 4 = 2 + (9 + 4)$ $(3 \cdot 6) \cdot 5 = 3 \cdot (6 \cdot 5)$
Commutative Property	
Words	The order in which you add or multiply numbers does not change their sum or product.
Symbols	For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Examples	$8 + 12 = 12 + 8$ $4 \cdot 9 = 9 \cdot 4$

28 **Module 1** • Expressions

Interactive Presentation

Learn

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Commutative and Associative Properties

Objective

Students evaluate numerical expressions by applying the Commutative and Associative Properties.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of the Commutative and Associative Properties in this Learn.

About the Key Concept

The Associative Property allows sums or products of three or more numbers to be regrouped without changing the value of the expression. The Commutative Property allows numbers being added or multiplied to be reordered without changing the value of the expression.

Common Misconception

Since the Commutative and Associative Properties apply to addition and multiplication, some students believe they will automatically apply to subtraction and division. They may rewrite or regroup subtraction or division expressions, which will lead to wrong answers. Remind students that subtraction and division must be performed in a certain order and these properties do not apply to those two operations.

Example 7 Evaluate Using the Associative Property

Teaching the Mathematical Practices

3 Construct Arguments In the Think About It! feature, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL When adding money without a calculator, what is one way to make adding easier? Sample answer: Add things in a different order.

OL Why should \$34.50 be grouped with \$32.50 and \$23.25 be grouped with \$31.75 when using the Associative Property? Sample answer: \$34.50 and \$32.50 both have 50 cents which, when added together, equals a whole dollar, and the same for the 25 cents and 75 cents.

BL If the amounts had been listed in a different order, could the expression be simplified using only the Associative Property? Explain. No; sample answer: If the amounts had been listed in a different order, then the Commutative Property would have been needed first.

DIFFERENTIATE

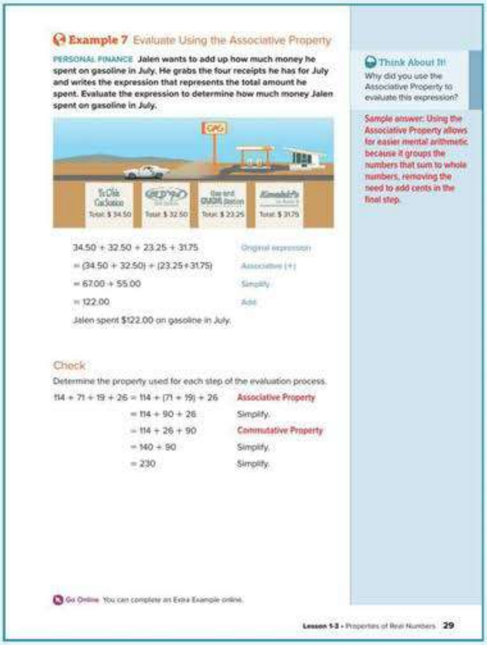
Reteaching Activity AL ELL

IF students are having a hard time distinguishing between the Commutative and Associative Properties,

THEN write the following examples of each property on the board and ask students to identify which property is being used.

- $12 + 16 + 8 + 14 = 12 + 8 + 16 + 14$
- $12 + 8 + 16 + 14 = (12 + 8) + (16 + 14)$
- $3 + 5 + 15 + 12 = 3 + 12 + 5 + 15$
- $3 + 12 + 5 + 15 = (3 + 12) + (5 + 15)$

Ask students what makes it so difficult to distinguish the properties. Discuss ways to tell the two properties apart. Explain that the Associative Property affects how numbers are grouped, or how they are *associated* with each other. The Commutative Property allows the numbers to be moved around or *commute*.



Example 7 Evaluate Using the Associative Property

PERSONAL FINANCE **Jalen wants to add up how much money he spent on gasoline in July. He grabs the four receipts he has for July and writes the expression that represents the total amount he spent. Evaluate the expression to determine how much money Jalen spent on gasoline in July.**

$34.50 + 32.50 + 23.25 + 31.75$	Original expression
$= (34.50 + 32.50) + (23.25 + 31.75)$	Associative (+)
$= 67.00 + 55.00$	Simplify
$= 122.00$	Add

Jalen spent \$122.00 on gasoline in July.

Think About It! Why did you use the Associative Property to evaluate this expression?

Sample answer: Using the Associative Property allows for easier mental arithmetic because it groups the numbers that sum to whole numbers, removing the need to add cents in the final step.

Check

Determine the property used for each step of the evaluation process.

$114 + 71 + 19 + 26 = 114 + (71 + 19) + 26$	Associative Property
$= 114 + 90 + 26$	Simplify.
$= 114 + 26 + 90$	Commutative Property
$= 140 + 90$	Simplify.
$= 230$	Simplify.

Go Online You can complete an Extra Example online.

Lesson 1-3 • Properties of Real Numbers 29

Interactive Presentation

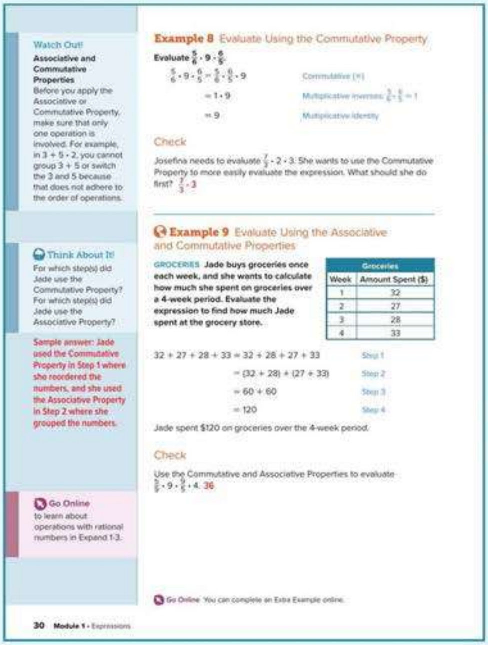
Example 7

TAP

Students move through the steps of evaluating the expression using the Associative Property.

TYPE

Students explain why the Associative Property could be used in the example.



Watch Out!

Associative and Commutative Properties Before you apply the Associative or Commutative Property, make sure that only one operation is involved. For example, in 3 + 5 · 2, you cannot group 3 + 5 or switch the 3 and 5 because that does not adhere to the order of operations.

Think About It! For which step(s) did Jade use the Commutative Property? For which step(s) did Jade use the Associative Property?

Sample answer: Jade used the Commutative Property in Step 1 where she reordered the numbers, and she used the Associative Property in Step 2 where she grouped the numbers.

Go Online to learn about operations with rational numbers in Expand 1-3.

Example 8 Evaluate Using the Commutative Property

Evaluate $\frac{5}{6} \cdot 9 \cdot \frac{6}{5}$.

$\frac{5}{6} \cdot 9 \cdot \frac{6}{5} = \frac{5}{6} \cdot \frac{6}{5} \cdot 9$ Commutative (×)

$= 1 \cdot 9$ Multiplicative inverses: $\frac{5}{6} \cdot \frac{6}{5} = 1$

$= 9$ Multiplicative identity

Check

Josefina needs to evaluate $\frac{7}{3} \cdot 2 \cdot 3$. She wants to use the Commutative Property to more easily evaluate the expression. What should she do first? $\frac{7}{3} \cdot 3$

Example 9 Evaluate Using the Associative and Commutative Properties

GROCERIES **Jade buys groceries once each week, and she wants to calculate how much she spent on groceries over a 4-week period. Evaluate the expression to find how much Jade spent at the grocery store.**

Groceries	
Week	Amount Spent ($)
1	32
2	27
3	28
4	33

$32 + 27 + 28 + 33 = 32 + 28 + 27 + 33$ Step 1

$= (32 + 28) + (27 + 33)$ Step 2

$= 60 + 60$ Step 3

$= 120$ Step 4

Jade spent $120 on groceries over the 4-week period.

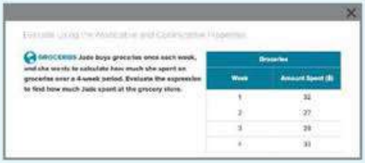
Check

Use the Commutative and Associative Properties to evaluate $\frac{5}{9} \cdot 9 \cdot \frac{9}{5} \cdot 4$. 36

Go Online You can complete an Extra Example online.

30 **Module 1** • Expressions

Interactive Presentation



Example 9

TYPE

Students identify when the Commutative and Associative Properties were used.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 8 Evaluate Using the Commutative Property

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the Commutative Property in this example.

Questions for Mathematical Discourse

AL When multiplying without a calculator, what is one way to make multiplying easier? Sample answer: Multiply things in a different order.

OL Is there a pair of numbers that multiply to a whole number? Explain. Yes; sample answer: $\frac{5}{6}$ and $\frac{6}{5}$ are reciprocals so they multiply to 1.

BL If the expression had been $\frac{5}{6} \cdot 9 \cdot \frac{6}{9}$, would you still use the Commutative Property to evaluate? Explain. No; sample answer: Because the fractions are not reciprocals, it does not make sense to switch 9 and $\frac{6}{9}$. Instead, the 9 and $\frac{6}{9}$ should be grouped to simplify to 6.

Example 9 Evaluate Using the Associative and Commutative Properties

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL In Step 1, what is the difference between the left side of the equation and the right side? The 27 and 28 changed places.

OL How does the expression in Step 2 differ from the right side of Step 1? Parentheses were added to indicate which additions should be performed first.

BL Why were the numbers grouped the way they were in Step 2? Sample answer: The numbers 32 and 28 add to a multiple of 10 as do the numbers 27 and 33.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Putting It All Together

Go Online to have students practice what they have learned about properties of real numbers in the Putting It All Together for Lessons 1-1 through 1-3.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–34
2	exercises that use a variety of skills from this lesson	35–48
3	exercises that emphasize higher-order and critical-thinking skills	49–56

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–47 odd, 49–56
- Extension: Properties of Operations
- **ALEKS** Properties of Real Numbers

OL

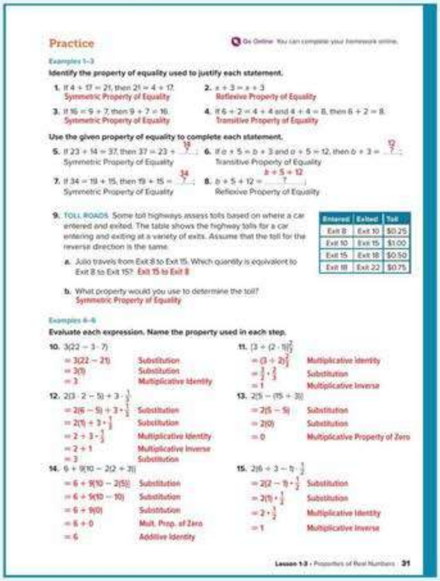
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–55 odd
- Remediation, Review Resources: Subtract Rational Numbers
- Personal Tutors
- BrainPOP Video: Commutative Property, Associative Property
- Extra Examples 1–9
- **ALEKS** Addition and Subtraction with Fractions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–33 odd
- Remediation, Review Resources: Subtract Rational Numbers
- *Quick Review Math Handbook*: Properties of Numbers
- Arrive**MATH** Take Another Look
- **ALEKS** Addition and Subtraction with Fractions



Practice

Go Online You can complete your homework online.

Examples 1–3

Identify the property of equality used to justify each statement.

1. If $4 + 17 = 21$, then $21 = 4 + 17$. Symmetric Property of Equality
2. $x + 3 = x + 3$ Reflexive Property of Equality
3. If $16 = 9 + 7$, then $9 + 7 = 16$. Symmetric Property of Equality
4. If $6 + 2 = 4 + 4$ and $4 + 4 = 8$, then $6 + 2 = 8$. Transitive Property of Equality

Use the given property of equality to complete each statement.

5. If $23 + 14 = 37$, then $37 = 23 +$ __?__ 14; Symmetric Property of Equality
6. If $a + 5 = b + 3$ and $a + 5 = 12$, then $b + 3 =$ __?__ 12; Transitive Property of Equality
7. If $34 = 19 + 15$, then $19 + 15 =$ __?__ 34; Symmetric Property of Equality
8. $b + 5 + 12 =$ __?__ $b + 5 + 12$; Reflexive Property of Equality

9. TOLL ROADS Some toll highways assess tolls based on where a car entered and exited. The table shows the highway tolls for a car entering and exiting at a variety of exits. Assume that the toll for the reverse direction is the same.

Entered	Exited	Toll
Exit 8	Exit 10	$0.25
Exit 10	Exit 15	$1.00
Exit 15	Exit 18	$0.50
Exit 18	Exit 22	$0.75

a. Julio travels from Exit 8 to Exit 15. Which quantity is equivalent to Exit 8 to Exit 15? Exit 15 to Exit 8

b. What property would you use to determine the toll? Symmetric Property of Equality

Examples 4–6

Evaluate each expression. Name the property used in each step.

10. $3(22 - 3 \cdot 7)$
$= 3(22 - 21)$ Substitution
$= 3(1)$ Substitution
$= 3$ Multiplicative Identity

11. $[3 \div (2 \cdot 1)]\frac{2}{3}$
$= (3 \div 2)\frac{2}{3}$ Multiplicative Identity
$= \frac{3}{2} \cdot \frac{2}{3}$ Substitution
$= 1$ Multiplicative Inverse

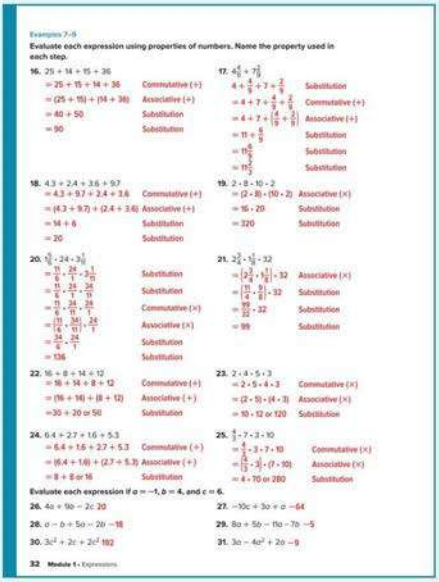
12. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$
$= 2(6 - 5) + 3 \cdot \frac{1}{3}$ Substitution
$= 2(1) + 3 \cdot \frac{1}{3}$ Substitution
$= 2 + 3 \cdot \frac{1}{3}$ Multiplicative Identity
$= 2 + 1$ Multiplicative Inverse
$= 3$ Substitution

13. $2(5 - (15 \div 3))$
$= 2(5 - 5)$ Substitution
$= 2(0)$ Substitution
$= 0$ Multiplicative Property of Zero

14. $6 + 9[10 - 2(2 + 3)]$
$= 6 + 9[10 - 2(5)]$ Substitution
$= 6 + 9(10 - 10)$ Substitution
$= 6 + 9(0)$ Substitution
$= 6 + 0$ Mult. Prop. of Zero
$= 6$ Additive Identity

15. $2(6 \div 3 - 1) \cdot \frac{1}{2}$
$= 2(2 - 1) \cdot \frac{1}{2}$ Substitution
$= 2(1) \cdot \frac{1}{2}$ Substitution
$= 2 \cdot \frac{1}{2}$ Multiplicative Identity
$= 1$ Multiplicative Inverse

Lesson 1-3 • Properties of Real Numbers 31



Examples 7–9

Evaluate each expression using properties of numbers. Name the property used in each step.

16. $25 + 14 + 15 + 36$
$= 25 + 15 + 14 + 36$ Commutative (+)
$= (25 + 15) + (14 + 36)$ Associative (+)
$= 40 + 50$ Substitution
$= 90$ Substitution

17. $4\frac{4}{9} + 7\frac{2}{9}$
$4 + \frac{4}{9} + 7 + \frac{2}{9}$ Substitution
$= 4 + 7 + \frac{4}{9} + \frac{2}{9}$ Commutative (+)
$= 4 + 7 + \left(\frac{4}{9} + \frac{2}{9}\right)$ Associative (+)
$= 11 + \frac{6}{9}$ Substitution
$= 11\frac{6}{9}$ Substitution
$= 11\frac{2}{3}$ Substitution

18. $4.3 + 2.4 + 3.6 + 9.7$
$= 4.3 + 9.7 + 2.4 + 3.6$ Commutative (+)
$= (4.3 + 9.7) + (2.4 + 3.6)$ Associative (+)
$= 14 + 6$ Substitution
$= 20$ Substitution

19. $2 \cdot 8 \cdot 10 \cdot 2$
$= (2 \cdot 8) \cdot (10 \cdot 2)$ Associative (×)
$= 16 \cdot 20$ Substitution
$= 320$ Substitution

20. $1\frac{5}{6} \cdot 24 \cdot 3\frac{1}{11}$
$= \frac{11}{6} \cdot \frac{24}{1} \cdot 3\frac{1}{11}$ Substitution
$= \frac{11}{6} \cdot \frac{24}{1} \cdot \frac{34}{11}$ Substitution
$= \frac{11}{6} \cdot \frac{34}{11} \cdot \frac{24}{1}$ Commutative (×)
$= \left(\frac{11}{6} \cdot \frac{34}{11}\right) \cdot \frac{24}{1}$ Associative (×)
$= \frac{34}{6} \cdot \frac{24}{1}$ Substitution
$= 136$ Substitution

21. $2\frac{3}{4} \cdot 1\frac{1}{8} \cdot 32$
$= \left(2\frac{3}{4} \cdot 1\frac{1}{8}\right) \cdot 32$ Associative (×)
$= \left(\frac{11}{4} \cdot \frac{9}{8}\right) \cdot 32$ Substitution
$= \frac{99}{32} \cdot 32$ Substitution
$= 99$ Substitution

22. $16 + 8 + 14 + 12$
$= 16 + 14 + 8 + 12$ Commutative (+)
$= (16 + 14) + (8 + 12)$ Associative (+)
$= 30 + 20$ or 50 Substitution

23. $2 \cdot 4 \cdot 5 \cdot 3$
$= 2 \cdot 5 \cdot 4 \cdot 3$ Commutative (×)
$= (2 \cdot 5) \cdot (4 \cdot 3)$ Associative (×)
$= 10 \cdot 12$ or 120 Substitution

24. $6.4 + 2.7 + 1.6 + 5.3$
$= 6.4 + 1.6 + 2.7 + 5.3$ Commutative (+)
$= (6.4 + 1.6) + (2.7 + 5.3)$ Associative (+)
$= 8 + 8$ or 16 Substitution

25. $\frac{4}{3} \cdot 7 \cdot 3 \cdot 10$
$= \frac{4}{3} \cdot 3 \cdot 7 \cdot 10$ Commutative (×)
$= \left(\frac{4}{3} \cdot 3\right) \cdot (7 \cdot 10)$ Associative (×)
$= 4 \cdot 70$ or 280 Substitution

Evaluate each expression if $a = -1$, $b = 4$, and $c = 6$.

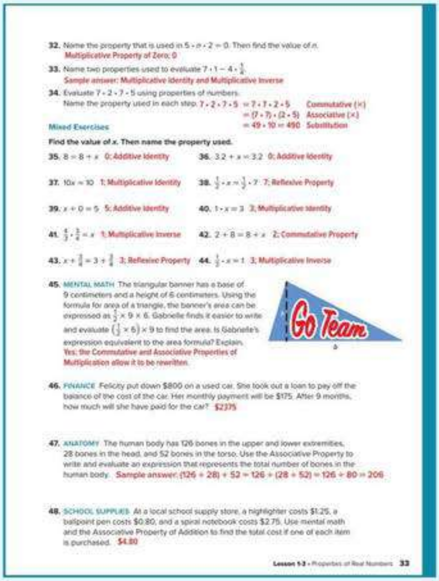
26. $4a + 9b - 2c$ 20
27. $-10c + 3a + a$ −64
28. $a - b + 5a - 2b$ −18
29. $8a + 5b - 11a - 7b$ −5
30. $3c^2 + 2c + 2c^2$ 192
31. $3a - 4a^2 + 2b$ −9

32 Module 1 • Expressions

3 REFLECT AND PRACTICE

A.SSE.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



32. Name the property that is used in $5 \cdot n \cdot 2 = 0$. Then find the value of n. Multiplicative Property of Zero; 0

33. Name two properties used to evaluate $7 \cdot 1 - 4 \cdot \frac{1}{4}$. Sample answer: Multiplicative Identity and Multiplicative Inverse

34. Evaluate $7 \cdot 2 \cdot 7 \cdot 5$ using properties of numbers. Name the property used in each step.

$7 \cdot 2 \cdot 7 \cdot 5 = 7 \cdot 7 \cdot 2 \cdot 5$ Commutative (×)
$= (7 \cdot 7) \cdot (2 \cdot 5)$ Associative (×)
$= 49 \cdot 10 = 490$ Substitution

Mixed Exercises

Find the value of x. Then name the property used.

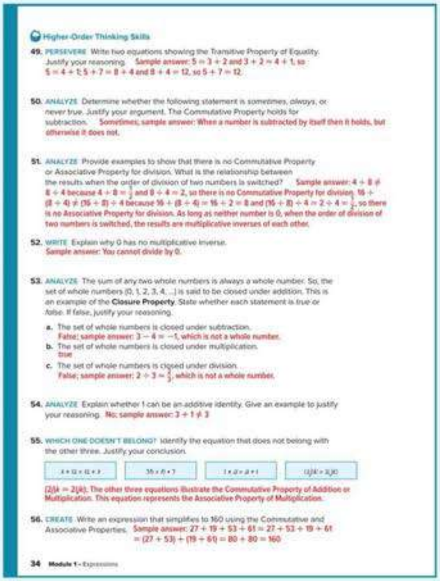
35. $8 = 8 + x$ 0; Additive Identity
36. $3.2 + x = 3.2$ 0; Additive Identity
37. $10x = 10$ 1; Multiplicative Identity
38. $\frac{1}{2} \cdot x = \frac{1}{2} \cdot 7$ 7; Reflexive Property
39. $x + 0 = 5$ 5; Additive Identity
40. $1 \cdot x = 3$ 3; Multiplicative Identity
41. $\frac{4}{3} \cdot \frac{3}{4} = x$ 1; Multiplicative Inverse
42. $2 + 8 = 8 + x$ 2; Commutative Property
43. $x + \frac{3}{4} = 3 + \frac{3}{4}$ 3; Reflexive Property
44. $\frac{1}{3} \cdot x = 1$ 3; Multiplicative Inverse

45. MENTAL MATH The triangular banner has a base of 9 centimeters and a height of 6 centimeters. Using the formula for area of a triangle, the banner's area can be expressed as $\frac{1}{2} \times 9 \times 6$. Gabrielle finds it easier to write and evaluate $\left(\frac{1}{2} \times 6\right) \times 9$ to find the area. Is Gabrielle's expression equivalent to the area formula? Explain. Yes; the Commutative and Associative Properties of Multiplication allow it to be rewritten.

46. FINANCE Felicity put down $800 on a used car. She took out a loan to pay off the balance of the cost of the car. Her monthly payment will be $175. After 9 months, how much will she have paid for the car? $2375

47. ANATOMY The human body has 126 bones in the upper and lower extremities, 28 bones in the head, and 52 bones in the torso. Use the Associative Property to write and evaluate an expression that represents the total number of bones in the human body. Sample answer: $(126 + 28) + 52 = 126 + (28 + 52) = 126 + 80 = 206$

48. SCHOOL SUPPLIES At a local school supply store, a highlighter costs $1.25, a ballpoint pen costs $0.80, and a spiral notebook costs $2.75. Use mental math and the Associative Property of Addition to find the total cost if one of each item is purchased. $4.80



Higher-Order Thinking Skills

49. PERSEVERE Write two equations showing the Transitive Property of Equality. Justify your reasoning. Sample answer: $5 = 3 + 2$ and $3 + 2 = 4 + 1$, so $5 = 4 + 1$; $5 + 7 = 8 + 4$ and $8 + 4 = 12$, so $5 + 7 = 12$.

50. ANALYZE Determine whether the following statement is sometimes, always, or never true. Justify your argument. The Commutative Property holds for subtraction. Sometimes; sample answer: When a number is subtracted by itself then it holds, but otherwise it does not.

51. ANALYZE Provide examples to show that there is no Commutative Property or Associative Property for division. What is the relationship between the results when the order of division of two numbers is switched? Sample answer: $4 \div 8 \neq 8 \div 4$ because $4 \div 8 = \frac{1}{2}$ and $8 \div 4 = 2$, so there is no Commutative Property for division. $16 \div (8 \div 4) \neq (16 \div 8) \div 4$ because $16 \div (8 \div 4) = 16 \div 2 = 8$ and $(16 \div 8) \div 4 = 2 \div 4 = \frac{1}{2}$, so there is no Associative Property for division. As long as neither number is 0, when the order of division of two numbers is switched, the results are multiplicative inverses of each other.

52. WRITE Explain why 0 has no multiplicative inverse. Sample answer: You cannot divide by 0.

53. ANALYZE The sum of any two whole numbers is always a whole number. So, the set of whole numbers {0, 1, 2, 3, 4, ...} is said to be closed under addition. This is an example of the **Closure Property**. State whether each statement is true or false. If false, justify your reasoning.
   a. The set of whole numbers is closed under subtraction. False; sample answer: $3 - 4 = -1$, which is not a whole number.
   b. The set of whole numbers is closed under multiplication. true
   c. The set of whole numbers is closed under division. False; sample answer: $2 \div 3 = \frac{2}{3}$, which is not a whole number.

54. ANALYZE Explain whether 1 can be an additive identity. Give an example to justify your reasoning. No; sample answer: $3 + 1 \neq 3$

55. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong with the other three. Justify your conclusion.

$x + 12 = 12 + x$	[illegible]	$1 \times a = a \times 1$	$(2j)k = 2(jk)$

$(2j)k = 2(jk)$; The other three equations illustrate the Commutative Property of Addition or Multiplication. This equation represents the Associative Property of Multiplication.

56. CREATE Write an expression that simplifies to 160 using the Commutative and Associative Properties. Sample answer: $27 + 19 + 53 + 61 = 27 + 53 + 19 + 61$
$= (27 + 53) + (19 + 61) = 80 + 80 = 160$

Lesson 1-4

Distributive Property

A.SSE.1a, A.SSE.2

LESSON GOAL

Students simplify expressions by using the Distributive Property.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore:

- Using Rectangles with the Distributive Property
- Modeling the Distributive Property

Develop:

Distributive Property with Numerical Expressions

- Use the Distributive Property
- Mental Math

Distributive Property with Algebraic Expressions

- Distribute an Algebraic Expression from the Left
- Distribute an Algebraic Expression from the Right
- Combine Like Terms
- Write and Simplify Expressions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Apply Rational Number Operations	●	●		●
Extension: Mayan Numerals		●	●	●

Language Development Handbook

Assign page 4 of the *Language Development Handbook* to help your students build mathematical language related to simplifying expressions using the Distributive Property.

ELL You can use the tips and suggestions on page T4 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.1a Identify parts of an expression, such as terms, factors, and coefficients.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students applied the properties of real numbers to simplify expressions.
6.EE.3, A.SSE.2

Now

Students use the Distributive Property to simplify expressions.
A.SSE.1a, A.SSE.2

Next

Students will apply the Distributive Property to construct and solve equations in one variable.
A.CED.1

Rigor

The Three Pillars of Rigor

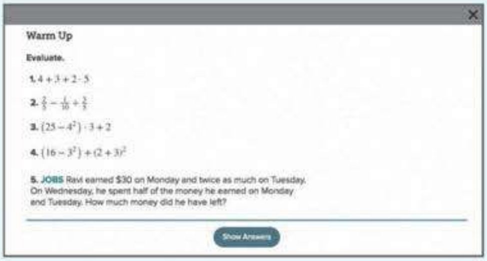
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their understanding of expressions to build fluency with using the Distributive Property to simplify expressions. They apply their understanding of the Distributive Property by solving real-world problems.

Mathematical Background

The Distributive Property can be used to evaluate and simplify expressions. The property permits a factor outside the parentheses to be distributed to each term inside the parentheses. When the Distributive Property is applied to algebraic expressions, the coefficients of like terms can be combined and the expressions can be simplified.

Interactive Presentation



Warm Up

Evaluate.

1. $4 + 3 + 2 \cdot 5$

2. $\frac{2}{5} - \frac{1}{10} \div \frac{3}{5}$

3. $(25 - 4^2) \cdot 3 + 2$

4. $(16 - 3^2) + (2 + 3)^2$

5. JOBS Ravi earned $30 on Monday and twice as much on Tuesday. On Wednesday, he spent half of the money he earned on Monday and Tuesday. How much money did he have left?

Warm Up

Warm Up

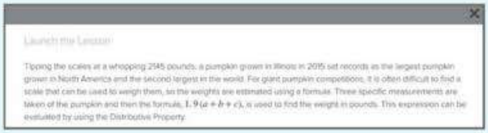
Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- using order of operations

Answers:

1. 17
2. $\frac{7}{30}$
3. 29
4. 32
5. $45



Launch the Lesson

Tipping the scales at a whopping 2145 pounds, a pumpkin grown in Illinois in 2015 set records as the largest pumpkin grown in North America and the second largest in the world. For giant pumpkin competitions, it is often difficult to find a scale that can be used to weigh them, so the weights are estimated using a formula. Three specific measurements are taken of the pumpkin and then the formula, $1.9(a + b + c)$, is used to find the weight in pounds. This expression can be evaluated by using the Distributive Property.

Launch

Launch the Lesson

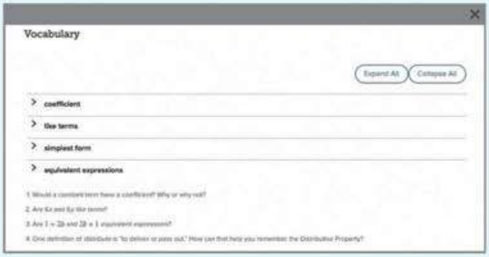
Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to analyze the given information about giant pumpkin competitions to evaluate an expression by using the Distributive Property.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.



Vocabulary

- coefficient
- like terms
- simplest form
- equivalent expressions

Today's Vocabulary

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Using Rectangles with the Distributive Property

Objective

Students use a sketch to model the Distributive Property.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use algebra tiles. Work with students to explore and deepen their understanding of the Distributive Property.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

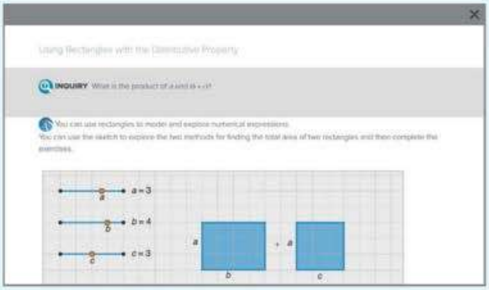
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students will use a sketch to explore two methods for finding the area of a rectangle, and then fill in the provided table with specific results. The guiding exercises will lead students to see the Distributive Property in action. Then, students will answer the Inquiry Question.

(continued on next page)

Interactive Presentation



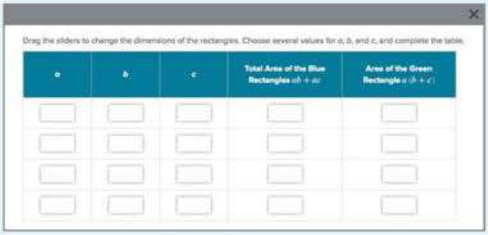
Using Rectangles with the Distributive Property

INQUIRY What is the product of a and $b + c$?

You can use rectangles to model and explore numerical expressions.

You can use the sketch to explore the two methods for finding the total area of two rectangles and then complete the exercises.

Explore



Drag the sliders to change the dimensions of the rectangles. Choose several values for a, b, and c, and complete the table.

a	b	c	Total Area of the Blue Rectangles $ab + ac$	Area of the Green Rectangle $a(b + c)$

Explore

WEB SKETCHPAD

Students use the sketch activity to complete an activity in which they explore finding the area of two rectangles.

TYPE

Students fill in a table with different values of a, b, and c from the sketch and then answer questions about the areas of rectangles.

Interactive Presentation

INQUIRY What is the product of a and $(b + c)$?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Rectangles with the Distributive Property (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What do you notice about the total area in each case? In each case, the combined area of the two rectangles is the same.

Inquiry

What is the product of a and $(b + c)$? $ab + ac$

Go Online to find additional teaching notes and answers for the Explore activity.

Explore Modeling the Distributive Property

Objective

Students use algebra tiles to model the Distributive Property.

MP Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in this Explore.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

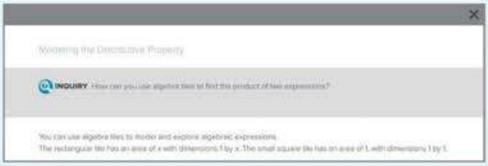
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students will use algebra tiles to model the expression 4(x + 2). The guiding exercises will lead students to see the Distributive Property in action. Then, students will answer the Inquiry Question.

(continued on next page)

Interactive Presentation

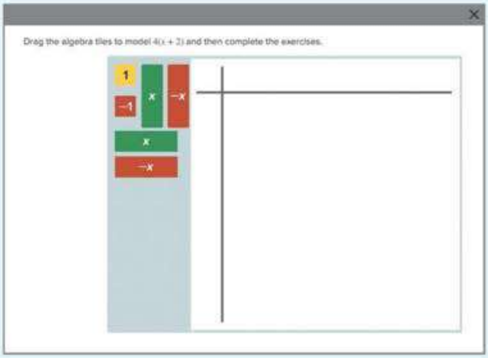


Modeling the Distributive Property

INQUIRY How can you use algebra tiles to find the product of two expressions?

You can use algebra tiles to model and explore algebraic expressions.
The rectangular tile has an area of x with dimensions 1 by x. The small square tile has an area of 1 with dimensions 1 by 1.

Explore



Drag the algebra tiles to model 4(x + 2) and then complete the exercises.

Explore

DRAG & DROP

Students drag and drop algebra tiles to model an expression.

TYPE

Students answer questions about the rectangles formed with the algebra tiles.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Modeling the Distributive Property (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Does it matter which expression you use as the length and which expression you use as the width? Explain. No; sample answer: Because you are multiplying to find the area, switching the expressions for the length and width will product the same area.

Inquiry

How can you use algebra tiles to find the product of two expressions? Sample answer: Form a rectangle with the algebra tiles where one expression is the length of the rectangle and one is the width. Then find the area of the rectangle.

Go Online to find additional teaching notes and answers to the Explore activity.

Learn Distributive Property with Numerical Expressions

Objective

Students identify and evaluate equivalent numerical expressions by using the Distributive Property.

MP Teaching the Mathematical Practices

7 Use the Distributive Property Point out that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

What Students Are Learning

When the Distributive Property is used, equivalent expressions are generated; 5(1 + 6) is equivalent to 5(1) + 5(6). The Distributive Property combines addition or subtraction with multiplication.

Common Misconception

Many times students will only multiply the coefficient of the sum or difference to the first term and ignore the second term. Reinforce to students that the Distributive Property is when the coefficient is multiplied to all terms of a sum or difference.

DIFFERENTIATE

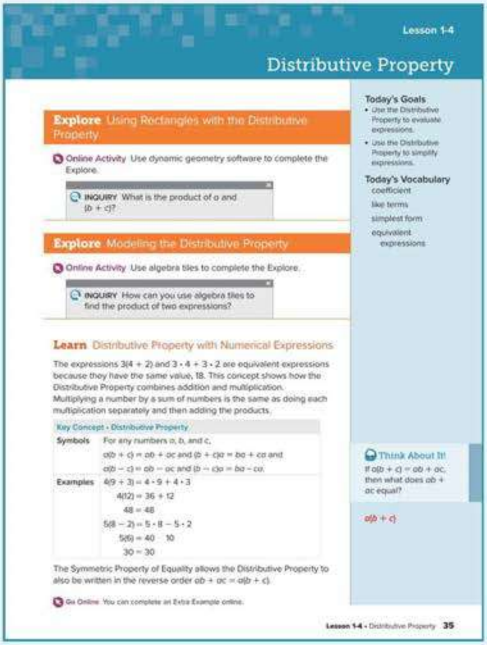
Language Development Activity ELL

Entering Before students read the lesson, provide examples or illustrations to introduce each vocabulary word. Have students repeat each word and point to the visual representation as you review vocabulary.

Emerging or Developing Have partners make and use flash cards to check each other's pronunciation and understanding of vocabulary.

Expanding Have students scan the lesson for content vocabulary words in context. Help them pronounce the vocabulary words correctly. Discuss vocabulary meanings with them.

Bridging After reading each example of the lesson, use an Interactive Question-Response to discuss it. Have students record the main idea and details of the paragraphs in their notes.



Lesson 1-4

Distributive Property

Explore Using Rectangles with the Distributive Property

Online Activity Use dynamic geometry software to complete the Explore.

INQUIRY What is the product of a and $(b + c)$?

Explore Modeling the Distributive Property

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you use algebra tiles to find the product of two expressions?

Learn Distributive Property with Numerical Expressions

The expressions $3(4 + 2)$ and $3 \cdot 4 + 3 \cdot 2$ are equivalent expressions because they have the same value, 18. This concept shows how the Distributive Property combines addition and multiplication. Multiplying a number by a sum of numbers is the same as doing each multiplication separately and then adding the products.

Key Concept • Distributive Property

Symbols	For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.
Examples	$4(9 + 3) = 4 \cdot 9 + 4 \cdot 3$ $4(12) = 36 + 12$ $48 = 48$ $5(8 - 2) = 5 \cdot 8 - 5 \cdot 2$ $5(6) = 40 - 10$ $30 = 30$

The Symmetric Property of Equality allows the Distributive Property to also be written in the reverse order $ab + ac = a(b + c)$.

Go Online You can complete an Extra Example online.

Today's Goals
- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify expressions.

Today's Vocabulary
coefficient
like terms
simplest form
equivalent expressions

Think About It!
If $a(b + c) = ab + ac$, then what does $ab + ac$ equal?

$a(b + c)$

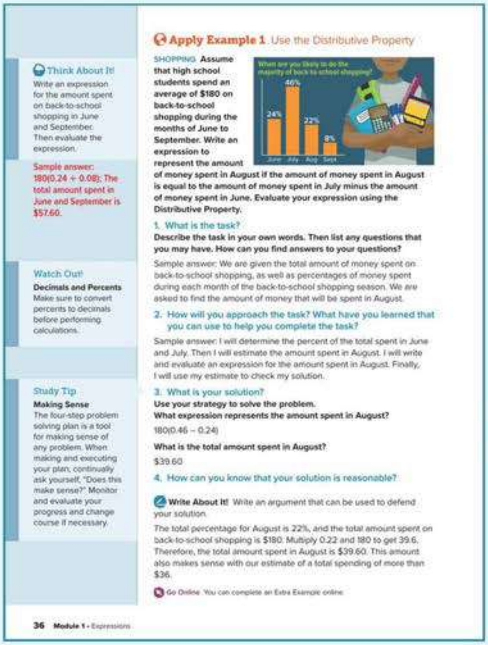
Lesson 1-4 • Distributive Property 35

Interactive Presentation

Learn

TYPE

Students consider how the Symmetric Property of Equality applies to the Distributive Property.



Think About It! Write an expression for the amount spent on back-to-school shopping in June and September. Then evaluate the expression.

Sample answer: 180(0.24 + 0.08); The total amount spent in June and September is $57.60.

Watch Out!

Decimals and Percents Make sure to convert percents to decimals before performing calculations.

Study Tip

Making Sense The four-step problem solving plan is a tool for making sense of any problem. When making and executing your plan, continually ask yourself, "Does this make sense?" Monitor and evaluate your progress and change course if necessary.

Apply Example 1 Use the Distributive Property

SHOPPING **Assume that high school students spend an average of $180 on back-to-school shopping during the months of June to September. Write an expression to represent the amount of money spent in August if the amount of money spent in August is equal to the amount of money spent in July minus the amount of money spent in June. Evaluate your expression using the Distributive Property.**

When are you likely to do the majority of back-to-school shopping?
24%
46%
22%
8%
June
July
Aug
Sept

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: We are given the total amount of money spent on back-to-school shopping, as well as percentages of money spent during each month of the back-to-school shopping season. We are asked to find the amount of money that will be spent in August.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will determine the percent of the total spent in June and July. Then I will estimate the amount spent in August. I will write and evaluate an expression for the amount spent in August. Finally, I will use my estimate to check my solution.

3. What is your solution?

Use your strategy to solve the problem.

What expression represents the amount spent in August?

180(0.46 − 0.24)

What is the total amount spent in August?

$39.60

4. How can you know that your solution is reasonable?

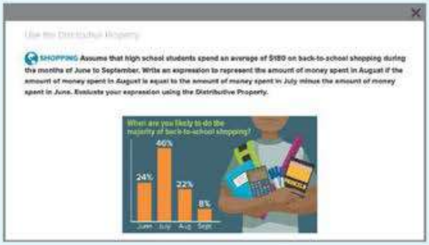
Write About It! Write an argument that can be used to defend your solution.

The total percentage for August is 22%, and the total amount spent on back-to-school shopping is $180. Multiply 0.22 and 180 to get 39.6. Therefore, the total amount spent in August is $39.60. This amount also makes sense with our estimate of a total spending of more than $36.

Go Online You can complete an Extra Example online.

36 **Module 1** • Expressions

Interactive Presentation



Use the Distributive Property

SHOPPING **Assume that high school students spend an average of $180 on back-to-school shopping during the months of June to September. Write an expression to represent the amount of money spent in August if the amount of money spent in August is equal to the amount of money spent in July minus the amount of money spent in June. Evaluate your expression using the Distributive Property.**

When are you likely to do the majority of back-to-school shopping?
24%
46%
22%
8%
June
July
Aug
Sept

Example 1

TAP

Students move through steps to solve a real-world problem.

TYPE

Students write and evaluate an expression for the back-to-school shopping in June and September.

Apply Example 1 Use the Distributive Property

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How is the amount spent in each month calculated?
- In what form should the percentages be written in order to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Mental Math

MP Teaching the Mathematical Practices

7 Use the Distributive Property Point out that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

Questions for Mathematical Discourse

AL In part **a**, what number is close to 99 that would be easier to multiply with using mental math? 100 In part **b**, what number is close to 1002 that would be easier to multiply with using mental math? 1000

OL In part **a**, why does it make sense to subtract 5 and not 1? Sample answer: Because you are multiplying by a number that is one less than 100, it makes sense to remove one multiple of 5 from 500.

BL Would you apply the Distributive Property to solve 4 • 1200? Why? No; sample answer: Because 1200 can just be multiplied as 12 hundreds.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

IF students are struggling to apply the Distributive Property with numerical expressions,

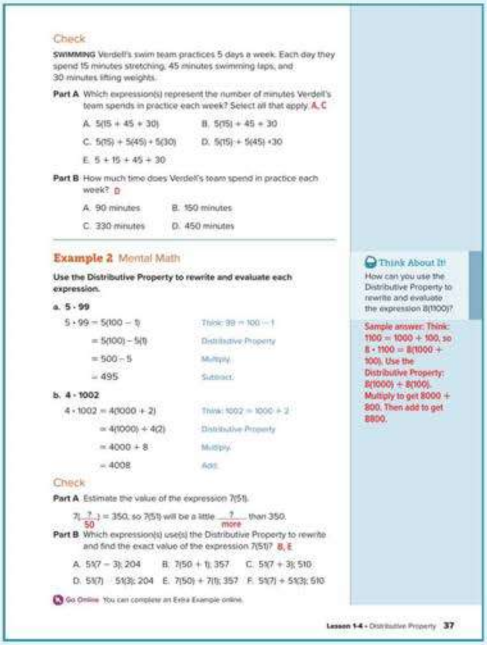
THEN let students model the expression using algebra tiles until they can successfully evaluate using the Distributive Property.

Enrichment Activity **BL**

Explain how the Distributive Property can be used to find the product 7 × 435 without the use of a calculator.

Sample answer: I can break 435 into a sum of its parts and then use the Distributive Property:

$$\begin{aligned} 7(400 + 30 + 5) &= 7(400) + 7(30) + 7(5) \\ &= 2800 + 210 + 35 \\ &= 3010 + 35 \\ &= 3045 \end{aligned}$$



Check

SWIMMING Verdell's swim team practices 5 days a week. Each day they spend 15 minutes stretching, 45 minutes swimming laps, and 30 minutes lifting weights.

Part A Which expression(s) represent the number of minutes Verdell's team spends in practice each week? Select all that apply. A, C

A. 5(15 + 45 + 30)
B. 5(15) + 45 + 30
C. 5(15) + 5(45) + 5(30)
D. 5(15) + 5(45) +30
E. 5 + 15 + 45 + 30

Part B How much time does Verdell's team spend in practice each week? D

A. 90 minutes
B. 150 minutes
C. 330 minutes
D. 450 minutes

Example 2 Mental Math

Use the Distributive Property to rewrite and evaluate each expression.

a. 5 · 99

$$\begin{aligned} 5 \cdot 99 &= 5(100 - 1) && \text{Think: } 99 = 100 - 1 \\ &= 5(100) - 5(1) && \text{Distributive Property} \\ &= 500 - 5 && \text{Multiply.} \\ &= 495 && \text{Subtract.} \end{aligned}$$

b. 4 · 1002

$$\begin{aligned} 4 \cdot 1002 &= 4(1000 + 2) && \text{Think: } 1002 = 1000 + 2 \\ &= 4(1000) + 4(2) && \text{Distributive Property} \\ &= 4000 + 8 && \text{Multiply.} \\ &= 4008 && \text{Add.} \end{aligned}$$

Check

Part A Estimate the value of the expression 7(51).

$7(\underset{50}{\underline{\ ?\ }}) = 350$, so 7(51) will be a little $\underset{\text{more}}{\underline{\ ?\ }}$ than 350.

Part B Which expression(s) use(s) the Distributive Property to rewrite and find the exact value of the expression 7(51)? B, E

A. 51(7 − 3); 204
B. 7(50 + 1); 357
C. 51(7 + 3); 510
D. 51(7) − 51(3); 204
E. 7(50) + 7(1); 357
F. 51(7) + 51(3); 510

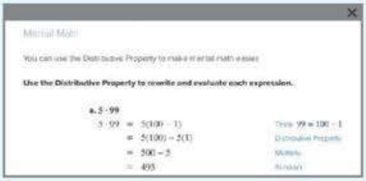
Go Online You can complete an Extra Example online.

Think About It! How can you use the Distributive Property to rewrite and evaluate the expression 8(1100)?

Sample answer: Think: 1100 = 1000 + 100, so 8 • 1100 = 8(1000 + 100). Use the Distributive Property: 8(1000) + 8(100). Multiply to get 8000 + 800. Then add to get 8800.

Lesson 1-4 • Distributive Property 37

Interactive Presentation



Mental Math

Use the Distributive Property to rewrite and evaluate each expression.

a. 5 · 99

$$\begin{aligned} 5 \cdot 99 &= 5(100 - 1) && \text{Think: } 99 = 100 - 1 \\ &= 5(100) - 5(1) \\ &= 500 - 5 \\ &= 495 \end{aligned}$$

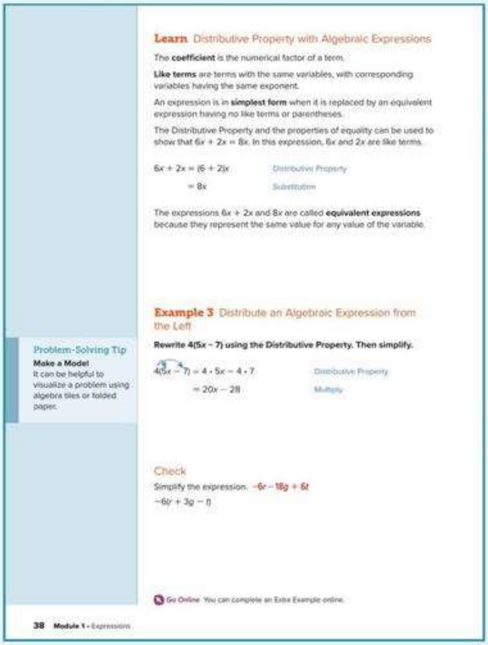
Example 2

TYPE

Students describe how to use the Distributive Property to evaluate a product of two numbers.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Learn Distributive Property with Algebraic Expressions

The **coefficient** is the numerical factor of a term.

Like terms are terms with the same variables, with corresponding variables having the same exponent.

An expression is in **simplest form** when it is replaced by an equivalent expression having no like terms or parentheses.

The Distributive Property and the properties of equality can be used to show that $6x + 2x = 8x$. In this expression, $6x$ and $2x$ are like terms.

$6x + 2x = (6 + 2)x$ Distributive Property
$= 8x$ Substitution

The expressions $6x + 2x$ and $8x$ are called **equivalent expressions** because they represent the same value for any value of the variable.

Example 3 Distribute an Algebraic Expression from the Left

Rewrite $4(5x - 7)$ using the Distributive Property. Then simplify.

$4(5x - 7) = 4 \cdot 5x - 4 \cdot 7$ Distributive Property
$= 20x - 28$ Multiply

Problem-Solving Tip
Make a Model
It can be helpful to visualize a problem using algebra tiles or folded paper.

Check
Simplify the expression. $-6r - 18g + 6t$
$-6(r + 3g - t)$

Go Online You can complete an Extra Example online.

38 Module 1 • Expressions

Interactive Presentation

Distributive Property with Algebraic Expressions

Learn

TAP

Students tap on each word or phrase to see an example and definition.

TYPE

Students determine if another student's claim is correct.

Learn Distributive Property with Algebraic Expressions

Objective

Students identify and simplify equivalent algebraic expressions by using the Distributive Property.

MP Teaching the Mathematical Practices

7 Use the Distributive Property Point out that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

Common Misconception

Students often believe that as long as variables are the same, they are like terms regardless of exponents. This causes problems not only in this lesson, but in future lessons as well. Reinforce that like terms have the same variables, but the corresponding variables must have the same exponents. The terms $5xy$ and $3xy^2$ are not alike, but $5xy^2$and $3xy$ are.

Example 3 Distribute an Algebraic Expression from the Left

MP Teaching the Mathematical Practices

7 Use the Distributive Property Point out that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

Questions for Mathematical Discourse

AL Which value is being distributed? 4

OL To which term does the subtraction sign belong? with the 7

BL Why is the expression $20x - 7$ not equivalent to the expression in the example? Sample answer: The 4 must be distributed to both terms in the difference, not just the first term.

Common Error

When distributing to a subtraction problem, many students will replace the subtraction sign with addition. Reinforce the rules of multiplying integers and that those rules are the only way signs can change.

Example 4 Distribute an Algebraic Expression from the Right

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. The Talk About It! feature asks students to respond to the arguments of others.

Questions for Mathematical Discourse

AL Does it matter that the 6 is to the right of the parentheses? Explain. No; sample answer: The 6 is multiplying the quantity inside the parentheses, and multiplication is commutative, so it doesn't matter whether it is in front of the parentheses or behind it.

OL Will the signs of the expression inside the parentheses change when 6 is distributed? Explain. No; sample answer: Since 6 is not a negative number, none of the signs will change.

BL How could the original expression be rewritten if a student does not like distributing from the right? $6(3y^2 + y - 8)$

Common Error

Students can get confused when the value to be distributed is on the right side of the parentheses, rather than the left. If students need to, they can rewrite the expression with the coefficient on the left.

Example 5 Combine Like T erms

Teaching the Mathematical Practices

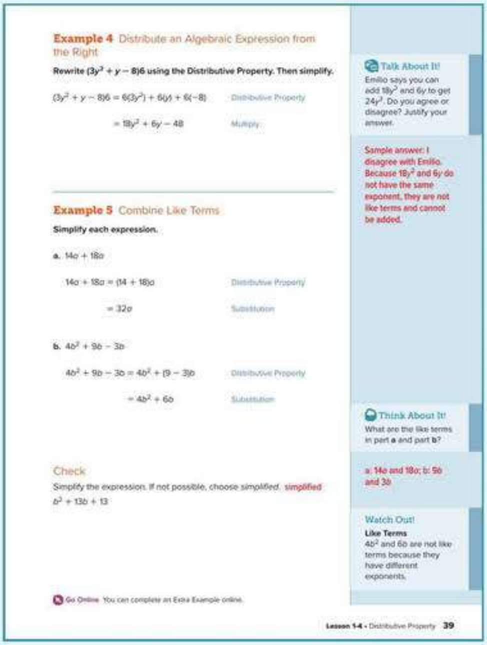
7 Use the Distributive Property Point out that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

Questions for Mathematical Discourse

AL How many terms are in part **a**? 2 part **b**? 3

OL In part **b**, why is $4b^2$ not a like term? because the exponent of b is different

BL How might algebra tiles help someone identify like terms in part **b**? Sample answer: Using square tiles for b^2 and rectangular tiles for the b terms allows you to see that the square tiles do not combine with the rectangular tiles.



Example 4 Distribute an Algebraic Expression from the Right

Rewrite $(3y^2 + y - 8)6$ using the Distributive Property. Then simplify.

$(3y^2 + y - 8)6 = 6(3y^2) + 6(y) + 6(-8)$ Distributive Property

$= 18y^2 + 6y - 48$ Multiply

Example 5 Combine Like Terms

Simplify each expression.

a. $14a + 18a$

$14a + 18a = (14 + 18)a$ Distributive Property

$= 32a$ Substitution

b. $4b^2 + 9b - 3b$

$4b^2 + 9b - 3b = 4b^2 + (9 - 3)b$ Distributive Property

$= 4b^2 + 6b$ Substitution

Check

Simplify the expression. If not possible, choose simplified. simplified

$b^2 + 13b + 13$

Go Online You can complete an Extra Example online.

Talk About It!

Emilio says you can add $18y^2$ and $6y$ to get $24y^3$. Do you agree or disagree? Justify your answer.

Sample answer: I disagree with Emilio. Because $18y^2$ and $6y$ do not have the same exponent, they are not like terms and cannot be added.

Think About It!

What are the like terms in part **a** and part **b**?

a: $14a$ and $18a$; b: $9b$ and $3b$

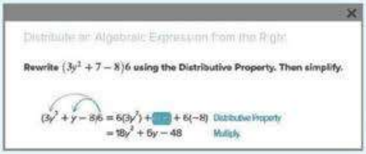
Watch Out!

Like Terms

$4b^2$ and $6b$ are not like terms because they have different exponents.

Lesson 1-4 • Distributive Property 39

Interactive Presentation



Distribute an Algebraic Expression from the Right

Rewrite $(3y^2 + 7 - 8)6$ using the Distributive Property. Then simplify.

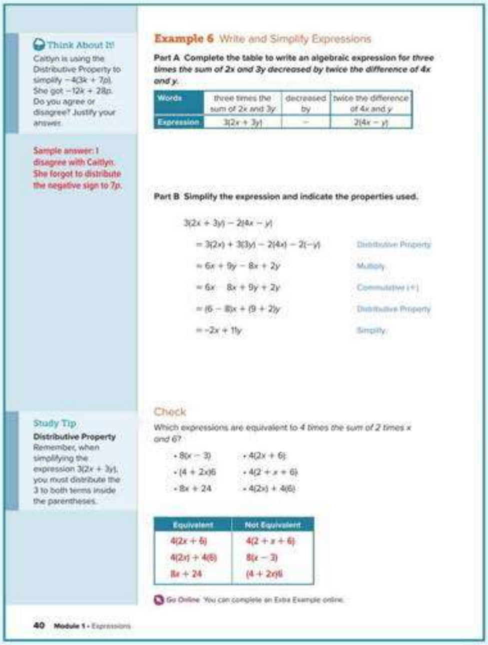
$(3y^2 + y - 8)6 = 6(3y^2) + \boxed{\quad} + 6(-8)$ Distributive Property

$= 18y^2 + 6y - 48$ Multiply

Example 4

TYPE

Students determine if another student's work is correct.



Think About It!
Caitlyn is using the Distributive Property to simplify $-4(3k + 7p)$. She got $-12k + 28p$. Do you agree or disagree? Justify your answer.

Sample answer: I disagree with Caitlyn. She forgot to distribute the negative sign to $7p$.

Example 6 Write and Simplify Expressions

Part A **Complete the table to write an algebraic expression for *three times the sum of 2x and 3y decreased by twice the difference of 4x and y*.**

Words	three times the sum of $2x$ and $3y$	decreased by	twice the difference of $4x$ and y
Expression	$3(2x + 3y)$	$-$	$2(4x - y)$

Part B **Simplify the expression and indicate the properties used.**

$3(2x + 3y) - 2(4x - y)$

$= 3(2x) + 3(3y) - 2(4x) - 2(-y)$ Distributive Property

$= 6x + 9y - 8x + 2y$ Multiply

$= 6x - 8x + 9y + 2y$ Commutative (+)

$= (6 - 8)x + (9 + 2)y$ Distributive Property

$= -2x + 11y$ Simplify

Study Tip
Distributive Property Remember, when simplifying the expression $3(2x + 3y)$, you must distribute the 3 to both terms inside the parentheses.

Check

Which expressions are equivalent to 4 times the sum of 2 times x and 6?

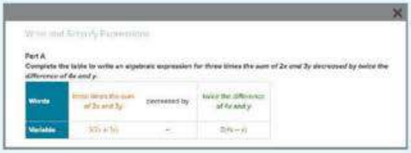
- $8(x - 3)$
- $(4 + 2x)6$
- $8x + 24$
- $4(2x + 6)$
- $4(2 + x + 6)$
- $4(2x) + 4(6)$

Equivalent	Not Equivalent
$4(2x + 6)$	$4(2 + x + 6)$
$4(2x) + 4(6)$	$8(x - 3)$
$8x + 24$	$(4 + 2x)6$

Go Online You can complete an Extra Example online.

40 Module 1 • Expressions

Interactive Presentation



Part A

Complete the table to write an algebraic expression for three times the sum of 2x and 3y decreased by twice the difference of 4x and y.

Words	three times the sum of $2x$ and $3y$	decreased by	twice the difference of $4x$ and y
Variable	$3(2x + 3y)$	$-$	$2(4x - y)$

Example 6

TYPE

Students determine if another student's work is correct.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 6 Write and Simplify Expressions

MP Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to respond to the arguments of others.

Questions for Mathematical Discourse

AL What does the word *sum* represent? addition What do *difference* and *decreased by* represent? subtraction

OL How do you know you need parentheses? because you are multiplying a constant by the sum or difference of two terms

BL Simplify the expression $4(a - 3b) - 3(2a + b)$ and indicate the properties used.

$4a - 12b - 6a - 3b$ Distributive Property

$= 4a - 6a - 12b - 3b$ Commutative Property

$= (4 - 6)a + (-12 - 3)b$ Distributive Property

$= -2a - 15b$ Simplify.

Common Error

With large algebraic expressions, students can make careless errors in translating, like forgetting parentheses or exponents. Encourage students to highlight or underline all words that imply a math operation or symbol to decrease the likelihood of a mistake.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

IF students are struggling to apply the Distributive Property with algebraic expressions,

THEN have students draw an arrow from the multiplier to each term in the parentheses as a visual reminder to multiply all terms.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–56
2	exercises that use a variety of skills from this lesson	57–67
3	exercises that emphasize higher-order and critical-thinking skills	68–72

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–67 odd, 68–72
- Extension: Mayan Numerals
- **ALEKS** Properties of Real Numbers

OL

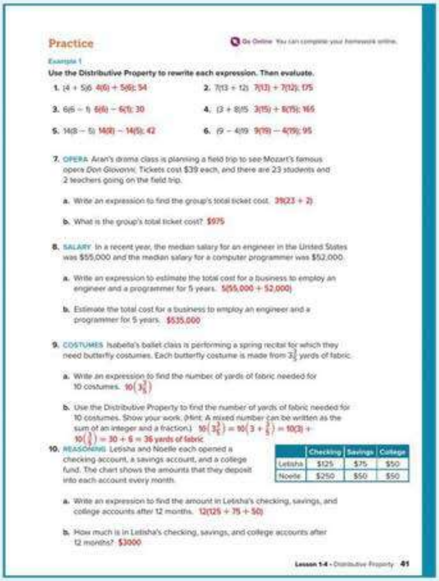
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–71 odd
- Remediation, Review Resources: Apply Rational Number Operations
- Personal Tutors
- BrainPOP Video: Distributive Property
- Extra Examples 1–6
- **ALEKS** Other Topics Available: Fractions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–55 odd
- Remediation, Review Resources: Apply Rational Number Operations
- *Quick Review Math Handbook*: Properties of Numbers
- Arrive**MATH** Take Another Look
- **ALEKS** Other Topics Available: Fractions



Practice

Go Online You can complete your homework online.

Example 1

Use the Distributive Property to rewrite each expression. Then evaluate.

1. $(4 + 5)6$ $4(6) + 5(6)$; 54
2. $7(13 + 12)$ $7(13) + 7(12)$; 175
3. $6(6 - 1)$ $6(6) - 6(1)$; 30
4. $(3 + 8)15$ $3(15) + 8(15)$; 165
5. $14(8 - 5)$ $14(8) - 14(5)$; 42
6. $(9 - 4)19$ $9(19) - 4(19)$; 95

7. OPERA Aran's drama class is planning a field trip to see Mozart's famous opera *Don Giovanni*. Tickets cost \$39 each, and there are 23 students and 2 teachers going on the field trip.
   a. Write an expression to find the group's total ticket cost. $39(23 + 2)$
   b. What is the group's total ticket cost? \$975

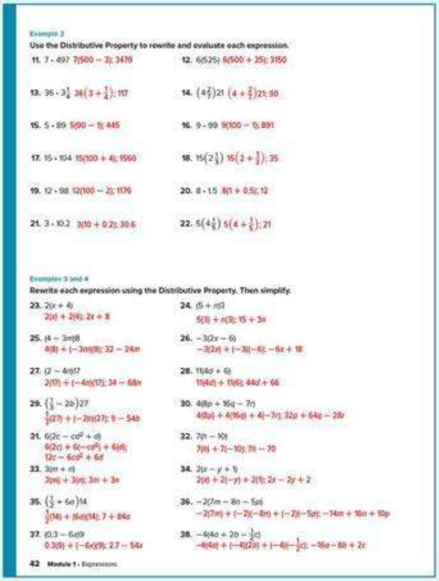
8. SALARY In a recent year, the median salary for an engineer in the United States was \$55,000 and the median salary for a computer programmer was \$52,000.
   a. Write an expression to estimate the total cost for a business to employ an engineer and a programmer for 5 years. $5(55{,}000 + 52{,}000)$
   b. Estimate the total cost for a business to employ an engineer and a programmer for 5 years. \$535,000

9. COSTUMES Isabella's ballet class is performing a spring recital for which they need butterfly costumes. Each butterfly costume is made from $3\frac{3}{5}$ yards of fabric.
   a. Write an expression to find the number of yards of fabric needed for 10 costumes. $10\left(3\frac{3}{5}\right)$
   b. Use the Distributive Property to find the number of yards of fabric needed for 10 costumes. Show your work. (Hint: A mixed number can be written as the sum of an integer and a fraction.) $10\left(3\frac{3}{5}\right) = 10\left(3 + \frac{3}{5}\right) = 10(3) + 10\left(\frac{3}{5}\right) = 30 + 6 = 36$ yards of fabric

10. REASONING Letisha and Noelle each opened a checking account, a savings account, and a college fund. The chart shows the amounts that they deposit into each account every month.

	Checking	Savings	College
Letisha	\$125	\$75	\$50
Noelle	\$250	\$50	\$50

   a. Write an expression to find the amount in Letisha's checking, savings, and college accounts after 12 months. $12(125 + 75 + 50)$
   b. How much is in Letisha's checking, savings, and college accounts after 12 months? \$3000



Example 2

Use the Distributive Property to rewrite and evaluate each expression.

11. $7 \cdot 497$ $7(500 - 3)$; 3479
12. $6(525)$ $6(500 + 25)$; 3150
13. $36 \cdot 3\frac{1}{4}$ $36\left(3 + \frac{1}{4}\right)$; 117
14. $\left(4\frac{2}{7}\right)21$ $\left(4 + \frac{2}{7}\right)21$; 90
15. $5 \cdot 89$ $5(90 - 1)$; 445
16. $9 \cdot 99$ $9(100 - 1)$; 891
17. $15 \cdot 104$ $15(100 + 4)$; 1560
18. $15\left(2\frac{1}{3}\right)$ $15\left(2 + \frac{1}{3}\right)$; 35
19. $12 \cdot 98$ $12(100 - 2)$; 1176
20. $8 \cdot 1.5$ $8(1 + 0.5)$; 12
21. $3 \cdot 10.2$ $3(10 + 0.2)$; 30.6
22. $5\left(4\frac{1}{5}\right)$ $5\left(4 + \frac{1}{5}\right)$; 21

Examples 3 and 4

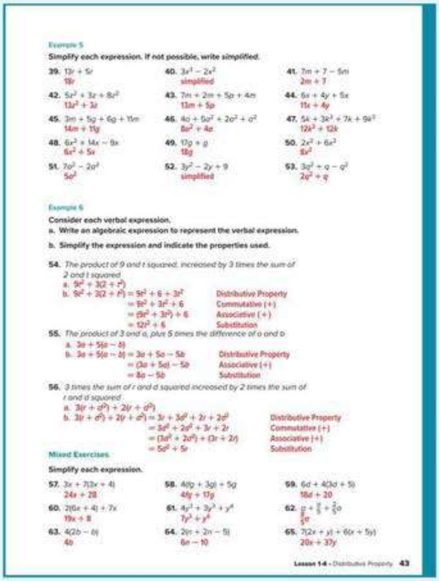
Rewrite each expression using the Distributive Property. Then simplify.

23. $2(x + 4)$
    $2(x) + 2(4)$; $2x + 8$
24. $(5 + n)3$
    $5(3) + n(3)$; $15 + 3n$
25. $(4 - 3m)8$
    $4(8) + (-3m)(8)$; $32 - 24m$
26. $-3(2x - 6)$
    $-3(2x) + (-3)(-6)$; $-6x + 18$
27. $(2 - 4n)17$
    $2(17) + (-4n)(17)$; $34 - 68n$
28. $11(4d + 6)$
    $11(4d) + 11(6)$; $44d + 66$
29. $\left(\frac{1}{3} - 2b\right)27$
    $\frac{1}{3}(27) + (-2b)(27)$; $9 - 54b$
30. $4(8p + 16q - 7r)$
    $4(8p) + 4(16q) + 4(-7r)$; $32p + 64q - 28r$
31. $6(2c - cd^2 + d)$
    $6(2c) + 6(-cd^2) + 6(d)$;
    $12c - 6cd^2 + 6d$
32. $7(h - 10)$
    $7(h) + 7(-10)$; $7h - 70$
33. $3(m + n)$
    $3(m) + 3(n)$; $3m + 3n$
34. $2(x - y + 1)$
    $2(x) + 2(-y) + 2(1)$; $2x - 2y + 2$
35. $\left(\frac{1}{2} + 6a\right)14$
    $\frac{1}{2}(14) + (6a)(14)$; $7 + 84a$
36. $-2(7m - 8n - 5p)$
    $-2(7m) + (-2)(-8n) + (-2)(-5p)$; $-14m + 16n + 10p$
37. $(0.3 - 6x)9$
    $0.3(9) + (-6x)(9)$; $2.7 - 54x$
38. $-4\left(4a + 2b - \frac{1}{2}c\right)$
    $-4(4a) + (-4)(2b) + (-4)\left(-\frac{1}{2}c\right)$; $-16a - 8b + 2c$

3 REFLECT AND PRACTICE

A.SSE.1a, A.SSE.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 5

Simplify each expression. If not possible, write simplified.

39. $13r + 5r$
$18r$

40. $3x^3 - 2x^2$
simplified

41. $7m + 7 - 5m$
$2m + 7$

42. $5z^2 + 3z + 8z^2$
$13z^2 + 3z$

43. $7m + 2m + 5p + 4m$
$13m + 5p$

44. $6x + 4y + 5x$
$11x + 4y$

45. $3m + 5g + 6g + 11m$
$14m + 11g$

46. $4a + 5a^2 + 2a^2 + a^2$
$8a^2 + 4a$

47. $5k + 3k^3 + 7k + 9k^3$
$12k^3 + 12k$

48. $6x^2 + 14x - 9x$
$6x^2 + 5x$

49. $17g + g$
$18g$

50. $2x^2 + 6x^2$
$8x^2$

51. $7a^2 - 2a^2$
$5a^2$

52. $3y^2 - 2y + 9$
simplified

53. $3q^2 + q - q^2$
$2q^2 + q$

Example 6

Consider each verbal expression.

a. Write an algebraic expression to represent the verbal expression.

b. Simplify the expression and indicate the properties used.

54. *The product of 9 and t squared, increased by 3 times the sum of 2 and t squared*
a. $9t^2 + 3(2 + t^2)$
b. $9t^2 + 3(2 + t^2) = 9t^2 + 6 + 3t^2$ Distributive Property
$= 9t^2 + 3t^2 + 6$ Commutative (+)
$= (9t^2 + 3t^2) + 6$ Associative (+)
$= 12t^2 + 6$ Substitution

55. *The product of 3 and a, plus 5 times the difference of a and b*
a. $3a + 5(a - b)$
b. $3a + 5(a - b) = 3a + 5a - 5b$ Distributive Property
$= (3a + 5a) - 5b$ Associative (+)
$= 8a - 5b$ Substitution

56. *3 times the sum of r and d squared increased by 2 times the sum of r and d squared*
a. $3(r + d^2) + 2(r + d^2)$
b. $3(r + d^2) + 2(r + d^2) = 3r + 3d^2 + 2r + 2d^2$ Distributive Property
$= 3d^2 + 2d^2 + 3r + 2r$ Commutative (+)
$= (3d^2 + 2d^2) + (3r + 2r)$ Associative (+)
$= 5d^2 + 5r$ Substitution

Mixed Exercises

Simplify each expression.

57. $3x + 7(3x + 4)$
$24x + 28$

58. $4(fg + 3g) + 5g$
$4fg + 17g$

59. $6d + 4(3d + 5)$
$18d + 20$

60. $2(6x + 4) + 7x$
$19x + 8$

61. $4y^3 + 3y^3 + y^4$
$7y^3 + y^4$

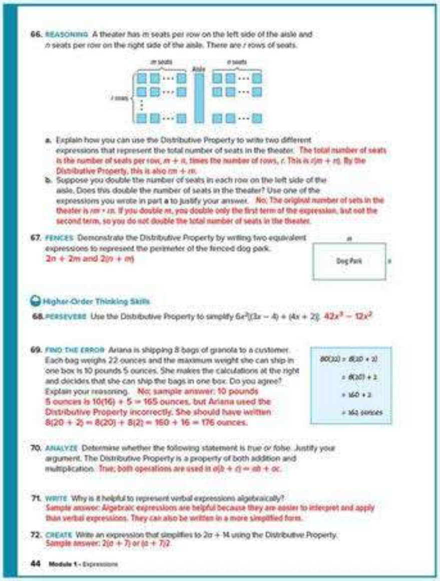
62. $\frac{a}{5} + \frac{a}{5} + \frac{2}{5}a$
$\frac{4}{5}a$

63. $4(2b - b)$
$4b$

64. $2(n + 2n - 5)$
$6n - 10$

65. $7(2x + y) + 6(x + 5y)$
$20x + 37y$

Lesson 1-4 • Distributive Property **43**



66. REASONING A theater has *m* seats per row on the left side of the aisle and *n* seats per row on the right side of the aisle. There are *r* rows of seats.

a. Explain how you can use the Distributive Property to write two different expressions that represent the total number of seats in the theater. The total number of seats is the number of seats per row, $m + n$, times the number of rows, r. This is $r(m + n)$. By the Distributive Property, this is also $rm + rn$.

b. Suppose you double the number of seats in each row on the left side of the aisle. Does this double the number of seats in the theater? Use one of the expressions you wrote in part **a** to justify your answer. No; The original number of sets in the theater is $rm + rn$. If you double m, you double only the first term of the expression, but not the second term, so you do not double the total number of seats in the theater.

67. FENCES Demonstrate the Distributive Property by writing two equivalent expressions to represent the perimeter of the fenced dog park.
$2n + 2m$ and $2(n + m)$

Higher-Order Thinking Skills

68. PERSEVERE Use the Distributive Property to simplify $6x^2[(3x - 4) + (4x + 2)]$. $42x^3 - 12x^2$

69. FIND THE ERROR Ariana is shipping 8 bags of granola to a customer. Each bag weighs 22 ounces and the maximum weight she can ship in one box is 10 pounds 5 ounces. She makes the calculations at the right and decides that she can ship the bags in one box. Do you agree? Explain your reasoning. No; sample answer: 10 pounds 5 ounces is $10(16) + 5 = 165$ ounces, but Ariana used the Distributive Property incorrectly. She should have written $8(20 + 2) = 8(20) + 8(2) = 160 + 16 = 176$ ounces.

$8(22) = 8(20 + 2)$
$= 8(20) + 2$
$= 160 + 2$
$= 162$ ounces

70. ANALYZE Determine whether the following statement is true or false. Justify your argument. The Distributive Property is a property of both addition and multiplication. True; both operations are used in $a(b + c) = ab + ac$.

71. WRITE Why is it helpful to represent verbal expressions algebraically?
Sample answer: Algebraic expressions are helpful because they are easier to interpret and apply than verbal expressions. They can also be written in a more simplified form.

72. CREATE Write an expression that simplifies to $2a + 14$ using the Distributive Property.
Sample answer: $2(a + 7)$ or $(a + 7)2$

44 **Module 1** • Expressions

Lesson 1-5

Expressions Involving Absolute Value

A.SSE.2

LESSON GOAL

Students evaluate absolute value expressions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Distance Between Points on a Number Line

Develop:

Evaluating Expressions Involving Absolute Value

- Write an Absolute Value Expression
- Evaluate the Absolute Value of an Algebraic Expression
- Evaluate an Expression Involving Absolute Value

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Represent Integers	●	●		●
Extension: Making Conjectures with Absolute Value Expressions		●	●	●

Language Development Handbook

Assign page 5 of the *Language Development Handbook* to help your students build mathematical language related to absolute value expressions.

ELL You can use the tips and suggestions on page T5 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice:

4 Model with mathematics.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous
Students used the Distributive Property to simplify expressions.
6.EE.3, A.SSE.1a, A.SSE.2

Now
Students evaluate absolute value expressions.
A.SSE.2

Next
Students will solve equations involving absolute value.
A.CED.1, A.REI.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of expressions to build fluency with simplifying expressions that involve absolute value. They apply their understanding of absolute value expressions by solving real-world problems.

Mathematical Background

The absolute value of a number is the distance the number is from zero on a number line. Absolute value is always greater than or equal to zero.

Explore Distance Between Points on a Number Line

Objective

Students use dynamic number lines to explore absolute value.

Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in this Explore

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

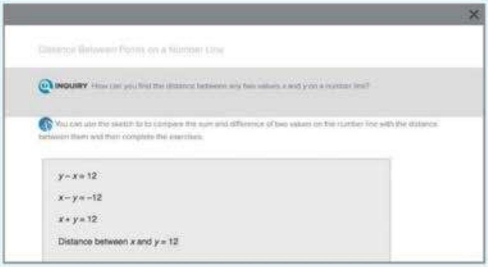
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will be presented with guiding exercises to complete at the end of the activity. Students will use a sketch to explore finding the distance between two points on the number line, and then fill in the provided table with specific results. The guiding exercises will lead students to determine how to calculate the distance between two values. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Distance Between Points on a Number Line

INQUIRY How can you find the distance between any two values x and y on a number line?

You can use the sketch to to compare the sum and difference of two values on the number line with the distance between them and then complete the exercises.

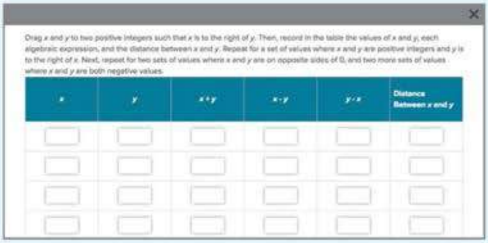
$y - x = 12$

$x - y = -12$

$x + y = 12$

Distance between x and $y = 12$

Explore



Drag x and y to two positive integers such that x is to the right of y. Then, record in the table the values of x and y, each algebraic expression, and the distance between x and y. Repeat for a set of values where x and y are positive integers and y is to the right of x. Next, repeat for two sets of values where x and y are on opposite sides of 0, and two more sets of values where x and y are both negative values.

x	y	$x + y$	$x - y$	$y - x$	Distance Between x and y
☐	☐	☐	☐	☐	☐
☐	☐	☐	☐	☐	☐
☐	☐	☐	☐	☐	☐
☐	☐	☐	☐	☐	☐

Explore

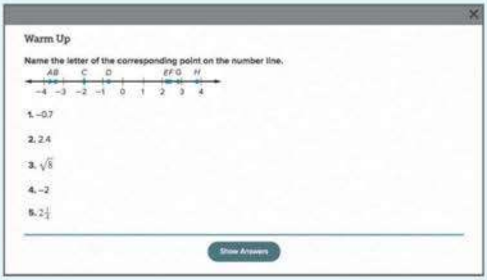
WEB SKETCHPAD

Students use the sketch activity to complete an activity in which they explore the distance between two points on the number line.

TYPE

Students fill in a table with different values of x and y from the sketch and then basic computations.

Interactive Presentation

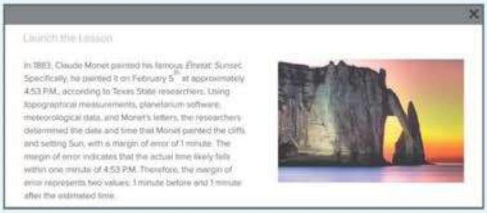


Warm Up

Name the letter of the corresponding point on the number line.

1. −0.7
2. 2.4
3. $\sqrt{8}$
4. −2
5. $2\frac{1}{4}$

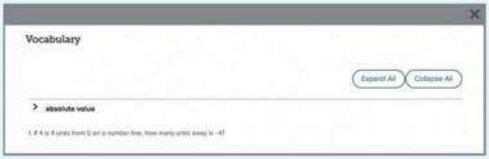
Warm Up



Launch the Lesson

In 1883, Claude Monet painted his famous *Étretat: Sunset*. Specifically, he painted it on February 5th at approximately 4:53 P.M., according to Texas State researchers. Using topographical measurements, planetarium software, meteorological data, and Monet's letters, the researchers determined the date and time that Monet painted the cliffs and setting Sun, with a margin of error of 1 minute. The margin of error indicates that the actual time likely falls within one minute of 4:53 P.M. Therefore, the margin of error represents two values: 1 minute before and 1 minute after the estimated time.

Launch



Vocabulary

> absolute value

1. If 4 is 4 units from 0 on a number line, how many units away is −4?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- naming points on number lines

Answers:

1. D
2. F
3. G
4. C
5. E

Launch the Lesson

Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to analyze the information about a Claude Monet painting to write an absolute value expression.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the question below with the class.

Interactive Presentation

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Distance Between Points on a Number Line (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Do any of the algebraic expressions equal the distance between x and y for every set of values? no
- What pattern do you notice when $x - y$ or $y - x$ does not equal the distance between x and y? Sample answer: When $x - y$ or $y - x$ does not equal the distance between x and y, the expression is the additive inverse of the distance.

Inquiry

How can you find the distance between any two values x and y on a number line? The distance between any two values on a number line can be found by taking the absolute value of the difference of the two numbers; i.e., $|x - y|$ or $|y - x|$.

Go Online to find additional teaching notes and answers for the Explore activity.

Learn Evaluating Expressions Involving Absolute Value

Objective

Students evaluate absolute value expressions by substituting values into and simplifying given expressions.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

Absolute value does not change the sign of the number (e.g. $|5| \neq -5$). Remind students that absolute value represents the distance from zero regardless of direction.

Essential Question Follow-Up

Students have begun evaluating expressions involving absolute value.

Ask:

When can absolute value model real-life situations? Sample answer: Absolute value can model real life situations when the distance is the only thing that matters, not the direction. For example, if the temperature of a cup of coffee went from 105°F to 90°F, and we wanted to know just the change in temperature, then we could say it changed 15°F.

Example 1 Write an Absolute Value Expression

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about absolute value expressions to solving a real-world problem.

Questions for Mathematical Discourse

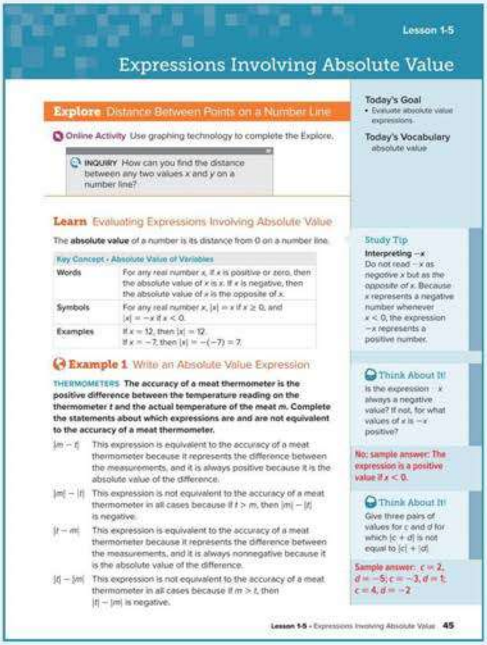
AL What do the variables t and m represent? t is the temperature reading on the thermometer and m is the actual temperature of the meat.

OL To find the accuracy of a meat thermometer, should you subtract the readings first and then take the absolute value, or take the absolute value of each reading and then subtract? Explain. Subtract the readings before taking the absolute value; sample answer: We want a nonnegative difference between the readings, so subtract them first and then take the absolute value.

BL Why is $|m - t|$ not equivalent to $|m| - |t|$? Sample answer: For the first expression, we would subtract and then take the absolute value. For the second expression, we would take the absolute value first and then subtract. If m were less than t, the second expression would yield a negative.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 1-5

Expressions Involving Absolute Value

Explore Distance Between Points on a Number Line

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you find the distance between any two values x and y on a number line?

Today's Goal
- Evaluate absolute value expressions.

Today's Vocabulary
absolute value

Learn Evaluating Expressions Involving Absolute Value

The **absolute value** of a number is its distance from 0 on a number line.

Key Concept • Absolute Value of Variables	
Words	For any real number x, if x is positive or zero, then the absolute value of x is x. If x is negative, then the absolute value of x is the opposite of x.
Symbols	For any real number x, $|x| = x$ if $x \geq 0$, and $|x| = -x$ if $x < 0$.
Examples	If $x = 12$, then $|x| = 12$. If $x = -7$, then $|x| = -(-7) = 7$.

Study Tip

Interpreting $-x$ Do not read $-x$ as negative x but as the opposite of x. Because x represents a negative number whenever $x < 0$, the expression $-x$ represents a positive number.

Example 1 Write an Absolute Value Expression

THERMOMETERS **The accuracy of a meat thermometer is the positive difference between the temperature reading on the thermometer t and the actual temperature of the meat m. Complete the statements about which expressions are and are not equivalent to the accuracy of a meat thermometer.**

$|m - t|$	This expression is equivalent to the accuracy of a meat thermometer because it represents the difference between the measurements, and it is always positive because it is the absolute value of the difference.
$|m| - |t|$	This expression is not equivalent to the accuracy of a meat thermometer in all cases because if $t > m$, then $|m| - |t|$ is negative.
$|t - m|$	This expression is equivalent to the accuracy of a meat thermometer because it represents the difference between the measurements, and it is always nonnegative because it is the absolute value of the difference.
$|t| - |m|$	This expression is not equivalent to the accuracy of a meat thermometer in all cases because if $m > t$, then $|t| - |m|$ is negative.

Think About It! Is the expression $-x$ always a negative value? If not, for what values of x is $-x$ positive?

No; sample answer: The expression is a positive value if $x < 0$.

Think About It! Give three pairs of values for c and d for which $|c + d|$ is not equal to $|c| + |d|$.

Sample answer: $c = 2$, $d = -5$; $c = -3$, $d = 1$; $c = 4$, $d = -2$

Lesson 1-5 • Expressions Involving Absolute Value 45

Interactive Presentation

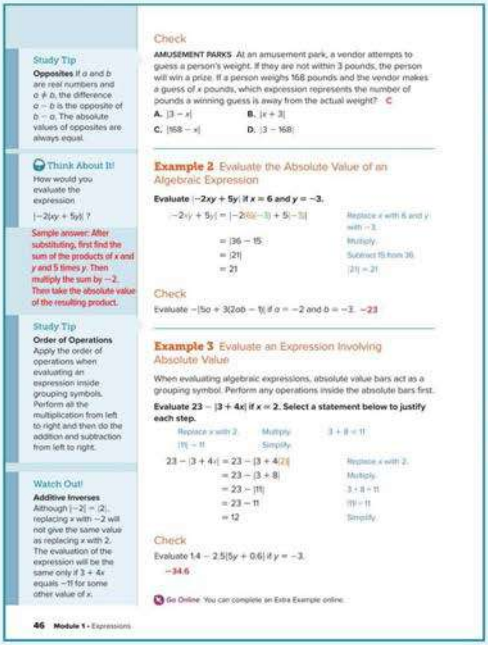
Learn

DRAG & DROP

Students drag and drop numbers to represent the given expressions.

TYPE

Students determine if an expression will always have a negative value.



Study Tip

Opposites If a and b are real numbers and $a \neq b$, the difference $a - b$ is the opposite of $b - a$. The absolute values of opposites are always equal.

Think About It!

How would you evaluate the expression $|-2(xy + 5y)|$?

Sample answer: After substituting, first find the sum of the products of x and y and 5 times y. Then multiply the sum by -2. Then take the absolute value of the resulting product.

Study Tip

Order of Operations Apply the order of operations when evaluating an expression inside grouping symbols. Perform all the multiplication from left to right and then do the addition and subtraction from left to right.

Watch Out!

Additive Inverses Although $|-2| = |2|$, replacing x with -2 will not give the same value as replacing x with 2. The evaluation of the expression will be the same only if $3 + 4x$ equals -11 for some other value of x.

Check

AMUSEMENT PARKS At an amusement park, a vendor attempts to guess a person's weight. If they are not within 3 pounds, the person will win a prize. If a person weighs 168 pounds and the vendor makes a guess of x pounds, which expression represents the number of pounds a winning guess is away from the actual weight? C

A. $|3 - x|$
B. $|x + 3|$
C. $|168 - x|$
D. $|3 - 168|$

Example 2 Evaluate the Absolute Value of an Algebraic Expression

Evaluate $|-2xy + 5y|$ if $x = 6$ and $y = -3$.

$|-2xy + 5y| = |-2(6)(-3) + 5(-3)|$	Replace x with 6 and y with -3.
$= |36 - 15|$	Multiply.
$= |21|$	Subtract 15 from 36.
$= 21$	$|21| = 21$

Check

Evaluate $-|5a + 3(2ab - 1)|$ if $a = -2$ and $b = -3$. -23

Example 3 Evaluate an Expression Involving Absolute Value

When evaluating algebraic expressions, absolute value bars act as a grouping symbol. Perform any operations inside the absolute bars first.

Evaluate $23 - |3 + 4x|$ if $x = 2$. Select a statement below to justify each step.

Replace x with 2. Multiply. $3 + 8 = 11$
$|11| = 11$ Simplify.

$23 - |3 + 4x| = 23 - |3 + 4(2)|$	Replace x with 2.
$= 23 - |3 + 8|$	Multiply.
$= 23 - |11|$	$3 + 8 = 11$
$= 23 - 11$	$|11| = 11$
$= 12$	Simplify.

Check

Evaluate $1.4 - 2.5|5y + 0.6|$ if $y = -3$.
-34.6

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 2

TYPE

Students describe how to evaluate the absolute value of an algebraic expression.

Example 2 Evaluate the Absolute Value of an Algebraic Expression

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How many times does x appear in the expression? 1 the y? 2

OL Why does $-2xy$ evaluate to positive 36? Sample answer: After substituting in the values for the variables, you get $-2(6)(-3)$, and a negative times a negative is a positive.

BL Explain how to evaluate the expression for $x = 1$ and $y = -7$. Sample answer: After substituting the values in for the variables, I got $|14 - 35|$, which simplifies to $|-21|$, which equals 21.

Example 3 Evaluate an Expression Involving Absolute Value

Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL What should be the first step in evaluating the absolute value of the algebraic expression? Replace x with 2.

OL Evaluate the expression for $x = -2$. Why is the answer for $x = 2$ not the same as $x = -2$? 18; sample answer: Substituting -2 for x gives $23 - |-5|$, which is not the same as $23 - |11|$.

BL What other value of x will yield the same result of 12? If $x = -3.5$, the expression will still be 12.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to evaluate absolute value expressions, **THEN** have them evaluate the inside of the absolute value using order of operations and plot the resulting value on the number line. Tell students the final answer is the distance that number is from zero. They can count the spaces from zero and see the answer is just the positive value.

Enrichment Activity BL

EVALUATE $|-3y| + |3y|$ for $y = 1$. Explain why even though it appears $-3y$ and $3y$ are additive inverses, they are in fact not.
Sample answer: $|3(-1)| + |3(1)| = |-3| + |3|$, which simplifies to $3 + 3 = 6$. The two terms appear to be additive inverses, but the absolute value converts the negative value to a nonnegative value. Thus the two are not additive inverses but like terms.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–34
2	exercises that use a variety of skills from this lesson	35–36
3	exercises that emphasize higher-order and critical-thinking skills	37–39

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–35 odd, 37–39
- Extension: Examining Cases
- **ALEKS** Operations with Signed Numbers

OL

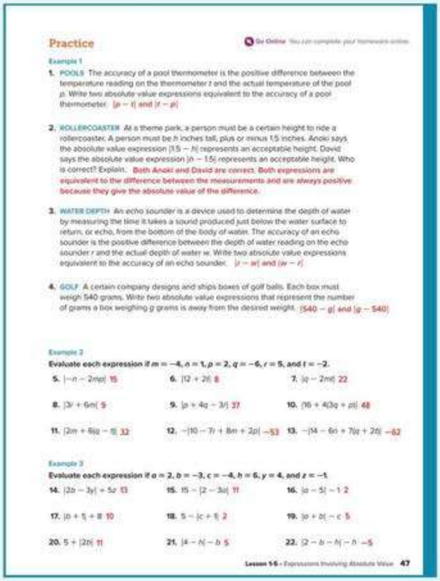
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–39 odd
- Remediation, Review Resources: Represent Integers
- Personal Tutors
- Extra Examples 1–3
- **ALEKS** Plotting and Comparing Signed Numbers

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–33 odd
- Remediation, Review Resources: Represent Integers
- Arrive**MATH** Take Another Look
- **ALEKS** Plotting and Comparing Signed Numbers



Practice

Go Online You can complete your homework online.

Example 1

1. POOLS The accuracy of a pool thermometer is the positive difference between the temperature reading on the thermometer t and the actual temperature of the pool p. Write two absolute value expressions equivalent to the accuracy of a pool thermometer. $|p - t|$ and $|t - p|$

2. ROLLERCOASTER At a theme park, a person must be a certain height to ride a rollercoaster. A person must be h inches tall, plus or minus 1.5 inches. Anoki says the absolute value expression $|1.5 - h|$ represents an acceptable height. David says the absolute value expression $|h - 1.5|$ represents an acceptable height. Who is correct? Explain. Both Anoki and David are correct. Both expressions are equivalent to the difference between the measurements and are always positive because they give the absolute value of the difference.

3. WATER DEPTH An echo sounder is a device used to determine the depth of water by measuring the time it takes a sound produced just below the water surface to return, or echo, from the bottom of the body of water. The accuracy of an echo sounder is the positive difference between the depth of water reading on the echo sounder r and the actual depth of water w. Write two absolute value expressions equivalent to the accuracy of an echo sounder. $|r - w|$ and $|w - r|$

4. GOLF A certain company designs and ships boxes of golf balls. Each box must weigh 540 grams. Write two absolute value expressions that represent the number of grams a box weighing g grams is away from the desired weight. $|540 - g|$ and $|g - 540|$

Example 2

Evaluate each expression if $m = -4$, $n = 1$, $p = 2$, $q = -6$, $r = 5$, and $t = -2$.

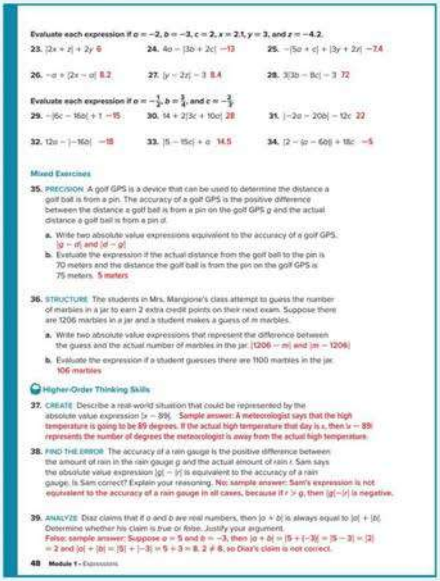
5. $|-n - 2mp|$ 15
6. $|12 + 2t|$ 8
7. $|q - 2mt|$ 22
8. $|3r + 6m|$ 9
9. $|p + 4q - 3r|$ 37
10. $|16 + 4(3q + p)|$ 48
11. $|2m| + 6|q - t|$ 32
12. $-|10 - 7r + 8m + 2p|$ −53
13. $-|14 - 6n + 7(q + 2t)|$ −62

Example 3

Evaluate each expression if $a = 2$, $b = -3$, $c = -4$, $h = 6$, $y = 4$, and $z = -1$.

14. $|2b - 3y| + 5z$ 13
15. $15 - |2 - 3a|$ 11
16. $|a - 5| - 1$ 2
17. $|b + 1| + 8$ 10
18. $5 - |c + 1|$ 2
19. $|a + b| - c$ 5
20. $5 + |2b|$ 11
21. $|4 - h| - b$ 5
22. $|2 - b - h| - h$ −5

Lesson 1-5 • Expressions Involving Absolute Value 47



Evaluate each expression if $a = -2$, $b = -3$, $c = 2$, $x = 2.1$, $y = 3$, and $z = -4.2$.

23. $|2x + z| + 2y$ 6

24. $4a - |3b + 2c|$ −13

25. $-|5a + c| + |3y + 2z|$ −7.4

26. $-a + |2x - a|$ 8.2

27. $|y - 2z| - 3$ 8.4

28. $3|3b - 8c| - 3$ 72

Evaluate each expression if $a = -\frac{1}{2}$, $b = \frac{3}{4}$, and $c = -\frac{2}{3}$.

29. $-|6c - 16b| + 1$ −15

30. $14 + 2|3c + 10a|$ 28

31. $|-2a - 20b| - 12c$ 22

32. $12a - |-16b|$ −18

33. $|5 - 15c| + a$ 14.5

34. $|2 - |a - 6b|| + 18c$ −5

Mixed Exercises

35. PRECISION A golf GPS is a device that can be used to determine the distance a golf ball is from a pin. The accuracy of a golf GPS is the positive difference between the distance a golf ball is from a pin on the golf GPS g and the actual distance a golf ball is from a pin d.

a. Write two absolute value expressions equivalent to the accuracy of a golf GPS. $|g - d|$ and $|d - g|$

b. Evaluate the expression if the actual distance from the golf ball to the pin is 70 meters and the distance the golf ball is from the pin on the golf GPS is 75 meters. 5 meters

36. STRUCTURE The students in Mrs. Mangione's class attempt to guess the number of marbles in a jar to earn 2 extra credit points on their next exam. Suppose there are 1206 marbles in a jar and a student makes a guess of m marbles.

a. Write two absolute value expressions that represent the difference between the guess and the actual number of marbles in the jar. $|1206 - m|$ and $|m - 1206|$

b. Evaluate the expression if a student guesses there are 1100 marbles in the jar. 106 marbles

Higher-Order Thinking Skills

37. CREATE Describe a real-world situation that could be represented by the absolute value expression $|x - 89|$. Sample answer: A meteorologist says that the high temperature is going to be 89 degrees. If the actual high temperature that day is x, then $|x - 89|$ represents the number of degrees the meteorologist is away from the actual high temperature.

38. FIND THE ERROR The accuracy of a rain gauge is the positive difference between the amount of rain in the rain gauge g and the actual amount of rain r. Sam says the absolute value expression $|g| - |r|$ is equivalent to the accuracy of a rain gauge. Is Sam correct? Explain your reasoning. No; sample answer: Sam's expression is not equivalent to the accuracy of a rain gauge in all cases, because if $r > g$, then $|g| - |r|$ is negative.

39. ANALYZE Diaz claims that if a and b are real numbers, then $|a + b|$ is always equal to $|a| + |b|$. Determine whether his claim is *true* or *false*. Justify your argument. False; sample answer: Suppose $a = 5$ and $b = -3$, then $|a + b| = |5 + (-3)| = |5 - 3| = |2| = 2$ and $|a| + |b| = |5| + |-3| = 5 + 3 = 8$. $2 \neq 8$, so Diaz's claim is not correct.

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Lesson 1-6

Descriptive Modeling and Accuracy

N.Q.2, N.Q.3

LESSON GOAL

Students use quantities for the purpose of descriptive modeling, and report solutions with an appropriate level of accuracy.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Descriptive Modeling

- Use Descriptive Modeling
- Compare Metrics

Accuracy

- Decide Where to Round
- Find an Appropriate Level of Accuracy
- Determine Accuracy

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Convert Customary Measurement Units	●	●		●
Extension: Appropriate Units		●	●	●

Language Development Handbook

Assign page 6 of the *Language Development Handbook* to help your students build mathematical language related to descriptive modeling.

ELL You can use the tips and suggestions on page T6 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

6 Attend to precision.

Coherence

Vertical Alignment

Previous

Students constructed simple equations to solve real-world problems.

7.EE.4

Now

Students use quantities for descriptive modeling.

N.Q.2, N.Q. 3

Next

Students will construct and solve more complicated equations in one variable.

A.CED.1

Rigor

The Three Pillars of Rigor

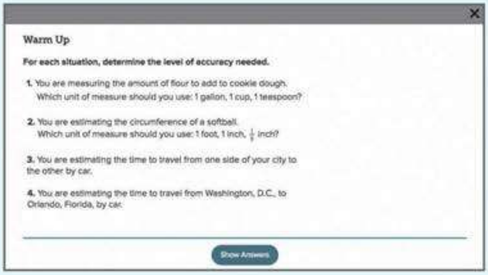
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of descriptive modeling and learn to use appropriate quantities when modeling and the correct level of accuracy to report those quantities. They apply their understanding by using descriptive modeling to solve problems.

Mathematical Background

Descriptive modeling is a way to mathematically describe real-world situations and the factors that cause them. Metrics are used to assign a number to some characteristic or attribute by creating a rule. Measurements are approximations, and thus will be rounded. Accuracy refers to the nearness of a measurement to the actual value of the measure. How measurements should be rounded depends on the limitation on the units.

Interactive Presentation



Warm Up

For each situation, determine the level of accuracy needed.

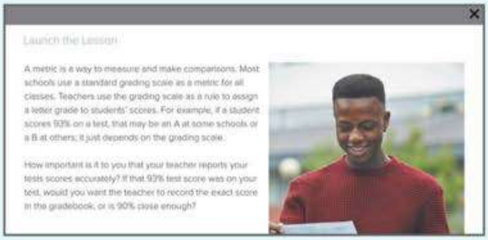
1. You are measuring the amount of flour to add to cookie dough. Which unit of measure should you use: 1 gallon, 1 cup, 1 teaspoon?

2. You are estimating the circumference of a softball. Which unit of measure should you use: 1 foot, 1 inch, $\frac{1}{8}$ inch?

3. You are estimating the time to travel from one side of your city to the other by car.

4. You are estimating the time to travel from Washington, D.C., to Orlando, Florida, by car.

Warm Up

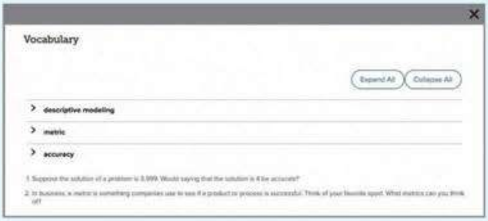


Launch the Lesson

A metric is a way to measure and make comparisons. Most schools use a standard grading scale as a metric for all classes. Teachers use the grading scale as a rule to assign a letter grade to students' scores. For example, if a student scores 93% on a test, that may be an A at some schools or a B at others; it just depends on the grading scale.

How important is it to you that your teacher reports your tests scores accurately? If that 93% test score was on your test, would you want the teacher to record the exact score in the gradebook, or is 90% close enough?

Launch



Vocabulary

- descriptive modeling
- metric
- accuracy

1. Suppose the solution of a problem is 3.999. Would saying that the solution is 4 be accurate?
2. In business, a metric is something companies use to see if a product or process is successful. Think of your favorite sport. What metrics can you think of?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- determining the level of accuracy needed in real-world situations

Answers:

1. 1 cup
2. 1 inch
3. minutes
4. hours

Launch the Lesson

MP Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to analyze the given information to understand how the metrics relate to grading scales.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Learn Descriptive Modeling

Objective

Students define and use appropriate quantities for the purpose of descriptive modeling.

MP Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of descriptive modeling in this Learn.

Common Misconception

Students often believe descriptive modeling is using the four-step problem-solving plan. The four-step plan is a good way to approach word problems, but descriptive modeling has more factors to consider. Encourage students to read problems carefully and consider any metrics provided.

Example 1 Use Descriptive Modeling

MP Teaching the Mathematical Practices

4 Analyze Relationships Mathematically Point out that to solve the problem in this example, students will need to analyze the mathematical relationships in the problem to draw a conclusion.

Questions for Mathematical Discourse

AL What scores will be substituted into the metric? reading, writing, and math scores, and GPA value

OL After substituting in the values, what operation should be performed first? the addition inside the parentheses

BL Suppose an athlete with a 3.3 GPA scored a 600 on reading and writing, and a 610 on math. Would the student qualify as an athlete at the university? Explain.

Yes; sample answer: $2\left[\frac{\left(\frac{600+600}{2}\right)+610}{20}\right]+70 = 121 + 70 = 191$, and $191 \geq 186$.

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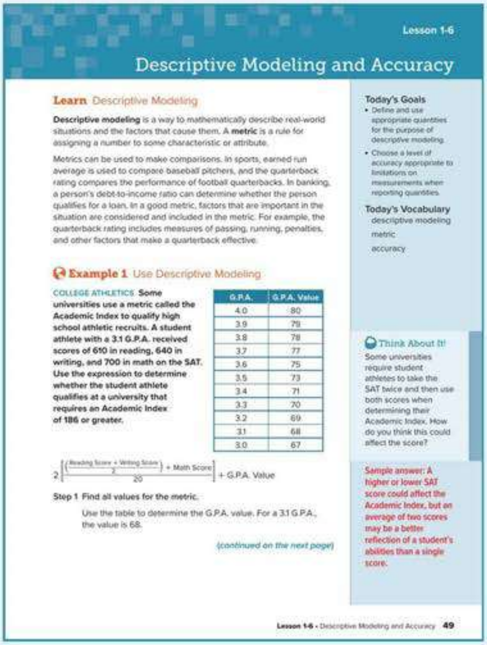

Lesson 1-6

Descriptive Modeling and Accuracy

Learn Descriptive Modeling

Descriptive modeling is a way to mathematically describe real-world situations and the factors that cause them. A **metric** is a rule for assigning a number to some characteristic or attribute.

Metrics can be used to make comparisons. In sports, earned run average is used to compare baseball pitchers, and the quarterback rating compares the performance of football quarterbacks. In banking, a person's debt-to-income ratio can determine whether the person qualifies for a loan. In a good metric, factors that are important in the situation are considered and included in the metric. For example, the quarterback rating includes measures of passing, running, penalties, and other factors that make a quarterback effective.

Example 1 Use Descriptive Modeling

COLLEGE ATHLETICS **Some universities use a metric called the Academic Index to qualify high school athletic recruits. A student athlete with a 3.1 G.P.A. received scores of 610 in reading, 640 in writing, and 700 in math on the SAT. Use the expression to determine whether the student athlete qualifies at a university that requires an Academic Index of 186 or greater.**

G.P.A.	G.P.A. Value
4.0	80
3.9	79
3.8	78
3.7	77
3.6	75
3.5	73
3.4	71
3.3	70
3.2	69
3.1	68
3.0	67

$$2\left[\frac{\left(\frac{\text{Reading Score} + \text{Writing Score}}{2}\right) + \text{Math Score}}{20}\right] + \text{G.P.A. Value}$$

Step 1 Find all values for the metric.

Use the table to determine the G.P.A. value. For a 3.1 G.P.A., the value is 68.

(continued on the next page)

Today's Goals
- Define and use appropriate quantities for the purpose of descriptive modeling.
- Choose a level of accuracy appropriate to limitations on measurements when reporting quantities.

Today's Vocabulary
descriptive modeling
metric
accuracy

Think About It!
Some universities require student athletes to take the SAT twice and then use both scores when determining their Academic Index. How do you think this could affect the score?

Sample answer: A higher or lower SAT score could affect the Academic Index, but an average of two scores may be a better reflection of a student's abilities than a single score.

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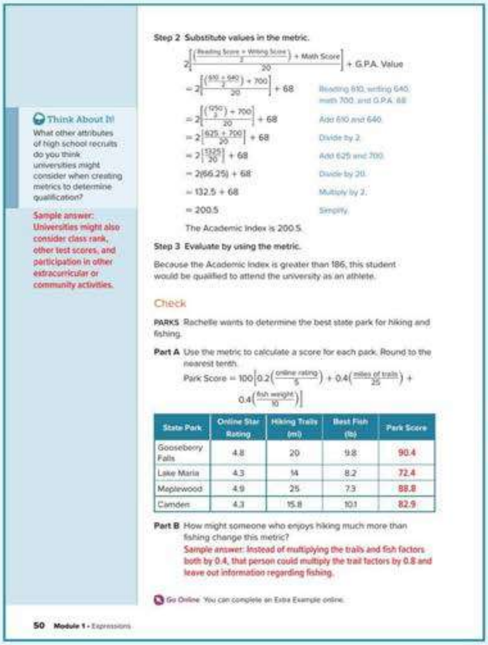
Interactive Presentation

Learn



Think About It!
What other attributes of high school recruits do you think universities might consider when creating metrics to determine qualification?

Sample answer: Universities might also consider class rank, other test scores, and participation in other extracurricular or community activities.

Step 2 Substitute values in the metric.

$$2\left[\frac{\left(\frac{\text{Reading Score}+\text{Writing Score}}{2}\right)+\text{Math Score}}{20}\right]+\text{G.P.A. Value}$$

$$=2\left[\frac{\left(\frac{610+640}{2}\right)+700}{20}\right]+68 \quad \text{Reading 610, writing 640, math 700, and G.P.A. 68}$$

$$=2\left[\frac{\left(\frac{1250}{2}\right)+700}{20}\right]+68 \quad \text{Add 610 and 640.}$$

$$=2\left[\frac{625+700}{20}\right]+68 \quad \text{Divide by 2.}$$

$$=2\left[\frac{1325}{20}\right]+68 \quad \text{Add 625 and 700.}$$

$$=2(66.25)+68 \quad \text{Divide by 20.}$$

$$=132.5+68 \quad \text{Multiply by 2.}$$

$$=200.5 \quad \text{Simplify.}$$

The Academic Index is 200.5.

Step 3 Evaluate by using the metric.

Because the Academic Index is greater than 186, this student would be qualified to attend the university as an athlete.

Check

PARKS Rachelle wants to determine the best state park for hiking and fishing.

Part A Use the metric to calculate a score for each park. Round to the nearest tenth.

$$\text{Park Score} = 100\left[0.2\left(\frac{\text{online rating}}{5}\right)+0.4\left(\frac{\text{miles of trails}}{25}\right)+0.4\left(\frac{\text{fish weight}}{10}\right)\right]$$

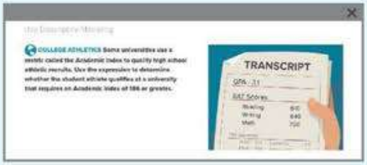
State Park	Online Star Rating	Hiking Trails (mi)	Best Fish (lb)	Park Score
Gooseberry Falls	4.8	20	9.8	90.4
Lake Maria	4.3	14	8.2	72.4
Maplewood	4.9	25	7.3	88.8
Camden	4.3	15.8	10.1	82.9

Part B How might someone who enjoys hiking much more than fishing change this metric?
Sample answer: Instead of multiplying the trails and fish factors both by 0.4, that person could multiply the trail factors by 0.8 and leave out information regarding fishing.

Go Online You can complete an Extra Example online.

50 Module 1 • Expressions

Interactive Presentation



COLLEGE ATHLETICS Some universities use a metric called the Academic Index to qualify high school athletic recruits. Use the expression to determine whether the student athlete qualifies at a university that requires an Academic Index of 186 or greater.

Example 1

TAP

Students move through the steps to solve descriptive modeling problem.

TYPE

Students determine how the Academic Index would be affected based on a proposed change, and identify attributes of high school recruits that should be considered.

Common Error

When students are presented with multiple grouping symbols inside an expression, they may make careless mistakes when evaluating. Remind students to start at the innermost set and work their way out.

DIFFERENTIATE

Language Development Activity ELL
Discuss grouping symbols, like nested parentheses or brackets, with your students. Point out that absolute value symbols can also act as grouping symbols, as in Example 3 of Lesson 1-5.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Compare Metrics

Teaching the Mathematical Practices

5 Use a Source Guide Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL What is the difference in predicted heights for the son using the two methods? 1.21 inches

OL Using the Doubling Method, what would be the predicted height of a girl who is 31 inches tall at age 18 months? $2(31) = 62$. Her predicted height is 62 inches.

BL Which metric seems to be a better predictor of height? Explain. The Gray Method; sample answer: The Gray Method takes into account the height of both parents, rather than just the height of the child. The parents' heights have a lot to do with how tall their children will be.

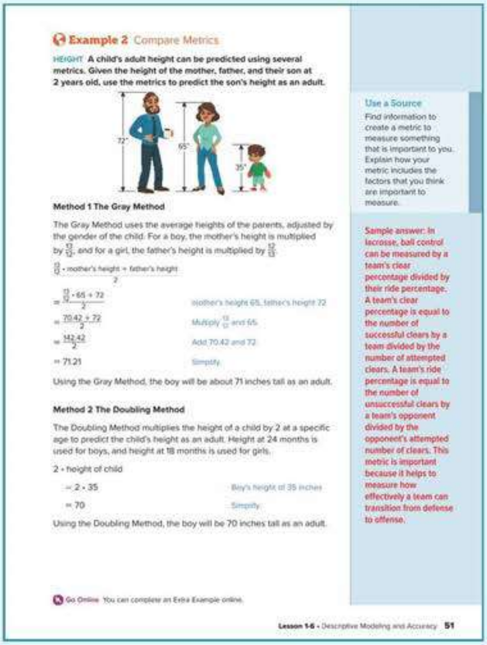
Common Error

Students can get overwhelmed with multiple solution pathways, and wonder which answer is correct. Remind students that the metrics can only predict height, so both solutions are possibilities.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

IF students are overwhelmed by descriptive modeling problems, **THEN** have them underline or highlight important information given, such as metrics and data, which can be used mathematically as a helpful first step when solving these problems. Then, have students discuss their solution methods with each other.



Example 2 Compare Metrics

HEIGHT **A child's adult height can be predicted using several metrics. Given the height of the mother, father, and their son at 2 years old, use the metrics to predict the son's height as an adult.**

Method 1 The Gray Method

The Gray Method uses the average heights of the parents, adjusted by the gender of the child. For a boy, the mother's height is multiplied by $\frac{13}{12}$, and for a girl, the father's height is multiplied by $\frac{12}{13}$.

$$\frac{\frac{13}{12}\cdot \text{mother's height} + \text{father's height}}{2}$$

$= \frac{\frac{13}{12}\cdot 65 + 72}{2}$ Mother's height 65, father's height 72

$= \frac{70.42 + 72}{2}$ Multiply $\frac{13}{12}$ and 65.

$= \frac{142.42}{2}$ Add 70.42 and 72.

$= 71.21$ Simplify.

Using the Gray Method, the boy will be about 71 inches tall as an adult.

Method 2 The Doubling Method

The Doubling Method multiplies the height of a child by 2 at a specific age to predict the child's height as an adult. Height at 24 months is used for boys, and height at 18 months is used for girls.

$2 \cdot \text{height of child}$

$= 2 \cdot 35$ Boy's height of 35 inches

$= 70$ Simplify.

Using the Doubling Method, the boy will be 70 inches tall as an adult.

Use a Source

Find information to create a metric to measure something that is important to you. Explain how your metric includes the factors that you think are important to measure.

Sample answer: In lacrosse, ball control can be measured by a team's clear percentage divided by their ride percentage. A team's clear percentage is equal to the number of successful clears by a team divided by the number of attempted clears. A team's ride percentage is equal to the number of unsuccessful clears by a team's opponent divided by the opponent's attempted number of clears. This metric is important because it helps to measure how effectively a team can transition from defense to offense.

Go Online You can complete an Extra Example online.

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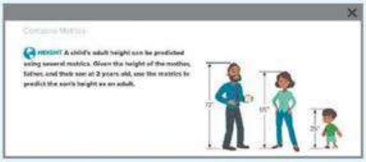
Interactive Presentation



HEIGHT A child's adult height can be predicted using several metrics. Given the height of the mother, father, and their son at 2 years old, use the metrics to predict the son's height as an adult.

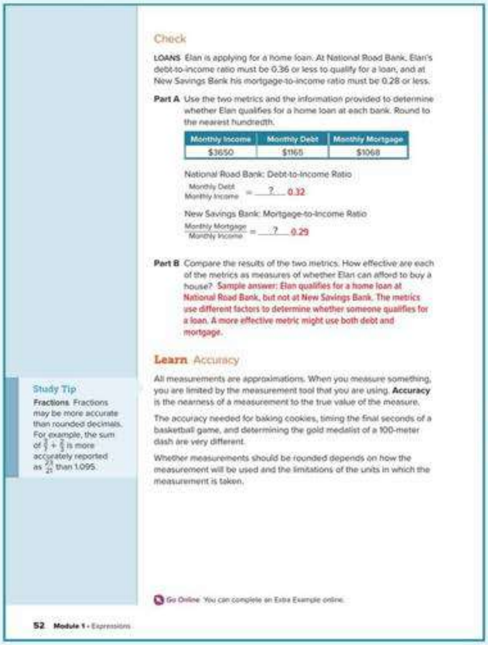
Example 2

TAP

Students select a method to predict the height of a child using his parents' heights.

TYPE

Students use a source to write a metric for something important to them, and explain their reasoning.



Check

LOANS Elan is applying for a home loan. At National Road Bank, Elan's debt-to-income ratio must be 0.36 or less to qualify for a loan, and at New Savings Bank his mortgage-to-income ratio must be 0.28 or less.

Part A Use the two metrics and the information provided to determine whether Elan qualifies for a home loan at each bank. Round to the nearest hundredth.

Monthly Income	Monthly Debt	Monthly Mortgage
$3650	$1165	$1068

National Road Bank: Debt-to-Income Ratio

$\frac{\text{Monthly Debt}}{\text{Monthly Income}} = \underline{\ ?\ }$ 0.32

New Savings Bank: Mortgage-to-Income Ratio

$\frac{\text{Monthly Mortgage}}{\text{Monthly Income}} = \underline{\ ?\ }$ 0.29

Part B Compare the results of the two metrics. How effective are each of the metrics as measures of whether Elan can afford to buy a house? **Sample answer: Elan qualifies for a home loan at National Road Bank, but not at New Savings Bank. The metrics use different factors to determine whether someone qualifies for a loan. A more effective metric might use both debt and mortgage.**

Learn Accuracy

All measurements are approximations. When you measure something, you are limited by the measurement tool that you are using. **Accuracy** is the nearness of a measurement to the true value of the measure.

The accuracy needed for baking cookies, timing the final seconds of a basketball game, and determining the gold medalist of a 100-meter dash are very different.

Whether measurements should be rounded depends on how the measurement will be used and the limitations of the units in which the measurement is taken.

Study Tip

Fractions Fractions may be more accurate than rounded decimals. For example, the sum of $\frac{3}{7} + \frac{2}{3}$ is more accurately reported as $\frac{23}{21}$ than 1.095.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Learn

CHECK

Students complete the Check online to determine whether they are ready to move on.

Learn Accuracy

Objective

Students choose a level of accuracy appropriate to limitations on measurements when reporting quantities.

MP Teaching the Mathematical Practices

6 Use Precision In this Learn, students learn how to calculate accurately and efficiently and to express numerical answers with a degree of precision appropriate to the problem context.

Common Misconception

Students often think that decimal answers are easier and better than fractional answers. When dealing with accuracy and measurement, it is important to pay attention to rounding. Decimal answers may be less accurate than fractional answers if they are not rounded appropriately. Remind students to analyze the context of the situation and the limitations on the tool of measurement when determining whether to use fractions or decimals.

Example 3 Decide Where to Round

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about accuracy to solving a real-world problem.

Questions for Mathematical Discourse

AL How many total miles will the three friends drive? 172 miles

OL Why does the answer $57.\overline{3}$ not make sense? Sample answer: Using a car's odometer, it would be impossible to measure $0\overline{333}$ of a mile while driving.

BL If Damien instead had three friends on the road trip, would the distance each person should drive need to be rounded? Explain. No; sample answer: They would each need to drive 43 miles. There is no decimal mileage, so the answer is exact.

Example 4 Find an Appropriate Level of Accuracy

Teaching the Mathematical Practices

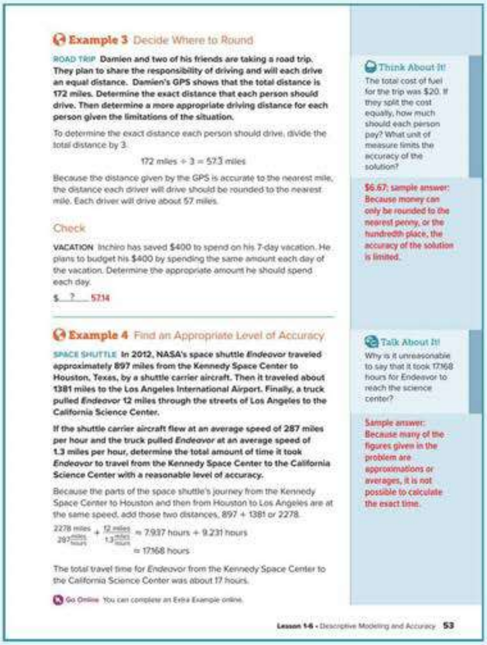
4 Make Assumptions Have students explain an assumption or approximation that was made to solve the problem.

Questions for Mathematical Discourse

AL What information is needed from the problem to calculate the total driving and flying time? driving time: 12 miles and 1.3 miles per hour; flying time: 897 miles, 1381 miles, and 287 miles per hour

OL What formula is used to relate distance, speed, and time? distance = speed • time, or speed = $\frac{\text{distance}}{\text{time}}$

BL How can you use the given units to help determine which operation needs to be done to solve the problem? Sample answer: You are given miles per hour, which tells you that a distance has been divided by a time. To cancel out distances, you will need to divide miles by miles per hour and, and you will be left with hours.



Example 3 Decide Where to Round

ROAD TRIP **Damien and two of his friends are taking a road trip. They plan to share the responsibility of driving and will each drive an equal distance. Damien's GPS shows that the total distance is 172 miles. Determine the exact distance that each person should drive. Then determine a more appropriate driving distance for each person given the limitations of the situation.**

To determine the exact distance each person should drive, divide the total distance by 3.

$$172 \text{ miles} \div 3 = 57.\overline{3} \text{ miles}$$

Because the distance given by the GPS is accurate to the nearest mile, the distance each driver will drive should be rounded to the nearest mile. Each driver will drive about 57 miles.

Check

VACATION Inchiro has saved $400 to spend on his 7-day vacation. He plans to budget his $400 by spending the same amount each day of the vacation. Determine the appropriate amount he should spend each day.

$? 57.14

Think About It! The total cost of fuel for the trip was $20. If they split the cost equally, how much should each person pay? What unit of measure limits the accuracy of the solution?

$6.67; sample answer: Because money can only be rounded to the nearest penny, or the hundredth place, the accuracy of the solution is limited.

Example 4 Find an Appropriate Level of Accuracy

SPACE SHUTTLE **In 2012, NASA's space shuttle *Endeavor* traveled approximately 897 miles from the Kennedy Space Center to Houston, Texas, by a shuttle carrier aircraft. Then it traveled about 1381 miles to the Los Angeles International Airport. Finally, a truck pulled *Endeavor* 12 miles through the streets of Los Angeles to the California Science Center.**

If the shuttle carrier aircraft flew at an average speed of 287 miles per hour and the truck pulled *Endeavor* at an average speed of 1.3 miles per hour, determine the total amount of time it took *Endeavor* to travel from the Kennedy Space Center to the California Science Center with a reasonable level of accuracy.

Because the parts of the space shuttle's journey from the Kennedy Space Center to Houston and then from Houston to Los Angeles are at the same speed, add those two distances, 897 + 1381 or 2278.

$$\frac{2278 \text{ miles}}{287\frac{\text{miles}}{\text{hours}}} + \frac{12 \text{ miles}}{1.3\frac{\text{miles}}{\text{hours}}} \approx 7.937 \text{ hours} + 9.231 \text{ hours}$$
$$\approx 17.168 \text{ hours}$$

The total travel time for *Endeavor* from the Kennedy Space Center to the California Science Center was about 17 hours.

Go Online You can complete an Extra Example online.

Talk About It! Why is it unreasonable to say that it took 17.168 hours for Endeavor to reach the science center?

Sample answer: Because many of the figures given in the problem are approximations or averages, it is not possible to calculate the exact time.

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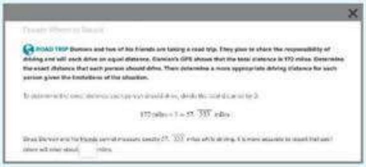
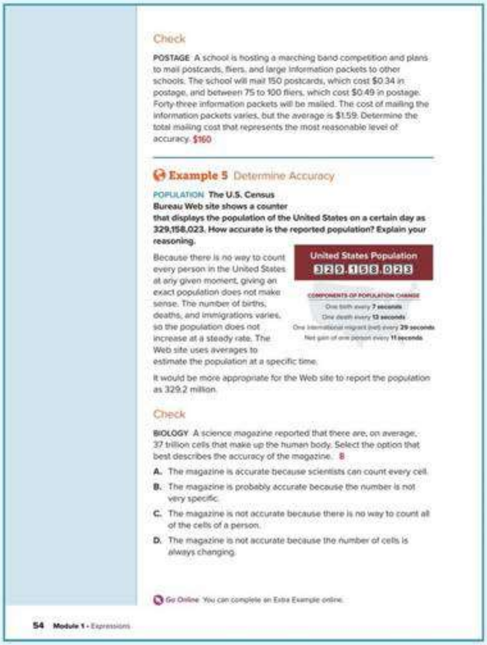
Interactive Presentation



Example 3

TYPE

Students decide how a total cost should be split among friends and discuss the limits of the accuracy.



Check

POSTAGE A school is hosting a marching band competition and plans to mail postcards, fliers, and large information packets to other schools. The school will mail 150 postcards, which cost $0.34 in postage, and between 75 to 100 fliers, which cost $0.49 in postage. Forty-three information packets will be mailed. The cost of mailing the information packets varies, but the average is $1.59. Determine the total mailing cost that represents the most reasonable level of accuracy. $160

Example 5 Determine Accuracy

POPULATION **The U.S. Census Bureau Web site shows a counter that displays the population of the United States on a certain day as 329,158,023. How accurate is the reported population? Explain your reasoning.**

United States Population
329.158.023

COMPONENTS OF POPULATION CHANGE
One birth every 7 seconds
One death every 12 seconds
One international migrant (net) every 29 seconds
Net gain of one person every 11 seconds

Because there is no way to count every person in the United States at any given moment, giving an exact population does not make sense. The number of births, deaths, and immigrations varies, so the population does not increase at a steady rate. The Web site uses averages to estimate the population at a specific time.

It would be more appropriate for the Web site to report the population as 329.2 million.

Check

BIOLOGY A science magazine reported that there are, on average, 37 trillion cells that make up the human body. Select the option that best describes the accuracy of the magazine. B

A. The magazine is accurate because scientists can count every cell.

B. The magazine is probably accurate because the number is not very specific.

C. The magazine is not accurate because there is no way to count all of the cells of a person.

D. The magazine is not accurate because the number of cells is always changing.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 5

SELECT

Students select the correct answer to show they understand the appropriate degree of precision.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 5 Determine Accuracy

Teaching the Mathematical Practices

6 Use Precision In this example, students learn how to calculate accurately and efficiently and to express numerical answers with a degree of precision appropriate to the problem context.

Questions for Mathematical Discourse

AL What are the different components of population change? births, deaths, and international migrants

OL Consider the birth and death rates; will the population change be positive or negative for each 15 seconds? Explain. Positive; sample answer: On average, 2 babies will be born in 15 seconds while only one person will die.

BL Why will we never know the exact population in the United States? Sample answer: The population changes so quickly with births and deaths and other factors, we cannot count the exact number of people at any one time. Even if we could, that number would change so quickly it wouldn't be accurate for long.

Essential Question Follow-Up

Students have begun learning about accuracy and rounding answers.

Ask:

Why is accuracy important with measurements? Sample answer: Accuracy is important because it allows us to represent measurements to the necessary degree of the actual measurement. In track events, time needs to be measured more accurately than in a basketball game.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are confused about rounding measurements,

THEN have them list potential rounded answers and discuss which answer makes the most sense. For example, if you need to make 8 goody bags with 17 treats divided evenly among the bags, how many treats would each bag receive? Students could write down 17/8, 2.125, 2.1, and 2. Discuss how the first three do not make sense since you cannot put part of a treat into a goody bag.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–20
2	exercises that use a variety of skills from this lesson	21–28
3	exercises that emphasize higher-order and critical-thinking skills	29–31

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–27 odd, 29–31
- Extension: Appropriate Units

OL

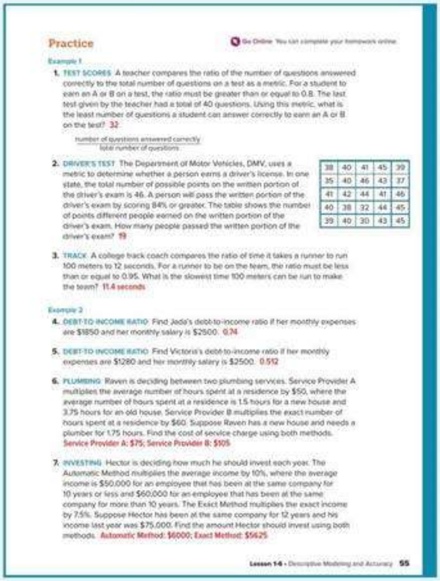
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–31 odd
- Remediation, Review Resources: Convert Customary Measurement Units
- Personal Tutors
- Extra Examples 1–5
- ALEKS U.S. Customary Units of Measurement

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–19 odd
- Remediation, Review Resources: Convert Customary Measurement Units
- *Quick Review Math Handbook*: Descriptive Modeling
- Arrive**MATH** Take Another Look
- ALEKS U.S. Customary Units of Measurement



Practice

Go Online You can complete your homework online.

Example 1

1. TEST SCORES A teacher compares the ratio of the number of questions answered correctly to the total number of questions on a test as a metric. For a student to earn an A or B on a test, the ratio must be greater than or equal to 0.8. The last test given by the teacher had a total of 40 questions. Using this metric, what is the least number of questions a student can answer correctly to earn an A or B on the test? 32

$$\frac{\text{number of questions answered correctly}}{\text{total number of questions}}$$

2. DRIVER'S TEST The Department of Motor Vehicles, DMV, uses a metric to determine whether a person earns a driver's license. In one state, the total number of possible points on the written portion of the driver's exam is 46. A person will pass the written portion of the driver's exam by scoring 84% or greater. The table shows the number of points different people earned on the written portion of the driver's exam. How many people passed the written portion of the driver's exam? 19

38	40	41	45	39
35	40	46	43	37
41	42	44	41	46
40	38	32	44	45
39	40	30	43	45

3. TRACK A college track coach compares the ratio of time it takes a runner to run 100 meters to 12 seconds. For a runner to be on the team, the ratio must be less than or equal to 0.95. What is the slowest time 100 meters can be run to make the team? 11.4 seconds

Example 2

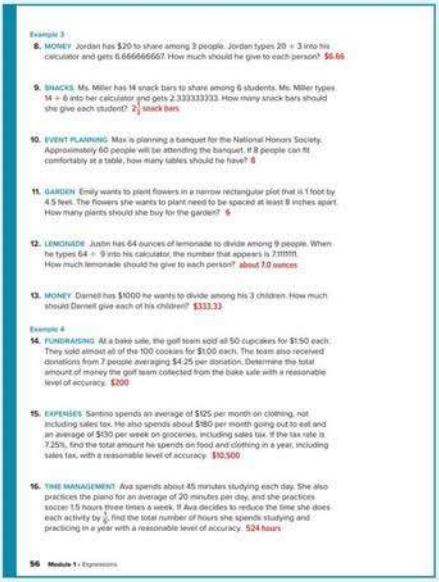
4. DEBT-TO-INCOME RATIO Find Jada's debt-to-income ratio if her monthly expenses are $1850 and her monthly salary is $2500. 0.74

5. DEBT-TO-INCOME RATIO Find Victoria's debt-to-income ratio if her monthly expenses are $1280 and her monthly salary is $2500. 0.512

6. PLUMBING Raven is deciding between two plumbing services. Service Provider A multiplies the average number of hours spent at a residence by $50, where the average number of hours spent at a residence is 1.5 hours for a new house and 3.75 hours for an old house. Service Provider B multiplies the exact number of hours spent at a residence by $60. Suppose Raven has a new house and needs a plumber for 1.75 hours. Find the cost of service charge using both methods. Service Provider A: $75; Service Provider B: $105

7. INVESTING Hector is deciding how much he should invest each year. The Automatic Method multiplies the average income by 10%, where the average income is $50,000 for an employee that has been at the same company for 10 years or less and $60,000 for an employee that has been at the same company for more than 10 years. The Exact Method multiplies the exact income by 7.5%. Suppose Hector has been at the same company for 12 years and his income last year was $75,000. Find the amount Hector should invest using both methods. Automatic Method: $6000; Exact Method: $5625

Lesson 1-6 • Descriptive Modeling and Accuracy 55



Example 3

8. MONEY Jordan has $20 to share among 3 people. Jordan types 20 ÷ 3 into his calculator and gets 6.666666667. How much should he give to each person? $6.66

9. SNACKS Ms. Miller has 14 snack bars to share among 6 students. Ms. Miller types 14 ÷ 6 into her calculator and gets 2.333333333. How many snack bars should she give each student? $2\frac{1}{3}$ snack bars

10. EVENT PLANNING Max is planning a banquet for the National Honors Society. Approximately 60 people will be attending the banquet. If 8 people can fit comfortably at a table, how many tables should he have? 8

11. GARDEN Emily wants to plant flowers in a narrow rectangular plot that is 1 foot by 4.5 feet. The flowers she wants to plant need to be spaced at least 8 inches apart. How many plants should she buy for the garden? 6

12. LEMONADE Justin has 64 ounces of lemonade to divide among 9 people. When he types 64 ÷ 9 into his calculator, the number that appears is 7.111111. How much lemonade should he give to each person? about 7.0 ounces

13. MONEY Darnell has $1000 he wants to divide among his 3 children. How much should Darnell give each of his children? $333.33

Example 4

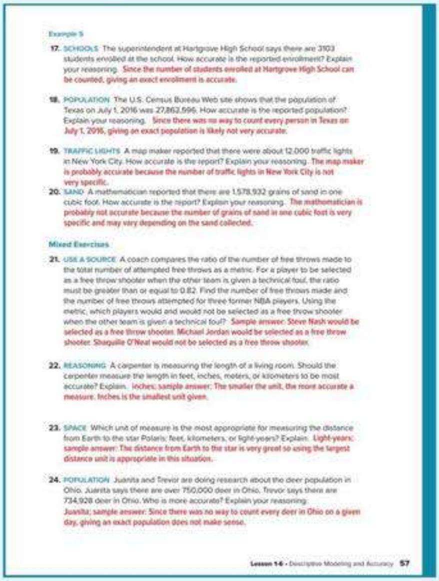
14. FUNDRAISING At a bake sale, the golf team sold all 50 cupcakes for $1.50 each. They sold almost all of the 100 cookies for $1.00 each. The team also received donations from 7 people averaging $4.25 per donation. Determine the total amount of money the golf team collected from the bake sale with a reasonable level of accuracy. $200

15. EXPENSES Santino spends an average of $125 per month on clothing, not including sales tax. He also spends about $180 per month going out to eat and an average of $130 per week on groceries, including sales tax. If the tax rate is 7.25%, find the total amount he spends on food and clothing in a year, including sales tax, with a reasonable level of accuracy. $10,500

16. TIME MANAGEMENT Ava spends about 45 minutes studying each day. She also practices the piano for an average of 20 minutes per day, and she practices soccer 1.5 hours three times a week. If Ava decides to reduce the time she does each activity by $\frac{1}{6}$, find the total number of hours she spends studying and practicing in a year with a reasonable level of accuracy. 524 hours

56 Module 1 • Expressions

3 REFLECT AND PRACTICE



Example 5

17. SCHOOLS The superintendent at Hartgrove High School says there are 3103 students enrolled at the school. How accurate is the reported enrollment? Explain your reasoning. **Since the number of students enrolled at Hartgrove High School can be counted, giving an exact enrollment is accurate.**

18. POPULATION The U.S. Census Bureau Web site shows that the population of Texas on July 1, 2016 was 27,862,596. How accurate is the reported population? Explain your reasoning. **Since there was no way to count every person in Texas on July 1, 2016, giving an exact population is likely not very accurate.**

19. TRAFFIC LIGHTS A map maker reported that there were about 12,000 traffic lights in New York City. How accurate is the report? Explain your reasoning. **The map maker is probably accurate because the number of traffic lights in New York City is not very specific.**

20. SAND A mathematician reported that there are 1,578,932 grains of sand in one cubic foot. How accurate is the report? Explain your reasoning. **The mathematician is probably not accurate because the number of grains of sand in one cubic foot is very specific and may vary depending on the sand collected.**

Mixed Exercises

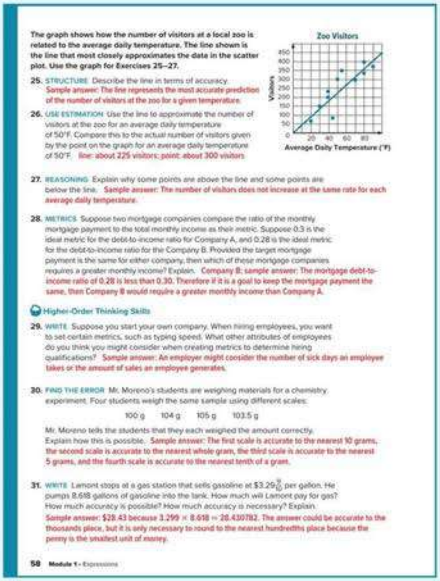
21. USE A SOURCE A coach compares the ratio of the number of free throws made to the total number of attempted free throws as a metric. For a player to be selected as a free throw shooter when the other team is given a technical foul, the ratio must be greater than or equal to 0.82. Find the number of free throws made and the number of free throws attempted for three former NBA players. Using the metric, which players would and would not be selected as a free throw shooter when the other team is given a technical foul? **Sample answer: Steve Nash would be selected as a free throw shooter. Michael Jordan would be selected as a free throw shooter. Shaquille O'Neal would not be selected as a free throw shooter.**

22. REASONING A carpenter is measuring the length of a living room. Should the carpenter measure the length in feet, inches, meters, or kilometers to be most accurate? Explain. **inches; sample answer: The smaller the unit, the more accurate a measure. Inches is the smallest unit given.**

23. SPACE Which unit of measure is the most appropriate for measuring the distance from Earth to the star Polaris: feet, kilometers, or light-years? Explain. **Light-years; sample answer: The distance from Earth to the star is very great so using the largest distance unit is appropriate in this situation.**

24. POPULATION Juanita and Trevor are doing research about the deer population in Ohio. Juanita says there are over 750,000 deer in Ohio. Trevor says there are 734,928 deer in Ohio. Who is more accurate? Explain your reasoning. **Juanita; sample answer: Since there was no way to count every deer in Ohio on a given day, giving an exact population does not make sense.**

Lesson 1-6 • Descriptive Modeling and Accuracy **57**



The graph shows how the number of visitors at a local zoo is related to the average daily temperature. The line shown is the line that most closely approximates the data in the scatter plot. Use the graph for Exercises 25–27.

25. STRUCTURE Describe the line in terms of accuracy. **Sample answer: The line represents the most accurate prediction of the number of visitors at the zoo for a given temperature.**

26. USE ESTIMATION Use the line to approximate the number of visitors at the zoo for an average daily temperature of 50°F. Compare this to the actual number of visitors given by the point on the graph for an average daily temperature of 50°F. **line: about 225 visitors; point: about 300 visitors**

27. REASONING Explain why some points are above the line and some points are below the line. **Sample answer: The number of visitors does not increase at the same rate for each average daily temperature.**

28. METRICS Suppose two mortgage companies compare the ratio of the monthly mortgage payment to the total monthly income as their metric. Suppose 0.3 is the ideal metric for the debt-to-income ratio for Company A, and 0.28 is the ideal metric for the debt-to-income ratio for the Company B. Provided the target mortgage payment is the same for either company, then which of these mortgage companies requires a greater monthly income? Explain. **Company B; sample answer: The mortgage debt-to-income ratio of 0.28 is less than 0.30. Therefore if it is a goal to keep the mortgage payment the same, then Company B would require a greater monthly income than Company A.**

Higher-Order Thinking Skills

29. WRITE Suppose you start your own company. When hiring employees, you want to set certain metrics, such as typing speed. What other attributes of employees do you think you might consider when creating metrics to determine hiring qualifications? **Sample answer: An employer might consider the number of sick days an employee takes or the amount of sales an employee generates.**

30. FIND THE ERROR Mr. Moreno's students are weighing materials for a chemistry experiment. Four students weigh the same sample using different scales:

100 g 104 g 105 g 103.5 g

Mr. Moreno tells the students that they each weighed the amount correctly. Explain how this is possible. **Sample answer: The first scale is accurate to the nearest 10 grams, the second scale is accurate to the nearest whole gram, the third scale is accurate to the nearest 5 grams, and the fourth scale is accurate to the nearest tenth of a gram.**

31. WRITE Lamont stops at a gas station that sells gasoline at $\$3.29\frac{9}{10}$ per gallon. He pumps 8.618 gallons of gasoline into the tank. How much will Lamont pay for gas? How much accuracy is possible? How much accuracy is necessary? Explain. **Sample answer: \$28.43 because $3.299 \times 8.618 = 28.430782$. The answer could be accurate to the thousands place, but it is only necessary to round to the nearest hundredths place because the penny is the smallest unit of money.**

58 **Module 1** • Expressions

Module 1 • Expressions

Review

Rate Yourself

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is order important when evaluating numerical expressions?
- Why is it important to use a variable in a real-world situation?
- Why may we need to evaluate algebraic expressions?
- When can absolute value model real-life situations?
- Why is accuracy important with measurements?

Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the Key Concepts related to writing and simplifying expressions, properties of real numbers, absolute value, descriptive modeling, and accuracy.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Relationships Between Quantities and Reasoning with Equations** and **Expressions and Equations**.

- Interpret the Structure of Expressions
- Use the Structure of an Expression to Rewrite It
- Write Expressions in Equivalent Forms to Solve Problems

Module 1 • Expressions

Review

Essential Question

How can mathematical expressions be represented and evaluated?

You can represent mathematical expressions verbally, numerically, and algebraically. They can be evaluated by applying properties and rules. For example, you can translate a sentence to a numerical or algebraic expression and use the order of operations to simplify or evaluate the expression.

Module Summary

Lessons 1-1 and 1-2

Numerical and Algebraic Expressions

- A numerical expression contains only numbers and mathematical operations.
- To evaluate an expression means to find its value. If a numerical expression contains more than one operation, the rule that lets you know which operation to perform first is called the *order of operations*.
- The Substitution Property allows you to evaluate an algebraic expression by replacing the variables with their values.

Lessons 1-3 and 1-4

Properties of Real Numbers and Distributive Property

- The Reflexive Property states that any quantity is equal to itself.
- The Symmetric Property states that if one quantity equals a second quantity, then the second quantity equals the first.
- The Transitive Property states that if one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.
- The Associative Property states that the way you group three or more numbers when adding or multiplying does not change their sum or product.
- The Commutative Property states that the order in which you add or multiply numbers does not change their sum or product.
- The Distributive Property states that multiplying a number by a sum of numbers is the same as doing each multiplication separately and then adding the products.

Lesson 1-5

Expressions Involving Absolute Value

- The absolute value of a number is its distance from 0 on the number line.
- Absolute value is always greater than or equal to zero.

Lesson 1-6

Descriptive Modeling and Accuracy

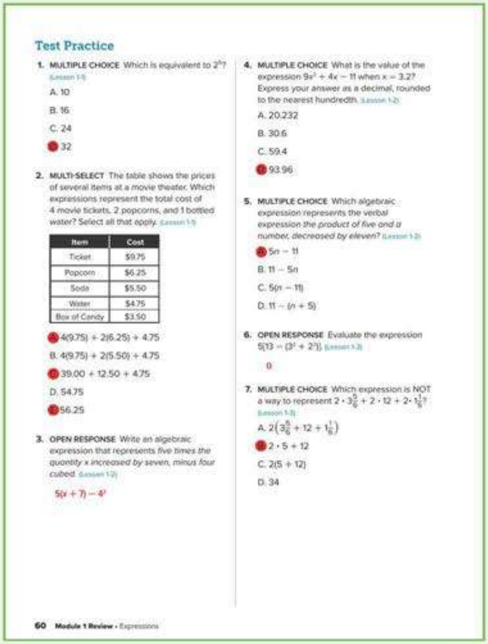
- Descriptive modeling is a way to mathematically describe real-world situations and the factors that cause them.
- A metric is a rule for assigning a number to a characteristic or attribute.
- Accuracy is the nearness of a measurement to the true value of the measure.

Study Organizer

Foldables

Use your Foldable to review the module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 1 Review • Expressions 59



Test Practice

1. **MULTIPLE CHOICE** Which is equivalent to 2^5? (Lesson 1-1)
   A. 10
   B. 16
   C. 24
   D. 32

2. **MULTI-SELECT** The table shows the prices of several items at a movie theater. Which expressions represent the total cost of 4 movie tickets, 2 popcorns, and 1 bottled water? Select all that apply. (Lesson 1-1)

Item	Cost
Ticket	$9.75
Popcorn	$6.25
Soda	$5.50
Water	$4.75
Box of Candy	$3.50

   A. $4(9.75) + 2(6.25) + 4.75$
   B. $4(9.75) + 2(5.50) + 4.75$
   C. $39.00 + 12.50 + 4.75$
   D. 54.75
   E. 56.25

3. **OPEN RESPONSE** Write an algebraic expression that represents *five times the quantity x increased by seven, minus four cubed.* (Lesson 1-2)

   $5(x + 7) - 4^3$

4. **MULTIPLE CHOICE** What is the value of the expression $9x^2 + 4x - 11$ when $x = 3.2$? Express your answer as a decimal, rounded to the nearest hundredth. (Lesson 1-2)
   A. 20.232
   B. 30.6
   C. 59.4
   D. 93.96

5. **MULTIPLE CHOICE** Which algebraic expression represents the verbal expression *the product of five and a number, decreased by eleven?* (Lesson 1-2)
   A. $5n - 11$
   B. $11 - 5n$
   C. $5(n - 11)$
   D. $11 - (n + 5)$

6. **OPEN RESPONSE** Evaluate the expression $5[13 - (3^2 + 2^2)]$. (Lesson 1-3)

   0

7. **MULTIPLE CHOICE** Which expression is NOT a way to represent $2 \cdot 3\frac{5}{6} + 2 \cdot 12 + 2 \cdot 1\frac{1}{6}$? (Lesson 1-3)
   A. $2\left(3\frac{5}{6} + 12 + 1\frac{1}{6}\right)$
   B. $2 \cdot 5 + 12$
   C. $2(5 + 12)$
   D. 34

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 1-1 through 1-3
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

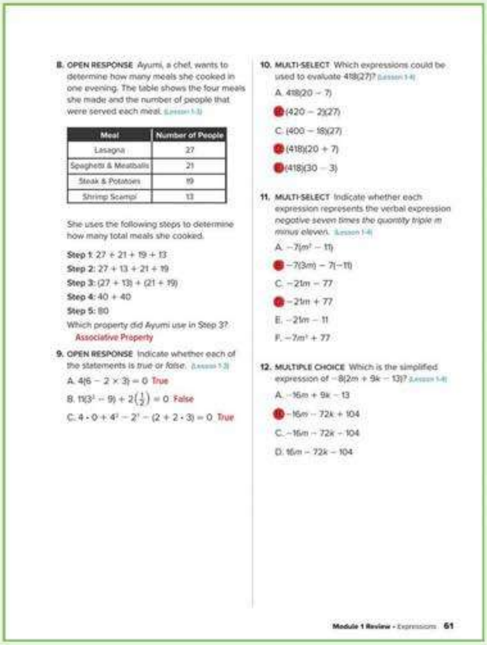
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–17 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 4, 5, 7, 12
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	2, 10, 13, 14
Table Item	Students complete a table by entering in the correct values.	9, 11
Open Response	Students construct their own response.	3, 6, 8, 15-17

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
N.Q.2, N.Q.3	1-6	16, 17
A.SSE.1	1-2, 1-3	3-6
A.SSE.1a	1-4	11, 13
A.SSE.1b	1-1	2
A.SSE.2	1-1, 1-3, 1-4, 1-5	1, 7-10, 12, 14, 15



8. **OPEN RESPONSE** Ayumi, a chef, wants to determine how many meals she cooked in one evening. The table shows the four meals she made and the number of people that were served each meal. (Lesson 1-3)

Meal	Number of People
Lasagna	27
Spaghetti & Meatballs	21
Steak & Potatoes	19
Shrimp Scampi	13

She uses the following steps to determine how many total meals she cooked.

Step 1: $27 + 21 + 19 + 13$
Step 2: $27 + 13 + 21 + 19$
Step 3: $(27 + 13) + (21 + 19)$
Step 4: $40 + 40$
Step 5: 80

Which property did Ayumi use in Step 3?
Associative Property

9. **OPEN RESPONSE** Indicate whether each of the statements is *true* or *false*. (Lesson 1-3)

A. $4(6 - 2 \times 3) = 0$ True
B. $11(3^2 - 9) + 2\left(\frac{1}{2}\right) = 0$ False
C. $4 \cdot 0 + 4^2 - 2^3 - (2 + 2 \cdot 3) = 0$ True

10. **MULTI-SELECT** Which expressions could be used to evaluate $418(27)$? (Lesson 1-4)

A. $418(20 - 7)$
● B. $(420 - 2)(27)$
C. $(400 - 18)(27)$
● D. $(418)(20 + 7)$
● E. $(418)(30 - 3)$

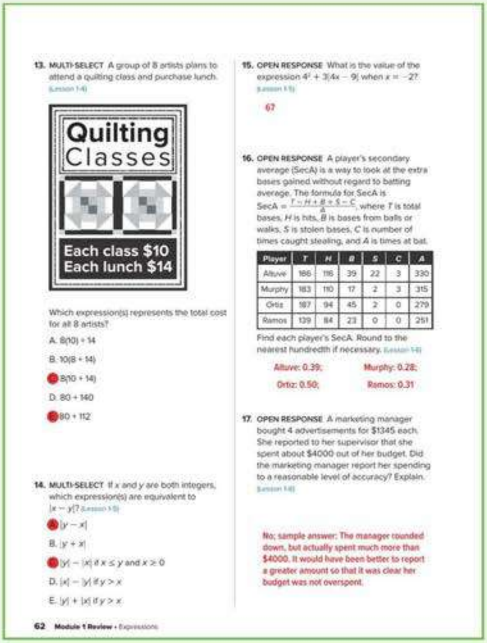
11. **MULTI-SELECT** Indicate whether each expression represents the verbal expression *negative seven times the quantity triple m minus eleven.* (Lesson 1-4)

A. $-7(m^3 - 11)$
● B. $-7(3m) - 7(-11)$
C. $-21m - 77$
● D. $-21m + 77$
E. $-21m - 11$
F. $-7m^3 + 77$

12. **MULTIPLE CHOICE** Which is the simplified expression of $-8(2m + 9k - 13)$? (Lesson 1-4)

A. $-16m + 9k - 13$
● B. $-16m - 72k + 104$
C. $-16m - 72k - 104$
D. $16m - 72k - 104$

Module 1 Review • Expressions 61



13. **MULTI-SELECT** A group of 8 artists plans to attend a quilting class and purchase lunch. (Lesson 1-4)

Quilting
Classes

Each class $10
Each lunch $14

Which expression(s) represents the total cost for all 8 artists?

A. 8(10) + 14

B. 10(8 + 14)

[✓ C.] 8(10 + 14)

D. 80 + 140

[✓ E.] 80 + 112

14. **MULTI-SELECT** If x and y are both integers, which expression(s) are equivalent to $|x - y|$? (Lesson 1-5)

[✓ A.] $|y - x|$

B. $|y + x|$

[✓ C.] $|y| - |x|$ if $x \leq y$ and $x \geq 0$

D. $|x| - |y|$ if $y > x$

E. $|y| + |x|$ if $y > x$

15. **OPEN RESPONSE** What is the value of the expression $4^2 + 3|4x - 9|$ when $x = -2$? (Lesson 1-5)

67

16. **OPEN RESPONSE** A player's secondary average (SecA) is a way to look at the extra bases gained without regard to batting average. The formula for SecA is $\text{SecA} = \frac{T - H + B + S - C}{A}$, where T is total bases, H is hits, B is bases from balls or walks, S is stolen bases, C is number of times caught stealing, and A is times at bat.

Player	T	H	B	S	C	A
Altuve	186	116	39	22	3	330
Murphy	183	110	17	2	3	315
Ortiz	187	94	45	2	0	279
Ramos	139	84	23	0	0	251

Find each player's SecA. Round to the nearest hundredth if necessary. (Lesson 1-6)

Altuve: 0.39; Murphy: 0.28;
Ortiz: 0.50; Ramos: 0.31

17. **OPEN RESPONSE** A marketing manager bought 4 advertisements for $1345 each. She reported to her supervisor that she spent about $4000 out of her budget. Did the marketing manager report her spending to a reasonable level of accuracy? Explain. (Lesson 1-6)

No; sample answer: The manager rounded down, but actually spent much more than $4000. It would have been better to report a greater amount so that it was clear her budget was not overspent.

62 **Module 1 Review** • Expressions

Module 2

Equations in One Variable

Module Goals

- Students solve linear equations in one variable.
- Students solve proportions.
- Students use formulas to solve real-world problems.

Focus

Domains: Number and Quantity, Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Also addresses A.CED.3, A.CED.4, and N.Q.1.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Coherence

Vertical Alignment

Previous

Students wrote and solved one-, two-, and multi-step equations in one variable.

6.EE.7, 7.EE.4a, 8.EE.7

Now

Students write and solve equations in one variable.

A.CED.1

Next

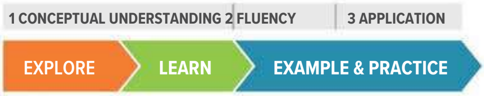
Students will construct equations in two variables.

A.CED.2

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.



Suggested Pacing

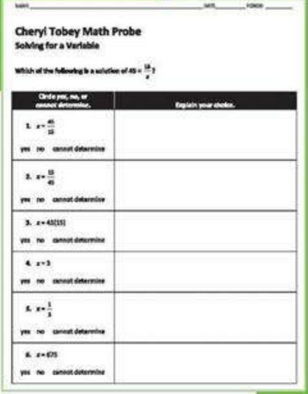
Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
2-1 Writing and Interpreting Equations	A.CED.1, A.CED.3	1	0.5
2-2 Solving One-Step Equations	A.CED.1, A.REI.1, A.REI.3	2	1
2-3 Solving Multi-Step Equations	A.CED.1, A.REI.3	1	0.5
2-4 Solving Equations with the Variable on Each Side	A.CED.1, A.REI.3	2	1
Put It All Together: Lessons 2-1 through 2-4		1	0.5
2-5 Solving Equations Involving Absolute Value	A.CED.1, A.REI.3	1	0.5
2-6 Solving Proportions	A.CED.1, A.REI.3	1	0.5
2-7 Using Formulas	A.CED.4, A.REI.3	2	1
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	14	7

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Solving for a Variable

Module Resource



Cheryl Tobey Math Probe
Solving for a Variable

Which of the following is a solution of $45 = \frac{15}{x}$?

Circle yes, no, or cannot determine.	Explain your choice.
1. $x = \frac{45}{15}$ yes no cannot determine	
2. $x = \frac{15}{45}$ yes no cannot determine	
3. $x = 45(15)$ yes no cannot determine	
4. $x = 3$ yes no cannot determine	
5. $x = \frac{1}{3}$ yes no cannot determine	
6. $x = 675$ yes no cannot determine	

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students determine which equation has been solved correctly and explain their choices.

Targeted Concepts Understand the relationships between numbers and variables in an equation.

Targeted Misconceptions

- When students do not understand the relationships between numbers and variables, they often incorrectly manipulate the numbers and/or variable by either
  - "flipping" only one side of the equality,
  - leaving the variable in the denominator and trying to "work around it," or
  - moving the variable to the numerator without maintaining the equality.
- Students may not recognize a simplified version or may not consider a nonsimplified version of the correct answer.

Use the Probe after Lesson 2-4.

Correct Answers:
1. no 2. yes 3. no
4. no 5. yes 6. no

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
1. yes **2.** no **4.** yes	did not use the proper steps to isolate *x*. Often students automatically divide the larger number by the smaller number without taking the relationship between the numbers into consideration.
2. no **5.** no	did not recognize the simplified form of $\frac{15}{45}$ (choice 5), or did not consider choice 2 as it is not in simplified form.
3. yes **6.** yes	realized there is division taking place on the right side of the equation and that multiplication is the inverse operation but did not take into consideration that the variable is in the denominator.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Multi-Step Linear Equations
- Lesson 2-3, Learn, Examples 1–2

Revisit the Probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

How can writing and solving equations help you solve problems in the real world? Sample answer: Equations can be written to describe the relationship between quantities in the real world. Solving these equations provides information about unknown quantities.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

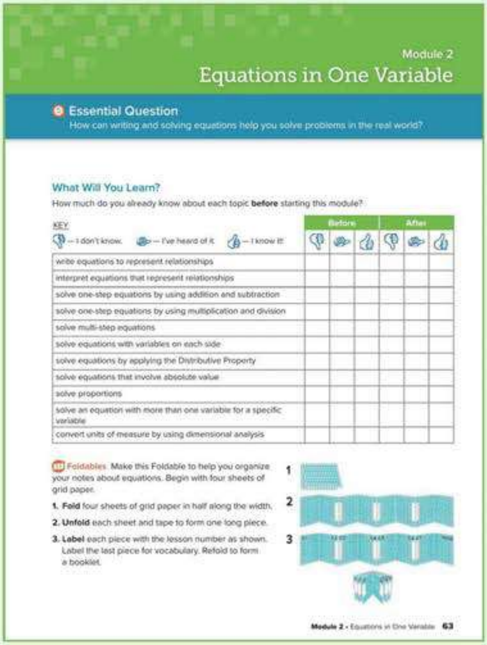
Focus Students read and learn about linear equations.

Teach Throughout the module, have students take notes from each lesson on the appropriate Foldables page. They should include definitions of terms and key concepts. Encourage students to record examples of the various linear equations from each lesson.

When to Use It Use the appropriate page in the Foldable booklet as students cover each lesson in this module.

Launch the Module

For this module, the Launch the Module video uses sports to demonstrate how equations can be written and solved in the real world. Students learn how equations, formulas, and absolute value can help calculate unknowns in the sports field.



Module 2

Equations in One Variable

Essential Question

How can writing and solving equations help you solve problems in the real world?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. 👉 — I've heard of it. 👍 — I know it!

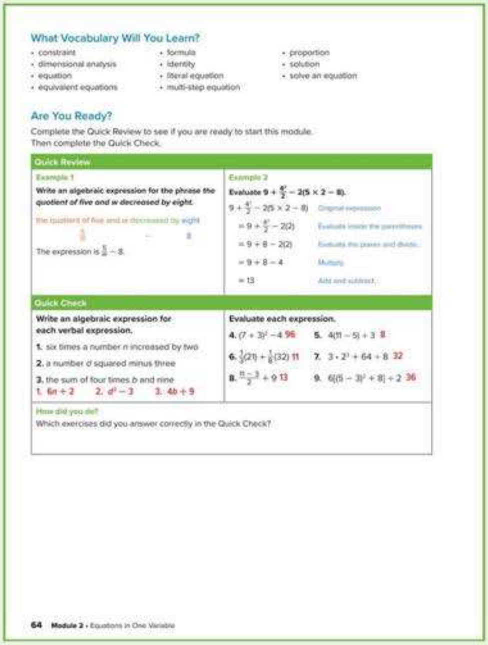
	Before			After		
	👎	👉	👍	👎	👉	👍
write equations to represent relationships						
interpret equations that represent relationships						
solve one-step equations by using addition and subtraction						
solve one-step equations by using multiplication and division						
solve multi-step equations						
solve equations with variables on each side						
solve equations by applying the Distributive Property						
solve equations that involve absolute value						
solve proportions						
solve an equation with more than one variable for a specific variable						
convert units of measure by using dimensional analysis						

Foldables Make this Foldable to help you organize your notes about equations. Begin with four sheets of grid paper.

1. **Fold** four sheets of grid paper in half along the width.
2. **Unfold** each sheet and tape to form one long piece.
3. **Label** each piece with the lesson number as shown. Label the last piece for vocabulary. Refold to form a booklet.

Module 2 • Equations in One Variable **63**

Interactive Presentation



What Vocabulary Will You Learn?

- constraint
- dimensional analysis
- equation
- equivalent equations
- formula
- identity
- literal equation
- multi-step equation
- proportion
- solution
- solve an equation

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Write an algebraic expression for the phrase *the quotient of five and w decreased by eight.*

the quotient of five and w decreased by eight

$\frac{5}{w}$ $-$ 8

The expression is $\frac{5}{w} - 8$.

Example 2

Evaluate $9 + \frac{4^2}{2} - 2(5 \times 2 - 8)$.

$9 + \frac{4^2}{2} - 2(5 \times 2 - 8)$ Original expression

$= 9 + \frac{4^2}{2} - 2(2)$ Evaluate inside the parentheses.

$= 9 + 8 - 2(2)$ Evaluate the power and divide.

$= 9 + 8 - 4$ Multiply.

$= 13$ Add and subtract.

Quick Check

Write an algebraic expression for each verbal expression.

1. six times a number n increased by two

2. a number d squared minus three

3. the sum of four times b and nine

1. $6n + 2$ 2. $d^2 - 3$ 3. $4b + 9$

Evaluate each expression.

4. $(7 + 3)^2 - 4$ 96

5. $4(11 - 5) \div 3$ 8

6. $\frac{1}{3}(21) + \frac{1}{8}(32)$ 11

7. $3 \cdot 2^3 + 64 \div 8$ 32

8. $\frac{11 - 3}{2} + 9$ 13

9. $6[(5 - 3)^2 + 8] \div 2$ 36

How did you do?

Which exercises did you answer correctly in the Quick Check?

64 **Module 2** • Equations in One Variable

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define An equation is a mathematical statement that contains two expressions and an equal sign, =.

Example $3p + 7 = 2$.

Ask Does this statement have two expressions and an equal sign? Yes, $3p + 7$ and 2 are both expressions with an equal sign between them.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- translating phrases into expressions
- multiplying and dividing rational numbers
- using the Distributive Property
- evaluating expressions
- calculating opposites and absolute values
- determining whether two ratios are equivalent
- translating sentences into equations

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the topics in the **Linear Equations** module—who is ready to learn these topics and who isn't quite ready to learn them yet—in order to adjust your instruction as appropriate.

Mindset Matters

View Challenges as Opportunities

Part of cultivating a growth mindset in math involves viewing challenging problems or tasks as an opportunity to learn and make new connections in your brain.

How Can I Apply It?

Encourage students to embrace challenges by trying problems that are thought provoking, such as the **Higher-Order Thinking Problems** in the practice section of each lesson. Remember to regularly remind students that each new challenge is an opportunity to grow!

Lesson 2-1

Writing and Interpreting Equations

A.CED.1, A.CED.3

LESSON GOAL

Students create and interpret equations that describe relationships.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Writing Equations by Modeling a Real-World Situation

Develop:

Writing Equations
- Write an Equation for a Sentence
- Write an Equation
- Write an Equation with Multiple Variables

Interpreting Equations
- Write a Sentence for an Equation
- Write a Sentence for an Equation with Grouping Symbols
- Interpret an Equation

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Write Algebraic Expressions	●	●		●
Extension: Guess the Number		●	●	●

Language Development Handbook

Assign page 7 of the *Language Development Handbook* to help your students build mathematical language related to creating and interpreting equations.

ELL You can use the tips and suggestions on page T7 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students evaluated and simplified algebraic expressions.
6.EE.2, 6.EE.3, A.SSE.1

Now
Students translate between sentences and equations and use them to solve problems.
A.CED.1, A.CED.3

Next
Students will solve one-step equations with the four operations.
A.REI.3

Rigor

The Three Pillars of Rigor

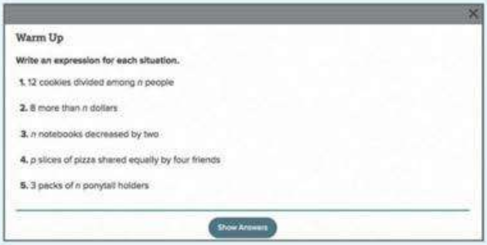
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of equations by expanding on what they have learned about expressions. They build fluency by writing and interpreting equations, and then they apply their understanding by interpreting solutions.

Mathematical Background

Variables are used to represent an unknown amount when writing equations from a verbal sentence. The ability to write an equation from a verbal sentence is needed when solving word problems. When a verbal sentence can be translated into an equation that states a rule for the relationship between certain quantities, the equation is then called a formula, and can be used to solve problems involving those quantities.

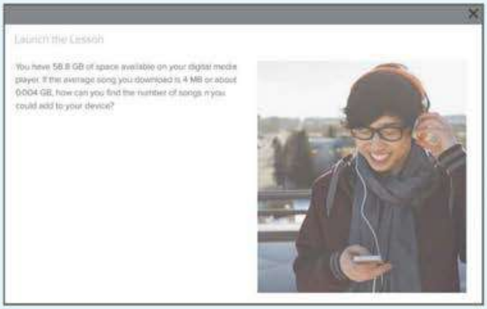
Interactive Presentation


Warm Up

Write an expression for each situation.

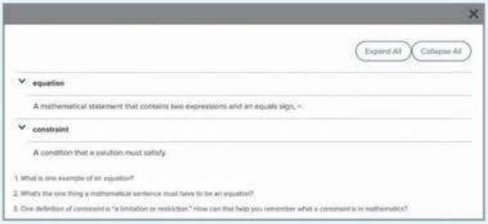
1. 12 cookies divided among *n* people
2. 8 more than *n* dollars
3. *n* notebooks decreased by two
4. *p* slices of pizza shared equally by four friends
5. 3 packs of *n* ponytail holders

Warm Up


Launch the Lesson

You have 58.8 GB of space available on your digital media player. If the average song you download is 4 MB or about 0.004 GB, how can you find the number of songs *n* you could add to your device?

Launch the Lesson



equation

A mathematical statement that contains two expressions and an equals sign, =.

constraint

A condition that a solution must satisfy.

1. What is one example of an equation?
2. What's the one thing a mathematical sentence must have to be an equation?
3. One definition of constraint is "a limitation or restriction." How can that help you remember what a constraint is in mathematics?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- translating phrases into expressions

Answers:

1. $12 \div n$
2. $n + 8$ or $8 + n$
3. $n - 2$
4. $p \div 4$
5. $3n$

Launch the Lesson

Teaching the Mathematical Practices

2 Represent a Situation Symbolically Encourage students to define all variables before writing an expression about the number of songs that can be downloaded on a digital media player.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class

Explore Writing Equations by Modeling a Real-World Situation

Objective

Students use a real-world situation to explore how to write equations.

Teaching the Mathematical Practices

2 Represent a Situation Symbolically Guide students to define variables to solve the problem in this Explore. Help students to identify the independent and dependent variables. Then work with them to find the other relationships in the problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

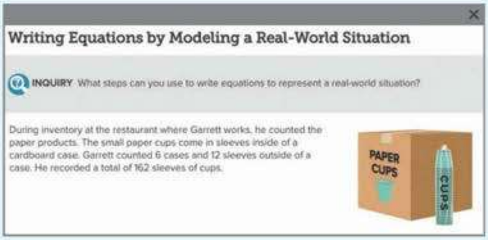
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use a drag and drop activity to help organize the presented information before considering algebraic representations. The guiding exercises will lead students to write an algebraic equation involving the given information. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

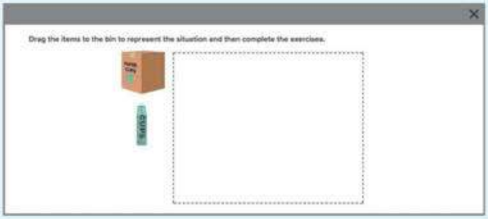


Writing Equations by Modeling a Real-World Situation

INQUIRY What steps can you use to write equations to represent a real-world situation?

During inventory at the restaurant where Garrett works, he counted the paper products. The small paper cups come in sleeves inside of a cardboard case. Garrett counted 6 cases and 12 sleeves outside of a case. He recorded a total of 162 sleeves of cups.

Explore



Drag the items to the bin to represent the situation and then complete the exercises.

Explore

DRAG & DROP

Students drag items to represent the given situation.

TYPE

Students answer questions to show they understand how to write an algebraic equation.

Interactive Presentation

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Writing Equations by Modeling a Real-World Situation (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does it help to define your variable when writing an expression? Sample answer: In order to define the variable, you have to think about the part of the problem that is unknown. Then you can see how the other values are related and come up with the expression.
- Would both sides of the equation change if Garrett used 7 sleeves of cups? What is the new equation? Yes; sample answer: Garrett would need to use 7 of the sleeves that are outside the box, which would change the expression to $6n + 5$. The total number of sleeves would also change, $162 - 7 = 155$. The new equation is $6n + 5 = 155$.

Inquiry

What steps can you use to write equations to represent a real-world situation? Sample answer: Identify the unknowns, write two equivalent expressions, and use the expressions with an equal sign to write the equation.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Writing Equations

Objective

Students create equations that describe relationships by interpreting words as mathematical symbols.

Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

Common Misconception

Students may believe that the order in which terms are placed in an equation does not matter Point out that subtraction and division are not commutative and the order in which terms are subtracted or divided does matter

Example 1 Write an Equation for a Sentence

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation that represents the sentence in this example.

Questions for Mathematical Discourse

AL How many operations are in this equation? 3

OL Why is the expression on the left not $\frac{7}{x} - 20$? Sample answer: The word *minus* lets you know that the quotient is being subtracted from 20.

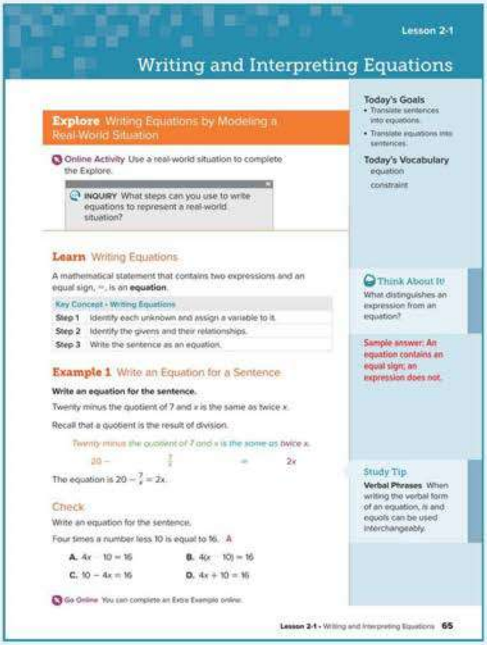
BL Why does the order matter in a division expression? Division is not commutative and the terms cannot be interchanged.

Common Error

Students may not realize the word *twice* means multiply by two. Point out to students that *twice* or *double* mean multiply by two while *triple* means multiply by three.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-1

Writing and Interpreting Equations

Explore Writing Equations by Modeling a Real-World Situation

Online Activity Use a real-world situation to complete the Explore.

INQUIRY What steps can you use to write equations to represent a real-world situation?

Today's Goals
- Translate sentences into equations.
- Translate equations into sentences.

Today's Vocabulary
equation
constraint

Learn Writing Equations

A mathematical statement that contains two expressions and an equal sign, =, is an **equation**.

Key Concept • Writing Equations

Step 1	Identify each unknown and assign a variable to it.
Step 2	Identify the givens and their relationships.
Step 3	Write the sentence as an equation.

Think About It! What distinguishes an expression from an equation?

Sample answer: An equation contains an equal sign; an expression does not.

Example 1 Write an Equation for a Sentence

Write an equation for the sentence.

Twenty minus the quotient of 7 and x is the same as twice x.

Recall that a quotient is the result of division.

Twenty minus the quotient of 7 and x is the same as twice x.

$20 - \quad \frac{7}{x} \quad = \quad 2x$

The equation is $20 - \frac{7}{x} = 2x$.

Study Tip
Verbal Phrases When writing the verbal form of an equation, *is* and *equals* can be used interchangeably.

Check

Write an equation for the sentence.

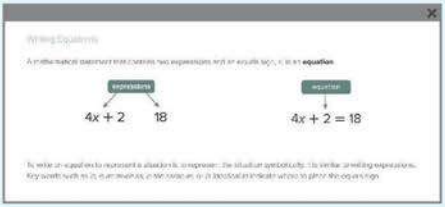
Four times a number less 10 is equal to 16. A

A. $4x - 10 = 16$
B. $4(x - 10) = 16$
C. $10 - 4x = 16$
D. $4x + 10 = 16$

Go Online You can complete an Extra Example online.

Lesson 2-1 • Writing and Interpreting Equations 65

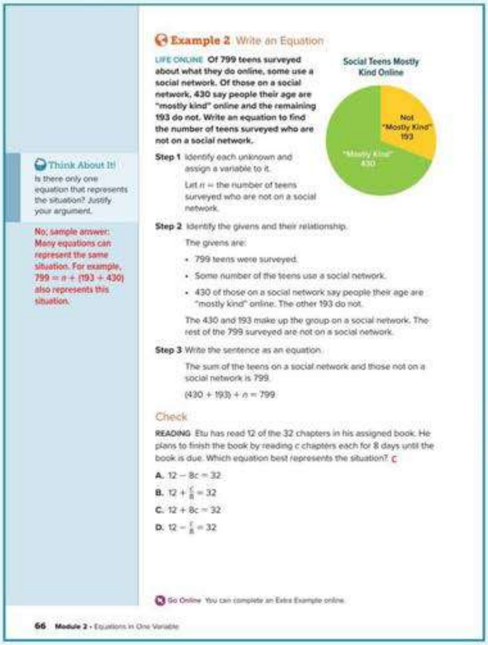
Interactive Presentation



Learn

TYPE

Students answer a question to show they understand the difference between an expression and an equation.



Example 2 Write an Equation

LIFE ONLINE **Of 799 teens surveyed about what they do online, some use a social network. Of those on a social network, 430 say people their age are "mostly kind" online and the remaining 193 do not. Write an equation to find the number of teens surveyed who are not on a social network.**

Step 1 Identify each unknown and assign a variable to it.

Let n = the number of teens surveyed who are not on a social network.

Step 2 Identify the givens and their relationship.

The givens are:

- 799 teens were surveyed.
- Some number of the teens use a social network.
- 430 of those on a social network say people their age are "mostly kind" online. The other 193 do not.

The 430 and 193 make up the group on a social network. The rest of the 799 surveyed are not on a social network.

Step 3 Write the sentence as an equation.

The sum of the teens on a social network and those not on a social network is 799.

$(430 + 193) + n = 799$

Think About It! Is there only one equation that represents the situation? Justify your argument.

No; sample answer: Many equations can represent the same situation. For example, $799 = n + (193 + 430)$ also represents this situation.

Check

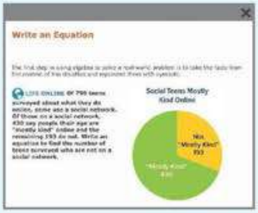
READING Etu has read 12 of the 32 chapters in his assigned book. He plans to finish the book by reading c chapters each for 8 days until the book is due. Which equation best represents the situation? C

A. $12 - 8c = 32$
B. $12 + \frac{c}{8} = 32$
C. $12 + 8c = 32$
D. $12 - \frac{c}{8} = 32$

Go Online You can complete an Extra Example online.

66 **Module 2** • Equations in One Variable

Interactive Presentation



Example 2

TAP

Students move through the steps to write an equation.

TYPE

Students answer a question to show they understand whether there is more than one possible equation that can be written.

Example 2 Write an Equation

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about writing equations to solve a real-world problem.

Questions for Mathematical Discourse

AL Is it possible for a surveyed teen to both use a social network and also not use a social network? no

OL Does the remaining "193 did not" refer to teens who do not use a social network or to teens who are not "mostly kind?" Explain. Teens who are not "mostly kind." Sample answer: The part of the sentence right before this phrase gave information about "mostly kind" teens, which means the remainder of the sentence would be about the same topic.

BL How would you solve for n in the equation? Explain the steps. Sample answer: First I would simplify the numbers inside the parentheses to get $623 + n = 799$. To solve for n, I would subtract 623 from both sides giving $n = 176$.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are having difficulty identifying the mathematical meaning of words,

THEN have students make a list of words or phrases they do know and put other words or phrases they do not know next to the ones they most closely match; discuss with another student, then rearrange, if needed.

Enrichment Activity BL

Write and simplify a variable expression to determine why the following works.

Think of a number. Multiply by 10. Add 5 to your result. Next, subtract 3. Then add 2. Next, subtract 4. Divide your result by 5. Finally, subtract your original number. Your result is your original number.

x = number   Simplify: $(10x + 2 + 2 - 4) \div 5 - x$ Subtract.
$10x$ $(10x + 4 - 4) \div 5 - x$ Add.
$10x + 5$ $(10x) \div 5 - x$ Subtract.
$10x + 5 - 3$ $2x - x$ Divide.
$10x + 5 - 3 + 2$ x Simplify.
$10x + 5 - 3 + 2 - 4$
$(10x + 5 - 3 + 2 - 4) \div 5$
$(10x + 5 - 3 + 2 - 4) \div 5 - x$

Example 3 Write an Equation with Multiple Variables

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

- **AL** What variables are used to represent the unknowns? P, ℓ, w
- **OL** What operations occur in the given relationship? multiplication and addition
- **BL** What is the perimeter of an equilateral triangle, which is a triangle whose sides are all the same measure? Sample answer: $P = 3s$

Essential Question Follow-Up

Students have begun writing equations including real-world situations.

Ask:

Why is it important for you to be able to write equations to help solve problems in the real world? Sample answer: I need to be able to write equations for problems in the real world because this will help me quickly set up and solve problems that I may face.

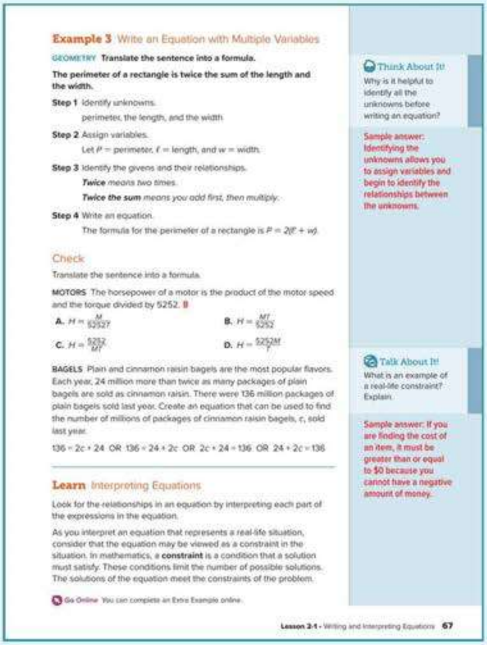
Learn Interpreting Equations

Objective

Students interpret equations that describe relationships by interpreting mathematical symbols as words.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.



Example 3 Write an Equation with Multiple Variables

GEOMETRY **Translate the sentence into a formula.**

The perimeter of a rectangle is twice the sum of the length and the width.

Step 1 Identify unknowns.

perimeter, the length, and the width

Step 2 Assign variables.

Let P = perimeter, ℓ = length, and w = width.

Step 3 Identify the givens and their relationships.

Twice *means two times.*

Twice the sum *means you add first, then multiply.*

Step 4 Write an equation.

The formula for the perimeter of a rectangle is $P = 2(\ell + w)$.

Think About It!

Why is it helpful to identify all the unknowns before writing an equation?

Sample answer: Identifying the unknowns allows you to assign variables and begin to identify the relationships between the unknowns.

Check

Translate the sentence into a formula.

MOTORS The horsepower of a motor is the product of the motor speed and the torque divided by 5252. B

A. $H = \frac{M}{5252T}$

B. $H = \frac{MT}{5252}$

C. $H = \frac{5252}{MT}$

D. $H = \frac{5252M}{T}$

BAGELS Plain and cinnamon raisin bagels are the most popular flavors. Each year, 24 million more than twice as many packages of plain bagels are sold as cinnamon raisin. There were 136 million packages of plain bagels sold last year. Create an equation that can be used to find the number of millions of packages of cinnamon raisin bagels, c, sold last year.

$136 = 2c + 24$ OR $136 = 24 + 2c$ OR $2c + 24 = 136$ OR $24 + 2c = 136$

Learn Interpreting Equations

Look for the relationships in an equation by interpreting each part of the expressions in the equation.

As you interpret an equation that represents a real-life situation, consider that the equation may be viewed as a constraint in the situation. In mathematics, a **constraint** is a condition that a solution must satisfy. These conditions limit the number of possible solutions. The solutions of the equation meet the constraints of the problem.

Talk About It!

What is an example of a real-life constraint? Explain.

Sample answer: If you are finding the cost of an item, it must be greater than or equal to $0 because you cannot have a negative amount of money.

Go Online You can complete an Extra Example online.

Lesson 2-1 • Writing and Interpreting Equations 67

Interactive Presentation

Example 3

TAP

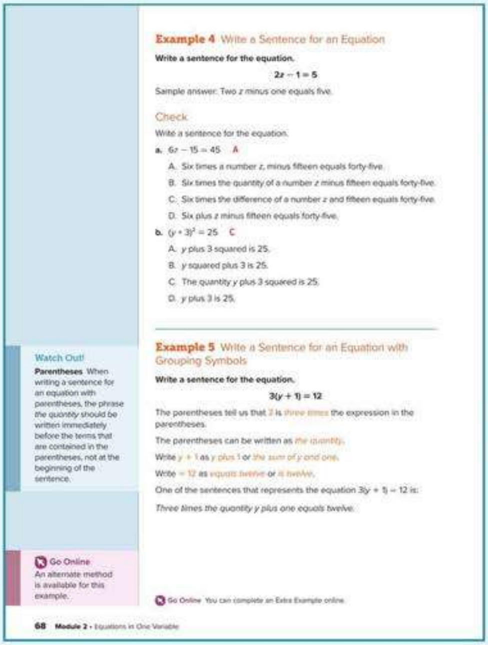
Students move through the steps to write an equation.

TYPE

Students answer a question to show they understand why defining a variable is an important first step.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 4 Write a Sentence for an Equation

Write a sentence for the equation.

$$2z - 1 = 5$$

Sample answer: Two z minus one equals five.

Check

Write a sentence for the equation.

a. $6z - 15 = 45$ A

- A. Six times a number z, minus fifteen equals forty-five.
- B. Six times the quantity of a number z minus fifteen equals forty-five.
- C. Six times the difference of a number z and fifteen equals forty-five.
- D. Six plus z minus fifteen equals forty-five.

b. $(y + 3)^2 = 25$ C

- A. y plus 3 squared is 25.
- B. y squared plus 3 is 25.
- C. The quantity y plus 3 squared is 25.
- D. y plus 3 is 25.

Example 5 Write a Sentence for an Equation with Grouping Symbols

Write a sentence for the equation.

$$3(y + 1) = 12$$

The parentheses tell us that 3 is three times the expression in the parentheses.

The parentheses can be written as the quantity.

Write $y + 1$ as y plus 1 or the sum of y and one.

Write $= 12$ as equals twelve or is twelve.

One of the sentences that represents the equation $3(y + 1) = 12$ is:

Three times the quantity y plus one equals twelve.

Watch Out!

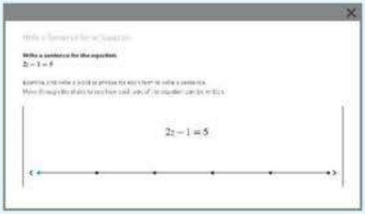
Parentheses When writing a sentence for an equation with parentheses, the phrase *the quantity* should be written immediately before the terms that are contained in the parentheses, not at the beginning of the sentence.

Go Online An alternate method is available for this example.

Go Online You can complete an Extra Example online.

68 **Module 2** • Equations in One Variable

Interactive Presentation



Example 4

TAP

Students move through the steps of writing a sentence.

TYPE

Students answer a question to show they understand how to decide which possible answer choice makes the most sense.

Example 4 Write a Sentence for an Equation

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What words could we use to represent the operations in the equation? Sample answer: for multiplication: *product of* or *times*; for subtraction: *difference of* or *minus*

OL Does the phrase *difference of* change the order of the terms in the equation? no

BL What is another way to write the sentence using different words? Sample answer: Five is one less than the product of two and z.

Common Error

Remind students that there is often more than one way to translate an equation into a verbal sentence. They should pick the words they are most comfortable using to ensure accuracy.

Example 5 Write a Sentence for an Equation with Grouping Symbols

Teaching the Mathematical Practices

1 Understand the Approaches of Others Work with students to look at the Alternate Method. Ask students to compare and contrast the original method and the alternate method.

Questions for Mathematical Discourse

AL What operations appear in the equation? multiplication and addition

OL What words could be used to represent the operation in the parentheses? sum, plus

BL How would the equation translate if there were no grouping symbols? Sample answer: One more than three times y is 12.

Example 6 Interpret an Equation

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about interpreting equations to solve a real-world problem.

Questions for Mathematical Discourse

AL What does the variable *S* represent in the formula? surface area

OL What do the other three variables in the equation represent? ℓ represents length, w represents width, and h represents height

BL Instead of writing two times length times height, how else could you represent the same expression? Sample answer: twice the length times the height

Essential Question Follow-Up

Students have begun to translate equations into sentences.

Ask:

Why would translating equations into verbal sentences be a helpful skill? Sample answer: I need to understand what an equation represents, and if I can write a sentence from an equation, I will understand the parts of the equation.

DIFFERENTIATE

Reteaching Activity AL ELL

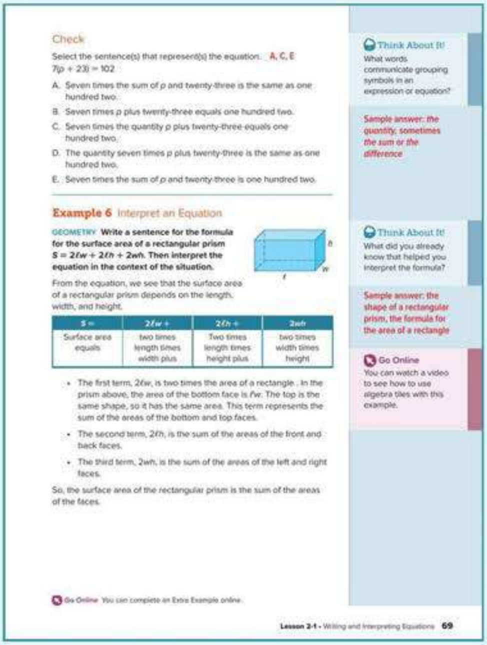
IF students are having difficulty translating equations into sentences, **THEN** have students work with a partner to take turns pretending to be the teacher who calls out a verbal expression for which the other student is to write a numerical or algebraic expression.

Language Development Activity ELL

Beginning Reinforce the use of visual context to derive meaning through examples of environmental print that can be used to write equations such as discount signs and road signs you find online. Pantomime or elicit one-word responses to the meaning derived from such images.

Intermediate Provide images to illustrate problems that students can work in pairs to solve. Move around the room to monitor progress.

Advanced High Provide an image and have students use it to write an equation. Have volunteers share their problems with their group.



Check

Select the sentence(s) that represent(s) the equation. A, C, E

$7(p + 23) = 102$

A. Seven times the sum of p and twenty-three is the same as one hundred two.

B. Seven times p plus twenty-three equals one hundred two.

C. Seven times the quantity p plus twenty-three equals one hundred two.

D. The quantity seven times p plus twenty-three is the same as one hundred two.

E. Seven times the sum of p and twenty-three is one hundred two.

Example 6 Interpret an Equation

GEOMETRY **Write a sentence for the formula for the surface area of a rectangular prism $S = 2\ell w + 2\ell h + 2wh$. Then interpret the equation in the context of the situation.**

From the equation, we see that the surface area of a rectangular prism depends on the length, width, and height.

$S =$	$2\ell w +$	$2\ell h +$	$2wh$
Surface area equals	two times length times width plus	Two times length times height plus	two times width times height

- The first term, $2\ell w$, is two times the area of a rectangle . In the prism above, the area of the bottom face is ℓw. The top is the same shape, so it has the same area. This term represents the sum of the areas of the bottom and top faces.
- The second term, $2\ell h$, is the sum of the areas of the front and back faces.
- The third term, $2wh$, is the sum of the areas of the left and right faces.

So, the surface area of the rectangular prism is the sum of the areas of the faces.

Go Online You can complete an Extra Example online.

Think About It! What words communicate grouping symbols in an expression or equation?

Sample answer: the quantity; sometimes the sum or the difference

Think About It! What did you already know that helped you interpret the formula?

Sample answer: the shape of a rectangular prism, the formula for the area of a rectangle

Go Online You can watch a video to see how to use algebra tiles with this example.

Lesson 2-1 • Writing and Interpreting Equations 69

Interactive Presentation

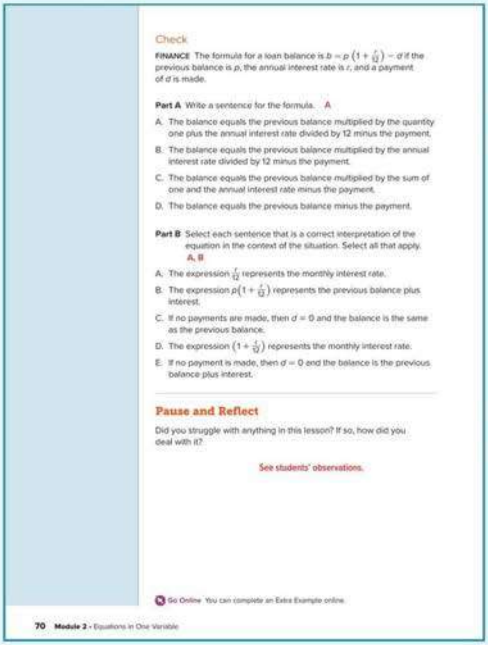
Example 6

TAP

Students move through the slides to write a sentence.

WATCH

Students can watch a video that explains how to use algebra tiles to represent expressions and equations.



Check

FINANCE The formula for a loan balance is $b = p\left(1 + \frac{r}{12}\right) - d$ if the previous balance is p, the annual interest rate is r, and a payment of d is made.

Part A Write a sentence for the formula. A

A. The balance equals the previous balance multiplied by the quantity one plus the annual interest rate divided by 12 minus the payment.
B. The balance equals the previous balance multiplied by the annual interest rate divided by 12 minus the payment.
C. The balance equals the previous balance multiplied by the sum of one and the annual interest rate minus the payment.
D. The balance equals the previous balance minus the payment.

Part B Select each sentence that is a correct interpretation of the equation in the context of the situation. Select all that apply. A, B

A. The expression $\frac{r}{12}$ represents the monthly interest rate.
B. The expression $p\left(1 + \frac{r}{12}\right)$ represents the previous balance plus interest.
C. If no payments are made, then $d = 0$ and the balance is the same as the previous balance.
D. The expression $\left(1 + \frac{r}{12}\right)$ represents the monthly interest rate.
E. If no payment is made, then $d = 0$ and the balance is the previous balance plus interest.

Pause and Reflect

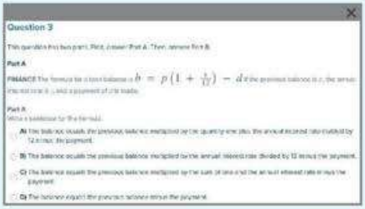
Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

Go Online You can complete an Extra Example online.

70 **Module 2** • Equations in One Variable

Interactive Presentation



Question 3

Part A

CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–36
2	exercises that use a variety of skills from this lesson	37–49
2	exercises that extend concept learned in this lesson to new contexts	50–53
3	exercises that emphasize higher-order and critical-thinking skills	54–61

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–53 odd, 54–61
- Extension: Guess the Number
- ALEKS Writing Expressions and Equations

OL

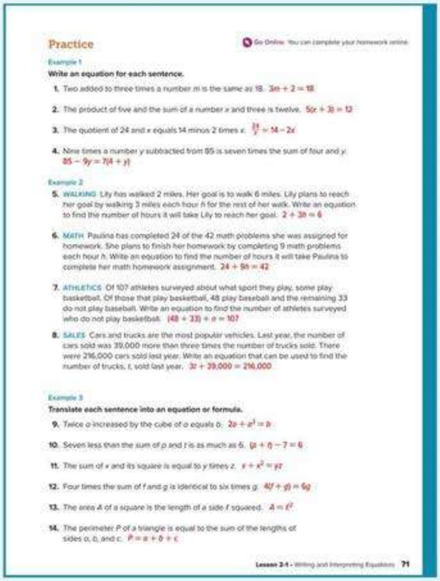
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–61 odd
- Remediation, Review Resources: Write Algebraic Expressions
- Personal Tutors
- Extra Examples 1–6
- ALEKS Evaluating and Writing Expressions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Write Algebraic Expressions
- *Quick Review Math Handbook*: Writing Equations
- Arrive**MATH** Take Another Look
- ALEKS Evaluating and Writing Expressions



Practice

Go Online You can complete your homework online.

Example 1

Write an equation for each sentence.

1. Two added to three times a number m is the same as 18. $3m + 2 = 18$
2. The product of five and the sum of a number x and three is twelve. $5(x + 3) = 12$
3. The quotient of 24 and x equals 14 minus 2 times x. $\frac{24}{x} = 14 - 2x$
4. Nine times a number y subtracted from 85 is seven times the sum of four and y. $85 - 9y = 7(4 + y)$

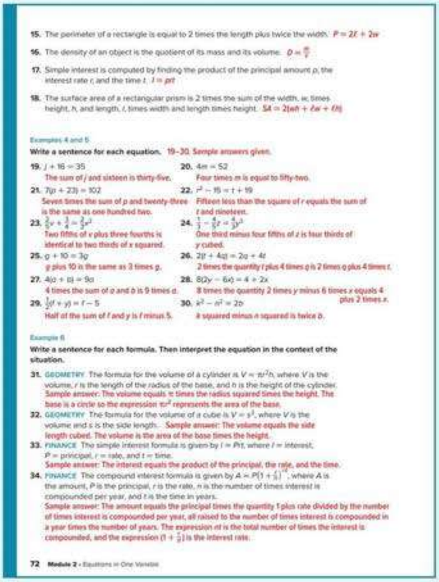
Example 2

5. WALKING Lily has walked 2 miles. Her goal is to walk 6 miles. Lily plans to reach her goal by walking 3 miles each hour h for the rest of her walk. Write an equation to find the number of hours it will take Lily to reach her goal. $2 + 3h = 6$
6. MATH Paulina has completed 24 of the 42 math problems she was assigned for homework. She plans to finish her homework by completing 9 math problems each hour h. Write an equation to find the number of hours it will take Paulina to complete her math homework assignment. $24 + 9h = 42$
7. ATHLETICS Of 107 athletes surveyed about what sport they play, some play basketball. Of those that play basketball, 48 play baseball and the remaining 33 do not play baseball. Write an equation to find the number of athletes surveyed who do not play basketball. $(48 + 33) + n = 107$
8. SALES Cars and trucks are the most popular vehicles. Last year, the number of cars sold was 39,000 more than three times the number of trucks sold. There were 216,000 cars sold last year. Write an equation that can be used to find the number of trucks, t, sold last year. $3t + 39{,}000 = 216{,}000$

Example 3

Translate each sentence into an equation or formula.

9. Twice a increased by the cube of a equals b. $2a + a^3 = b$
10. Seven less than the sum of p and t is as much as 6. $(p + t) - 7 = 6$
11. The sum of x and its square is equal to y times z. $x + x^2 = yz$
12. Four times the sum of f and g is identical to six times g. $4(f + g) = 6g$
13. The area A of a square is the length of a side ℓ squared. $A = \ell^2$
14. The perimeter P of a triangle is equal to the sum of the lengths of sides a, b, and c. $P = a + b + c$



15. The perimeter of a rectangle is equal to 2 times the length plus twice the width. $P = 2\ell + 2w$
16. The density of an object is the quotient of its mass and its volume. $D = \frac{m}{V}$
17. Simple interest is computed by finding the product of the principal amount p, the interest rate r, and the time t. $I = prt$
18. The surface area of a rectangular prism is 2 times the sum of the width, w, times height, h, and length, ℓ, times width and length times height. $SA = 2(wh + \ell w + \ell h)$

Examples 4 and 5

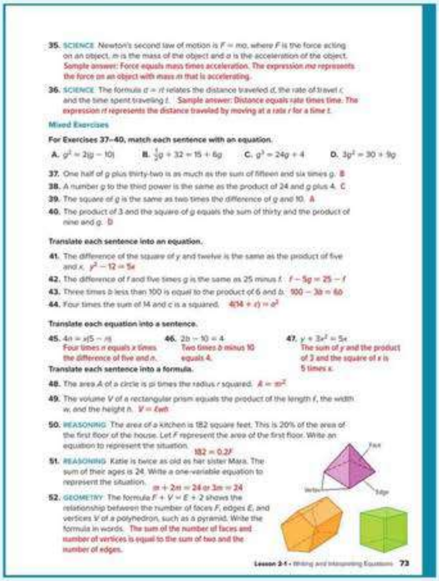
Write a sentence for each equation. 19–30. Sample answers given.

19. $j + 16 = 35$
The sum of j and sixteen is thirty-five.
20. $4m = 52$
Four times m is equal to fifty-two.
21. $7(p + 23) = 102$
Seven times the sum of p and twenty-three is the same as one hundred two.
22. $r^2 - 15 = r + 19$
Fifteen less than the square of r equals the sum of r and nineteen.
23. $\frac{2}{5}v + \frac{3}{4} = \frac{2}{3}x^2$
Two fifths of v plus three fourths is identical to two thirds of x squared.
24. $\frac{1}{3} - \frac{4}{5}z = \frac{4}{3}y^3$
One third minus four fifths of z is four thirds of y cubed.
25. $g + 10 = 3g$
g plus 10 is the same as 3 times g.
26. $2(t + 4q) = 2q + 4t$
2 times the quantity t plus 4 times q is 2 times q plus 4 times t.
27. $4(a + b) = 9a$
4 times the sum of a and b is 9 times a.
28. $8(2y - 6x) = 4 + 2x$
8 times the quantity 2 times y minus 6 times x equals 4 plus 2 times x.
29. $\frac{1}{2}(f + y) = f - 5$
Half of the sum of f and y is f minus 5.
30. $k^2 - n^2 = 2b$
k squared minus n squared is twice b.

Example 6

Write a sentence for each formula. Then interpret the equation in the context of the situation.

31. GEOMETRY The formula for the volume of a cylinder is $V = \pi r^2 h$, where V is the volume, r is the length of the radius of the base, and h is the height of the cylinder. Sample answer: The volume equals π times the radius squared times the height. The base is a circle so the expression πr^2 represents the area of the base.
32. GEOMETRY The formula for the volume of a cube is $V = s^3$, where V is the volume and s is the side length. Sample answer: The volume equals the side length cubed. The volume is the area of the base times the height.
33. FINANCE The simple interest formula is given by $I = Prt$, where I = interest, P = principal, r = rate, and t = time.
Sample answer: The interest equals the product of the principal, the rate, and the time.
34. FINANCE The compound interest formula is given by $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the amount, P is the principal, r is the rate, n is the number of times interest is compounded per year, and t is the time in years.
Sample answer: The amount equals the principal times the quantity 1 plus rate divided by the number of times interest is compounded per year, all raised to the number of times interest is compounded in a year times the number of years. The expression nt is the total number of times the interest is compounded, and the expression $\left(1 + \frac{r}{n}\right)$ is the interest rate.



35. SCIENCE Newton's second law of motion is $F = ma$, where F is the force acting on an object, m is the mass of the object and a is the acceleration of the object. Sample answer: Force equals mass times acceleration. The expression ma represents the force on an object with mass m that is accelerating.

36. SCIENCE The formula $d = rt$ relates the distance traveled d, the rate of travel r, and the time spent traveling t. Sample answer: Distance equals rate times time. The expression rt represents the distance traveled by moving at a rate r for a time t.

Mixed Exercises

For Exercises 37–40, match each sentence with an equation.

A. $g^2 = 2(g - 10)$ B. $\frac{1}{2}g + 32 = 15 + 6g$ C. $g^3 = 24g + 4$ D. $3g^2 = 30 + 9g$

37. One half of g plus thirty-two is as much as the sum of fifteen and six times g. B
38. A number g to the third power is the same as the product of 24 and g plus 4. C
39. The square of g is the same as two times the difference of g and 10. A
40. The product of 3 and the square of g equals the sum of thirty and the product of nine and g. D

Translate each sentence into an equation.

41. The difference of the square of y and twelve is the same as the product of five and x. $y^2 - 12 = 5x$
42. The difference of f and five times g is the same as 25 minus f. $f - 5g = 25 - f$
43. Three times b less than 100 is equal to the product of 6 and b. $100 - 3b = 6b$
44. Four times the sum of 14 and c is a squared. $4(14 + c) = a^2$

Translate each equation into a sentence.

45. $4n = x(5 - n)$ Four times n equals x times the difference of five and n.
46. $2b - 10 = 4$ Two times b minus 10 equals 4.
47. $y + 3x^2 = 5x$ The sum of y and the product of 3 and the square of x is 5 times x.

Translate each sentence into a formula.

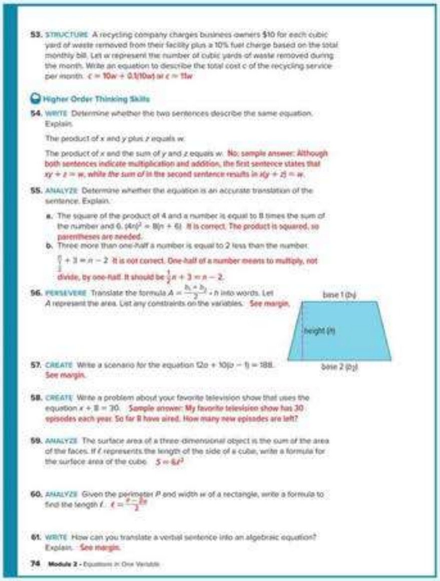
48. The area A of a circle is pi times the radius r squared. $A = \pi r^2$
49. The volume V of a rectangular prism equals the product of the length ℓ, the width w, and the height h. $V = \ell wh$

50. REASONING The area of a kitchen is 182 square feet. This is 20% of the area of the first floor of the house. Let F represent the area of the first floor. Write an equation to represent the situation. $182 = 0.2F$

51. REASONING Katie is twice as old as her sister Mara. The sum of their ages is 24. Write a one-variable equation to represent the situation. $m + 2m = 24$ or $3m = 24$

52. GEOMETRY The formula $F + V = E + 2$ shows the relationship between the number of faces F, edges E, and vertices V of a polyhedron, such as a pyramid. Write the formula in words. The sum of the number of faces and number of vertices is equal to the sum of two and the number of edges.

Lesson 2-1 • Writing and Interpreting Equations 73



53. STRUCTURE A recycling company charges business owners \$10 for each cubic yard of waste removed from their facility plus a 10% fuel charge based on the total monthly bill. Let w represent the number of cubic yards of waste removed during the month. Write an equation to describe the total cost c of the recycling service per month. $c = 10w + 0.1(10w)$ or $c = 11w$

Higher Order Thinking Skills

54. WRITE Determine whether the two sentences describe the same equation. Explain.

The product of x and y plus z equals w.

The product of x and the sum of y and z equals w. No; sample answer: Although both sentences indicate multiplication and addition, the first sentence states that $xy + z = w$, while the sum of in the second sentence results in $x(y + z) = w$.

55. ANALYZE Determine whether the equation is an accurate translation of the sentence. Explain.

a. The square of the product of 4 and a number is equal to 8 times the sum of the number and 6. $(4n)^2 = 8(n + 6)$ It is correct. The product is squared, so parentheses are needed.

b. Three more than one-half a number is equal to 2 less than the number. $\frac{n}{\frac{1}{2}} + 3 = n - 2$ It is not correct. One-half of a number means to multiply, not divide, by one-half. It should be $\frac{1}{2}n + 3 = n - 2$.

56. PERSEVERE Translate the formula $A = \frac{b_1 + b_2}{2} \cdot h$ into words. Let A represent the area. List any constraints on the variables. See margin.

57. CREATE Write a scenario for the equation $12a + 10(a - 1) = 188$. See margin.

58. CREATE Write a problem about your favorite television show that uses the equation $x + 8 = 30$. Sample answer: My favorite television show has 30 episodes each year. So far 8 have aired. How many new episodes are left?

59. ANALYZE The surface area of a three-dimensional object is the sum of the area of the faces. If ℓ represents the length of the side of a cube, write a formula for the surface area of the cube. $S = 6\ell^2$

60. ANALYZE Given the perimeter P and width w of a rectangle, write a formula to find the length ℓ. $\ell = \frac{P - 2w}{2}$

61. WRITE How can you translate a verbal sentence into an algebraic equation? Explain. See margin.

74 Module 2 • Equations in One Variable

Answers

56. Sample answer: The area of a trapezoid is equal to the height times the quotient of the sum of the bases and 2. All of the variables must be positive.

57. Sample answer: Mr. Rhoads ordered 188 math books. The algebra books were packed in boxes of 12. The geometry books were packed in boxes of 10. He ordered one more box of algebra books than geometry books. How many books of each type book did he order? Let a = number of algebra books.

61. Sample answer: First you should identify the unknown quantity or quantities for which you are trying to solve, and assign variables. Then you should look for key words or phrases that can help you to determine operations that are being used. You can then write the equation using the numbers that you are given and the variables and operations that you assigned.

Lesson 2-2

Solving One-Step Equations

A.CED.1, A.REI.1, A.REI.3

LESSON GOAL

Students solve equations by using addition, subtraction, multiplication, and division.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore:

- Using Algebra Tiles to Solve One-Step Equations Involving Addition or Subtraction
- Using Algebra Tiles to Solve One-Step Equations Involving Multiplication

Develop:

Solving One-Step Equations Involving Addition or Subtraction

- Solve by Adding
- Solve by Subtracting
- Write a One-Step Equation

Solving One-Step Equations Involving Multiplication or Division

- Solve Equations by Multiplying or Dividing
- Solve by Multiplying

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Divide Rational Numbers	●	●		●
Extension: Generalized One-Step Equations		●	●	●

Language Development Handbook

Assign page 8 of the *Language Development Handbook* to help your students build mathematical language related to solving equations.

ELL You can use the tips and suggestions on page T8 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students translated between sentences and equations and used them to solve problems.

6.EE.6, 7.EE.4, A.SSE.1, A.SSE.2

Now

Students solve one-step equations.

A.CED.1, A.REI.1, A.REI.3

Next

Students will solve multi-step equations.

A.CED.1, A.REI.1, A.REI.3

Rigor

The Three Pillars of Rigor

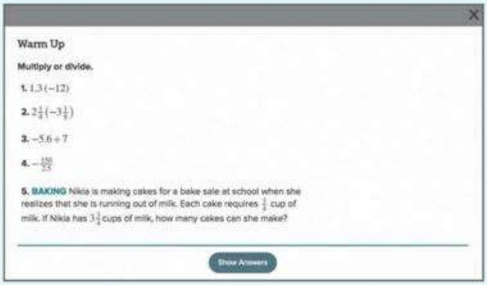
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of writing equations and use it to build fluency with solving one-step equations. They apply their understanding of one-step equations by solving real-world problems.

Mathematical Background

Solving an equation means finding the value of the variable in the equation that makes the equation true. To solve a one-step equation, isolate the variable with a coefficient of one by applying the correct property of equality to maintain equivalent expressions in each step of the process.

Interactive Presentation

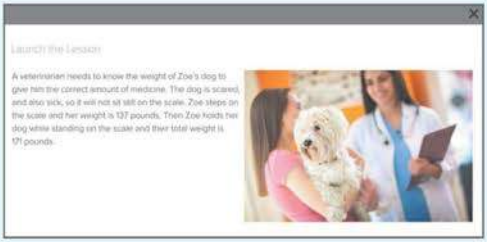


Warm Up

Multiply or divide.

1. $1.3(-12)$
2. $2\frac{1}{4}\left(-3\frac{1}{8}\right)$
3. $-5.6 \div 7$
4. $-\frac{150}{2.5}$
5. BAKING Nikia is making cakes for a bake sale at school when she realizes that she is running out of milk. Each cake requires $\frac{3}{4}$ cup of milk. If Nikia has $3\frac{3}{4}$ cups of milk, how many cakes can she make?

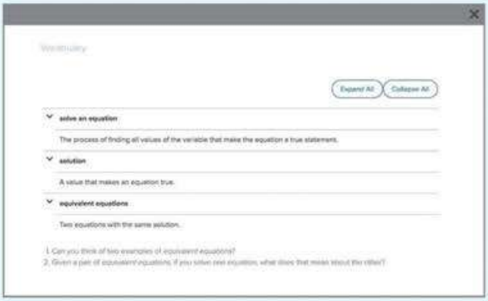
Warm Up



Launch the Lesson

A veterinarian needs to know the weight of Zoe's dog to give him the correct amount of medicine. The dog is scared, and also sick, so it will not sit still on the scale. Zoe steps on the scale and her weight is 137 pounds. Then Zoe holds her dog while standing on the scale and their total weight is 171 pounds.

Launch the Lesson



Vocabulary

solve an equation
The process of finding all values of the variable that make the equation a true statement.

solution
A value that makes an equation true.

equivalent equations
Two equations with the same solution.

1. Can you think of two examples of equivalent equations?
2. Given a pair of equivalent equations, if you solve one equation, what does that mean about the other?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- multiplying and dividing rational numbers

Answers:

1. -15.6
2. $-7\frac{1}{32}$
3. -0.8
4. -60
5. 5 cakes

Launch the Lesson

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation that models the situation and use it to find the weight of a dog.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Using Algebra Tiles to Solve One-Step Equations Involving Addition or Subtraction

Objective

Students use algebra tiles to explore solving one-step equations involving addition or subtraction.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use algebra tiles. Work with students to explore and deepen their understanding of addition and subtraction equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

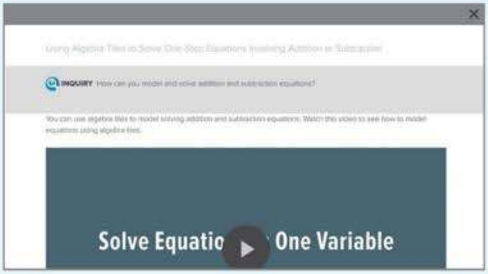
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use a drag and drop algebra tile mat to visualize the given equations before solving. The guiding exercises will lead students to solve one-step equations. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

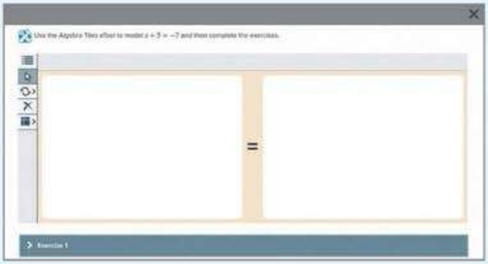


Using Algebra Tiles to Solve One-Step Equations Involving Addition or Subtraction

INQUIRY How can you model and solve addition and subtraction equations?

You can use algebra tiles to model solving addition and subtraction equations. Watch this video to see how to model equations using algebra tiles.

Explore



Use the Algebra Tiles eTool to model $x + 5 = -7$ and then complete the exercises.

Explore

DRAG & DROP	Students drag algebra tiles to represent the given equation.
TYPE	Students answer a series of questions to show they understand how to solve a one-step equation.

Interactive Presentation

INQUIRY How can you model and solve addition and subtraction equations?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Algebra Tiles to Solve One-Step Equations Involving Addition or Subtraction (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How does using zero pairs help you solve the equation? Sample answer: Zero pairs allow you to cancel values when you have the same number of positives and negatives. We can isolate *x*, by adding either positive or negative tiles to create zero pairs for any 1-tiles that are with the *x*-tile.
- Which tiles would you use to model $x - 3 = -8$? Which tiles do you need to place on the mat to solve for *x*? Sample answer: To model $x - 3$, place one *x*-tile and three negative 1-tiles on the left side of the mat. Place eight negative 1-tiles on the right side of the mat to model -8. To solve for *x*, add three 1-tiles to both sides of the mat and remove zero pairs so that the *x*-tile is alone.

Inquiry

How can you model and solve addition and subtraction equations? Sample answer: Use algebra tiles to model the equation. Isolate the *x*-tiles on one side of the mat and then remove any zero pairs.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Explore Using Algebra Tiles to Solve One-Step Equations Involving Multiplication

Objective

Students use algebra tiles to explore solving one-step equations involving multiplication.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use algebra tiles. Work with students to explore and deepen their understanding of one-step equations involving multiplication or division.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

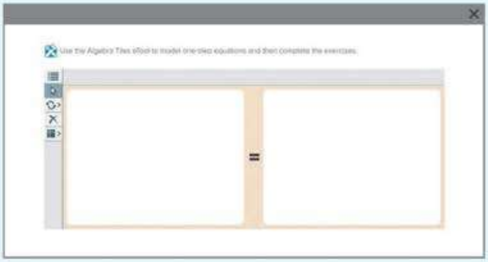
Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use a drag and drop algebra tile mat to visualize the given equation before solving. The guiding exercises will lead students to solve the one-step equations. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

DRAG & DROP

Students drag algebra tiles to represent the given equation.

TYPE

Students answer a series of questions to show they understand how to solve a one-step equation.

Interactive Presentation

INQUIRY How can you use algebra tiles to solve multiplication equations?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Algebra Tiles to Solve One-Step Equations Involving Multiplication (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does it not it make sense to use algebra tiles to model $\frac{x}{4} = 5$? Sample answer: There are only 1-tiles, x-tiles and x^2-tiles. Because we can't break the tiles into smaller pieces, it's not possible to model one-fourth of the x-tile.
- Draw a square to represent x. How could you use this model to solve $\frac{x}{4} = 5$? Sample answer: Draw lines to divide the square into 4 even pieces and shade one piece. Write a 5 into the shaded piece to model $\frac{x}{4} = 5$. If one piece is 5, you can write a 5 into each of the remaining pieces and see that the full square has 5 written 4 times, or 20, so $x = 20$.

Inquiry

How can you model and solve multiplication equations?
Sample answer: Use algebra tiles to model the equation. Isolate the x-tiles on one side of the mat and then separate the tiles into equal groups.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving One-Step Equations Involving Addition or Subtraction

Objective

Students solve one-step equations by applying the Addition Property of Equality or Subtraction Property of Equality.

Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

About the Key Concept

The Addition Property of Equality states that if a number is added to each side of a true equation, the resulting equivalent equation is also true. The Subtraction Property of Equality states that if a number is subtracted from each side of a true equation, the resulting equivalent equation is also true.

Common Misconception

Students often add or subtract the same number that is with the variable, instead of using the opposite operation. Remind students that to solve an equation, we must use zero pairs.

DIFFERENTIATE

Reteaching Activity AL ELL

IF some students are struggling to solve an equation involving addition or subtraction,

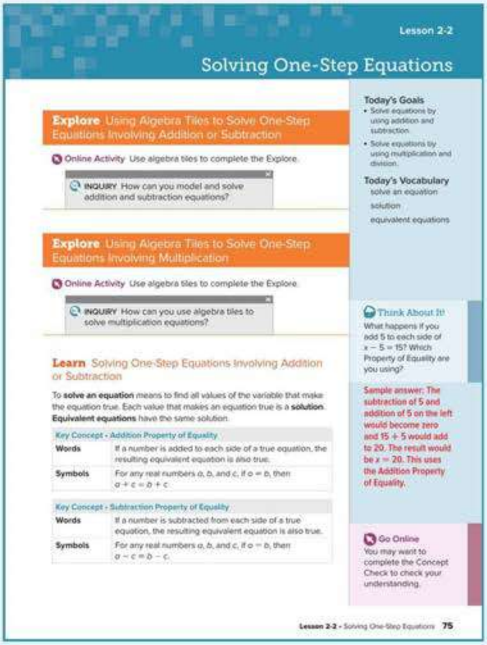
THEN have them list pairs of opposites, such as 4 and -4 or -23 and 23. Remind students that they should be using the opposite operation when solving an equation.

Enrichment Activity BL

A city bus is picking up and dropping off passengers. There are already some passengers on the bus. At the first stop, 5 passengers board the bus. At the second stop, 7 passengers exit the bus. At the third stop, 2 passengers exit the bus. At the final stop, there are 14 passengers on the bus. How many were on the bus before the first stop? Write and solve an equation.

$$\begin{aligned} x + 5 - 7 - 2 &= 14 \\ x - 4 &= 14 \\ x - 4 + 4 &= 14 + 4 \\ x &= 18 \end{aligned}$$

There were 18 passengers on the bus before the first stop.



Lesson 2-2

Solving One-Step Equations

Explore Using Algebra Tiles to Solve One-Step Equations Involving Addition or Subtraction

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you model and solve addition and subtraction equations?

Explore Using Algebra Tiles to Solve One-Step Equations Involving Multiplication

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you use algebra tiles to solve multiplication equations?

Learn Solving One-Step Equations Involving Addition or Subtraction

To **solve an equation** means to find all values of the variable that make the equation true. Each value that makes an equation true is a **solution**. **Equivalent equations** have the same solution.

Key Concept • Addition Property of Equality	
Words	If a number is added to each side of a true equation, the resulting equivalent equation is also true.
Symbols	For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$.

Key Concept • Subtraction Property of Equality	
Words	If a number is subtracted from each side of a true equation, the resulting equivalent equation is also true.
Symbols	For any real numbers a, b, and c, if $a = b$, then $a - c = b - c$.

Today's Goals
- Solve equations by using addition and subtraction.
- Solve equations by using multiplication and division.

Today's Vocabulary
solve an equation
solution
equivalent equations

Think About It!
What happens if you add 5 to each side of $x - 5 = 15$? Which Property of Equality are you using?

Sample answer: The subtraction of 5 and addition of 5 on the left would become zero and 15 + 5 would add to 20. The result would be $x = 20$. This uses the Addition Property of Equality.

Go Online
You may want to complete the Concept Check to check your understanding.

Lesson 2-2 • Solving One-Step Equations 75

Interactive Presentation

Learn

TYPE

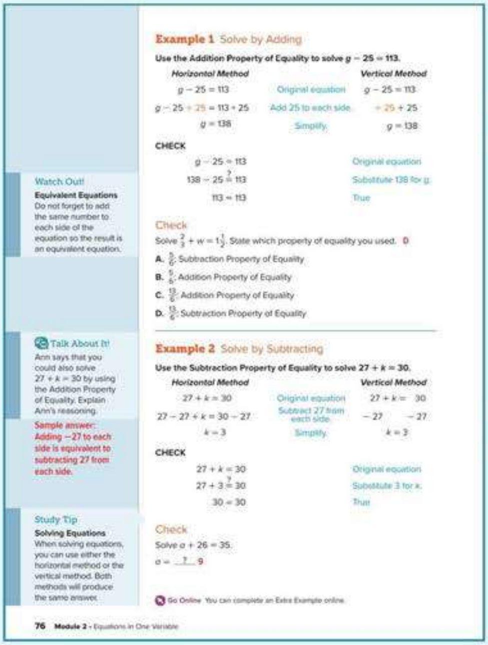
a|

Students complete a concept check with two different cases to deepen their understanding of the Addition and Subtraction Properties of Equality.

TYPE

a|

Students answer a question to determine whether they understand the Addition and Subtraction Properties of Equality.



Example 1 Solve by Adding

Use the Addition Property of Equality to solve $g - 25 = 113$.

Horizontal Method		*Vertical Method*
$g - 25 = 113$	Original equation	$g - 25 = 113$
$g - 25 + 25 = 113 + 25$	Add 25 to each side.	$+25 \quad +25$
$g = 138$	Simplify.	$g = 138$

CHECK

$g - 25 = 113$	Original equation
$138 - 25 \overset{?}{=} 113$	Substitute 138 for g.
$113 = 113$	True

Check

Solve $\frac{2}{3} + w = 1\frac{1}{2}$. State which property of equality you used. D

A. $\frac{5}{6}$; Subtraction Property of Equality

B. $\frac{5}{5}$; Addition Property of Equality

C. $\frac{13}{6}$; Addition Property of Equality

D. $\frac{13}{6}$; Subtraction Property of Equality

Example 2 Solve by Subtracting

Use the Subtraction Property of Equality to solve $27 + k = 30$.

Horizontal Method		*Vertical Method*
$27 + k = 30$	Original equation	$27 + k = 30$
$27 - 27 + k = 30 - 27$	Subtract 27 from each side.	$-27 \quad -27$
$k = 3$	Simplify.	$k = 3$

CHECK

$27 + k = 30$	Original equation
$27 + 3 \overset{?}{=} 30$	Substitute 3 for k.
$30 = 30$	True

Check

Solve $a + 26 = 35$.

$a =$ __?__ 9

Go Online You can complete an Extra Example online.

Watch Out!

Equivalent Equations Do not forget to add the same number to each side of the equation so the result is an equivalent equation.

Talk About It!

Ann says that you could also solve $27 + k = 30$ by using the Addition Property of Equality. Explain Ann's reasoning.

Sample answer: Adding -27 to each side is equivalent to subtracting 27 from each side.

Study Tip

Solving Equations When solving equations, you can use either the horizontal method or the vertical method. Both methods will produce the same answer.

76 **Module 2** • Equations in One Variable

Interactive Presentation

Example 1

TAP

Students tap to check their work.

TYPE

Students answer a question to show they understand how to solve a second equation using the Addition Property of Equality.

TAP

Students tap the Watch Out button for a helpful reminder.

Example 1 Solve by Adding

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What value are we trying to determine? the value of g

OL Why do we add 25 to both sides? Sample answer: to isolate the variable g

BL Explain why we can add 25 to both sides of the equation. Sample answer: The Addition Property of Equality states that if the same number is added to both sides of an equation, the resulting equation is equivalent to the original equation.

Example 2 Solve by Subtracting

Teaching the Mathematical Practices

1 Understand the Approaches of Others Mathematically proficient students can explain the methods used to solve a problem. The Talk About It! asks students to justify the reasoning of Ann.

Questions for Mathematical Discourse

AL What value is being added to k? 27

OL Why would the Subtraction Property of Equality be the best choice when solving the equation? Sample answer: We need to undo the addition of 27 and k, so we will subtract 27 from both sides.

BL How would the answer have changed if the equation were $k - (-27) = 30$? Sample answer: The answer would not have changed because $k - (-27)$ can be simplified to $k + 27$To solve, we would still subtract 27 from both sides.

Common Error

Students may subtract in the wrong order when using the Subtraction Property of Equality. Reinforce to students that subtraction is not commutative and the order matters.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Write a One-Step Equation

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL For which tennis player are we interested in finding the number of Grand Slam single titles won? Rafael Nadal

OL Which property of equality would be the best choice when solving the equation? Explain. Sample answer: The Subtraction Property of Equality would be the best choice because we need to subtract 3 from both sides to make a zero pair.

BL Pete Sampras has 3 fewer Grand Slam singles titles than Roger Federer did at that point in his career. Set up and solve an equation to find the number of Grand Slam singles titles Pete Sampras has. $a + 3 = 17$, $a + 3 - 3 = 17 - 3$, $a = 14$

Common Error

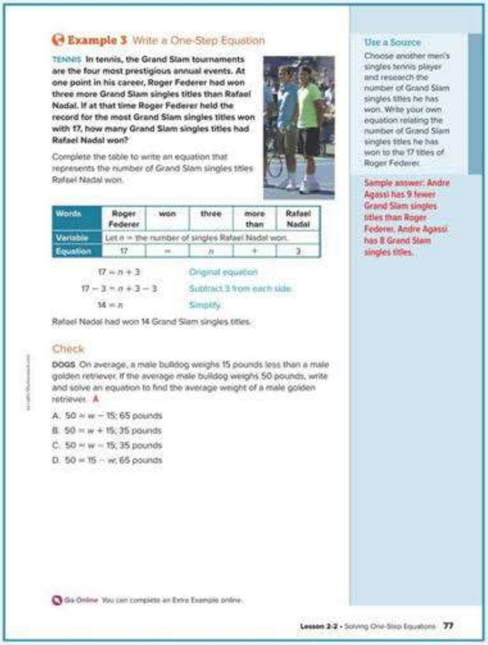
Students may confuse which tennis player is the variable and set up an incorrect equation. Remind students to carefully read the problem and determine for which tennis player we seem to lack information.

Essential Question Follow-Up

Students have begun solving one-step equations involving addition or subtraction.

Ask:

Why is it important to create equivalent equations when solving an equation? Sample answer: It is important to create equivalent equations because if you start with a true equation, then any equivalent equation will also be true. Therefore, the solution will be correct.



Example 3 Write a One-Step Equation

TENNIS **In tennis, the Grand Slam tournaments are the four most prestigious annual events. At one point in his career, Roger Federer had won three more Grand Slam singles titles than Rafael Nadal. If at that time Roger Federer held the record for the most Grand Slam singles titles won with 17, how many Grand Slam singles titles had Rafael Nadal won?**

Complete the table to write an equation that represents the number of Grand Slam singles titles Rafael Nadal won.

Words	Roger Federer	won	three	more than	Rafael Nadal
Variable	Let n = the number of singles Rafael Nadal won.				
Equation	17	=	n	+	3

$17 = n + 3$ Original equation

$17 - 3 = n + 3 - 3$ Subtract 3 from each side.

$14 = n$ Simplify.

Rafael Nadal had won 14 Grand Slam singles titles.

Check

DOGS On average, a male bulldog weighs 15 pounds less than a male golden retriever. If the average male bulldog weighs 50 pounds, write and solve an equation to find the average weight of a male golden retriever. A

A. $50 = w - 15$; 65 pounds

B. $50 = w + 15$; 35 pounds

C. $50 = w - 15$; 35 pounds

D. $50 = 15 - w$; 65 pounds

Go Online You can complete an Extra Example online.

Use a Source

Choose another men's singles tennis player and research the number of Grand Slam singles titles he has won. Write your own equation relating the number of Grand Slam singles titles he has won to the 17 titles of Roger Federer.

Sample answer: Andre Agassi has 9 fewer Grand Slam singles titles than Roger Federer, Andre Agassi has 8 Grand Slam singles titles.

Lesson 2-2 • Solving One-Step Equations 77

Interactive Presentation

Write a One-Step Equation

TENNIS In tennis, the Grand Slam tournaments are the four most prestigious annual events. At one point in his career, Roger Federer had won three more Grand Slam singles titles than Rafael Nadal. If at that time Roger Federer held the record for the most Grand Slam singles titles won with 17, how many Grand Slam singles titles had Rafael Nadal won?

Example 3

SELECT

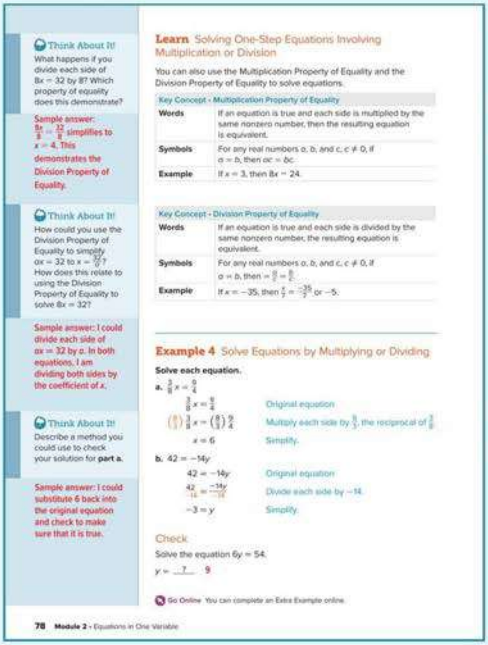
Students select an answer to complete a statement.

TYPE

Students research a men's singles tennis player to write their own equation.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Think About It!
What happens if you divide each side of $8x = 32$ by 8? Which property of equality does this demonstrate?

Sample answer: $\frac{8x}{8} = \frac{32}{8}$ simplifies to $x = 4$. This demonstrates the Division Property of Equality.

Think About It!
How could you use the Division Property of Equality to simplify $ax = 32$ to $x = \frac{32}{a}$? How does this relate to using the Division Property of Equality to solve $8x = 32$?

Sample answer: I could divide each side of $ax = 32$ by a. In both equations, I am dividing both sides by the coefficient of x.

Think About It!
Describe a method you could use to check your solution for **part a.**

Sample answer: I could substitute 6 back into the original equation and check to make sure that it is true.

Learn Solving One-Step Equations Involving Multiplication or Division

You can also use the Multiplication Property of Equality and the Division Property of Equality to solve equations.

Key Concept • Multiplication Property of Equality

Words	If an equation is true and each side is multiplied by the same nonzero number, then the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $ac = bc$.
Example	If $x = 3$, then $8x = 24$.

Key Concept • Division Property of Equality

Words	If an equation is true and each side is divided by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
Example	If $x = -35$, then $\frac{x}{7} = \frac{-35}{7}$ or -5.

Example 4 Solve Equations by Multiplying or Dividing

Solve each equation.

a. $\frac{3}{8}x = \frac{9}{4}$

$\frac{3}{8}x = \frac{9}{4}$ Original equation

$\left(\frac{8}{3}\right)\frac{3}{8}x = \left(\frac{8}{3}\right)\frac{9}{4}$ Multiply each side by $\frac{8}{3}$, the reciprocal of $\frac{3}{8}$.

$x = 6$ Simplify.

b. $42 = -14y$

$42 = -14y$ Original equation

$\frac{42}{-14} = \frac{-14y}{-14}$ Divide each side by -14.

$-3 = y$ Simplify.

Check

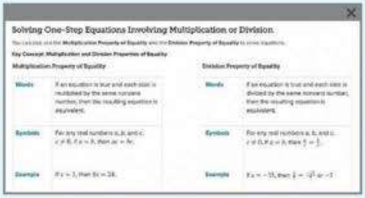
Solve the equation $6y = 54$.

$y = \underline{\ ?\ }$ 9

Go Online You can complete an Extra Example online.

78 **Module 2** • Equations in One Variable

Interactive Presentation



Learn

TYPE

Students answer a question to show they understand how to practice the new concept with an equation and explanation.

TYPE

Students answer a question to show they understand how to solve an equation with a variable coefficient.

Learn Solving One-Step Equations Involving Multiplication or Division

Objective

Students solve one-step equations by applying the Multiplication Property of Equality or the Division Property of Equality.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 4 Solve Equations by Multiplying or Dividing

MP Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, “Does this make sense?” Point out that in this example, students need to check their answer. They should ask themselves whether their answer makes sense and whether they have answered the question asked.

Questions for Mathematical Discourse

AL What is the reciprocal of $\frac{3}{8}$? $\frac{8}{3}$

OL Which property of equality will allow us to isolate the variable? Sample answer: The Multiplication Property of Equality would allow me to isolate the variable because I can multiply both sides of the equation by the reciprocal.

BL Why does the answer, 6, make sense as the solution? Sample answer: $\frac{9}{4}$ can be rounded to 2, and $\frac{3}{8}$ can be rounded to one half. We can estimate that 2 is half of 4. So, the answer of 6 makes sense.

Common Error

Students often make mistakes when dealing with fractional coefficients. Many try to divide both sides by the fraction and then get stuck. Remind students that to divide fractions, you must multiply by the reciprocal of the divisor fraction.

Apply Example 5 Solve by Multiplying

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

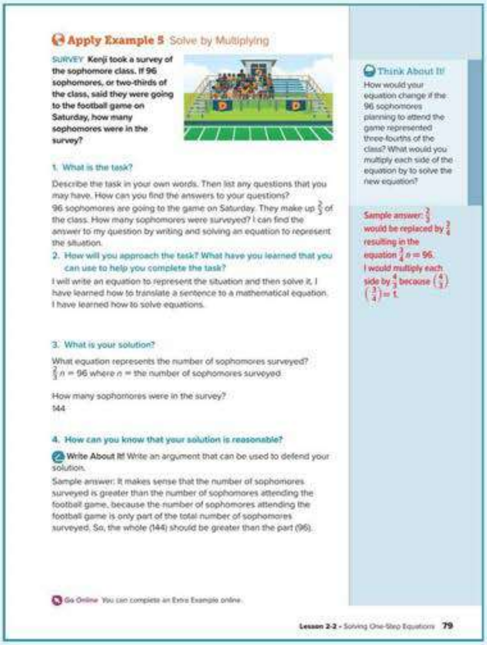
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What other mathematical process can be used to solve this equation?
- How can the answer be proven correct?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Example 5 Solve by Multiplying

SURVEY **Kenji took a survey of the sophomore class. If 96 sophomores, or two-thirds of the class, said they were going to the football game on Saturday, how many sophomores were in the survey?**

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find the answers to your questions?

96 sophomores are going to the game on Saturday. They make up $\frac{2}{3}$ of the class. How many sophomores were surveyed? I can find the answer to my question by writing and solving an equation to represent the situation.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

I will write an equation to represent the situation and then solve it. I have learned how to translate a sentence to a mathematical equation. I have learned how to solve equations.

3. What is your solution?

What equation represents the number of sophomores surveyed?

$\frac{2}{3}n = 96$ where n = the number of sophomores surveyed

How many sophomores were in the survey?

144

4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: It makes sense that the number of sophomores surveyed is greater than the number of sophomores attending the football game, because the number of sophomores attending the football game is only part of the total number of sophomores surveyed. So, the whole (144) should be greater than the part (96).

Go Online You can complete an Extra Example online.

Think About It!

How would your equation change if the 96 sophomores planning to attend the game represented three-fourths of the class? What would you multiply each side of the equation by to solve the new equation?

Sample answer: $\frac{2}{3}$ would be replaced by $\frac{3}{4}$ resulting in the equation $\frac{3}{4}n = 96$. I would multiply each side by $\frac{4}{3}$ because $\left(\frac{4}{3}\right)\left(\frac{3}{4}\right) = 1$.

Lesson 2-2 • Solving One-Step Equations 79

Interactive Presentation

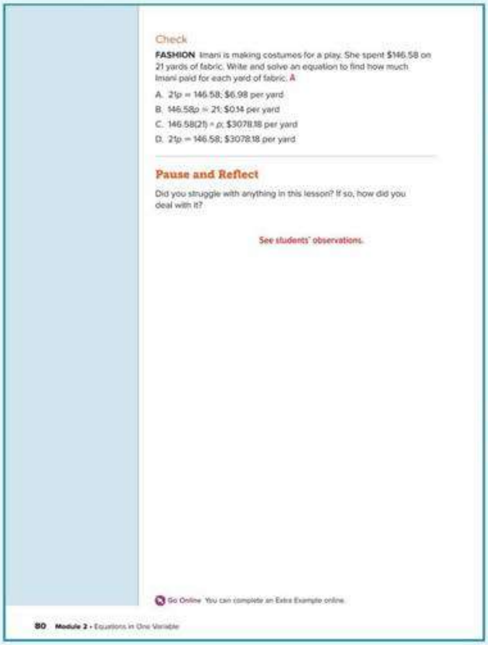
Solve by Multiplying

SURVEY Kenji took a survey of the sophomore class. If 96 sophomores, or two-thirds of the class, said they were going to the football game on Saturday, how many sophomores were in the survey?

Apply Example 5

TYPE

Students answer questions to show they understand how to set up and solve an equation.



Check

FASHION Imani is making costumes for a play. She spent $146.58 on 21 yards of fabric. Write and solve an equation to find how much Imani paid for each yard of fabric. A

A. 21p = 146.58; $6.98 per yard
B. 146.58p = 21; $0.14 per yard
C. 146.58(21) = p; $3078.18 per yard
D. 21p = 146.58; $3078.18 per yard

Pause and Reflect

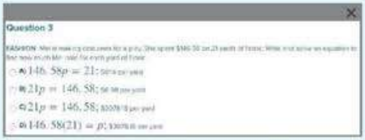
Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

Go Online You can complete an Extra Example online.

80 Module 2 • Equations in One Variable

Interactive Presentation



Question 3

○ A) $146.58p = 21$;
○ B) $21p = 146.58$;
○ C) $21p = 146.58$;
○ D) $146.58(21) = p$;

Check

CHECK

Students complete the Check online to determine whether they are ready to move on.

Essential Question Follow-Up

Students have begun solving one-step equations involving multiplication or division.

Ask:

Why should we be able to set up and solve one-step equations in the real world? Sample answer: I need to be able to set up and solve equations in the real world because I may know only part of the information for a situation and can solve for the missing part. For example, I may know how much something costs after a discount so I can find the original price.

DIFFERENTIATE

Reteaching Activity AL ELL

IF some students are struggling to use the appropriate property of equality to solve an equation,

THEN have students write down the operation given in the equation and then the property of equality for the opposite operation.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–59
2	exercises that use a variety of skills from this lesson	60–89, 93–98
2	exercises that extend concepts learned in this lesson to new contexts	90–92
3	exercises that emphasize higher-order and critical-thinking skills	99–104

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–97 odd, 99–104
- Extension: Generalized One-Step Equations
- **ALEKS** One-Step Linear Equations

OL

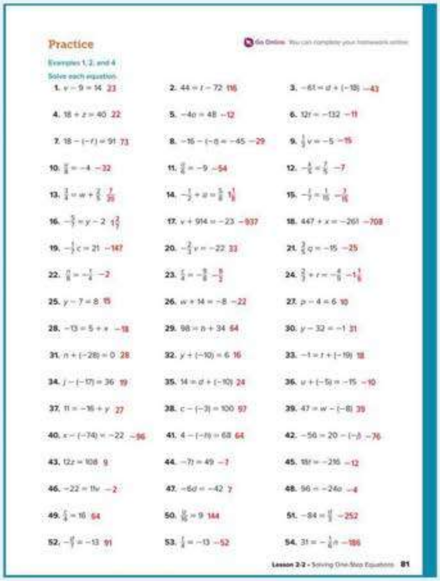
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–103 odd
- Remediation, Review Resources: Divide Rational Numbers
- BrainPOP Video: Solving Equations
- Extra Examples 1–5
- **ALEKS** Multiplication and Division with Fractions; Multiplication and Division with Mixed Numbers; Decimals: Division

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–65 odd
- Remediation, Review Resources: Divide Rational Numbers
- *Quick Review Math Handbook*: Solving One-Step Equations
- Arrive**MATH** Take Another Look
- **ALEKS** Multiplication and Division with Fractions; Multiplication and Division with Mixed Numbers; Decimals: Division



Practice

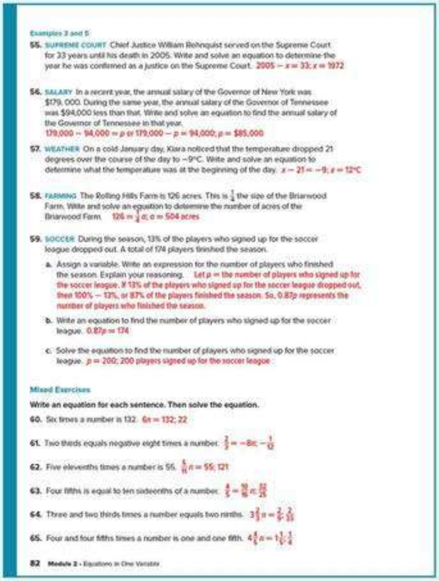
Go Online You can complete your homework online.

Examples 1, 2, and 4

Solve each equation.

1. $v - 9 = 14$ 23
2. $44 = t - 72$ 116
3. $-61 = d + (-18)$ −43
4. $18 + z = 40$ 22
5. $-4a = 48$ −12
6. $12f = -132$ −11
7. $18 - (-f) = 91$ 73
8. $-16 - (-t) = -45$ −29
9. [illegible] −15
10. [illegible] −32
11. [illegible] −54
12. [illegible] −7
13. [illegible]
14. [illegible]
15. [illegible]
16. [illegible]
17. $v + 914 = -23$ −937
18. $447 + x = -261$ −708
19. [illegible] −147
20. [illegible] 33
21. [illegible] −25
22. [illegible] −2
23. [illegible]
24. [illegible]
25. $y - 7 = 8$ 15
26. $w + 14 = -8$ −22
27. $p - 4 = 6$ 10
28. $-13 = 5 + x$ −18
29. $98 = n + 34$ 64
30. $y - 32 = -1$ 31
31. $n + (-28) = 0$ 28
32. $y + (-10) = 6$ 16
33. $-1 = t + (-19)$ 18
34. $j - (-17) = 36$ 19
35. $14 = d + (-10)$ 24
36. $u + (-5) = -15$ −10
37. $11 = -16 + y$ 27
38. $c - (-3) = 100$ 97
39. $47 = w - (-8)$ 39
40. $x - (-74) = -22$ −96
41. $4 - (-h) = 68$ 64
42. $-56 = 20 - (-j)$ −76
43. $12z = 108$ 9
44. $-7t = 49$ −7
45. $18f = -216$ −12
46. $-22 = 11v$ −2
47. $-6d = -42$ 7
48. $96 = -24a$ −4
49. [illegible] 64
50. [illegible] 144
51. [illegible] −252
52. [illegible] 91
53. [illegible] −52
54. [illegible] −186

Lesson 2-2 • Solving One-Step Equations 81



Examples 3 and 5

55. SUPREME COURT Chief Justice William Rehnquist served on the Supreme Court for 33 years until his death in 2005. Write and solve an equation to determine the year he was confirmed as a justice on the Supreme Court. $2005 - x = 33$; $x = 1972$

56. SALARY In a recent year, the annual salary of the Governor of New York was \$179,000. During the same year, the annual salary of the Governor of Tennessee was \$94,000 less than that. Write and solve an equation to find the annual salary of the Governor of Tennessee in that year.
$179{,}000 - 94{,}000 = p$ or $179{,}000 - p = 94{,}000$; $p = \$85{,}000$

57. WEATHER On a cold January day, Kiara noticed that the temperature dropped 21 degrees over the course of the day to −9°C. Write and solve an equation to determine what the temperature was at the beginning of the day. $x - 21 = -9$; $x = 12$°C

58. FARMING The Rolling Hills Farm is 126 acres. This is $\frac{1}{4}$ the size of the Briarwood Farm. Write and solve an equation to determine the number of acres of the Briarwood Farm. $126 = \frac{1}{4}a$; $a = 504$ acres

59. SOCCER During the season, 13% of the players who signed up for the soccer league dropped out. A total of 174 players finished the season.
    a. Assign a variable. Write an expression for the number of players who finished the season. Explain your reasoning. Let p = the number of players who signed up for the soccer league. If 13% of the players who signed up for the soccer league dropped out, then 100% − 13%, or 87% of the players finished the season. So, $0.87p$ represents the number of players who finished the season.
    b. Write an equation to find the number of players who signed up for the soccer league. $0.87p = 174$
    c. Solve the equation to find the number of players who signed up for the soccer league. $p = 200$; 200 players signed up for the soccer league

Mixed Exercises

Write an equation for each sentence. Then solve the equation.

60. Six times a number is 132. $6n = 132$; 22
61. Two thirds equals negative eight times a number. $\frac{2}{3} = -8x$; $-\frac{1}{12}$
62. Five elevenths times a number is 55. $\frac{5}{11}n = 55$; 121
63. Four fifths is equal to ten sixteenths of a number. $\frac{4}{5} = \frac{10}{16}n$; $\frac{32}{25}$
64. Three and two thirds times a number equals two ninths. $3\frac{2}{3}n = \frac{2}{9}$; $\frac{2}{33}$
65. Four and four fifths times a number is one and one fifth. $4\frac{4}{5}n = 1\frac{1}{5}$; $\frac{1}{4}$

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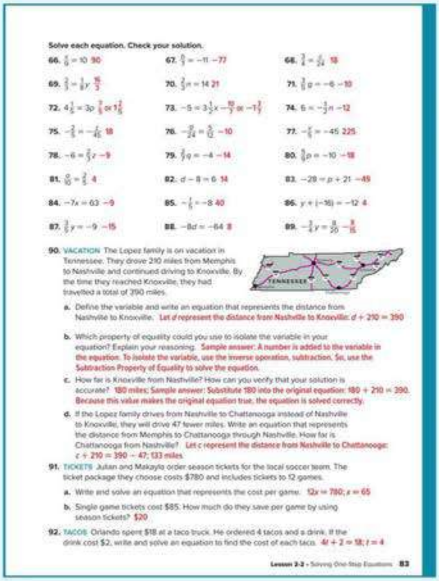
3 REFLECT AND PRACTICE

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Solve each equation. Check your solution.

66. $\frac{x}{9}=10$ 90
67. $\frac{a}{7}=-11$ −77
68. $\frac{3}{4}=\frac{c}{24}$ 18
69. $\frac{2}{3}=\frac{1}{8}y$ $\frac{16}{3}$
70. $\frac{2}{3}n=14$ 21
71. $\frac{3}{5}g=-6$ −10
72. $4\frac{1}{5}=3p$ $\frac{7}{5}$ or $1\frac{2}{5}$
73. $-5=3\frac{1}{2}x$ $-\frac{10}{7}$ or $-1\frac{3}{7}$
74. $6=-\frac{1}{2}n$ −12
75. $-\frac{2}{5}=-\frac{z}{45}$ 18
76. $-\frac{v}{24}=\frac{5}{12}$ −10
77. $-\frac{r}{5}=-45$ 225
78. $-6=\frac{2}{3}r$ −9
79. $\frac{2}{7}q=-4$ −14
80. $\frac{5}{9}p=-10$ −18
81. $\frac{a}{10}=\frac{2}{5}$ 4
82. $d-8=6$ 14
83. $-28=p+21$ −49
84. $-7x=63$ −9
85. $-\frac{t}{5}=-8$ 40
86. $y+(-16)=-12$ 4
87. $\frac{3}{5}y=-9$ −15
88. $-8d=-64$ 8
89. $-\frac{3}{4}y=\frac{8}{20}$ $-\frac{8}{15}$

90. VACATION The Lopez family is on vacation in Tennessee. They drove 210 miles from Memphis to Nashville and continued driving to Knoxville. By the time they reached Knoxville, they had travelled a total of 390 miles.

TENNESSEE

a. Define the variable and write an equation that represents the distance from Nashville to Knoxville. Let d represent the distance from Nashville to Knoxville; $d+210=390$

b. Which property of equality could you use to isolate the variable in your equation? Explain your reasoning. Sample answer: A number is added to the variable in the equation. To isolate the variable, use the inverse operation, subtraction. So, use the Subtraction Property of Equality to solve the equation.

c. How far is Knoxville from Nashville? How can you verify that your solution is accurate? 180 miles; Sample answer: Substitute 180 into the original equation: $180+210=390$. Because this value makes the original equation true, the equation is solved correctly.

d. If the Lopez family drives from Nashville to Chattanooga instead of Nashville to Knoxville, they will drive 47 fewer miles. Write an equation that represents the distance from Memphis to Chattanooga through Nashville. How far is Chattanooga from Nashville? Let c represent the distance from Nashville to Chattanooga: $c+210=390-47$; 133 miles

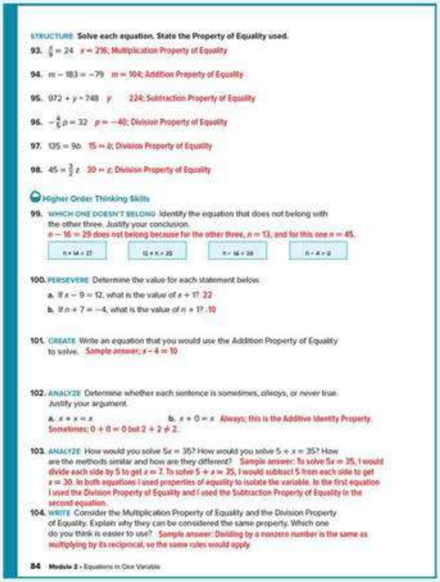
91. TICKETS Julian and Makayla order season tickets for the local soccer team. The ticket package they choose costs $780 and includes tickets to 12 games.

a. Write and solve an equation that represents the cost per game. $12x=780$; $x=65$

b. Single game tickets cost $85. How much do they save per game by using season tickets? $20

92. TACOS Orlando spent $18 at a taco truck. He ordered 4 tacos and a drink. If the drink cost $2, write and solve an equation to find the cost of each taco. $4t+2=18$; $t=4$

Lesson 2-2 • Solving One-Step Equations 83



STRUCTURE **Solve each equation. State the Property of Equality used.**

93. $\frac{x}{9}=24$ $x=216$; Multiplication Property of Equality

94. $m-183=-79$ $m=104$; Addition Property of Equality

95. $972+y=748$ y 224; Subtraction Property of Equality

96. $-\frac{4}{5}p=32$ $p=-40$; Division Property of Equality

97. $135=9b$ $15=b$; Division Property of Equality

98. $45=\frac{3}{2}z$ $30=z$; Division Property of Equality

Higher Order Thinking Skills

99. WHICH ONE DOESN'T BELONG Identify the equation that does not belong with the other three. Justify your conclusion.
$n-16=29$ does not belong because for the other three, $n=13$, and for this one $n=45$.

$n+14=27$	$12+n=25$	$n-16=29$	$n-4=9$

100. PERSEVERE Determine the value for each statement below.

a. If $x-9=12$, what is the value of $x+1$? 22

b. If $n+7=-4$, what is the value of $n+1$? −10

101. CREATE Write an equation that you would use the Addition Property of Equality to solve. Sample answer: $x-4=10$

102. ANALYZE Determine whether each sentence is sometimes, always, or *never* true. Justify your argument.

a. $x+x=x$ Sometimes; $0+0=0$ but $2+2\neq2$.

b. $x+0=x$ Always; this is the Additive Identity Property.

103. ANALYZE How would you solve $5x=35$? How would you solve $5+x=35$? How are the methods similar and how are they different? Sample answer: To solve $5x=35$, I would divide each side by 5 to get $x=7$. To solve $5+x=35$, I would subtract 5 from each side to get $x=30$. In both equations I used properties of equality to isolate the variable. In the first equation I used the Division Property of Equality and I used the Subtraction Property of Equality in the second equation.

104. WRITE Consider the Multiplication Property of Equality and the Division Property of Equality. Explain why they can be considered the same property. Which one do you think is easier to use? Sample answer: Dividing by a nonzero number is the same as multiplying by its reciprocal, so the same rules would apply.

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Lesson 2-3

Solving Multi-Step Equations

A.CED.1, A.REI.1, A.REI.3

LESSON GOAL

Students solve multi-step equations and equations for specific lettered coefficients by applying properties of equality.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Using Algebra Tiles to Model Multi-Step Equations

Develop:

Solving Multi-Step Equations

- Solve Multi-Step Equations
- Write and Solve a Multi-Step Equation
- Solve Multi-Step Equations with Letters as Coefficients

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Simplify Algebraic Expressions	●	●		●
Extension: Angles of a Triangle		●	●	●

Language Development Handbook

Assign page 9 of the *Language Development Handbook* to help your students build mathematical language related to solving multi-step equations.

ELL You can use the tips and suggestions on page T9 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students solved one-step equations with the four operations.
A.REI.3

Now
Students solve multi-step equations.
A.CED.1, A.REI.1, A.REI.3

Next
Students will solve equations with the variable on each side.
A.CED.1, A.REI.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

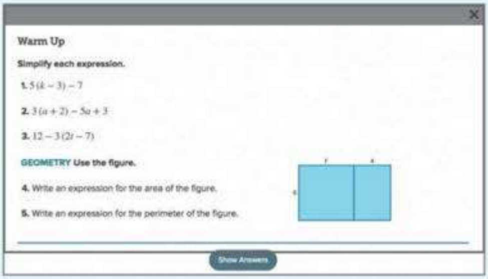
Conceptual Bridge In this lesson, students draw on their understanding of solving one-step equations and build fluency with solving multi-step equations. They apply their understanding of multi-step equations by solving real-world problems.

Mathematical Background

Multi-step equations involve more than one operation. These equations can be solved using the properties of equality and the strategy of undoing each operation by working backward.

In number theory, multi-step equations are written and solved to understand the relationship between numbers, such as consecutive integers.

Interactive Presentation



Warm Up

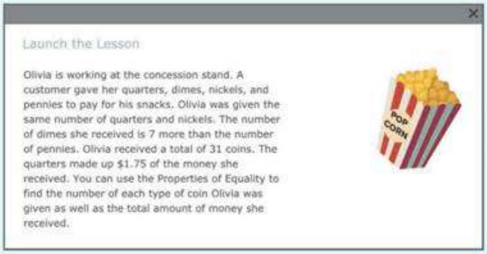
Simplify each expression.

1. $5(k-3)-7$
2. $3(a+2)-5a+3$
3. $12-3(2t-7)$

GEOMETRY Use the figure.

4. Write an expression for the area of the figure.
5. Write an expression for the perimeter of the figure.

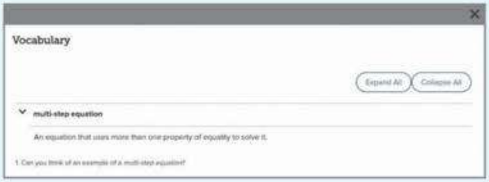
Warm Up



Launch the Lesson

Olivia is working at the concession stand. A customer gave her quarters, dimes, nickels, and pennies to pay for his snacks. Olivia was given the same number of quarters and nickels. The number of dimes she received is 7 more than the number of pennies. Olivia received a total of 31 coins. The quarters made up $1.75 of the money she received. You can use the Properties of Equality to find the number of each type of coin Olivia was given as well as the total amount of money she received.

Launch the Lesson



Vocabulary

multi-step equation

An equation that uses more than one property of equality to solve it.

1. Can you think of an example of a multi-step equation?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- using the Distributive Property

Answers:

1. $5k-22$
2. $-2a+9$
3. $33-6t$
4. $6y+24$
5. $2y+20$

Launch the Lesson

Teaching the Mathematical Practices

2 Represent a Situation Symbolically Encourage students to define variables, recognize relationships between given quantities, and write an equation to model the amount of money Olivia received.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the question below with the class.

Explore Using Algebra Tiles to Model Multi-Step Equations

Objective

Students use algebra tiles to explore solving multi-step equations.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use algebra tiles. Work with students to explore and deepen their understanding of multi-step equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

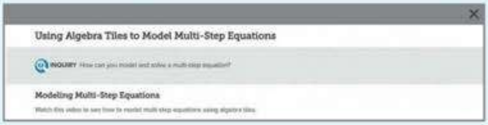
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use the Algebra Tiles eTool to help visualize the equation before solving. The guiding exercises will lead students to solve the multi-step equation. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



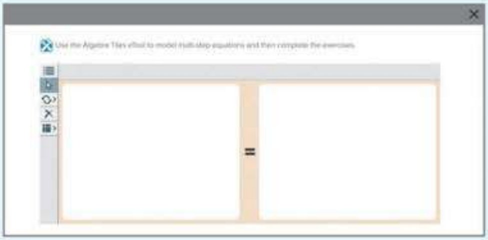
Using Algebra Tiles to Model Multi-Step Equations

INQUIRY How can you model and solve a multi-step equation?

Modeling Multi-Step Equations

Watch this video to see how to model multi-step equations using algebra tiles.

Explore



Use the Algebra Tiles eTool to model multi-step equations and then complete the exercises.

Explore

SELECT

Students select the correct answer to two questions regarding solving a multi-step equation.

Interactive Presentation

INQUIRY How can you model and solve a multi-step equation?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Algebra Tiles to Model Multi-Step Equations (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How is solving a multi-step equation similar to solving a one-step equation? Sample answer: You still need to perform the opposite operation in order to isolate x.
- Can you use algebra tiles to model and solve $5x + 3 = -13$? Why or why not? Sample answer: You can model $5x + 3 = -13$ with five x-tiles and three 1-tiles on the left side of the mat and thirteen negative 1-tiles on the right side. When you place three negative 1-tiles onto each side of the mat to isolate $5x$, you're left with $5x = -16$. Because you can't divide -16 into 5 equal groups, you can't solve with algebra tiles.

Inquiry

How can you model and solve a multi-step equation? Sample answer: Use algebra tiles to model the equation. Then add positive or negative 1-tiles to form zero pairs to isolate the x-tiles. Finally, separate the remaining tiles into equal groups.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Multi-Step Equations

Objective

Students solve multi-step equations by applying properties of equality.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students are Learning

In this section, students learn the definition of a multi-step equation and how to solve using two properties of equality. Students see that solving a multi-step equation is like working backward from the order of operations.

Example 1 Solve Multi-Step Equations

MP Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students need to check their answer. They should ask themselves whether their answer makes sense and whether they have answered the question asked.

Questions for Mathematical Discourse

AL In part **a,** what operations are being performed on the variable? multiplication and subtraction

OL In part **a,** what operation will you undo first? Explain. subtraction; Sample answer: It is necessary to reverse the order of operations.

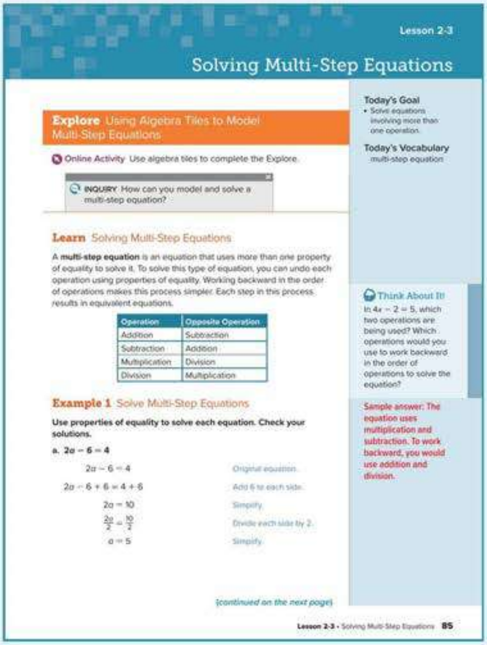
BL In part **b,** why would we not undo the addition first? Sample answer: Because the fraction bar is a grouping symbol, we must undo the division. Grouping symbols would be last when reversing the order of operations.

Common Error

Students often undo addition and subtraction first no matter what operations or grouping symbols are present. Remind students that fraction bars group terms together just like parentheses and these are always one of the last operations to undo when working backward.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-3

Solving Multi-Step Equations

Explore Using Algebra Tiles to Model Multi-Step Equations

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you model and solve a multi-step equation?

Learn Solving Multi-Step Equations

A **multi-step equation** is an equation that uses more than one property of equality to solve it. To solve this type of equation, you can undo each operation using properties of equality. Working backward in the order of operations makes this process simpler. Each step in this process results in equivalent equations.

Operation	Opposite Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

Example 1 Solve Multi-Step Equations

Use properties of equality to solve each equation. Check your solutions.

a. $2a - 6 = 4$

$2a - 6 = 4$ Original equation

$2a - 6 + 6 = 4 + 6$ Add 6 to each side.

$2a = 10$ Simplify.

$\frac{2a}{2} = \frac{10}{2}$ Divide each side by 2.

$a = 5$ Simplify.

(continued on the next page)

Today's Goal
- Solve equations involving more than one operation.

Today's Vocabulary
multi-step equation

Think About It!
In $4x - 2 = 5$, which two operations are being used? Which operations would you use to work backward in the order of operations to solve the equation?

Sample answer: The equation uses multiplication and subtraction. To work backward, you would use addition and division.

Lesson 2-3 • Solving Multi-Step Equations 85

Interactive Presentation

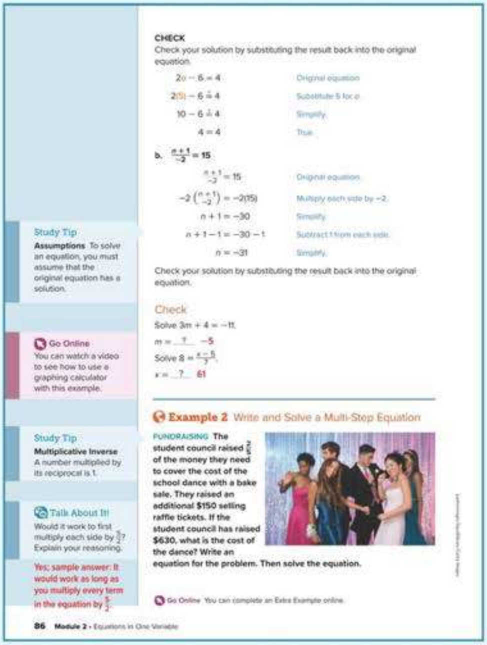
Learn

DRAG & DROP

Students drag the correct operation to complete a table of operations.

TYPE

Students answer a question to show they understand opposite operations.



CHECK

Check your solution by substituting the result back into the original equation.

$2a - 6 = 4$ Original equation

$2(5) - 6 \overset{?}{=} 4$ Substitute 5 for a.

$10 - 6 \overset{?}{=} 4$ Simplify.

$4 = 4$ True

b. $\frac{n+1}{-2} = 15$

$\frac{n+1}{-2} = 15$ Original equation

$-2\left(\frac{n+1}{-2}\right) = -2(15)$ Multiply each side by -2.

$n + 1 = -30$ Simplify.

$n + 1 - 1 = -30 - 1$ Subtract 1 from each side.

$n = -31$ Simplify.

Check your solution by substituting the result back into the original equation.

Check

Solve $3m + 4 = -11$.

$m = \underline{\ ?\ }$ -5

Solve $8 = \frac{x-5}{7}$.

$x = \underline{\ ?\ }$ 61

Study Tip

Assumptions To solve an equation, you must assume that the original equation has a solution.

Go Online You can watch a video to see how to use a graphing calculator with this example.

Study Tip

Multiplicative Inverse A number multiplied by its reciprocal is 1.

Talk About It! Would it work to first multiply each side by $\frac{5}{2}$? Explain your reasoning.

Yes; sample answer: It would work as long as you multiply every term in the equation by $\frac{5}{2}$.

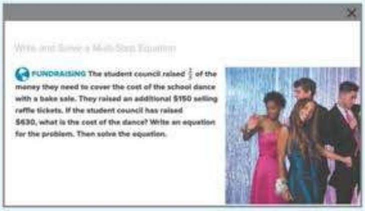
Example 2 Write and Solve a Multi-Step Equation

FUNDRAISING The student council raised $\frac{2}{5}$ of the money they need to cover the cost of the school dance with a bake sale. They raised an additional $150 selling raffle tickets. If the student council has raised $630, what is the cost of the dance? Write an equation for the problem. Then solve the equation.

Go Online You can complete an Extra Example online.

86 Module 2 • Equations in One Variable

Interactive Presentation



Write and Solve a Multi-Step Equation

FUNDRAISING The student council raised $\frac{2}{5}$ of the money they need to cover the cost of the school dance with a bake sale. They raised an additional $150 selling raffle tickets. If the student council has raised $630, what is the cost of the dance? Write an equation for the problem. Then solve the equation.

Example 2

SELECT

Students select the correct phrase to define the variable.

TYPE

Students answer a question to show they understand how to determine if a proposed step is correct in the solving process.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to work backward in the order of operations, **THEN** have them circle the variable and its coefficient. Suggest they undo the operation on the outside of the circle first.

Enrichment Activity BL

An odd integer can be written as $2n + 1$, where n is any integer.
If $2n + 1$ is the first integer, then add 2 to get the next largest odd integer, and so on.
What two consecutive odd integers have a sum of 144? Write an equation and solve.

$2n + 1 + 2n + 1 + 2 = 144$
$4n + 4 = 144$
$4n + 4 - 4 = 144 - 4$
$4n = 140$
$n = 35$ The two integers would be 71 and 73.

Example 2 Write and Solve a Multi-Step Equation

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation that models the situation in this example. Then use the equation to solve the problem.

Questions for Mathematical Discourse

AL What information is the question asking us to find? the cost of the dance

OL How much was raised by the bake sale? $480 How can you use this to estimate the cost of the dance? Sample answer: Because the bake sale raised enough to cover less than half the cost of the dance, the dance costs more than 2($480) or $960.

BL What percent of the cost of the dance was raised through the bake sale and raffle ticket sales? 52.5%

Common Error

When given a real-world situation, students may struggle to understand the relationship of the numbers given. They may write the numbers in an equation as they are presented in the problem. Have students underline or highlight important words or phrases that represent mathematical operations. This will help when writing the equation.

Example 3 Solve Multi-Step Equations with Letters as Coefficients

MP Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

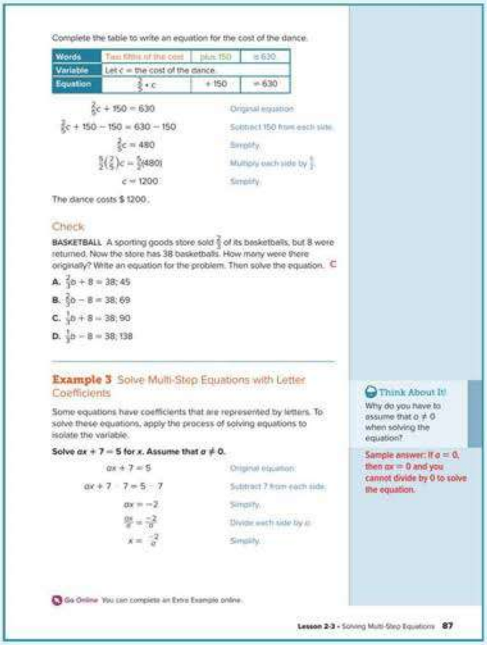
AL What is the variable and what is the coefficient? The variable is x and the coefficient is a.

OL What is the reciprocal of a? $\frac{1}{a}$

BL Would the answer change if we multiplied by $\frac{1}{a}$ instead of dividing by a? Explain. No; sample answer: $\frac{1}{a} \cdot ax$ and $\frac{ax}{a}$ are equivalent expressions, as are $\frac{1}{a} \cdot (-2)$ and $-\frac{2}{a}$.

Common Error

Students may have trouble remembering which letter is the variable and which is the coefficient. Encourage students to highlight or underline the variable so they will solve for the correct letter.



Complete the table to write an equation for the cost of the dance.

Words	Two fifths of the cost	plus 150	is 630
Variable	Let c = the cost of the dance.		
Equation	$\frac{2}{5} \cdot c$	$+ 150$	$= 630$

$\frac{2}{5}c + 150 = 630$ Original equation

$\frac{2}{5}c + 150 - 150 = 630 - 150$ Subtract 150 from each side.

$\frac{2}{5}c = 480$ Simplify.

$\frac{5}{2}\left(\frac{2}{5}\right)c = \frac{5}{2}(480)$ Multiply each side by $\frac{5}{2}$.

$c = 1200$ Simplify.

The dance costs $ 1200.

Check

BASKETBALL A sporting goods store sold $\frac{2}{3}$ of its basketballs, but 8 were returned. Now the store has 38 basketballs. How many were there originally? Write an equation for the problem. Then solve the equation. C

A. $\frac{2}{3}b + 8 = 38; 45$

B. $\frac{2}{3}b - 8 = 38; 69$

C. $\frac{1}{3}b + 8 = 38; 90$

D. $\frac{1}{3}b - 8 = 38; 138$

Example 3 Solve Multi-Step Equations with Letter Coefficients

Some equations have coefficients that are represented by letters. To solve these equations, apply the process of solving equations to isolate the variable.

Solve $ax + 7 = 5$ for x. Assume that $a \neq 0$.

$ax + 7 = 5$ Original equation

$ax + 7 - 7 = 5 - 7$ Subtract 7 from each side.

$ax = -2$ Simplify.

$\frac{ax}{a} = \frac{-2}{a}$ Divide each side by a.

$x = \frac{-2}{a}$ Simplify.

Think About It!

Why do you have to assume that $a \neq 0$ when solving the equation?

Sample answer: If $a = 0$, then $ax = 0$ and you cannot divide by 0 to solve the equation.

Go Online You can complete an Extra Example online.

Lesson 2-3 • Solving Multi-Step Equations 87

Interactive Presentation

Solve Multi-Step Equations with Letters as Coefficients

Some equations have coefficients that are represented by letters. To solve these equations, apply the process of solving equations to isolate the variable.

Solve $ax + 7 = 5$ for x. Assume that $a \neq 0$.

$ax + 7 = 5$ Original equation

Example 3

TAP

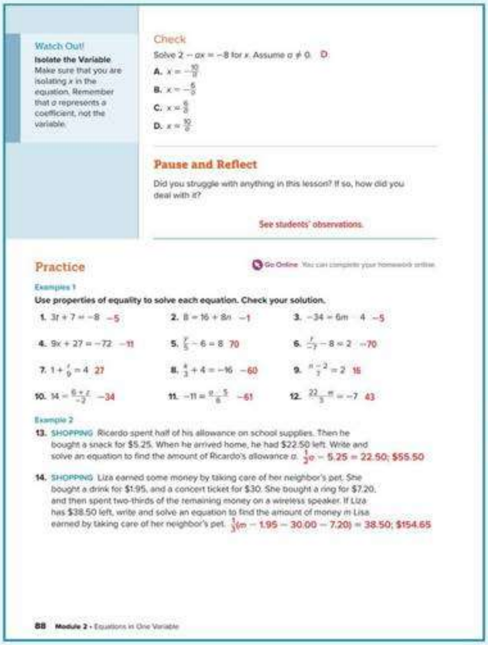
Students move through the steps to solve the equation.

TYPE

Students answer a question to show they understand why $a \neq 0$ when solving equations with letter coefficients.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Watch Out!

Isolate the Variable Make sure that you are isolating x in the equation. Remember that a represents a coefficient, not the variable.

Check

Solve $2 - ax = -8$ for x. Assume $a \neq 0$. D

A. $x = -\frac{10}{a}$

B. $x = -\frac{6}{a}$

C. $x = \frac{6}{a}$

D. $x = \frac{10}{a}$

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

Practice

Go Online You can complete your homework online.

Examples 1

Use properties of equality to solve each equation. Check your solution.

1. $3r + 7 = -8$ -5
2. $8 = 16 + 8n$ -1
3. $-34 = 6m - 4$ -5
4. $9x + 27 = -72$ -11
5. $\frac{y}{5} - 6 = 8$ 70
6. $\frac{f}{-7} - 8 = 2$ -70
7. $1 + \frac{r}{9} = 4$ 27
8. $\frac{k}{3} + 4 = -16$ -60
9. $\frac{n-2}{7} = 2$ 16
10. $14 = \frac{6+z}{-2}$ -34
11. $-11 = \frac{a-5}{6}$ -61
12. $\frac{22-m}{3} = -7$ 43

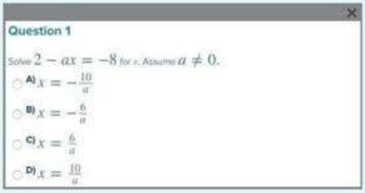
Example 2

13. SHOPPING Ricardo spent half of his allowance on school supplies. Then he bought a snack for \$5.25. When he arrived home, he had \$22.50 left. Write and solve an equation to find the amount of Ricardo's allowance a. $\frac{1}{2}a - 5.25 = 22.50$; \$55.50

14. SHOPPING Liza earned some money by taking care of her neighbor's pet. She bought a drink for \$1.95, and a concert ticket for \$30. She bought a ring for \$7.20, and then spent two-thirds of the remaining money on a wireless speaker. If Liza has \$38.50 left, write and solve an equation to find the amount of money m Lisa earned by taking care of her neighbor's pet. $\frac{1}{3}(m - 1.95 - 30.00 - 7.20) = 38.50$; \$154.65

88 Module 2 • Equations in One Variable

Interactive Presentation



Question 1

Solve $2 - ax = -8$ for x. Assume $a \neq 0$.

○ A) $x = -\frac{10}{a}$

○ B) $x = -\frac{6}{a}$

○ C) $x = \frac{6}{a}$

○ D) $x = \frac{10}{a}$

Check

CHECK

Students complete the Check online to determine whether they are ready to move on.

Essential Question Follow-Up

Students have begun thinking about working backward to solve multi-step equations.

Ask:

Why is working backward an important part of the solving process? Sample answer: Working backward is important because it undoes the operations in the equation in the correct order to isolate the variable.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–35
2	exercises that extend concepts learned in this lesson to new contexts	36, 37
3	exercises that emphasize higher-order and critical-thinking skills	38–44

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

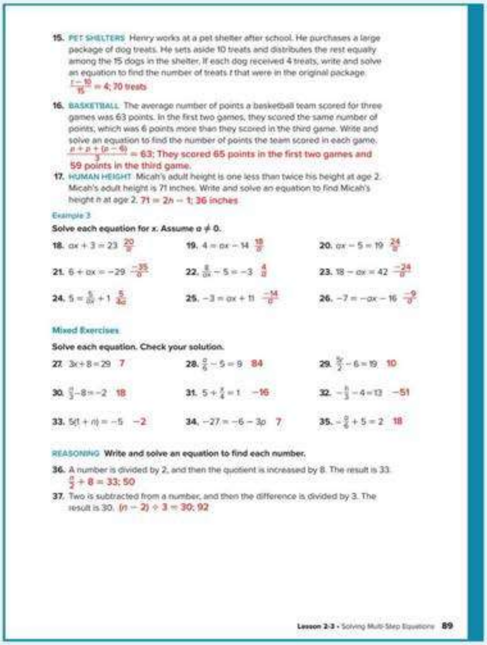
- Practice, Exercises 1–37 odd, 38–44
- Extension: Angles of a Triangle
- ALEKS Multi-Step Linear Equations

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–43 odd
- Remediation, Review Resources: Simplify Algebraic Expressions
- BrainPOP Video: Solving Equations
- Extra Examples 1–3
- ALEKS Equations and Inequalities

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Simplify Algebraic Expressions
- *Quick Review Math Handbook*: Solving Multi-Step Equations
- Arrive**MATH** Take Another Look
- ALEKS Equations and Inequalities



15. PET SHELTERS Henry works at a pet shelter after school. He purchases a large package of dog treats. He sets aside 10 treats and distributes the rest equally among the 15 dogs in the shelter. If each dog received 4 treats, write and solve an equation to find the number of treats t that were in the original package. $\frac{t-10}{15} = 4$; 70 treats

16. BASKETBALL The average number of points a basketball team scored for three games was 63 points. In the first two games, they scored the same number of points, which was 6 points more than they scored in the third game. Write and solve an equation to find the number of points the team scored in each game. $\frac{p+p+(p-6)}{3} = 63$; They scored 65 points in the first two games and 59 points in the third game.

17. HUMAN HEIGHT Micah's adult height is one less than twice his height at age 2. Micah's adult height is 71 inches. Write and solve an equation to find Micah's height h at age 2. $71 = 2h - 1$; 36 inches

Example 3

Solve each equation for x. Assume $a \neq 0$.

18. $ax + 3 = 23$ $\frac{20}{a}$
19. $4 = ax - 14$ $\frac{18}{a}$
20. $ax - 5 = 19$ $\frac{24}{a}$
21. $6 + ax = -29$ $\frac{-35}{a}$
22. $\frac{8}{ax} - 5 = -3$ $\frac{4}{a}$
23. $18 - ax = 42$ $\frac{-24}{a}$
24. $5 = \frac{5}{ax} + 1$ $\frac{5}{4a}$
25. $-3 = ax + 11$ $\frac{-14}{a}$
26. $-7 = -ax - 16$ $\frac{-9}{a}$

Mixed Exercises

Solve each equation. Check your solution.

27. $3x + 8 = 29$ 7
28. $\frac{a}{6} - 5 = 9$ 84
29. $\frac{5y}{2} - 6 = 19$ 10
30. $\frac{a}{3} - 8 = -2$ 18
31. $5 + \frac{z}{4} = 1$ −16
32. $-\frac{h}{3} - 4 = 13$ −51
33. $5(1 + n) = -5$ −2
34. $-27 = -6 - 3p$ 7
35. $-\frac{a}{6} + 5 = 2$ 18

REASONING **Write and solve an equation to find each number.**

36. A number is divided by 2, and then the quotient is increased by 8. The result is 33. $\frac{n}{2} + 8 = 33$; 50

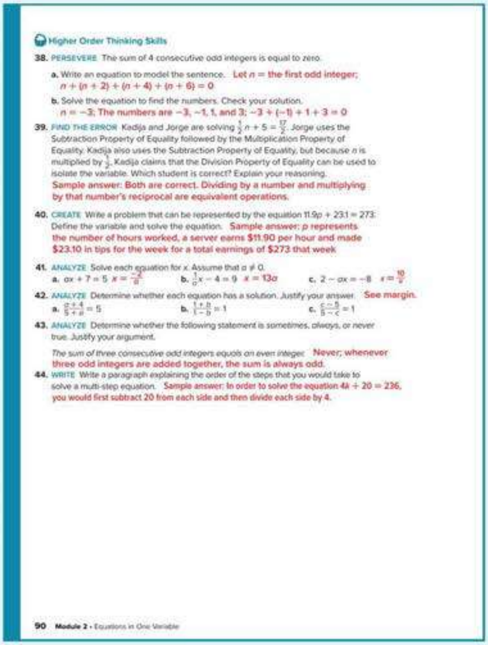
37. Two is subtracted from a number, and then the difference is divided by 3. The result is 30. $(n - 2) \div 3 = 30$; 92

Lesson 2-3 • Solving Multi-Step Equations 89

3 REFLECT AND PRACTICE

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Higher Order Thinking Skills

38. PERSEVERE The sum of 4 consecutive odd integers is equal to zero.

a. Write an equation to model the sentence. Let n = the first odd integer; $n + (n + 2) + (n + 4) + (n + 6) = 0$

b. Solve the equation to find the numbers. Check your solution. $n = -3$; The numbers are $-3, -1, 1$, and 3; $-3 + (-1) + 1 + 3 = 0$

39. FIND THE ERROR Kadija and Jorge are solving $\frac{1}{2}n + 5 = \frac{17}{2}$. Jorge uses the Subtraction Property of Equality followed by the Multiplication Property of Equality. Kadija also uses the Subtraction Property of Equality, but because n is multiplied by $\frac{1}{2}$, Kadija claims that the Division Property of Equality can be used to isolate the variable. Which student is correct? Explain your reasoning. Sample answer: Both are correct. Dividing by a number and multiplying by that number's reciprocal are equivalent operations.

40. CREATE Write a problem that can be represented by the equation $11.9p + 23.1 = 273$. Define the variable and solve the equation. Sample answer: p represents the number of hours worked, a server earns $11.90 per hour and made $23.10 in tips for the week for a total earnings of $273 that week

41. ANALYZE Solve each equation for x. Assume that $a \neq 0$.

a. $ax + 7 = 5$ $x = \frac{-2}{a}$ b. $\frac{1}{a}x - 4 = 9$ $x = 13a$ c. $2 - ax = -8$ $x = \frac{10}{a}$

42. ANALYZE Determine whether each equation has a solution. Justify your answer. See margin.

a. $\frac{a + 4}{5 + a} = 5$ b. $\frac{1 + b}{1 - b} = 1$ c. $\frac{c - 5}{5 - c} = 1$

43. ANALYZE Determine whether the following statement is sometimes, always, or never true. Justify your argument.

The sum of three consecutive odd integers equals an even integer. Never; whenever three odd integers are added together, the sum is always odd.

44. WRITE Write a paragraph explaining the order of the steps that you would take to solve a multi-step equation. Sample answer: In order to solve the equation $4k + 20 = 236$, you would first subtract 20 from each side and then divide each side by 4.

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Answers

42a. No; for there to be a solution there must be a number for which $a + 4 = a + 5$.

42b. Yes; for $b = 0$, $\frac{1 + b}{1 - b} = \frac{1 + 0}{1 - 0}$ or 1.

42c. No; $c - 5 = 5 - c$ when $c = 5$. However, $\frac{c - 5}{5 - c}$ is undefined for $c = 5$ since the fraction represents division by 0.

Lesson 2-4

A.CED.1, A.REI.1, A.REI.3

Solving Equations with the Variable on Each Side

LESSON GOAL

Students solve equations with the variable on each side by applying the properties of equality and the Distributive Property.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Modeling Equations with the Variable on Each Side

Develop:

Solving Equations with the Variable on Each Side
- Solve an Equation with the Variable on Each Side
- Write an Equation with the Variable on Each Side

Solving Equations Involving the Distributive Property
- Solve an Equation with Grouping Symbols
- Solve an Equation with a Fraction Bar
- Write an Equation with Grouping Symbols

Identities and Equations with No Solutions
- Solve an Equation with No Solution
- Solve an Identity

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Simplify Algebraic Expressions	●	●		●
Extension: Finding Unknowns in Identities		●	●	●

Language Development Handbook

Assign page 10 of the *Language Development Handbook* to help your students build mathematical language related to the properties of equality and the Distributive Property.

ELL You can use the tips and suggestions on page T10 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students solved multi-step equations.
8.EE.7, A.CED.1, A.REI.1, A.REI.3

Now
Students solve equations with the variable on each side.
A.CED.1, A.REI.1, A.REI.3

Next
Students will solve equations involving absolute value.
A.CED.1, A.REI.3

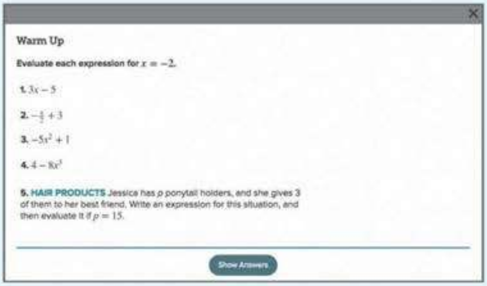
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their understanding of solving multi-step equations and build fluency with solving equations with the variable on each side. They apply their understanding of equations with the variable on each side by solving real-world problems.

Interactive Presentation

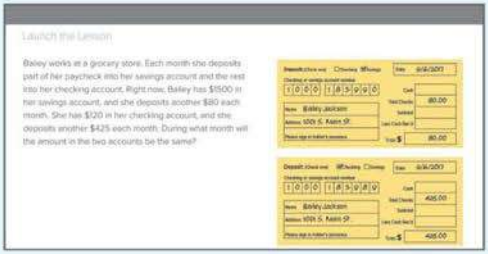


Warm Up

Evaluate each expression for $x = -2$.

1. $3x - 5$
2. $-\frac{x}{2} + 3$
3. $-5x^2 + 1$
4. $4 - 8x^3$
5. **HAIR PRODUCTS** Jessica has p ponytail holders, and she gives 3 of them to her best friend. Write an expression for this situation, and then evaluate it if $p = 15$.

Warm Up



Launch the Lesson

Bailey works at a grocery store. Each month she deposits part of her paycheck into her savings account and the rest into her checking account. Right now, Bailey has $1500 in her savings account, and she deposits another $80 each month. She has $120 in her checking account, and she deposits another $425 each month. During what month will the amount in the two accounts be the same?

Launch the Lesson

Vocabulary

identity

An equation that is true for every value of its variables.

1. What is the difference between an identity and an equation that has no solution?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- evaluating expressions

Answers:

1. −11
2. 4
3. −19
4. 68
5. $p - 3$; 12 ponytail holders

Launch the Lesson

Teaching the Mathematical Practices

2 Make Sense of Quantities Encourage students to define the variables and write equations relating the balances of bank account over time.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the question below with the class.

Mathematical Background

When the variable is on each side of the equation, first use the Distributive Property when appropriate to simplify before using the properties of equality to solve. If all of the variables are eliminated during the solving process, and both sides of the equation are different, then the equation is not true; there is no solution. If both sides of the equation are the same, then the equation is an identity; all values are solutions.

Explore Modeling Equations with the Variable on Each Side

Objective

Students use a sketch to explore solving equations with the variable on each side.

Teaching the Mathematical Practices

3 Construct Arguments In this Explore, students will use stated assumptions, definitions, and previously established results to construct an argument.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

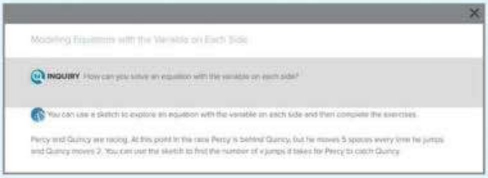
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use a sketch to determine the relationship between two racing rabbits. They will also write an expression for the distance each rabbit travels which leads to an equation with the variable on each side. The guiding exercises will help students write and solve an equation with the variable on each side. Then, students will answer the Inquiry Question.

(continued on next page)

Interactive Presentation



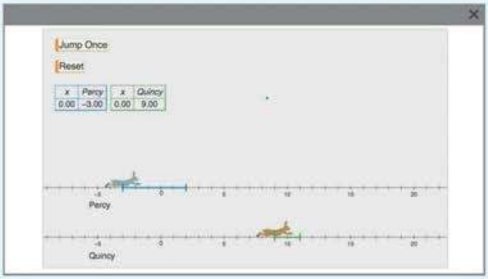
Modeling Equations with the Variable on Each Side

INQUIRY How can you solve an equation with the variable on each side?

You can use a sketch to explore an equation with the variable on each side and then complete the exercises.

Percy and Quincy are racing. At this point in the race Percy is behind Quincy, but he moves 5 spaces every time he jumps and Quincy moves 2. You can use the sketch to find the number of x jumps it takes for Percy to catch Quincy.

Explore



x	Percy	x	Quincy
0.00	-3.00	0.00	9.00

Explore

WEB SKETCHPAD

Students use a sketch to complete an activity in which they explore an equation with the variable on each side.

TYPE

Students answer a series of questions to show they understand how to write and solve an equation with the variable on both sides.

Interactive Presentation

INQUIRY How can you solve an equation with the variable on each side?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Modeling Equations with the Variable on Each Side (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What would happen if you eliminated the variable from the left side of the equation? Sample answer: The coefficient for x would be negative instead of positive, $-3 = 9 - 3x$. Then you would subtract 9 from both sides to have $-12 = -3x$, so you would have to divide by -3. Because $4 = x$, the answer is still $x = 4$.
- What should be the first step in solving $2(x - 5) = 3x + 4$? Sample answer: Before you can move the variables to one side, you have to use the Distributive Property for $2(x - 5)$ to give you $2x - 10$. Then you could solve like before.

Inquiry

How can you solve an equation with the variable on each side? Sample answer: Isolate the variables on one side. Then solve the equation like a one-step or multi-step equation.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Equations with the Variable on Each Side

Objective

Students solve equations with the variable on each side by applying the properties of equality.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Solve an Equation with the Variable on Each Side

Teaching the Mathematical Practices

3 Compare Arguments Mathematically proficient students can compare arguments, determine which one is flawed, and explain the flaw. In this example, students have to identify the flawed argument and choose the correct one.

Questions for Mathematical Discourse

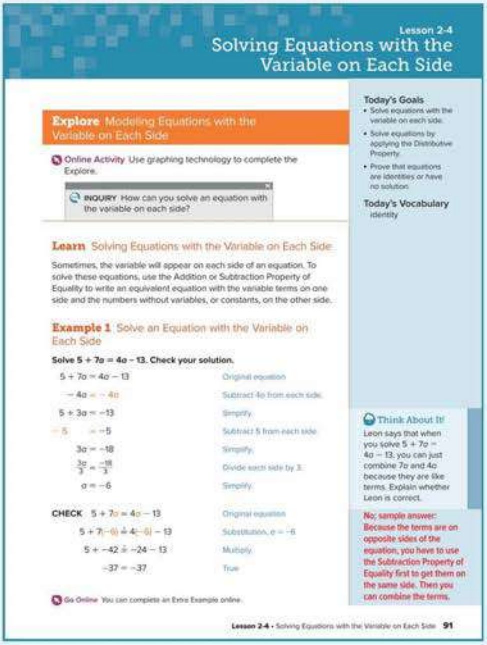
AL What are the variable terms given in the equation? $7a$ and $4a$

OL Does the variable in the solution always have to be located on the left side of the equation? Explain. No; sample answer: The variable can be on either side of the equation. A solution of $-6 = a$ is equivalent because of the Symmetric Property of Equality.

BL Explain a second method to solve the equation. Sample answer: We could have subtracted $7a$ from both sides of the equation and then added 13 to both sides. The variable term would then be on the right instead of the left. The solution would stay the same.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-4

Solving Equations with the Variable on Each Side

Explore Modeling Equations with the Variable on Each Side

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you solve an equation with the variable on each side?

Learn Solving Equations with the Variable on Each Side

Sometimes, the variable will appear on each side of an equation. To solve these equations, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side and the numbers without variables, or constants, on the other side.

Example 1 Solve an Equation with the Variable on Each Side

Solve $5 + 7a = 4a - 13$. Check your solution.

$5 + 7a = 4a - 13$ Original equation

$-4a = -4a$ Subtract $4a$ from each side.

$5 + 3a = -13$ Simplify.

$-5 = -5$ Subtract 5 from each side.

$3a = -18$ Simplify.

$\frac{3a}{3} = \frac{-18}{3}$ Divide each side by 3.

$a = -6$ Simplify.

CHECK $5 + 7a = 4a - 13$ Original equation

$5 + 7(-6) \stackrel{?}{=} 4(-6) - 13$ Substitution, $a = -6$

$5 + -42 \stackrel{?}{=} -24 - 13$ Multiply.

$-37 = -37$ True

Go Online You can complete an Extra Example online.

Today's Goals
- Solve equations with the variable on each side.
- Solve equations by applying the Distributive Property.
- Prove that equations are identities or have no solution.

Today's Vocabulary
identity

Think About It!
Leon says that when you solve $5 + 7a = 4a - 13$, you can just combine $7a$ and $4a$ because they are like terms. Explain whether Leon is correct.

No; sample answer: Because the terms are on opposite sides of the equation, you have to use the Subtraction Property of Equality first to get them on the same side. Then you can combine the terms.

Lesson 2-4 • Solving Equations with the Variable on Each Side 91

Interactive Presentation

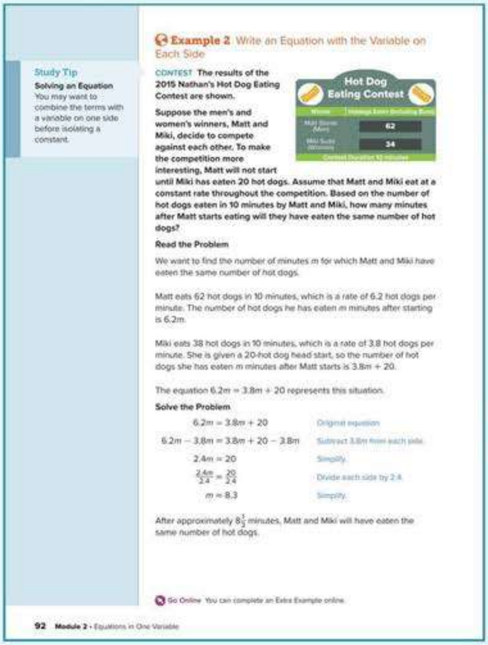
Learn

DRAG & DROP

Students drag shapes to identify variables and constants in an equation.

TYPE

Students answer questions to show they understand how to decide which property of equality is appropriate to use when solving.



Study Tip

Solving an Equation You may want to combine the terms with a variable on one side before isolating a constant.

Example 2 Write an Equation with the Variable on Each Side

CONTEST **The results of the 2015 Nathan's Hot Dog Eating Contest are shown.**

Suppose the men's and women's winners, Matt and Miki, decide to compete against each other. To make the competition more interesting, Matt will not start until Miki has eaten 20 hot dogs. Assume that Matt and Miki eat at a constant rate throughout the competition. Based on the number of hot dogs eaten in 10 minutes by Matt and Miki, how many minutes after Matt starts eating will they have eaten the same number of hot dogs?

Read the Problem

We want to find the number of minutes m for which Matt and Miki have eaten the same number of hot dogs.

Matt eats 62 hot dogs in 10 minutes, which is a rate of 6.2 hot dogs per minute. The number of hot dogs he has eaten m minutes after starting is $6.2m$.

Miki eats 38 hot dogs in 10 minutes, which is a rate of 3.8 hot dogs per minute. She is given a 20-hot dog head start, so the number of hot dogs she has eaten m minutes after Matt starts is $3.8m + 20$.

The equation $6.2m = 3.8m + 20$ represents this situation.

Solve the Problem

$6.2m = 3.8m + 20$ Original equation

$6.2m - 3.8m = 3.8m + 20 - 3.8m$ Subtract 3.8m from each side.

$2.4m = 20$ Simplify.

$\frac{2.4m}{2.4} = \frac{20}{2.4}$ Divide each side by 2.4.

$m \approx 8.3$ Simplify.

After approximately $8\frac{1}{3}$ minutes, Matt and Miki will have eaten the same number of hot dogs.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Write an Equation with the Variable on Each Side

CONTEST The Nathan's Hot Dog Eating Contest is a competition that takes place every year at New York's Coney Island on July 4th. The results of the 2015 Nathan's Hot Dog Eating Contest are shown below.

Example 2

WATCH

Students can watch a video clip from the Nathan's Hot Dog Eating Contest.

TAP

Students tap on each part of the equation to see what it represents.

Example 2 Write an Equation with the Variable on Each Side

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about writing equations to solve a real-world problem.

Questions for Mathematical Discourse

AL What does the variable m represent? the number of minutes Matt has been eating hot dogs

OL Why is it more difficult to subtract $6.2m$ from each side than $3.8m$? Sample answer: It is more difficult because the only constant is on the right side, so if $6.2m$ is subtracted, one side of the equation will equal zero.

BL Suppose Miki had been given a 48-hot dog head start instead. What is the new equation and solution? The new equation is $6.2m = 3.8m + 48$, and the new solution is $m = 20$ minutes.

Essential Question Follow-Up

Students have begun to solve equations with the variable on each side.

Ask:

Why might equations have variables on each side in the real world? Sample answer: When two quantities are the same, you can set them equal and solve. Consider two athletes running, each with a different expression representing their distance at a certain time. They can be set equal to find the time when the distances are equal.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to solve an equation with the variable on each side,

THEN they should use the algebra tiles to model and solve the equation. Have students remove x-tiles from one side of the equation in the same manner as removing a number using the +/− 1 tiles.

Enrichment Activity BL

Paul wants to buy a plaque for his football coach. One store charges $15 for the plaque and then $0.50 per word engraved. A second store charges $20 for the plaque but only $0.30 per word engraved. How many words could Paul have engraved on the plaques for the same cost? Write and solve an equation. Be sure to define your variable.

Let w = number of words engraved.

$$15 + 0.50w = 20 + 0.30w$$
$$15 + 0.50w - 0.30w = 20 + 0.30w - 0.30w$$
$$15 - 15 + 0.20w = 20 - 15$$
$$0.20w = 5$$
$$w = 25$$

Paul could have 25 words engraved on each plaque for the same cost.

Learn Solving Equations Involving the Distributive Property

Objective

Students solve equations by applying the Distributive Property.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

When an equation contains parentheses, some students still try to get the variable terms together first before applying the Distributive Property. Remind students that grouping symbols should always be handled first because of the order of operations.

Essential Question Follow-Up

Students have begun to solve equations involving the Distributive Property.

Ask:

> Why is it important to follow the order of operations even when solving equations? Sample answer: The order of operations is the correct order to simplify expressions, and equations that have expressions to simplify must be simplified consistently. Without the order of operations, different solutions could be found for one equation.

DIFFERENTIATE

Reteaching Activity AL ELL

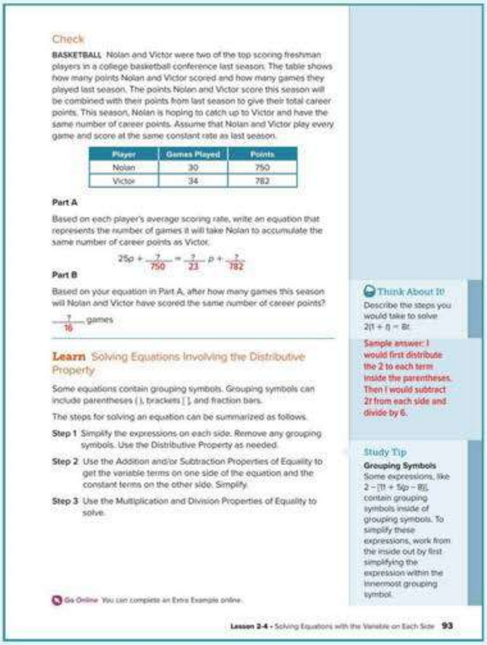
IF students are struggling to use the Distributive Property,

THEN have them draw an arrow from the coefficient of the parentheses to each term inside the parentheses as a visual reminder of "handing out" the coefficient to each term.

Enrichment Activity BL

Three times the sum of a number and 4 subtracted from 10 is the same as two times the same number increased by 8. Write and solve an equation.

$$10 - 3(n + 4) = 2n + 8$$
$$10 - 3n - 12 = 2n + 8$$
$$-3n + 3n - 2 = 2n + 8 + 3n$$
$$-2 - 8 = 8 + 5n - 8$$
$$-10 = 5n$$
$$-2 = n$$



Check

BASKETBALL Nolan and Victor were two of the top scoring freshman players in a college basketball conference last season. The table shows how many points Nolan and Victor scored and how many games they played last season. The points Nolan and Victor score this season will be combined with their points from last season to give their total career points. This season, Nolan is hoping to catch up to Victor and have the same number of career points. Assume that Nolan and Victor play every game and score at the same constant rate as last season.

Player	Games Played	Points
Nolan	30	750
Victor	34	782

Part A

Based on each player's average scoring rate, write an equation that represents the number of games it will take Nolan to accumulate the same number of career points as Victor.

$$25p + \underset{750}{\underline{\ ?\ }} = \underset{23}{\underline{\ ?\ }}\,p + \underset{782}{\underline{\ ?\ }}$$

Part B

Based on your equation in Part A, after how many games this season will Nolan and Victor have scored the same number of career points?

$\underset{16}{\underline{\ ?\ }}$ games

Learn Solving Equations Involving the Distributive Property

Some equations contain grouping symbols. Grouping symbols can include parentheses ( ), brackets [ ], and fraction bars.

The steps for solving an equation can be summarized as follows.

Step 1 Simplify the expressions on each side. Remove any grouping symbols. Use the Distributive Property as needed.

Step 2 Use the Addition and/or Subtraction Properties of Equality to get the variable terms on one side of the equation and the constant terms on the other side. Simplify.

Step 3 Use the Multiplication and Division Properties of Equality to solve.

Think About It!

Describe the steps you would take to solve $2(1 + t) = 8t$.

Sample answer: I would first distribute the 2 to each term inside the parentheses. Then I would subtract $2t$ from each side and divide by 6.

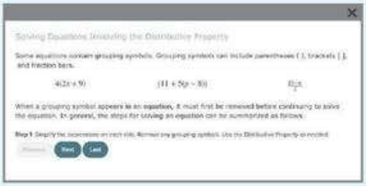
Study Tip

Grouping Symbols Some expressions, like $2 - [11 + 5(p - 8)]$, contain grouping symbols inside of grouping symbols. To simplify these expressions, work from the inside out by first simplifying the expression within the innermost grouping symbol.

Go Online You can complete an Extra Example online.

Lesson 2-4 • Solving Equations with the Variable on Each Side 93

Interactive Presentation



Solving Equations Involving the Distributive Property

Some equations contain grouping symbols. Grouping symbols can include parentheses ( ), brackets [ ], and fraction bars.

Learn

TAP

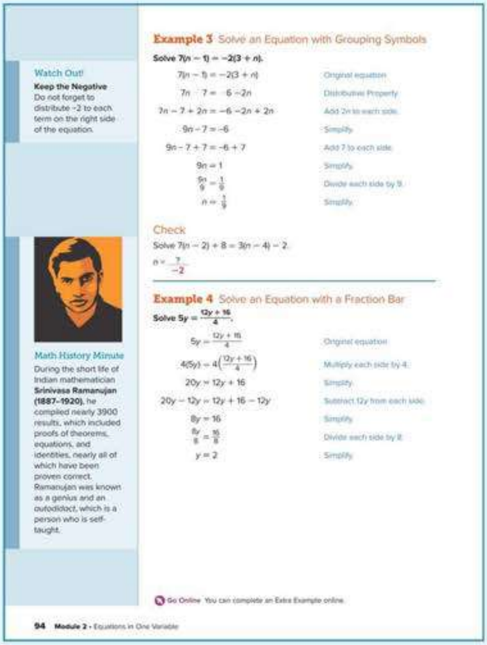
Students move through the steps to solve an equation involving the Distributive Property.

TYPE

Students answer a question to show they understand how to solve a given equation.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Watch Out!

Keep the Negative Do not forget to distribute −2 to each term on the right side of the equation.

Math History Minute

During the short life of Indian mathematician **Srinivasa Ramanujan (1887–1920)**, he compiled nearly 3900 results, which included proofs of theorems, equations, and identities, nearly all of which have been proven correct. Ramanujan was known as a genius and an autodidact, which is a person who is self-taught.

Example 3 Solve an Equation with Grouping Symbols

Solve $7(n-1) = -2(3+n)$.

$7(n-1) = -2(3+n)$	Original equation
$7n-7 = -6-2n$	Distributive Property
$7n-7+2n = -6-2n+2n$	Add 2n to each side.
$9n-7 = -6$	Simplify.
$9n-7+7 = -6+7$	Add 7 to each side.
$9n = 1$	Simplify.
$\frac{9n}{9} = \frac{1}{9}$	Divide each side by 9.
$n = \frac{1}{9}$	Simplify.

Check

Solve $7(n-2)+8 = 3(n-4)-2$.

$n = -2$

Example 4 Solve an Equation with a Fraction Bar

Solve $5y = \frac{12y+16}{4}$.

$5y = \frac{12y+16}{4}$	Original equation
$4(5y) = 4\left(\frac{12y+16}{4}\right)$	Multiply each side by 4.
$20y = 12y+16$	Simplify.
$20y-12y = 12y+16-12y$	Subtract 12y from each side.
$8y = 16$	Simplify.
$\frac{8y}{8} = \frac{16}{8}$	Divide each side by 8.
$y = 2$	Simplify.

Go Online You can complete an Extra Example online.

94 Module 2 • Equations in One Variable

Interactive Presentation

Example 3

Example 3 Solve an Equation with Grouping Symbols

MP Teaching the Mathematical Practices

7 Use the Distributive Property Point out to students that the Distributive Property is one of the most-used properties in algebra. Students should know that whenever they see a number outside of a sum or difference within parentheses, they should apply the Distributive Property.

Questions for Mathematical Discourse

AL What operation is assumed between the 7 and the $(n - 1)$? multiplication

OL What number is distributed to $(3 + n)$? −2

BL What would a student have done incorrectly to get the equation $7n - 7 = -6 + 2n$? Sample answer: The student would have not distributed the −2 correctly in the second parentheses, instead multiplying the constant by −2 but the variable by 2.

Common Error

Students tend to forget to multiply the second term inside the parentheses by the number out front. Remind students that the number outside of the parentheses must be multiplied to all terms within the parentheses.

Example 4 Solve an Equation with a Fraction Bar

MP Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students should check their answer. They should ask themselves whether their answer makes sense and whether they have answered the question asked.

Questions for Mathematical Discourse

AL What operation, addition, subtraction, multiplication, or division, does the denominator of a fraction represent? division

OL Why does each side of the equation need to be multiplied by 4? Sample answer: Multiplying each side of the equation by 4 eliminates the denominator of the fraction on the right side of the equation.

BL How could the equation have been solved if the large fraction was split into two fractions? Sample answer: If the fraction was broken into $\frac{12y}{4} + \frac{16}{4}$, then the terms could have been simplified to $3y + 4$. Then the equation could have been solved by subtracting $3y$ from each side, and then dividing each side by 2.

Example 5 Write an Equation with Grouping Symbols

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation that models the situation in this example. Then use the equation to solve the problem.

Questions for Mathematical Discourse

AL Write a verbal sentence that states the areas of the figures are the same. Sample answer: The area of the rectangle is equal to the area of the triangle.

OL Why does $\frac{1}{2}(12)(2x - 6)$ simplify to $6(2x - 6)$ rather than $6 \cdot \frac{1}{2}(2x - 6)$? Sample answer: The operation between $\frac{1}{2}$, 12, and $(2x - 6)$ is multiplication, so the $\frac{1}{2}$ does not need to be distributed to $(2x - 6)$. By simplifying to 6 outside the parentheses, the multiplication of $\frac{1}{2}$ has been completed.

BL What would be the value of x if twice the area of the rectangle were equal to the area of the triangle?

$$2(5)(x + 4) = 6(2x - 6)$$
$$10(x + 4) = 6(2x - 6)$$
$$10x + 40 = 12x - 36$$
$$76 = 2x$$
$$38 = x$$

Common Error

When writing an equation, many students do not read well enough to determine the relationship of all the values. Encourage students to read the problem, underline or highlight important words, numbers, or phrases, and then write the equation.

Learn Identities and Equations with No Solutions

Objective

Students prove that equations are identities or have no solution by applying the properties of equality.

Teaching the Mathematical Practices

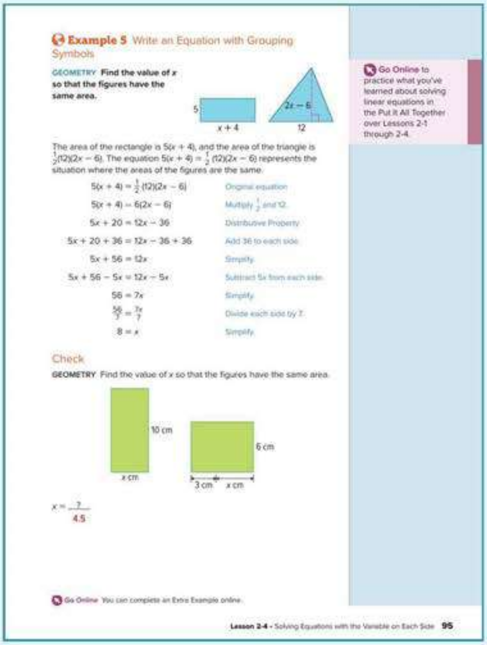
3 Analyze Cases Work with students to look at the three types of equations. Encourage students to familiarize themselves with all of the cases.

Important to Know

An identity is an equation that is true for all values of the variable. An equation that does not have a value that makes it true will have no solution. An equation can have one solution, no solution, or is an identity.

Common Misconception

Students confuse equations whose solution is zero with equations that have no solution or are identities. Remind students that zero is a valid solution, but when the variable is eliminated, then the equation has no solution or is an identity.



Example 5 Write an Equation with Grouping Symbols

GEOMETRY **Find the value of x so that the figures have the same area.**

The area of the rectangle is $5(x + 4)$, and the area of the triangle is $\frac{1}{2}(12)(2x - 6)$. The equation $5(x + 4) = \frac{1}{2}(12)(2x - 6)$ represents the situation where the areas of the figures are the same.

$5(x + 4) = \frac{1}{2}(12)(2x - 6)$	Original equation
$5(x + 4) = 6(2x - 6)$	Multiply $\frac{1}{2}$ and 12.
$5x + 20 = 12x - 36$	Distributive Property
$5x + 20 + 36 = 12x - 36 + 36$	Add 36 to each side.
$5x + 56 = 12x$	Simplify.
$5x + 56 - 5x = 12x - 5x$	Subtract 5x from each side.
$56 = 7x$	Simplify.
$\frac{56}{7} = \frac{7x}{7}$	Divide each side by 7.
$8 = x$	Simplify.

Check

GEOMETRY Find the value of x so that the figures have the same area.

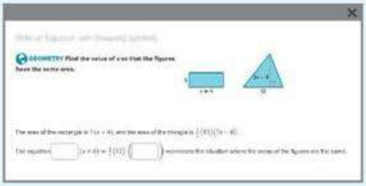
$x =$? 4.5

Go Online You can complete an Extra Example online.

Go Online to practice what you've learned about solving linear equations in the Put It All Together over Lessons 2-1 through 2-4.

Lesson 2-4 • Solving Equations with the Variable on Each Side 95

Interactive Presentation



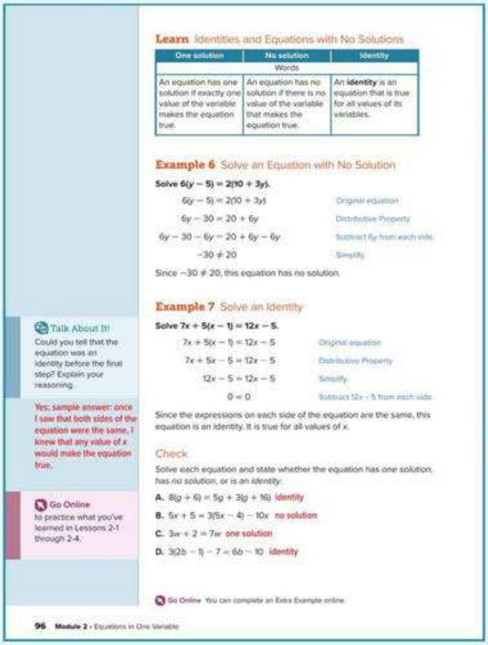
Example 5

TYPE

Students complete the table to create the equation representing the areas.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Learn Identities and Equations with No Solutions

One solution	No solution	Identity
Words		
An equation has one solution if exactly one value of the variable makes the equation true.	An equation has no solution if there is no value of the variable that makes the equation true.	An **identity** is an equation that is true for all values of its variables.

Example 6 Solve an Equation with No Solution

Solve $6(y - 5) = 2(10 + 3y)$.

$6(y - 5) = 2(10 + 3y)$	Original equation
$6y - 30 = 20 + 6y$	Distributive Property
$6y - 30 - 6y = 20 + 6y - 6y$	Subtract 6y from each side
$-30 \neq 20$	Simplify.

Since $-30 \neq 20$, this equation has no solution.

Example 7 Solve an Identity

Solve $7x + 5(x - 1) = 12x - 5$.

$7x + 5(x - 1) = 12x - 5$	Original equation
$7x + 5x - 5 = 12x - 5$	Distributive Property
$12x - 5 = 12x - 5$	Simplify.
$0 = 0$	Subtract 12x − 5 from each side

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of x.

Talk About It!
Could you tell that the equation was an identity before the final step? Explain your reasoning.

Yes; sample answer: once I saw that both sides of the equation were the same, I knew that any value of x would make the equation true.

Check

Solve each equation and state whether the equation has one solution, has no solution, or is an identity.

A. $8(g + 6) = 5g + 3(g + 16)$ identity

B. $5x + 5 = 3(5x - 4) - 10x$ no solution

C. $3w + 2 = 7w$ one solution

D. $3(2b - 1) - 7 = 6b - 10$ identity

Go Online to practice what you've learned in Lessons 2-1 through 2-4.

Go Online You can complete an Extra Example online.

96 **Module 2** • Equations in One Variable

Interactive Presentation

Learn

TAP

Students move through the categories to compare three types of equations.

DRAG & DROP

Students drag the given words, examples, and written forms to identify the correct type of equation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 6 Solve an Equation with No Solution

Questions for Mathematical Discourse

AL Why is using the Distributive Property the first step in the solution? Sample answer: We need to undo the grouping symbols first.

OL Why is there no solution to the equation? Sample answer: The variable was eliminated and the remaining numbers are not equal.

BL Before subtracting $6y$ from each side, how can you determine that there is no solution? Sample answer: After applying the Distributive Property, the two sides of the equation have the same variable term, but different constants. This means there is no solution.

Example 7 Solve an Identity

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What type of equation simplifies to $0 = 0$? an identity

OL Why is the equation an identity? Sample answer: The variable terms are eliminated and the remaining numbers are equal.

BL If the expression on the right side of the equation was $12x + 5$, would this still be an identity? Explain. No; sample answer: After simplifying, all that remains is $-5 = 5$.

DIFFERENTIATE

Enrichment Activity **BL**
We have focused on equations that have no solution, have one solution, or are identities. Give an example of an equation that has two solutions. Sample answer: $|x| = 5$ or $x^2 = 16$

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–36
2	exercises that use a variety of skills from this lesson	37–48
2	exercises that extend concepts learned in this lesson to new contexts	49–52
3	exercises that emphasize higher-order and critical-thinking skills	53–60

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–51 odd, 53–60
- Extension: Finding Unknowns in Identities
- ALEKS Multi-Step Linear Equations

OL

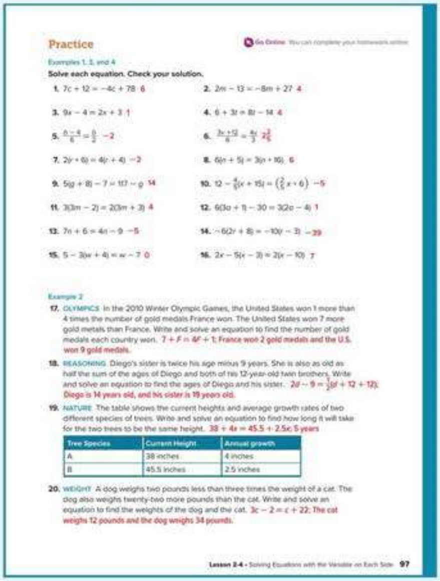
IF students score 66–89% on the Checks,
THEN assign:

- Practice, Exercises 1–59 odd
- Remediation, Review Resources: Evaluate Algebraic Expressions
- BrainPOP Video: Solving Equations
- Extra Examples 1–7
- ALEKS Evaluating and Writing Expressions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Evaluate Algebraic Expressions
- *Quick Review Math Handbook*: Solving Equations with the Variable on Each Side
- Arrive**MATH** Take Another Look
- ALEKS Evaluating and Writing Expressions



Practice

Go Online You can complete your homework online.

Examples 1, 3, and 4

Solve each equation. Check your solution.

1. $7c + 12 = -4c + 78$ 6
2. $2m - 13 = -8m + 27$ 4
3. $9x - 4 = 2x + 3$ 1
4. $6 + 3t = 8t - 14$ 4
5. $\frac{b-4}{6} = \frac{b}{2}$ −2
6. $\frac{3x+12}{6} = \frac{4x}{3}$ $2\frac{2}{5}$
7. $2(r + 6) = 4(r + 4)$ −2
8. $6(n + 5) = 3(n + 16)$ 6
9. $5(g + 8) - 7 = 117 - g$ 14
10. $12 - \frac{4}{5}(x + 15) = \left(\frac{2}{5}x + 6\right)$ −5
11. $3(3m - 2) = 2(3m + 3)$ 4
12. $6(3a + 1) - 30 = 3(2a - 4)$ 1
13. $7n + 6 = 4n - 9$ −5
14. $-6(2r + 8) = -10(r - 3)$ −39
15. $5 - 3(w + 4) = w - 7$ 0
16. $2x - 5(x - 3) = 2(x - 10)$ 7

Example 2

17. OLYMPICS In the 2010 Winter Olympic Games, the United States won 1 more than 4 times the number of gold medals France won. The United States won 7 more gold metals than France. Write and solve an equation to find the number of gold medals each country won. $7 + F = 4F + 1$; France won 2 gold medals and the U.S. won 9 gold medals.

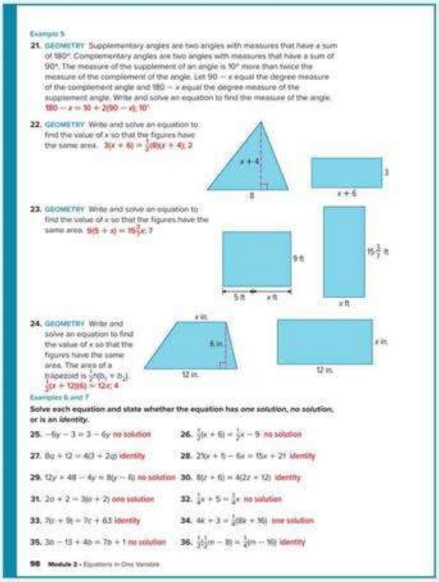
18. REASONING Diego's sister is twice his age minus 9 years. She is also as old as half the sum of the ages of Diego and both of his 12-year-old twin brothers. Write and solve an equation to find the ages of Diego and his sister. $2d - 9 = \frac{1}{2}(d + 12 + 12)$; Diego is 14 years old, and his sister is 19 years old.

19. NATURE The table shows the current heights and average growth rates of two different species of trees. Write and solve an equation to find how long it will take for the two trees to be the same height. $38 + 4x = 45.5 + 2.5x$; 5 years

Tree Species	Current Height	Annual growth
A	38 inches	4 inches
B	45.5 inches	2.5 inches

20. WEIGHT A dog weighs two pounds less than three times the weight of a cat. The dog also weighs twenty-two more pounds than the cat. Write and solve an equation to find the weights of the dog and the cat. $3c - 2 = c + 22$; The cat weighs 12 pounds and the dog weighs 34 pounds.

Lesson 2-4 • Solving Equations with the Variable on Each Side 97



Example 5

21. GEOMETRY Supplementary angles are two angles with measures that have a sum of 180°. Complementary angles are two angles with measures that have a sum of 90°. The measure of the supplement of an angle is 10° more than twice the measure of the complement of the angle. Let $90 - x$ equal the degree measure of the complement angle and $180 - x$ equal the degree measure of the supplement angle. Write and solve an equation to find the measure of the angle. $180 - x = 10 + 2(90 - x)$; 10°

22. GEOMETRY Write and solve an equation to find the value of x so that the figures have the same area. $3(x + 6) = \frac{1}{2}(8)(x + 4)$; 2

23. GEOMETRY Write and solve an equation to find the value of x so that the figures have the same area. $9(5 + x) = 15\frac{3}{7}x$; 7

24. GEOMETRY Write and solve an equation to find the value of x so that the figures have the same area. The area of a trapezoid is $\frac{1}{2}h(b_1 + b_2)$. $\frac{1}{2}(x + 12)(6) = 12x$; 4

Examples 6 and 7

Solve each equation and state whether the equation has one solution, no solution, or is an identity.

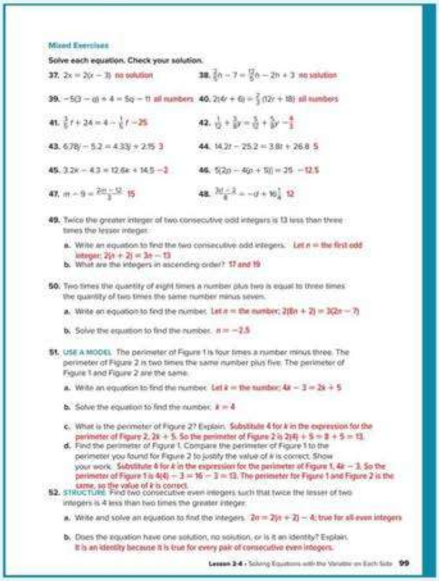
25. $-6y - 3 = 3 - 6y$ no solution
26. $\frac{1}{2}(x + 6) = \frac{1}{2}x - 9$ no solution
27. $8q + 12 = 4(3 + 2q)$ identity
28. $21(x + 1) - 6x = 15x + 21$ identity
29. $12y + 48 - 4y = 8(y - 6)$ no solution
30. $8(z + 6) = 4(2z + 12)$ identity
31. $2a + 2 = 3(a + 2)$ one solution
32. $\frac{1}{4}x + 5 = \frac{1}{4}x$ no solution
33. $7(c + 9) = 7c + 63$ identity
34. $4x + 3 = \frac{1}{4}(8x + 16)$ one solution
35. $3b - 13 + 4b = 7b + 1$ no solution
36. $\frac{1}{2}\left(\frac{1}{2}m - 8\right) = \frac{1}{4}(m - 16)$ identity

98 Module 2 • Equations in One Variable

3 REFLECT AND PRACTICE

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Mixed Exercises

Solve each equation. Check your solution.

37. $2x = 2(x - 3)$ no solution

38. $\frac{2}{5}n - 7 = \frac{12}{5}n - 2n + 3$ no solution

39. $-5(3 - q) + 4 = 5q - 11$ all numbers

40. $2(4r + 6) = \frac{2}{3}(12r + 18)$ all numbers

41. $\frac{3}{5}f + 24 = 4 - \frac{1}{5}f$ -25

42. $\frac{1}{12} + \frac{3}{8}y = \frac{5}{12} + \frac{5}{8}y$ $-\frac{4}{3}$

43. $6.78j - 5.2 = 4.33j + 2.15$ 3

44. $14.2t - 25.2 = 3.8t + 26.8$ 5

45. $3.2k - 4.3 = 12.6k + 14.5$ -2

46. $5(2p - 4(p + 5)) = 25$ -12.5

47. $m - 9 = \frac{2m - 12}{3}$ 15

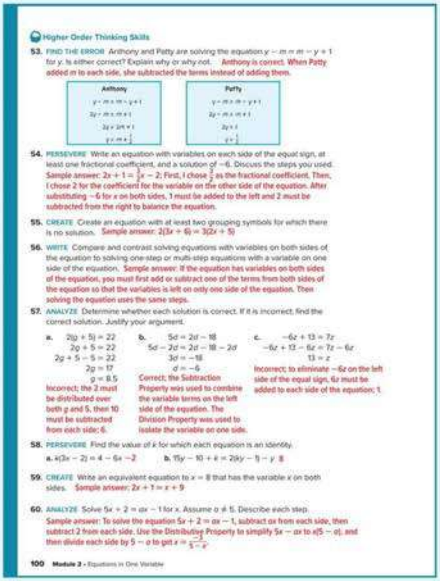
48. $\frac{3d - 2}{8} = -d + 16\frac{1}{4}$ 12

49. Twice the greater integer of two consecutive odd integers is 13 less than three times the lesser integer.
    a. Write an equation to find the two consecutive odd integers. Let n = the first odd integer; $2(n + 2) = 3n - 13$
    b. What are the integers in ascending order? 17 and 19

50. Two times the quantity of eight times a number plus two is equal to three times the quantity of two times the same number minus seven.
    a. Write an equation to find the number. Let n = the number; $2(8n + 2) = 3(2n - 7)$
    b. Solve the equation to find the number. $n = -2.5$

51. USE A MODEL The perimeter of Figure 1 is four times a number minus three. The perimeter of Figure 2 is two times the same number plus five. The perimeter of Figure 1 and Figure 2 are the same.
    a. Write an equation to find the number. Let k = the number; $4k - 3 = 2k + 5$
    b. Solve the equation to find the number. $k = 4$
    c. What is the perimeter of Figure 2? Explain. Substitute 4 for k in the expression for the perimeter of Figure 2, $2k + 5$. So the perimeter of Figure 2 is $2(4) + 5 = 8 + 5 = 13$.
    d. Find the perimeter of Figure 1. Compare the perimeter of Figure 1 to the perimeter you found for Figure 2 to justify the value of k is correct. Show your work. Substitute 4 for k in the expression for the perimeter of Figure 1, $4k - 3$. So the perimeter of Figure 1 is $4(4) - 3 = 16 - 3 = 13$. The perimeter for Figure 1 and Figure 2 is the same, so the value of k is correct.

52. STRUCTURE Find two consecutive even integers such that twice the lesser of two integers is 4 less than two times the greater integer.
    a. Write and solve an equation to find the integers. $2n = 2(n + 2) - 4$; true for all even integers
    b. Does the equation have one solution, no solution, or is it an identity? Explain. It is an identity because it is true for every pair of consecutive even integers.



Higher Order Thinking Skills

53. FIND THE ERROR Anthony and Patty are solving the equation $y - m = m - y + 1$ for y. Is either correct? Explain why or why not. Anthony is correct. When Patty added m to each side, she subtracted the terms instead of adding them.

Anthony

$y - m = m - y + 1$
$2y - m = m + 1$
$2y = 2m + 1$
$y = m + \frac{1}{2}$

Patty

$y - m = m - y + 1$
$2y - m = m + 1$
$2y = 1$
$y = \frac{1}{2}$

54. PERSEVERE Write an equation with variables on each side of the equal sign, at least one fractional coefficient, and a solution of -6. Discuss the steps you used. Sample answer: $2x + 1 = \frac{3}{2}x - 2$; First, I chose $\frac{3}{2}$ as the fractional coefficient. Then, I chose 2 for the coefficient for the variable on the other side of the equation. After substituting -6 for x on both sides, 1 must be added to the left and 2 must be subtracted from the right to balance the equation.

55. CREATE Create an equation with at least two grouping symbols for which there is no solution. Sample answer: $2(3x + 6) = 3(2x + 5)$

56. WRITE Compare and contrast solving equations with variables on both sides of the equation to solving one-step or multi-step equations with a variable on one side of the equation. Sample answer: If the equation has variables on both sides of the equation, you must first add or subtract one of the terms from both sides of the equation so that the variables is left on only one side of the equation. Then solving the equation uses the same steps.

57. ANALYZE Determine whether each solution is correct. If it is incorrect, find the correct solution. Justify your argument.

a.
$2(g + 5) = 22$
$2g + 5 = 22$
$2g + 5 - 5 = 22$
$2g = 17$
$g = 8.5$
Incorrect; the 2 must be distributed over both g and 5, then 10 must be subtracted from each side; 6.

b.
$5d = 2d - 18$
$5d - 2d = 2d - 18 - 2d$
$3d = -18$
$d = -6$
Correct; the Subtraction Property was used to combine the variable terms on the left side of the equation. The Division Property was used to isolate the variable on one side.

c.
$-6z + 13 = 7z$
$-6z + 13 - 6z = 7z - 6z$
$13 = z$
Incorrect; to eliminate $-6z$ on the left side of the equal sign, $6z$ must be added to each side of the equation; 1

58. PERSEVERE Find the value of k for which each equation is an identity.
    a. $k(3x - 2) = 4 - 6x$ -2
    b. $11y - 10 + k = 2(6y - 1) - y$ 8

59. CREATE Write an equivalent equation to $x = 8$ that has the variable x on both sides. Sample answer: $2x + 1 = x + 9$

60. ANALYZE Solve $5x + 2 = ax - 1$ for x. Assume $a \neq 5$. Describe each step. Sample answer: To solve the equation $5x + 2 = ax - 1$, subtract ax from each side, then subtract 2 from each side. Use the Distributive Property to simplify $5x - ax$ to $x(5 - a)$, and then divide each side by $5 - a$ to get $x = \frac{-3}{5 - a}$.

Lesson 2-5

A.CED.1, A.REI.3

Solving Equations Involving Absolute Value

LESSON GOAL

Students solve absolute value equations.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Modeling Absolute Value

Develop:

Solving Equations Involving Absolute Value

- Solve an Absolute Value Equation When $n > 0$
- Solve an Absolute Value Equation When $n < 0$
- Solve an Absolute Value Equation
- Write an Absolute Value Equation

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Integers: Opposites and Absolute Value	●	●		●
Extension: Solving Absolute Value Equations with Variables on Both Sides		●	●	●

Language Development Handbook

Assign page 11 of the *Language Development Handbook* to help your students build mathematical language related to equations involving absolute values.

ELL You can use the tips and suggestions on page T11 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable anduse them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students evaluated expressions with absolute value and solved multi-step equations.

8.EE.7, A.REI.3

Now

Students solve equations involving absolute value.

A.CED.1, A.REI.3

Next

Students will solve equations involving proportions.

A.CED.1, A.REI.3

Rigor

The Three Pillars of Rigor

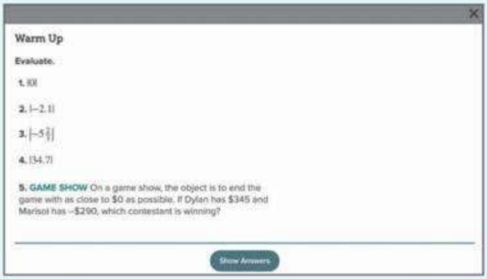
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of absolute value expressions to build fluency with solving equations that involve absolute value. They apply their understanding of solving absolute value equations by solving real-world problems.

Mathematical Background

The absolute value of a number is the distance the number is from zero on the number line. To solve an equation involving absolute value, first isolate the absolute value on one side of the equation and rewrite the equation as a compound sentence using the word *or*. The solution set of an absolute value equation can be graphed on a number line or written in set notation.

Interactive Presentation



Warm Up

Evaluate.

1. $|0|$
2. $|-2.1|$
3. $\left|-5\frac{2}{3}\right|$
4. $|34.7|$
5. **GAME SHOW** On a game show, the object is to end the game with as close to \$0 as possible. If Dylan has \$345 and Marisol has −\$290, which contestant is winning?

Warm Up

Launch the Lesson

All of the Major League Baseball stadiums are equipped with triangulated tracking cameras to track the baseball's position as it crosses home plate. This technology is said to have a margin of error of 1 inch. You can use an absolute value equation to represent the relationship between the position indicated by the tracking cameras and the actual location of the ball.

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- calculating opposites and absolute values

Answers:

1. 0
2. 2.1
3. $5\frac{2}{3}$
4. 34.7
5. Marisol

Launch the Lesson

Teaching the Mathematical Practices

1 Seek Information Encourage students to identify the given information and find the relationship between the reported location and actual location of a baseball.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Explore Modeling Absolute Value

Objective

Students use survey data to explore the absolute value of a number.

Teaching the Mathematical Practices

4 Apply Mathematics In this Explore, students apply what they have learned about margin of error to solve a real-world problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

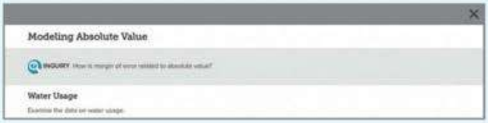
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Using an informational handout, students will determine the bounds of a margin of error and plot on a number line. They will answer a series of questions about the margin of error before examining absolute value. The guiding exercises will help students determine the similarities of margin of error and absolute value. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



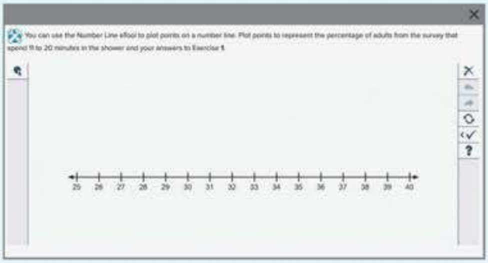
Modeling Absolute Value

INQUIRY How is margin of error related to absolute value?

Water Usage

Examine the data on water usage.

Explore



You can use the Number Line eTool to plot points on a number line. Plot points to represent the percentage of adults from the survey that spend 11 to 20 minutes in the shower and your answers to Exercise 1.

Explore

TYPE

Students answer a series of questions to show they understand absolute value.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Modeling Absolute Value (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What does the notation ±3 mean in words? Why do you think this is used for the margin of error? Sample answer: The plus/minus notation means to add and subtract the value, so in words it's three more and three less than the starting value. This is used for margin of error to show that there could be some variation, but within a specific range.
- If the margin of error for the survey was ±4, what are the lowest and highest percentages for a shower that is less than 5 minutes? Sample answer: The survey says 10% of adults shower for less than 5 minutes. With the margin of error, that means the percentages could be 6% to 14% of adults.

Inquiry

How is margin of error related to absolute value? Sample answer: Both represent distances, where margin of error is the distance from a given value and absolute value is the distance from zero.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Equations Involving Absolute Value

Objective

Students solve and graph equations involving absolute values by constructing two cases for the equation.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to set up the two equations for absolute value, **THEN** have students write the two cases with the positive and negative signs placed on the side of the absolute value, rather than the other side. For example, $|x| = 4$ would be written as $x = 4$ and $-x = 4$, which yields $x = -4$.

Enrichment Activity BL

Consider the equation $|3q - 1| = 2q + 3$.

a. Set up the equations for the two cases needed to solve the absolute value equation.

b. Solve each case for the value of q.

c. Check each answer and write the solution set to the absolute value equation.

a. Case 1: $3q - 1 = 2q + 3$ Case 2: $3q - 1 = -2q - 3$

b. Case 1: $q = 4$ Case 2: $q = -\frac{2}{5}$

c. Case 1: $|3(4) - 1| = 2(4) + 3$

$|12 - 1| = 8 + 3$

$|11| = 11$

Case 2: $\left|3\left(-\frac{2}{5}\right) - 1\right| = 2\left(-\frac{2}{5}\right) + 3$

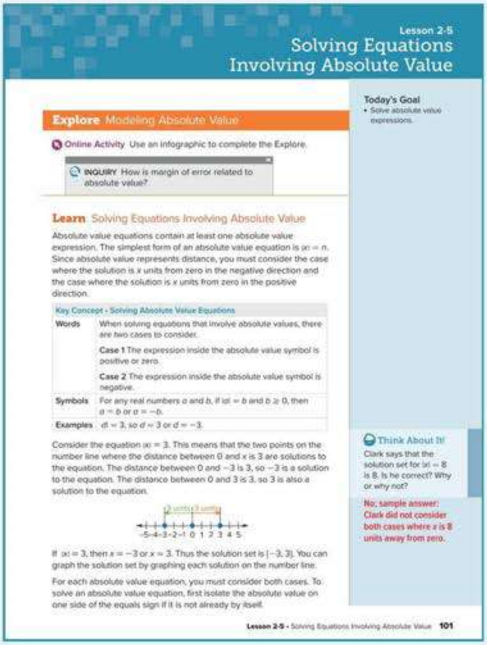
$\left|-\frac{6}{5} - 1\right| = -\frac{4}{5} + 3$

$\left|-\frac{11}{5}\right| = \frac{11}{5}$

Solution Set: $\left\{-\frac{2}{5}, 4\right\}$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-5

Solving Equations Involving Absolute Value

Today's Goal
- Solve absolute value expressions.

Explore Modeling Absolute Value

Online Activity Use an infographic to complete the Explore.

INQUIRY How is margin of error related to absolute value?

Learn Solving Equations Involving Absolute Value

Absolute value equations contain at least one absolute value expression. The simplest form of an absolute value equation is $|x| = n$. Since absolute value represents distance, you must consider the case where the solution is x units from zero in the negative direction and the case where the solution is x units from zero in the positive direction.

Key Concept • Solving Absolute Value Equations

Words	When solving equations that involve absolute values, there are two cases to consider. **Case 1** The expression inside the absolute value symbol is positive or zero. **Case 2** The expression inside the absolute value symbol is negative.
Symbols	For any real numbers a and b, if $\|a\| = b$ and $b \geq 0$, then $a = b$ or $a = -b$.
Examples	$\|d\| = 3$, so $d = 3$ or $d = -3$.

Consider the equation $|x| = 3$. This means that the two points on the number line where the distance between 0 and x is 3 are solutions to the equation. The distance between 0 and −3 is 3, so −3 is a solution to the equation. The distance between 0 and 3 is 3, so 3 is also a solution to the equation.

If $|x| = 3$, then $x = -3$ or $x = 3$. Thus the solution set is $\{-3, 3\}$. You can graph the solution set by graphing each solution on the number line.

For each absolute value equation, you must consider both cases. To solve an absolute value equation, first isolate the absolute value on one side of the equals sign if it is not already by itself.

Think About It!
Clark says that the solution set for $|x| = 8$ is 8. Is he correct? Why or why not?

No; sample answer: Clark did not consider both cases where x is 8 units away from zero.

Lesson 2-5 • Solving Equations Involving Absolute Value 101

Interactive Presentation

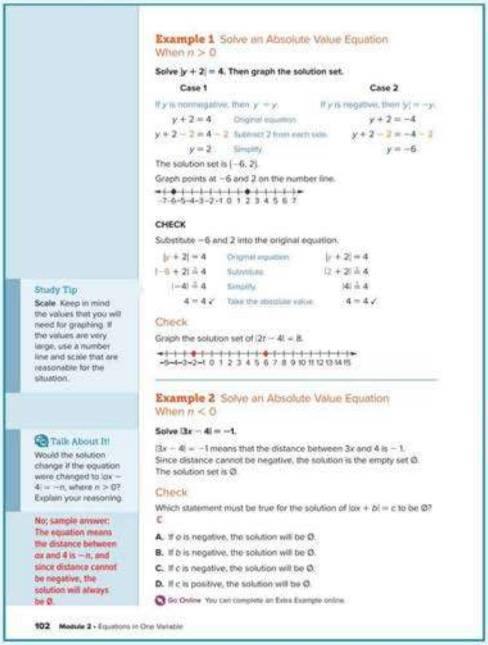
Learn

TAP

Students tap on each marker to learn about absolute value.

TYPE

Students answer questions to show they understand if the solution presented is correct.



Example 1 Solve an Absolute Value Equation When $n > 0$

Solve $|y + 2| = 4$. Then graph the solution set.

Case 1		Case 2
If y is nonnegative, then $\lvert y\rvert = y$.		If y is negative, then $\lvert y\rvert = -y$.
$y + 2 = 4$	Original equation	$y + 2 = -4$
$y + 2 - 2 = 4 - 2$	Subtract 2 from each side.	$y + 2 - 2 = -4 - 2$
$y = 2$	Simplify.	$y = -6$

The solution set is $\{-6, 2\}$.

Graph points at -6 and 2 on the number line.

CHECK

Substitute -6 and 2 into the original equation.

$\lvert y + 2\rvert = 4$	Original equation	$\lvert y + 2\rvert = 4$
$\lvert -6 + 2\rvert \stackrel{?}{=} 4$	Substitute.	$\lvert 2 + 2\rvert \stackrel{?}{=} 4$
$\lvert -4\rvert \stackrel{?}{=} 4$	Simplify.	$\lvert 4\rvert \stackrel{?}{=} 4$
$4 = 4$ ✓	Take the absolute value.	$4 = 4$ ✓

Check

Graph the solution set of $|2r - 4| = 8$.

Study Tip

Scale Keep in mind the values that you will need for graphing. If the values are very large, use a number line and scale that are reasonable for the situation.

Example 2 Solve an Absolute Value Equation When $n < 0$

Solve $|3x - 4| = -1$.

$|3x - 4| = -1$ means that the distance between $3x$ and 4 is -1. Since distance cannot be negative, the solution is the empty set $\varnothing$. The solution set is $\varnothing$.

Check

Which statement must be true for the solution of $|ax + b| = c$ to be $\varnothing$? C

A. If a is negative, the solution will be $\varnothing$.

B. If b is negative, the solution will be $\varnothing$.

C. If c is negative, the solution will be $\varnothing$.

D. If c is positive, the solution will be $\varnothing$.

Go Online You can complete an Extra Example online.

Talk About It!

Would the solution change if the equation were changed to $|ax - 4| = -n$, where $n > 0$? Explain your reasoning.

No; sample answer: The equation means the distance between ax and 4 is $-n$, and since distance cannot be negative, the solution will always be $\varnothing$.

102 Module 2 • Equations in One Variable

Interactive Presentation

Example 1

Example 1 Solve an Absolute Value Equation When $n > 0$

Teaching the Mathematical Practices

4 Analyze Cases This example asks students to examine the two cases when solving an absolute value equation. Encourage students to familiarize themselves with all of the cases.

Questions for Mathematical Discourse

AL In **Case 1,** when $y + 2 = 4$, do you expect y to be positive or negative? Explain. positive; sample answer: I know that $2 + 2 = 4$.

OL In **Case 2,** when $y + 2 = -4$, do you expect y to be positive or negative? Explain. negative; sample answer: When adding a positive number with a negative result, the starting number must be negative and have a greater absolute value.

BL Explain why both solutions work, even though one solution is negative. Because the absolute value expression is equal to a positive number, both values of y will work in the equation.

Example 2 Solve an Absolute Value Equation When $n < 0$

Teaching the Mathematical Practices

7 Interpret Complicated Expressions Mathematically proficient students can see complicated expressions as single objects or as being composed of several objects. In this example, guide students to see what information they can gather about the expression just from looking at it.

Questions for Mathematical Discourse

AL What does the absolute value of a number represent? the distance between a number and zero on the number line

OL Why should the two cases not be considered for the equation? The absolute value is always positive, so there is no solution.

BL Why can absolute value not equal a negative number? Sample answer: Because absolute value represents a distance, it cannot be negative because there is no such thing as a negative distance.

Common Error

Students may not pay attention when an equation involving absolute value is set equal to a negative number, and solve the equation as usual. Remind students to always check the given equation before solving.

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Solve an Absolute Value Equation

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL What is meant by the phrase *give or take 250 songs*? Sample answer: The number of songs on her phone will range from 250 songs less than 2000 up to 250 songs more than 2000.

OL How would you describe the situation in terms of distance on a number line? Sample answer: We are looking for values that are 250 units from 2000.

BL Why does this situation model an absolute value equation? Sample answer: The number of songs could be in the positive or negative direction from 2000 songs, which is an absolute value problem.

Common Error

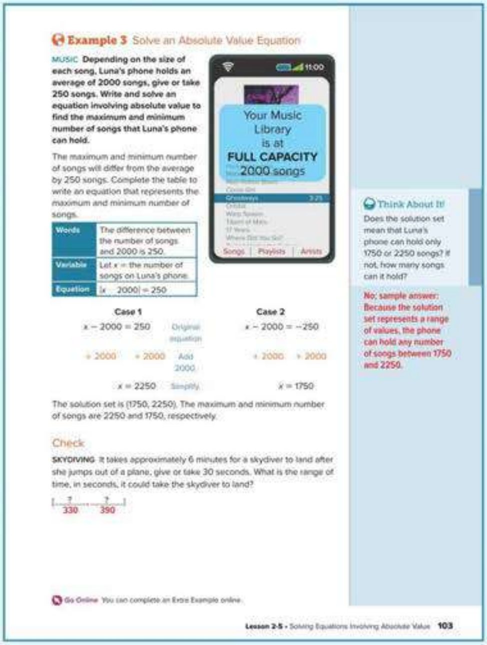
Students are used to writing equations without absolute value, so there is a good chance they will not write an equation involving absolute value. Remind students that when a scenario is within a certain amount, this is represented by absolute value.

Essential Question Follow-Up

Students have begun solving absolute value equations.

Ask:

Why is absolute value an important concept for real-world equations? Sample answer: Many quantities are within certain bounds, and absolute value equations allow real-world concepts to be bounded.



Example 3 Solve an Absolute Value Equation

MUSIC **Depending on the size of each song, Luna's phone holds an average of 2000 songs, give or take 250 songs. Write and solve an equation involving absolute value to find the maximum and minimum number of songs that Luna's phone can hold.**

The maximum and minimum number of songs will differ from the average by 250 songs. Complete the table to write an equation that represents the maximum and minimum number of songs.

Words	The difference between the number of songs and 2000 is 250.
Variable	Let x = the number of songs on Luna's phone.
Equation	$\lvert x - 2000\rvert = 250$

Case 1

$x - 2000 = 250$ Original equation

$+2000 \quad +2000$ Add 2000.

$x = 2250$ Simplify.

Case 2

$x - 2000 = -250$

$+2000 \quad +2000$

$x = 1750$

The solution set is {1750, 2250}. The maximum and minimum number of songs are 2250 and 1750, respectively.

Think About It!

Does the solution set mean that Luna's phone can hold only 1750 or 2250 songs? If not, how many songs can it hold?

No; sample answer: Because the solution set represents a range of values, the phone can hold any number of songs between 1750 and 2250.

Check

SKYDIVING It takes approximately 6 minutes for a skydiver to land after she jumps out of a plane, give or take 30 seconds. What is the range of time, in seconds, it could take the skydiver to land?

[? 330 , ? 390]

Go Online You can complete an Extra Example online.

Lesson 2-5 • Solving Equations Involving Absolute Value 103

Interactive Presentation

MUSIC Depending on the size of each song, Luna's phone holds an average of 2000 songs, give or take 250 songs. Write and solve an equation involving absolute value to find the maximum and minimum number of songs that Luna's phone can hold.

Example 3

TYPE

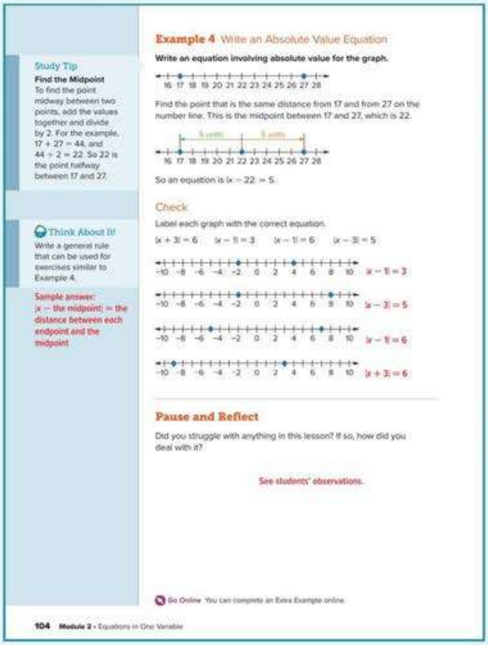
Students complete a sentence by entering the correct word and complete an equation by entering the correct number.

TYPE

Students answer a question to show they understand the solution set.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 4 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

Find the point that is the same distance from 17 and from 27 on the number line. This is the midpoint between 17 and 27, which is 22.

So an equation is $|x - 22| = 5$.

Check

Label each graph with the correct equation.

$|x + 3| = 6$ $|x - 1| = 3$ $|x - 1| = 6$ $|x - 3| = 5$

$|x - 1| = 3$

$|x - 3| = 5$

$|x - 1| = 6$

$|x + 3| = 6$

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

Study Tip

Find the Midpoint To find the point midway between two points, add the values together and divide by 2. For the example, $17 + 27 = 44$, and $44 \div 2 = 22$. So 22 is the point halfway between 17 and 27.

Think About It!

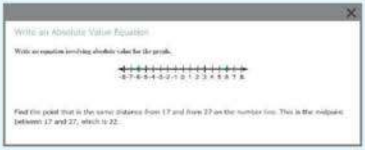
Write a general rule that can be used for exercises similar to Example 4.

Sample answer: |x − the midpoint| = the distance between each endpoint and the midpoint

Go Online You can complete an Extra Example online.

104 Module 2 • Equations in One Variable

Interactive Presentation



Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

Find the point that is the same distance from 17 and from 27 on the number line. This is the midpoint between 17 and 27, which is 22.

Example 4

TYPE

Students complete calculations to write an equation involving absolute value given a graph.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Write an Absolute Value Equation

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How can you use the number line to find the value in the middle of the two points? Sample answer: I can start with a finger at each point and move both in one point at a time until they meet.

OL How is the distance between the points related to the value on the right side of the equation? Sample answer: The distance between the points, 10, is twice the value on the right side of the equation, 5.

BL How do the numbers in the equation relate to the number line? Sample answer: The number being subtracted from *x* is the midpoint between the two plotted points on the number line. The number on the right side of the equation is the distance from each plotted point to the midpoint.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–31
2	exercises that use a variety of skills from this lesson	32–43
2	exercises that extend concepts learned in this lesson to new contexts	44–48
3	exercises that emphasize higher-order and critical-thinking skills	49–55

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–45 odd, 47–55
- Extension: Solving Absolute Value Equations with Variables on Both Sides
- **ALEKS** Absolute Value Equations

OL

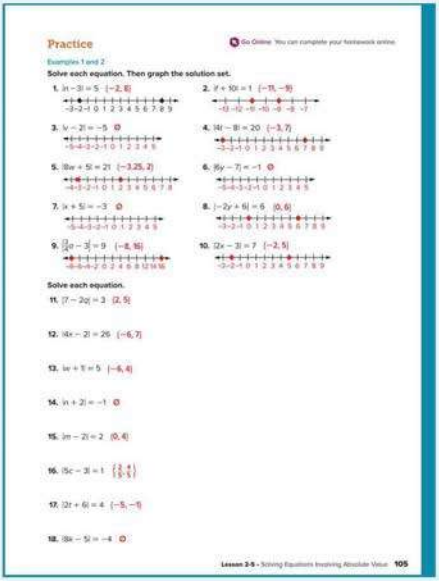
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–55 odd
- Remediation, Review Resources: Integers: Opposites and Absolute Value
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** Plotting and Comparing Signed Numbers

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–31 odd
- Remediation, Review Resources: Integers: Opposites and Absolute Value
- *Quick Review Math Handbook*: Solving Equations Involving Absolute Value
- Arrive**MATH** Take Another Look
- **ALEKS** Plotting and Comparing Signed Numbers



Practice

Go Online You can complete your homework online.

Examples 1 and 2

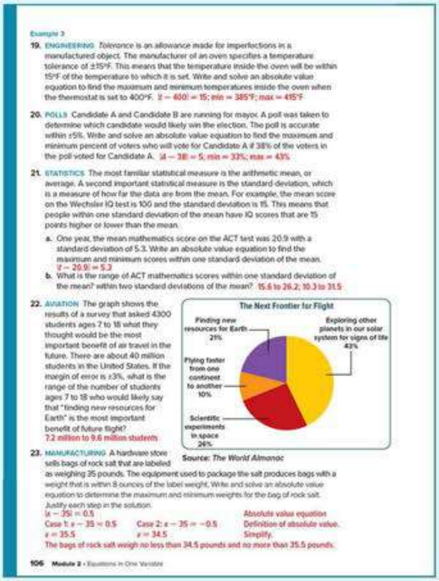
Solve each equation. Then graph the solution set.

1. $|n - 3| = 5$ $\{-2, 8\}$
2. $|f + 10| = 1$ $\{-11, -9\}$
3. $|v - 2| = -5$ $\emptyset$
4. $|4t - 8| = 20$ $\{-3, 7\}$
5. $|8w + 5| = 21$ $\{-3.25, 2\}$
6. $|6y - 7| = -1$ $\emptyset$
7. $|x + 5| = -3$ $\emptyset$
8. $|-2y + 6| = 6$ $\{0, 6\}$
9. $\left|\frac{3}{4}a - 3\right| = 9$ $\{-8, 16\}$
10. $|2x - 3| = 7$ $\{-2, 5\}$

Solve each equation.

11. $|7 - 2q| = 3$ $\{2, 5\}$
12. $|4x - 2| = 26$ $\{-6, 7\}$
13. $|w + 1| = 5$ $\{-6, 4\}$
14. $|n + 2| = -1$ $\emptyset$
15. $|m - 2| = 2$ $\{0, 4\}$
16. $|5c - 3| = 1$ $\left\{\frac{2}{5}, \frac{4}{5}\right\}$
17. $|2t + 6| = 4$ $\{-5, -1\}$
18. $|8x - 5| = -4$ $\emptyset$

Lesson 2-5 • Solving Equations Involving Absolute Value **105**



Example 3

19. ENGINEERING *Tolerance* is an allowance made for imperfections in a manufactured object. The manufacturer of an oven specifies a temperature tolerance of ±15°F. This means that the temperature inside the oven will be within 15°F of the temperature to which it is set. Write and solve an absolute value equation to find the maximum and minimum temperatures inside the oven when the thermostat is set to 400°F. $|t - 400| = 15$; min = 385°F; max = 415°F

20. POLLS Candidate A and Candidate B are running for mayor. A poll was taken to determine which candidate would likely win the election. The poll is accurate within ±5%. Write and solve an absolute value equation to find the maximum and minimum percent of voters who will vote for Candidate A if 38% of the voters in the poll voted for Candidate A. $|a - 38| = 5$; min = 33%; max = 43%

21. STATISTICS The most familiar statistical measure is the arithmetic mean, or average. A second important statistical measure is the standard deviation, which is a measure of how far the data are from the mean. For example, the mean score on the Wechsler IQ test is 100 and the standard deviation is 15. This means that people within one standard deviation of the mean have IQ scores that are 15 points higher or lower than the mean.
    a. One year, the mean mathematics score on the ACT test was 20.9 with a standard deviation of 5.3. Write an absolute value equation to find the maximum and minimum scores within one standard deviation of the mean. $|t - 20.9| = 5.3$
    b. What is the range of ACT mathematics scores within one standard deviation of the mean? within two standard deviations of the mean? 15.6 to 26.2; 10.3 to 31.5

22. AVIATION The graph shows the results of a survey that asked 4300 students ages 7 to 18 what they thought would be the most important benefit of air travel in the future. There are about 40 million students in the United States. If the margin of error is ±3%, what is the range of the number of students ages 7 to 18 who would likely say that "finding new resources for Earth" is the most important benefit of future flight? 7.2 million to 9.6 million students

The Next Frontier for Flight

Finding new resources for Earth 21%

Exploring other planets in our solar system for signs of life 43%

Flying faster from one continent to another 10%

Scientific experiments in space 26%

Source: *The World Almanac*

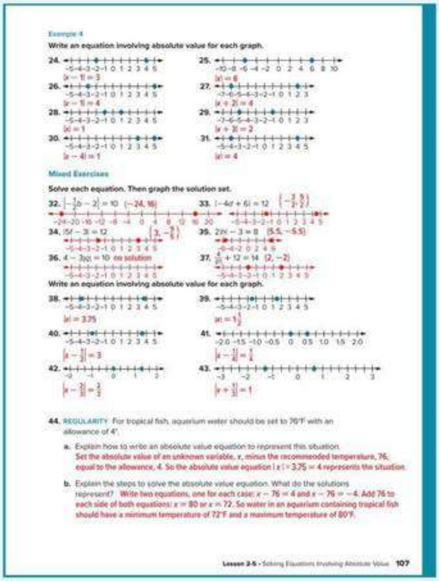
23. MANUFACTURING A hardware store sells bags of rock salt that are labeled as weighing 35 pounds. The equipment used to package the salt produces bags with a weight that is within 8 ounces of the label weight. Write and solve an absolute value equation to determine the maximum and minimum weights for the bag of rock salt. Justify each step in the solution.
    $|x - 35| = 0.5$ Absolute value equation
    Case 1: $x - 35 = 0.5$ Case 2: $x - 35 = -0.5$ Definition of absolute value.
    $x = 35.5$ $x = 34.5$ Simplify.
    The bags of rock salt weigh no less than 34.5 pounds and no more than 35.5 pounds.

106 **Module 2** • Equations in One Variable

3 REFLECT AND PRACTICE

A.CED.1, A.REI.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 4

Write an equation involving absolute value for each graph.

24. $|x - 1| = 3$

25. $|x| = 6$

26. $|x - 1| = 4$

27. $|x + 2| = 4$

28. $|x| = 1$

29. $|x + 3| = 2$

30. $|x - 4| = 1$

31. $|x| = 4$

Mixed Exercises

Solve each equation. Then graph the solution set.

32. $\left|-\frac{1}{2}b - 2\right| = 10$ $\{-24, 16\}$

33. $|-4d + 6| = 12$ $\left\{-\frac{3}{2}, \frac{9}{2}\right\}$

34. $|5f - 3| = 12$ $\left\{3, -\frac{9}{5}\right\}$

35. $2|x| - 3 = 8$ $\{5.5, -5.5\}$

36. $4 - 3|q| = 10$ no solution

37. $\frac{4}{|z|} + 12 = 14$ $\{2, -2\}$

Write an equation involving absolute value for each graph.

38. $|x| = 3.75$

39. $|x| = 1\frac{1}{2}$

40. $\left|x - \frac{1}{2}\right| = 3$

41. $\left|x - \frac{1}{4}\right| = \frac{1}{4}$

42. $\left|x - \frac{2}{3}\right| = \frac{2}{3}$

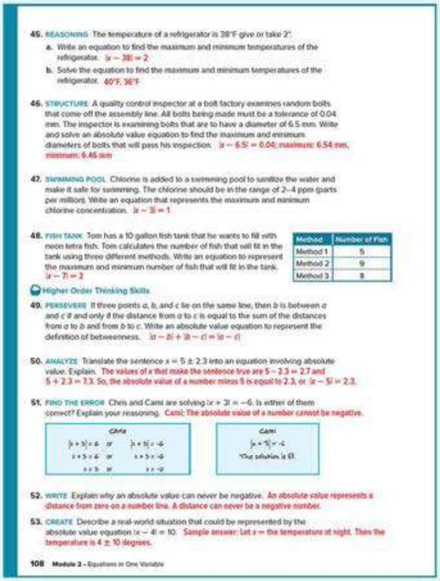
43. $\left|x + \frac{1}{3}\right| = 1$

44. REGULARITY For tropical fish, aquarium water should be set to 76°F with an allowance of 4°.

a. Explain how to write an absolute value equation to represent this situation. Set the absolute value of an unknown variable, x, minus the recommended temperature, 76, equal to the allowance, 4. So the absolute value equation $|x| - 3.75 = 4$ represents the situation.

b. Explain the steps to solve the absolute value equation. What do the solutions represent? Write two equations, one for each case: $x - 76 = 4$ and $x - 76 = -4$. Add 76 to each side of both equations: $x = 80$ or $x = 72$. So water in an aquarium containing tropical fish should have a minimum temperature of 72°F and a maximum temperature of 80°F.

Lesson 2-5 • Solving Equations Involving Absolute Value 107



45. REASONING The temperature of a refrigerator is 38°F give or take 2°.

a. Write an equation to find the maximum and minimum temperatures of the refrigerator. $|x - 38| = 2$

b. Solve the equation to find the maximum and minimum temperatures of the refrigerator. 40°F, 36°F

46. STRUCTURE A quality control inspector at a bolt factory examines random bolts that come off the assembly line. All bolts being made must be a tolerance of 0.04 mm. The inspector is examining bolts that are to have a diameter of 6.5 mm. Write and solve an absolute value equation to find the maximum and minimum diameters of bolts that will pass his inspection. $|x - 6.5| = 0.04$; maximum: 6.54 mm, minimum: 6.46 mm

47. SWIMMING POOL Chlorine is added to a swimming pool to sanitize the water and make it safe for swimming. The chlorine should be in the range of 2–4 ppm (parts per million). Write an equation that represents the maximum and minimum chlorine concentration. $|x - 3| = 1$

48. FISH TANK Tom has a 10 gallon fish tank that he wants to fill with neon tetra fish. Tom calculates the number of fish that will fit in the tank using three different methods. Write an equation to represent the maximum and minimum number of fish that will fit in the tank. $|x - 7| = 2$

Method	Number of Fish
Method 1	5
Method 2	9
Method 3	8

Higher Order Thinking Skills

49. PERSEVERE If three points a, b, and c lie on the same line, then b is between a and c if and only if the distance from a to c is equal to the sum of the distances from a to b and from b to c. Write an absolute value equation to represent the definition of betweenness. $|a - b| + |b - c| = |a - c|$

50. ANALYZE Translate the sentence $x = 5 \pm 2.3$ into an equation involving absolute value. Explain. The values of x that make the sentence true are $5 - 2.3 = 2.7$ and $5 + 2.3 = 7.3$. So, the absolute value of a number minus 5 is equal to 2.3, or $|x - 5| = 2.3$.

51. FIND THE ERROR Chris and Cami are solving $|x + 3| = -6$. Is either of them correct? Explain your reasoning. Cami; The absolute value of a number cannot be negative.

Chris

$|x + 3| = 6$ or $|x + 3| = -6$

$x + 3 = 6$ or $x + 3 = -6$

$x = 3$ or $x = -9$

Cami

$|x + 3| = -6$

The solution is ∅.

52. WRITE Explain why an absolute value can never be negative. An absolute value represents a distance from zero on a number line. A distance can never be a negative number.

53. CREATE Describe a real-world situation that could be represented by the absolute value equation $|x - 4| = 10$. Sample answer: Let x = the temperature at night. Then the temperature is 4 ± 10 degrees.

108 Module 2 • Equations in One Variable

Lesson 2-6

Solving Proportions

A.CED.1, A.REI.3

LESSON GOAL

Students solve equations involving proportions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Comparing Two Quantities

Develop:

Solving Proportions

- Solve a Proportion
- Solve a Proportion with Two Missing Quantities
- Solve a Proportion by Using a Constant Rate
- Solve a Percent Problem by Using a Proportion

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Equivalent Ratios and Rates	●	●		●
Extension: Scale Models		●	●	●

Language Development Handbook

Assign page 12 of the *Language Development Handbook* to help your students build mathematical language related to solving proportions.

ELL You can use the tips and suggestions on page T12 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students solved equations involving absolute value.
A.CED.1, A.REI.3

Now
Students solve equations involving proportions.
A.CED.1, A.REI.3

Next
Students will use a process of reasoning to rearrange formulas to highlight a quantity of interest.
A.CED.4

Rigor

The Three Pillars of Rigor

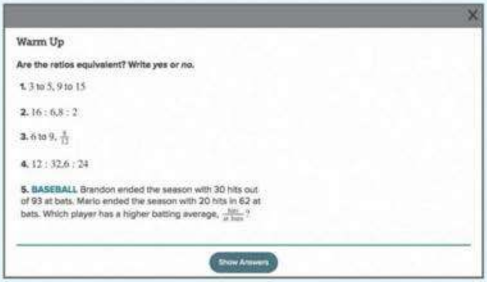
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of solving equations with the variable on each side to build fluency with solving proportions. They apply their understanding of solving proportions by solving real-world problems.

Mathematical Background

A ratio is a comparison of two numbers by division. A ratio is called a rate if the two numbers of a ratio represent measurements with different units, such as miles and hours. A proportion is an equation stating that two ratios are equal. Proportions are useful in finding missing values in a ratio relationship.

Interactive Presentation

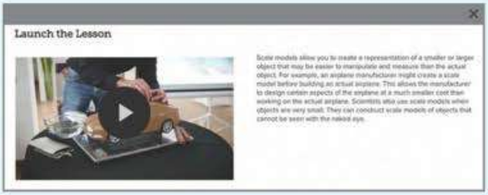


Warm Up

Are the ratios equivalent? Write *yes* or *no*.

1. 3 to 5, 9 to 15
2. 16 : 6.8 : 2
3. 6 to 9, [illegible]
4. 12 : 32.6 : 24
5. BASEBALL Brandon ended the season with 30 hits out of 93 at bats. Mario ended the season with 20 hits in 62 at bats. Which player has a higher batting average, $\frac{\text{hits}}{\text{at bats}}$?

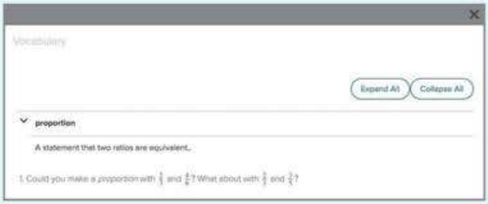
Warm Up



Launch the Lesson

Scale models allow you to create a representation of a smaller or larger object that may be easier to manipulate and measure than the actual object. For example, an airplane manufacturer might create a scale model before building an actual airplane. This allows the manufacturer to design certain aspects of the airplane at a much smaller cost than working on the actual airplane. Scientists also use scale models when objects are very small. They can conduct scale models of objects that cannot be seen with the naked eye.

Launch the Lesson



Vocabulary

proportion

A statement that two ratios are equivalent.

1. Could you make a proportion with [illegible] and [illegible]? What about with [illegible] and [illegible]?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- determining whether two ratios are equivalent

Answers:

1. yes
2. no
3. yes
4. no
5. They are the same because the ratios are equivalent.

Launch the Lesson

MP Teaching the Mathematical Practices

2 Make Sense of Quantities Mathematically proficient students need to be able to make sense of quantities and their relationships while considering units. Encourage students to consider how units are important in scale models.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the question below with the class.

Explore Comparing Two Quantities

Objective

Students use a sketch to explore how to compare two quantities using ratios and proportions.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

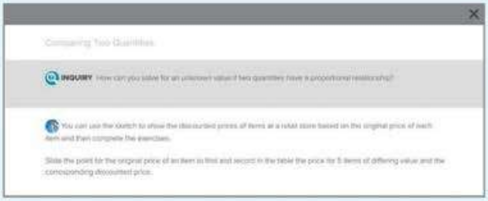
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to find five pairs of original and discounted prices. The guiding exercises will help students understand ratios and how proportions can be used to solve for missing variables. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



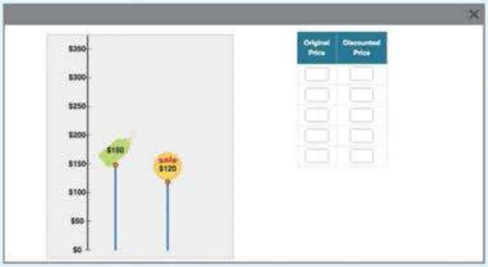
Comparing Two Quantities

INQUIRY How can you solve for an unknown value if two quantities have a proportional relationship?

You can use the sketch to show the discounted prices of items at a retail store based on the original price of each item and then complete the exercises.

Slide the point for the original price of an item to find and record in the table the price for 5 items of differing value and the corresponding discounted price.

Explore



Explore

WEB SKETCHPAD

Students use a sketch to complete an activity in which they explore a proportion.

TYPE

Students answer a series of questions to show they understand the concept of ratios and proportions.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Comparing Two Quantities *(continued)*

Questions

Have students complete the Explore activity.

Ask:

- Why is the ratio of the discounted price to the original price a fraction less than 1? Sample answer: The discounted price is less than the original amount, so the ratio should be less than 1. If the ratio was greater than 1, then the price would be going up instead.
- How could you use the ratio $\frac{4}{5}$ and a sale price of $36 to find the original price? Sample answer: Set up a ratio with 36 over the unknown price, then set that equal to $\frac{4}{5}$ and solve.

Inquiry

How can you solve for an unknown value if two quantities have a proportional relationship? Sample answer: Write a ratio relating the two quantities using a variable for the unknown value. Then set the ratio equal to the original ratio and solve.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Proportions

Objective

Students solve proportions by using the Distributive Property.

Common Error

When setting up proportions, students overlook that the ratios must be equal. The units of each ratio must match in order to set them equal. Reinforce during examples that the units of each ratio are in the same position.

Example 1 Solve a Proportion

MP Teaching the Mathematical Practices

6 Use Precision In this example, students learn how to calculate accurately and efficiently and to express numerical answers with a degree of precision appropriate to the problem context.

Questions for Mathematical Discourse

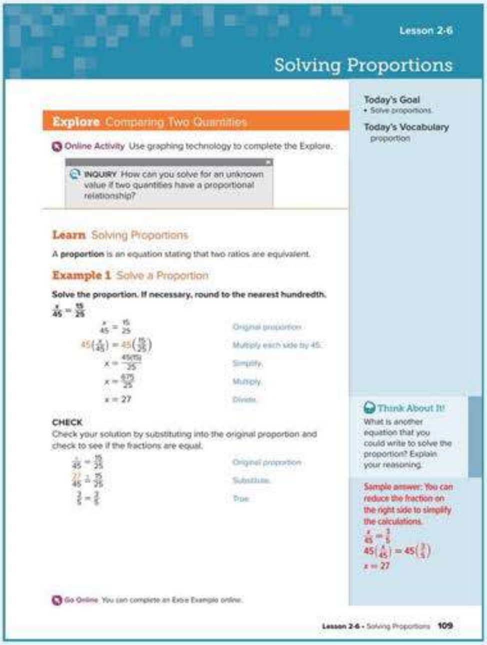
AL What does the term *proportion* tell us about these two fractions? The two fractions are equal to each other.

OL Could we have made the calculations easier to do? Explain. Yes; sample answer: We could have simplified $\frac{15}{25}$ to $\frac{3}{5}$. When multiplying $\frac{3}{5}$ by 45, we could simplify $\frac{45}{5}$ to be 9 and then multiply by 3. The result would be the same.

BL Would the solution be different if the proportion was $\frac{45}{x} = \frac{25}{15}$? Explain. No; sample answer: Both fractions have been flipped. To solve, we would need to multiply by x first, then multiply by $\frac{15}{25}$. The solution would still be $x = 27$.

Go Online

- F ind additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-6

Solving Proportions

Today's Goal
- Solve proportions.

Today's Vocabulary
proportion

Explore Comparing Two Quantities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you solve for an unknown value if two quantities have a proportional relationship?

Learn Solving Proportions

A **proportion** is an equation stating that two ratios are equivalent.

Example 1 Solve a Proportion

Solve the proportion. If necessary, round to the nearest hundredth.

$\frac{x}{45} = \frac{15}{25}$

$\frac{x}{45} = \frac{15}{25}$ Original proportion

$45\left(\frac{x}{45}\right) = 45\left(\frac{15}{25}\right)$ Multiply each side by 45.

$x = \frac{45(15)}{25}$ Simplify.

$x = \frac{675}{25}$ Multiply.

$x = 27$ Divide.

CHECK

Check your solution by substituting into the original proportion and check to see if the fractions are equal.

$\frac{x}{45} = \frac{15}{25}$ Original proportion

$\frac{27}{45} \stackrel{?}{=} \frac{15}{25}$ Substitute.

$\frac{3}{5} = \frac{3}{5}$ True.

Think About It! What is another equation that you could write to solve the proportion? Explain your reasoning.

Sample answer: You can reduce the fraction on the right side to simplify the calculations.

$\frac{x}{45} = \frac{3}{5}$

$45\left(\frac{x}{45}\right) = 45\left(\frac{3}{5}\right)$

$x = 27$

Go Online You can complete an Extra Example online.

Lesson 2-6 • Solving Proportions 109

Interactive Presentation

Solving Proportions

Key Concept Cross Products Property of Proportions

Words: The cross products of a proportion are equal.

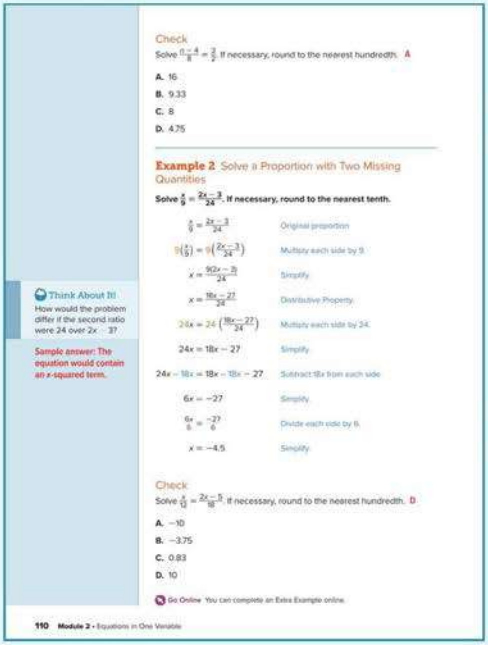
Learn

DRAG & DROP

Students drag and drop numbers to complete an equation.

TYPE

Students answer a question to show they understand that writing a proportion differently yields the same solution.



Check

Solve $\frac{n-4}{8} = \frac{3}{2}$. If necessary, round to the nearest hundredth. A

A. 16
B. 9.33
C. 8
D. 4.75

Example 2 Solve a Proportion with Two Missing Quantities

Solve $\frac{x}{9} = \frac{2x-3}{24}$. If necessary, round to the nearest tenth.

$$\frac{x}{9} = \frac{2x-3}{24} \quad \text{Original proportion}$$
$$9\left(\frac{x}{9}\right) = 9\left(\frac{2x-3}{24}\right) \quad \text{Multiply each side by 9.}$$
$$x = \frac{9(2x-3)}{24} \quad \text{Simplify.}$$
$$x = \frac{18x-27}{24} \quad \text{Distributive Property}$$
$$24x = 24\left(\frac{18x-27}{24}\right) \quad \text{Multiply each side by 24.}$$
$$24x = 18x - 27 \quad \text{Simplify.}$$
$$24x - 18x = 18x - 18x - 27 \quad \text{Subtract 18x from each side.}$$
$$6x = -27 \quad \text{Simplify.}$$
$$\frac{6x}{6} = \frac{-27}{6} \quad \text{Divide each side by 6.}$$
$$x = -4.5 \quad \text{Simplify.}$$

Think About It!
How would the problem differ if the second ratio were 24 over $2x - 3$?

Sample answer: The equation would contain an x-squared term.

Check

Solve $\frac{x}{12} = \frac{2x-5}{18}$. If necessary, round to the nearest hundredth. D

A. −10
B. −3.75
C. 0.83
D. 10

Go Online You can complete an Extra Example online.

110 **Module 2** • Equations in One Variable

Interactive Presentation

Example 2

TYPE

Students answer a question to show they understand how the problem would differ if one ratio were reversed.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Solve a Proportion with Two Missing Quantities

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to justify their conclusions.

Questions for Mathematical Discourse

AL Which numerator is a binomial? $2x - 3$ What property do we need to use when we multiply by a binomial? the Distributive Property

OL Describe another way to solve the proportion. Sample answer: Multiply each side by 24 to eliminate the denominator on the right side of the equation first. Then solve the resulting equation.

BL Write the proportion in a different way that will still yield the same solution. Sample answer: $\frac{9}{x} = \frac{24}{2x-3}$

Common Error

With the introduction of new concepts, students tend to forget previously learned material. Students know how to solve an equation with the variable on each side, however they may temporarily forget while solving proportions. Remind students to use properties of equality to solve the equation.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are not mastering the concept of proportions,
THEN place students in small groups to work through the Check problems. Have one student per group report back on the group's progress and any areas where they may need assistance.

Enrichment Activity BL

A school holds a 24-hour dance-a-thon to raise money for a local charity. During the event, participants and spectators can purchase raffle tickets as part of the fundraiser. Every 45 minutes a ticket is selected and a winner called. How many winning tickets will be called during the dance-a-thon? Show your work.

24 hours = 1440 minutes
$$\frac{x}{1440} = \frac{1}{45}$$
$$45x = 1440$$
$$x = 32$$
There will be 32 winning tickets drawn.

Example 3 Solve a Proportion by Using a Constant Rate

Teaching the Mathematical Practices

5 Use Estimation Point out that in this example, students need to include an estimate and check against the estimate at the end.

Questions for Mathematical Discourse

AL Real-world problems involve units. What units are used in this scenario? centimeters and years

OL Step 3 shows us a proportion in which centimeters are in the numerator, years are in the denominator, the known rate is on the left, and the unknown rate is on the right. How else could you set up the proportion? Sample answer: centimeters on the left, years on the right, the known rate on top, and the unknown rate on the bottom

BL Why can you not put centimeters over years in one fraction and years over centimeters in the other? Sample answer: A proportion is made by comparing two fractions. In order to compare these, you need to set them up using the same method. So, either both sides have centimeters over years, or both sides have years over centimeters.

Common Error

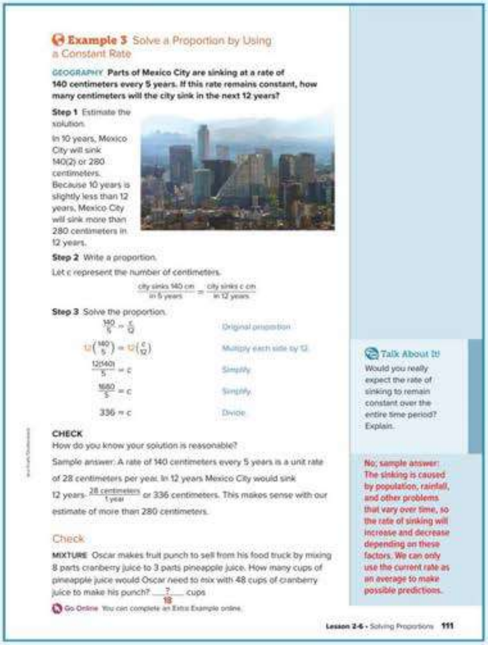
When students set up the rate ratio, the second ratio must be in the same unit order. Have students write units with each number as a visual check.

Essential Question Follow-Up

Students have begun to set up and solve proportions.

Ask:

Why is it important to set up and solve proportions in the real world? Sample answer: Many real-life situations are proportional, such as gallons of gasoline used for a number of miles, the price of apples for the weight, and the number of girls in a group of people. Being able to set up and solve a proportion allows us to find missing quantities.



Example 3 Solve a Proportion by Using a Constant Rate

GEOGRAPHY **Parts of Mexico City are sinking at a rate of 140 centimeters every 5 years. If this rate remains constant, how many centimeters will the city sink in the next 12 years?**

Step 1 Estimate the solution.

In 10 years, Mexico City will sink 140(2) or 280 centimeters. Because 10 years is slightly less than 12 years, Mexico City will sink more than 280 centimeters in 12 years.

Step 2 Write a proportion.

Let c represent the number of centimeters.

$$\frac{\text{city sinks 140 cm}}{\text{in 5 years}} = \frac{\text{city sinks } c \text{ cm}}{\text{in 12 years}}$$

Step 3 Solve the proportion.

$\frac{140}{5} = \frac{c}{12}$ Original proportion

$12\left(\frac{140}{5}\right) = 12\left(\frac{c}{12}\right)$ Multiply each side by 12.

$\frac{12(140)}{5} = c$ Simplify.

$\frac{1680}{5} = c$ Simplify.

$336 = c$ Divide.

CHECK

How do you know your solution is reasonable?

Sample answer: A rate of 140 centimeters every 5 years is a unit rate of 28 centimeters per year. In 12 years Mexico City would sink 12 years $\cdot \frac{28 \text{ centimeters}}{1 \text{ year}}$ or 336 centimeters. This makes sense with our estimate of more than 280 centimeters.

Check

MIXTURE Oscar makes fruit punch to sell from his food truck by mixing 8 parts cranberry juice to 3 parts pineapple juice. How many cups of pineapple juice would Oscar need to mix with 48 cups of cranberry juice to make his punch? $\underset{18}{\underline{\ \ ?\ \ }}$ cups

Go Online You can complete an Extra Example online.

Talk About It!

Would you really expect the rate of sinking to remain constant over the entire time period? Explain.

No; sample answer: The sinking is caused by population, rainfall, and other problems that vary over time, so the rate of sinking will increase and decrease depending on these factors. We can only use the current rate as an average to make possible predictions.

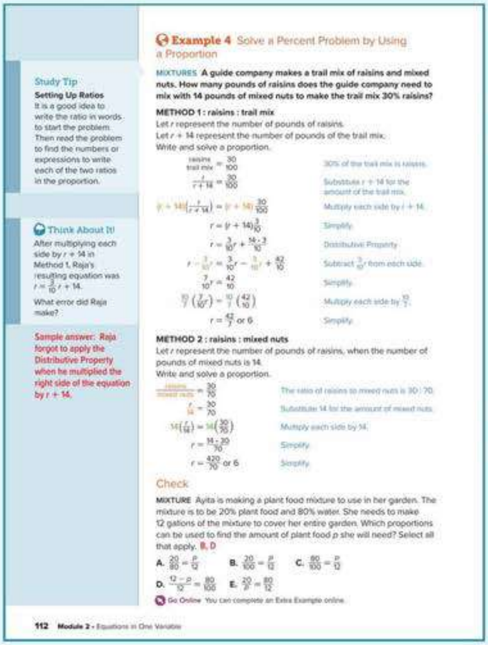
Lesson 2-6 • Solving Proportions **111**

Interactive Presentation

Example 3

TYPE

Students answer a question to show they understand the reasonableness of the problem's claim.



Study Tip

Setting Up Ratios It is a good idea to write the ratio in words to start the problem. Then read the problem to find the numbers or expressions to write each of the two ratios in the proportion.

Think About It!

After multiplying each side by $r + 14$ in Method 1, Raja's resulting equation was $r = \frac{3}{10}r + 14$.

What error did Raja make?

Sample answer: Raja forgot to apply the Distributive Property when he multiplied the right side of the equation by $r + 14$.

Example 4 Solve a Percent Problem by Using a Proportion

MIXTURES **A guide company makes a trail mix of raisins and mixed nuts. How many pounds of raisins does the guide company need to mix with 14 pounds of mixed nuts to make the trail mix 30% raisins?**

METHOD 1 : raisins : trail mix

Let r represent the number of pounds of raisins.

Let $r + 14$ represent the number of pounds of the trail mix.

Write and solve a proportion.

$\frac{\text{raisins}}{\text{trail mix}} = \frac{30}{100}$ — 30% of the trail mix is raisins.

$\frac{r}{r+14} = \frac{30}{100}$ — Substitute $r + 14$ for the amount of the trail mix.

$(r+14)\left(\frac{r}{r+14}\right) = (r+14)\frac{30}{100}$ — Multiply each side by $r + 14$.

$r = (r+14)\frac{3}{10}$ — Simplify.

$r = \frac{3}{10}r + \frac{14 \cdot 3}{10}$ — Distributive Property

$r - \frac{3}{10}r = \frac{3}{10}r - \frac{3}{10}r + \frac{42}{10}$ — Subtract $\frac{3}{10}r$ from each side.

$\frac{7}{10}r = \frac{42}{10}$ — Simplify.

$\frac{10}{7}\left(\frac{7}{10}r\right) = \frac{10}{7}\left(\frac{42}{10}\right)$ — Multiply each side by $\frac{10}{7}$.

$r = \frac{42}{7}$ or 6 — Simplify.

METHOD 2 : raisins : mixed nuts

Let r represent the number of pounds of raisins, when the number of pounds of mixed nuts is 14.

Write and solve a proportion.

$\frac{\text{raisins}}{\text{mixed nuts}} = \frac{30}{70}$ — The ratio of raisins to mixed nuts is 30 : 70.

$\frac{r}{14} = \frac{30}{70}$ — Substitute 14 for the amount of mixed nuts.

$14\left(\frac{r}{14}\right) = 14\left(\frac{30}{70}\right)$ — Multiply each side by 14.

$r = \frac{14 \cdot 30}{70}$ — Simplify.

$r = \frac{420}{70}$ or 6 — Simplify.

Check

MIXTURE Ayita is making a plant food mixture to use in her garden. The mixture is to be 20% plant food and 80% water. She needs to make 12 gallons of the mixture to cover her entire garden. Which proportions can be used to find the amount of plant food p she will need? Select all that apply. B, D

A. $\frac{20}{80} = \frac{p}{12}$

B. $\frac{20}{100} = \frac{p}{12}$

C. $\frac{80}{100} = \frac{p}{12}$

D. $\frac{12-p}{12} = \frac{80}{100}$

E. $\frac{20}{p} = \frac{80}{12}$

Go Online You can complete an Extra Example online.

112 Module 2 • Equations in One Variable

Interactive Presentation

Solve a Percent Problem by Using a Proportion

MIXTURES A guide company makes a trail mix of raisins and mixed nuts. How many pounds of raisins does the guide company need to mix with 14 pounds of mixed nuts to make the trail mix 30% raisins?

Example 4

SELECT

Students select a method to graph the data.

TYPE

Students answer a question to show they understand the error made when the proportion is solved.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Solve a Percent Problem by Using a Proportion

Teaching the Mathematical Practices

3 Find the Error This example requires students to read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Questions for Mathematical Discourse

AL If 30% of the trail mix is raisins, what percentage of the trail mix is mixed nuts? 70%

OL Which method do you prefer? Why? Sample answer: I prefer Method 2 because I do not have to distribute.

BL Suppose the trail mix was to be 55% raisins. How would the proportions change?
The proportions would be $\frac{r}{r+14} = \frac{55}{100}$ and $\frac{r}{14} = \frac{55}{45}$.

Common Error

Proportions do not always have just one variable; there are times the unknown value is used twice. Encourage students to underline or highlight the relationship of the given information before setting up the proportion.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–48
2	exercises that use a variety of skills from this lesson	49–69
2	exercises that extend concepts learned in this lesson to new contexts	70–74
3	exercises that emphasize higher-order and critical-thinking skills	75–80

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

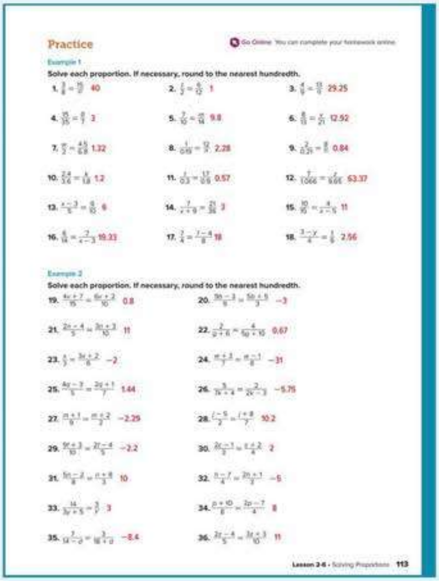
- Practice, Exercises 1–73 odd, 74–80
- Extension: Scale Models
- **ALEKS** Proportions

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–79 odd
- Remediation, Review Resources: Equivalent Ratios and Rates
- BrainPOP Video: Using Proportions
- Extra Examples 1–4
- **ALEKS** Ratios and Unit Rates

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–47 odd
- Remediation, Review Resources: Equivalent Ratios and Rates
- ArriveMATH Take Another Look
- **ALEKS** Ratios and Unit Rates



Practice

Go Online You can complete your homework online.

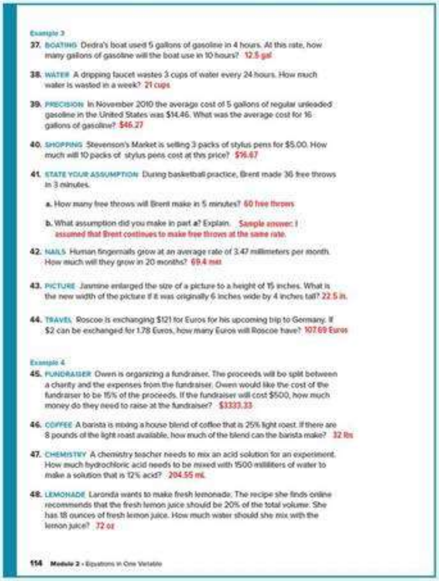
Example 1

Solve each proportion. If necessary, round to the nearest hundredth.

Example 2

Solve each proportion. If necessary, round to the nearest hundredth.

Lesson 2-6 • Solving Proportions 113



Example 3

37. BOATING Dedra's boat used 5 gallons of gasoline in 4 hours. At this rate, how many gallons of gasoline will the boat use in 10 hours? 12.5 gal

38. WATER A dripping faucet wastes 3 cups of water every 24 hours. How much water is wasted in a week? 21 cups

39. PRECISION In November 2010 the average cost of 5 gallons of regular unleaded gasoline in the United States was $14.46. What was the average cost for 16 gallons of gasoline? $46.27

40. SHOPPING Stevenson's Market is selling 3 packs of stylus pens for $5.00. How much will 10 packs of stylus pens cost at this price? $16.67

41. STATE YOUR ASSUMPTION During basketball practice, Brent made 36 free throws in 3 minutes.
    a. How many free throws will Brent make in 5 minutes? 60 free throws
    b. What assumption did you make in part a? Explain. Sample answer: I assumed that Brent continues to make free throws at the same rate.

42. NAILS Human fingernails grow at an average rate of 3.47 millimeters per month. How much will they grow in 20 months? 69.4 mm

43. PICTURE Jasmine enlarged the size of a picture to a height of 15 inches. What is the new width of the picture if it was originally 6 inches wide by 4 inches tall? 22.5 in.

44. TRAVEL Roscoe is exchanging $121 for Euros for his upcoming trip to Germany. If $2 can be exchanged for 1.78 Euros, how many Euros will Roscoe have? 107.69 Euros

Example 4

45. FUNDRAISER Owen is organizing a fundraiser. The proceeds will be split between a charity and the expenses from the fundraiser. Owen would like the cost of the fundraiser to be 15% of the proceeds. If the fundraiser will cost $500, how much money do they need to raise at the fundraiser? $3333.33

46. COFFEE A barista is mixing a house blend of coffee that is 25% light roast. If there are 8 pounds of the light roast available, how much of the blend can the barista make? 32 lbs

47. CHEMISTRY A chemistry teacher needs to mix an acid solution for an experiment. How much hydrochloric acid needs to be mixed with 1500 milliliters of water to make a solution that is 12% acid? 204.55 mL

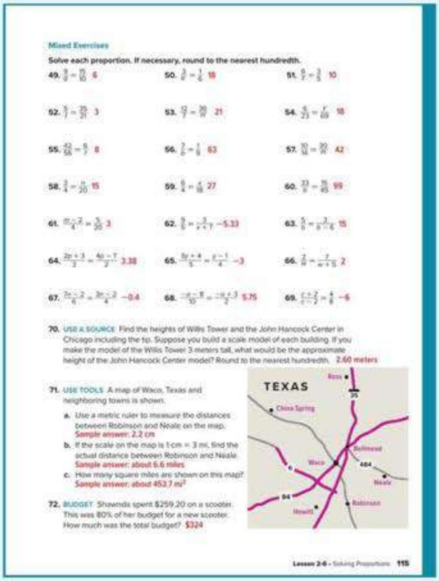
48. LEMONADE Laronda wants to make fresh lemonade. The recipe she finds online recommends that the fresh lemon juice should be 20% of the total volume. She has 18 ounces of fresh lemon juice. How much water should she mix with the lemon juice? 72 oz

114 Module 2 • Equations in One Variable

3 REFLECT AND PRACTICE

A.CED.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Mixed Exercises

Solve each proportion. If necessary, round to the nearest hundredth.

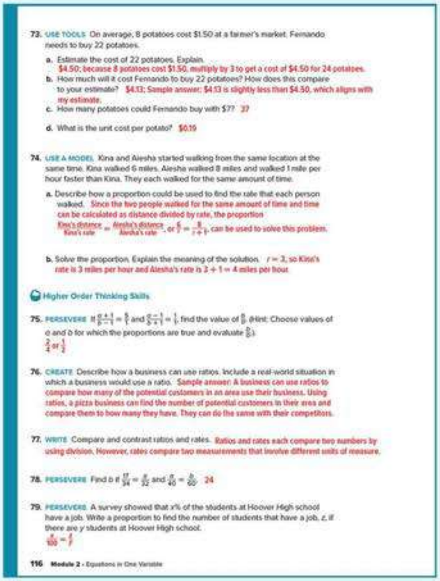
49. [illegible] 6
50. [illegible] 18
51. [illegible] 10
52. [illegible] 3
53. [illegible] 21
54. [illegible] 18
55. [illegible] 8
56. [illegible] 63
57. [illegible] 42
58. [illegible] 15
59. [illegible] 27
60. [illegible] 99
61. [illegible] 3
62. [illegible] −5.33
63. [illegible] 15
64. [illegible] 3.38
65. [illegible] −3
66. [illegible] 2
67. [illegible] −0.4
68. [illegible] 5.75
69. [illegible] −6

70. **USE A SOURCE** Find the heights of Willis Tower and the John Hancock Center in Chicago including the tip. Suppose you build a scale model of each building. If you make the model of the Willis Tower 3 meters tall, what would be the approximate height of the John Hancock Center model? Round to the nearest hundredth. 2.60 meters

71. **USE TOOLS** A map of Waco, Texas and neighboring towns is shown.
    a. Use a metric ruler to measure the distances between Robinson and Neale on the map. Sample answer: 2.2 cm
    b. If the scale on the map is 1 cm = 3 mi, find the actual distance between Robinson and Neale. Sample answer: about 6.6 miles
    c. How many square miles are shown on this map? Sample answer: about 453.7 mi²

72. **BUDGET** Shawnda spent $259.20 on a scooter. This was 80% of her budget for a new scooter. How much was the total budget? $324

Lesson 2-6 • Solving Proportions 115



73. **USE TOOLS** On average, 8 potatoes cost $1.50 at a farmer's market. Fernando needs to buy 22 potatoes.
    a. Estimate the cost of 22 potatoes. Explain. $4.50; because 8 potatoes cost $1.50, multiply by 3 to get a cost of $4.50 for 24 potatoes.
    b. How much will it cost Fernando to buy 22 potatoes? How does this compare to your estimate? $4.13; Sample answer: $4.13 is slightly less than $4.50, which aligns with my estimate.
    c. How many potatoes could Fernando buy with $7? 37
    d. What is the unit cost per potato? $0.19

74. **USE A MODEL** Kina and Alesha started walking from the same location at the same time. Kina walked 6 miles. Alesha walked 8 miles and walked 1 mile per hour faster than Kina. They each walked for the same amount of time.
    a. Describe how a proportion could be used to find the rate that each person walked. Since the two people walked for the same amount of time and time can be calculated as distance divided by rate, the proportion $\frac{\text{Kina's distance}}{\text{Kina's rate}} = \frac{\text{Alesha's distance}}{\text{Alesha's rate}}$, or $\frac{6}{r} = \frac{8}{r+1}$, can be used to solve this problem.
    b. Solve the proportion. Explain the meaning of the solution. $r = 3$, so Kina's rate is 3 miles per hour and Alesha's rate is $3 + 1 = 4$ miles per hour.

Higher Order Thinking Skills

75. **PERSEVERE** If [illegible] and [illegible], find the value of $\frac{a}{b}$. (Hint: Choose values of a and b for which the proportions are true and evaluate $\frac{a}{b}$.)
    $\frac{2}{4}$ or $\frac{1}{2}$

76. **CREATE** Describe how a business can use ratios. Include a real-world situation in which a business would use a ratio. Sample answer: A business can use ratios to compare how many of the potential customers in an area use their business. Using ratios, a pizza business can find the number of potential customers in their area and compare them to how many they have. They can do the same with their competitors.

77. **WRITE** Compare and contrast ratios and rates. Ratios and rates each compare two numbers by using division. However, rates compare two measurements that involve different units of measure.

78. **PERSEVERE** Find b if [illegible] and [illegible]. 24

79. **PERSEVERE** A survey showed that x% of the students at Hoover High school have a job. Write a proportion to find the number of students that have a job, z, if there are y students at Hoover High school.
    $\frac{x}{100} = \frac{z}{y}$

116 Module 2 • Equations in One Variable

Lesson 2-7

Using Formulas

N.Q.1, A.CED.3, A.CED.4, A.REI.3

LESSON GOAL

Students solve equations for specific variables and convert units of measure by applying the properties of equality.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Centripetal Force

Develop:

Solving Equations for Given Variables

- Solve for a Specific Variable
- Solve for a Specific Variable When the Variable Is on Each Side
- Solve Literal Equations for a Given Variable
- Use Literal Equations

Explore: Using Dimensional Analysis

Develop:

Dimensional Analysis

- Multiply by a Conversion Factor
- Use Dimensional Analysis to Convert Units
- Use Dimensional Analysis to Convert Rates

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Write and Solve One-Step Equations	●	●		●
Extension: Dimensional Analysis with Area and Volume		●	●	●

Language Development Handbook

Assign page 13 of the *Language Development Handbook* to help your students build mathematical language related to solving equations for specific variables.

ELL You can use the tips and suggestions on page T13 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

6 Attend to precision.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous

Students solved equations involving proportions.

7.RP.3, A.CED.1, A.REI.3

Now

Students use formulas to solve problems.

N.Q.1, A.CED.3, A.CED.4, A.REI.3

Next

Students will use formulas to solve problems involving sequences and geometric measurement.

F.BF.2 (Course 1), G.GMD.4 (Course 3)

Rigor

The Three Pillars of Rigor

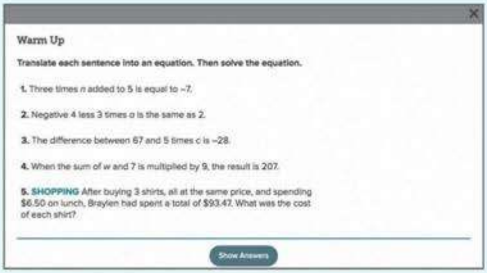
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of solving equations to build fluency with solving formulas for a particular variable. They apply their understanding of formulas by solving real-world problems.

Mathematical Background

Equations or formulas containing more than one variable are called literal equations. These equations can be solved for a specific variable in terms of the other variable(s). Many formulas require using dimensional analysis, which is converting units or rates.

Interactive Presentation



Warm Up

Translate each sentence into an equation. Then solve the equation.

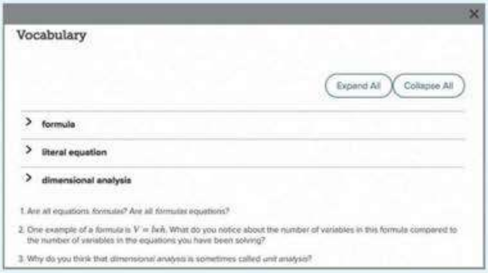
1. Three times *n* added to 5 is equal to –7.
2. Negative 4 less 3 times *a* is the same as 2.
3. The difference between 67 and 5 times *c* is –28.
4. When the sum of *w* and 7 is multiplied by 9, the result is 207.
5. SHOPPING After buying 3 shirts, all at the same price, and spending \$6.50 on lunch, Braylen had spent a total of \$93.47. What was the cost of each shirt?

Warm Up

Launch the Lesson

Watch this video to learn about the importance of using correct units of measure.

Launch the Lesson



Vocabulary

1. Are all equations *formulas*? Are all *formulas* equations?
2. One example of a *formula* is $V = \ell wh$. What do you notice about the number of variables in this formula compared to the number of variables in the equations you have been solving?
3. Why do you think that *dimensional analysis* is sometimes called *unit analysis*?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- translating sentences into equations

Answers:

1. $3n + 5 = -7; -4$
2. $-4 - 3a = 2; -2$
3. $67 - 5c = -28; 19$
4. $9(w + 7) = 207; 16$
5. $3s + 6.50 = 93.47$; \$28.99

Launch the Lesson

Teaching the Mathematical Practices

6 Use Quantities Explain to students the importance of specifying units of measure as they work through problems, as illustrated by this video.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Centripetal Force

Objective

Students explore how to rearrange a formula to solve for specific variables.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will watch a video about objects that move in a circular path and the concept of centripetal force. They will complete a chart using given formulas. The guiding exercises will lead students to consider literal equations and solving equations for a specific variable. Then, students will answer the Inquiry Question.

(continued on the next page)

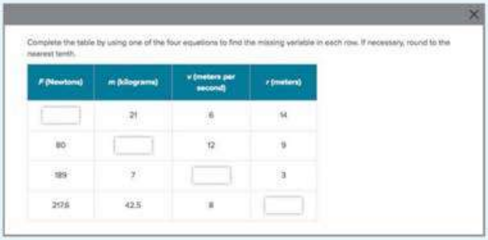
Interactive Presentation

Centripetal Force

INQUIRY Why might you want to solve a formula for a specified variable?

Watch the video to learn about centripetal force.

Explore



Complete the table by using one of the four equations to find the missing variable in each row. If necessary, round to the nearest tenth.

F (Newtons)	m (kilograms)	v (meters per second)	r (meters)
	21	6	14
80		12	9
189	7		3
217.6	42.5	8	

Explore

TYPE	Students complete a table by using the given formulas.
TYPE	Students answer a series of questions to show they understand solving literal equations.
SELECT	Students select whether a different equation yields the same answer for *m*.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Centripetal Force (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Which equation is easiest to use to find the velocity? Why? Sample answer: The equation that is solved for v. The variable is already isolated, so you just have to substitute the other values and evaluate.
- The formula for velocity involves a square root. If you were given $F = ma$, do you think the other forms of the equation would have a square root? Why or why not? Sample answer: The other formulas had v^2, so the equation for v had to use a square root. Because none of the variables have an exponent in $F = ma$, there would not be a square root in any form of the equations.

Inquiry

Why might you want to solve a formula for a specified variable? Sample answer: I might solve a formula for a specified variable to make it easier to use the formula to find a specific value.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Explore Using Dimensional Analysis

Objective

Students explore how to use dimensional analysis to compare two quantities.

MP Teaching the Mathematical Practices

2 Different Properties Mathematically proficient students look for different ways to solve problems. Encourage them to work through both ways to solve the problem and to choose the method that works best for them.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

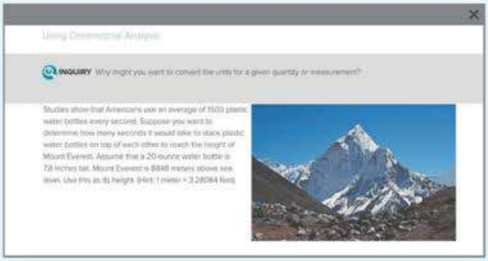
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will read about the rate in which Americans use water bottles, and consider how many bottles would need to be stacked on top of each other to reach the height of Mount Everest. The guiding exercises will lead students to consider units and unit conversions. Then, students will answer the Inquiry Question.

(continued on the next page)

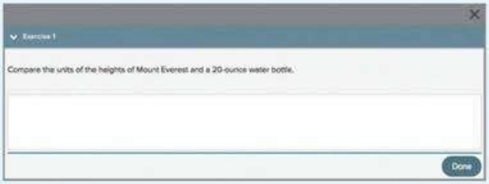
Interactive Presentation


Using Dimensional Analysis

INQUIRY Why might you want to convert the units for a given quantity or measurement?

Studies show that Americans use an average of 1500 plastic water bottles every second. Suppose you want to determine how many seconds it would take to stack plastic water bottles on top of each other to reach the height of Mount Everest. Assume that a 20-ounce water bottle is 7.8 inches tall. Mount Everest is 8848 meters above sea level. Use this as its height. (Hint: 1 meter ≈ 3.28084 feet)

Explore



Compare the units of the heights of Mount Everest and a 20-ounce water bottle.

Explore

TYPE

Students answer questions to show they understand converting units.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Dimensional Analysis (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is it difficult to compare the measurements 7.8 inches and 8848 meters? Sample answer: The units are in completely different systems. It would be easier if inches were centimeters, or if meters were given in feet.
- How would the calculations change if the height of Mount Everest were given as 8.848 km? Sample answer: You would also need to change from km into m by multiplying by the conversion $\frac{1000\text{ m}}{1\text{ km}}$ before completing the rest of the calculations.

Inquiry

Why might you want to convert the units for a given quantity or measurement? Sample answer: Converting units for a given quantity or measurement can make it easier to compare two different quantities or measurements.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Equations for Given Variables

Objective

Students solve equations for specified variables by applying the properties of equality.

Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

Common Misconception

Students often believe they can combine variables as they do numerical values when solving equations in one variable. Reinforce that only like terms can be combined, otherwise the variables must be left separate.

Example 1 Solve for a Specific Value

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in this example.

Questions for Mathematical Discourse

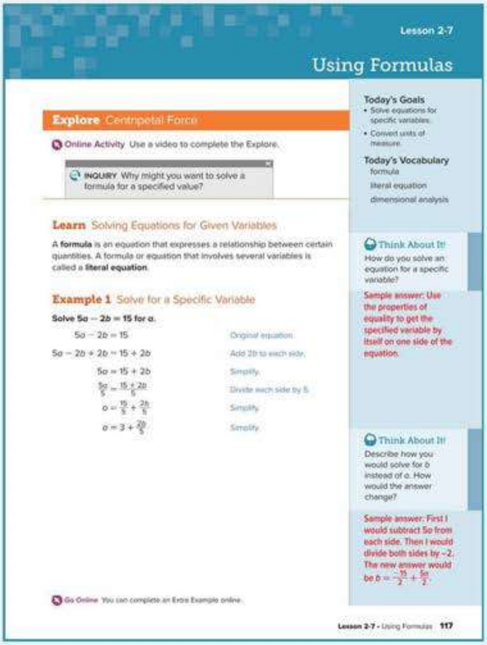
AL Why can $5a$ and $2b$ not be combined when solving the equation? Sample answer: because they are not like terms

OL Is solving an equation with one variable different than solving a literal equation for a specific variable? Explain. No; sample answer: You still use the properties of equality to reverse the order of operations, which undoes the equation and solves for the variable.

BL Why do you not have to state that the variables are not equal to zero? Sample answer: When solving for a, you do not have to divide by either a or b, so it is possible for either variable to equal zero.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-7

Using Formulas

Explore Centripetal Force

Online Activity Use a video to complete the Explore.

INQUIRY Why might you want to solve a formula for a specified value?

Learn Solving Equations for Given Variables

A **formula** is an equation that expresses a relationship between certain quantities. A formula or equation that involves several variables is called a **literal equation**.

Example 1 Solve for a Specific Variable

Solve $5a - 2b = 15$ for a.

$5a - 2b = 15$	Original equation
$5a - 2b + 2b = 15 + 2b$	Add $2b$ to each side.
$5a = 15 + 2b$	Simplify.
$\frac{5a}{5} = \frac{15 + 2b}{5}$	Divide each side by 5.
$a = \frac{15}{5} + \frac{2b}{5}$	Simplify.
$a = 3 + \frac{2b}{5}$	Simplify.

Go Online You can complete an Extra Example online.

Today's Goals
- Solve equations for specific variables.
- Convert units of measure.

Today's Vocabulary
formula
literal equation
dimensional analysis

Think About It!
How do you solve an equation for a specific variable?

Sample answer: Use the properties of equality to get the specified variable by itself on one side of the equation.

Think About It!
Describe how you would solve for b instead of a. How would the answer change?

Sample answer: First I would subtract $5a$ from each side. Then I would divide both sides by -2. The new answer would be $b = \frac{-15}{2} + \frac{5a}{2}$.

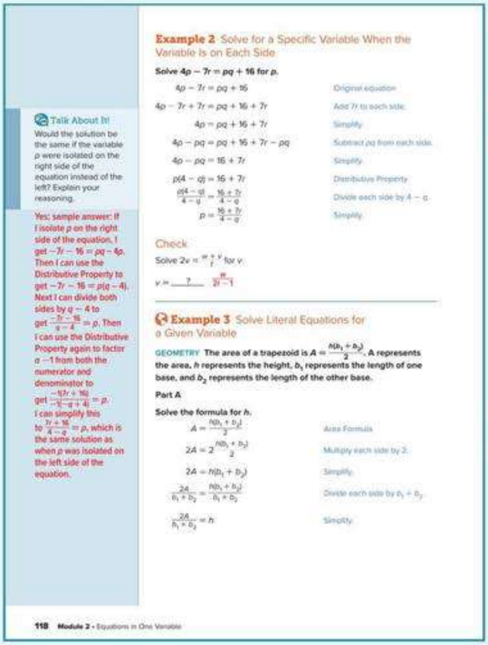
Lesson 2-7 • Using Formulas 117

Interactive Presentation

Learn

TYPE

Students answer a question to show they understand how to solve an equation for a specific variable.



Example 2 Solve for a Specific Variable When the Variable Is on Each Side

Solve $4p - 7r = pq + 16$ for p.

$4p - 7r = pq + 16$	Original equation
$4p - 7r + 7r = pq + 16 + 7r$	Add 7r to each side.
$4p = pq + 16 + 7r$	Simplify.
$4p - pq = pq + 16 + 7r - pq$	Subtract pq from each side.
$4p - pq = 16 + 7r$	Simplify.
$p(4 - q) = 16 + 7r$	Distributive Property
$\frac{p(4-q)}{4-q} = \frac{16+7r}{4-q}$	Divide each side by 4 − q.
$p = \frac{16+7r}{4-q}$	Simplify.

Talk About It!

Would the solution be the same if the variable p were isolated on the right side of the equation instead of the left? Explain your reasoning.

Yes; sample answer: If I isolate p on the right side of the equation, I get $-7r - 16 = pq - 4p$. Then I can use the Distributive Property to get $-7r - 16 = p(q - 4)$. Next I can divide both sides by $q - 4$ to get $\frac{-7r-16}{q-4} = p$. Then I can use the Distributive Property again to factor $a - 1$ from both the numerator and denominator to get $\frac{-1(7r+16)}{-1(-q+4)} = p$. I can simplify this to $\frac{7r+16}{4-q} = p$, which is the same solution as when p was isolated on the left side of the equation.

Check

Solve $2v = \frac{w+v}{t}$ for v.

$v =$ __?__ $\frac{w}{2t-1}$

Example 3 Solve Literal Equations for a Given Variable

GEOMETRY **The area of a trapezoid is $A = \frac{h(b_1 + b_2)}{2}$. A represents the area, h represents the height, b_1 represents the length of one base, and b_2 represents the length of the other base.**

Part A

Solve the formula for h.

$A = \frac{h(b_1 + b_2)}{2}$	Area Formula
$2A = 2\frac{h(b_1 + b_2)}{2}$	Multiply each side by 2.
$2A = h(b_1 + b_2)$	Simplify.
$\frac{2A}{b_1 + b_2} = \frac{h(b_1 + b_2)}{b_1 + b_2}$	Divide each side by $b_1 + b_2$.
$\frac{2A}{b_1 + b_2} = h$	Simplify.

118 **Module 2** • Equations in One Variable

Interactive Presentation

Example 2

SELECT

Students select the correct term or expression to solve the literal equation.

TYPE

Students answer a question to show they understand whether the solution would be the same if the variable ended up on the right side rather than the left.

Example 2 Solve for a Specific Variable When the Variable Is on Each Side

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What is the first goal when solving the equation for p? Get all terms with a p on the same side of the equal sign.

OL Which property allows us to get p by itself by factoring it from the expression $4p - pq$? the Distributive Property

BL Why do we have to divide by $4 - q$ instead of just 4 or $-q$? Sample answer: The quantity $(4 - q)$ is multiplied by p, so the entire expression must be used when dividing to isolate the variable.

Common Error

Students often struggle with reversing the Distributive Property, or factoring out a common factor. When two or more terms have the same variable, students may try to solve for only one of the desired terms, rather than factor out the desired variable. Reinforce that solving equations is undoing the equation, which can mean reversing the Distributive Property.

Example 3 Solve Literal Equations for a Given Variable

Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in this example.

Questions for Mathematical Discourse

AL What operation is applied to h in the formula? multiplication

OL Should we distribute the h to b_1 and b_2? Explain. No; sample answer: We want to isolate the h, and it is already isolated as is.

BL What would be the height of a trapezoid whose area is 87 square feet with bases of 10.5 feet and 11.25 feet? Show all work. $h = \frac{2(87)}{(10.5 + 11.25)}, h = \frac{174}{21.75} = 8$ft

Common Error

When an equation contains a fractional coefficient, many students divide by the denominator, which puts the fraction on the other side. Reinforce that the Multiplication Property of Equality should be used and the reciprocal should be multiplied to both sides of the equation.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to solve literal equations for a specified variable,

THEN turn the literal equation into an equation in one variable by substituting numbers. Have the students solve the equation when there is only one variable, then have them solve the literal equation. The process is the same, but the variables cannot be combined like numbers.

Enrichment Activity BL

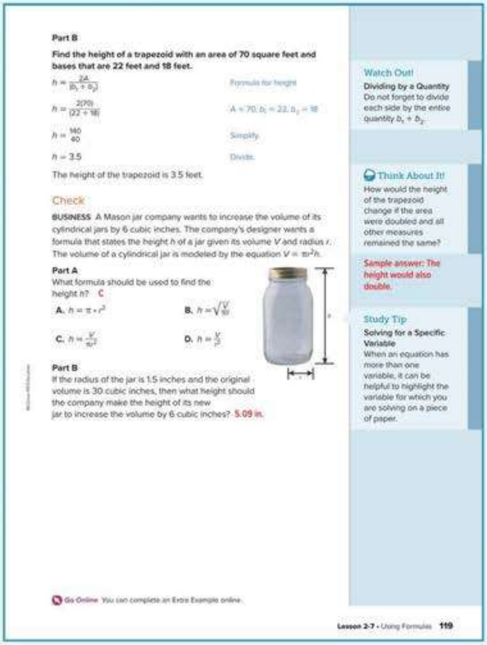
The formula for area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Solve the equation for b_1.

Sample answer: $2 \times A = \frac{1}{2}h(b_1 + b_2) \times 2$

$$\frac{2A}{h} = \frac{h(b_1 + b_2)}{h}$$

$$\frac{2A}{h} - b_2 = b_1 + b_2 - b_2$$

$$\frac{2A}{h} - b_2 = b_1$$



Part B

Find the height of a trapezoid with an area of 70 square feet and bases that are 22 feet and 18 feet.

$h = \frac{2A}{(b_1 + b_2)}$ Formula for height

$h = \frac{2(70)}{(22 + 18)}$ $A = 70, b_1 = 22, b_2 = 18$

$h = \frac{140}{40}$ Simplify.

$h = 3.5$ Divide.

The height of the trapezoid is 3.5 feet.

Check

BUSINESS A Mason jar company wants to increase the volume of its cylindrical jars by 6 cubic inches. The company's designer wants a formula that states the height h of a jar given its volume V and radius r. The volume of a cylindrical jar is modeled by the equation $V = \pi r^2 h$.

Part A

What formula should be used to find the height h? C

A. $h = \pi \cdot r^2$

B. $h = \sqrt{\frac{V}{\pi r}}$

C. $h = \frac{V}{\pi r^2}$

D. $h = \frac{V}{r^2}$

Part B

If the radius of the jar is 1.5 inches and the original volume is 30 cubic inches, then what height should the company make the height of its new jar to increase the volume by 6 cubic inches? 5.09 in.

Go Online You can complete an Extra Example online.

Watch Out!

Dividing by a Quantity Do not forget to divide each side by the entire quantity $b_1 + b_2$.

Think About It!

How would the height of the trapezoid change if the area were doubled and all other measures remained the same?

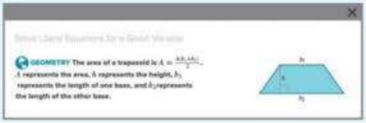
Sample answer: The height would also double.

Study Tip

Solving for a Specific Variable When an equation has more than one variable, it can be helpful to highlight the variable for which you are solving on a piece of paper.

Lesson 2-7 • Using Formulas 119

Interactive Presentation



GEOMETRY The area of a trapezoid is $A = \frac{h(b_1+b_2)}{2}$. A represents the area, h represents the height, b_1 represents the length of one base, and b_2 represents the length of the other base.

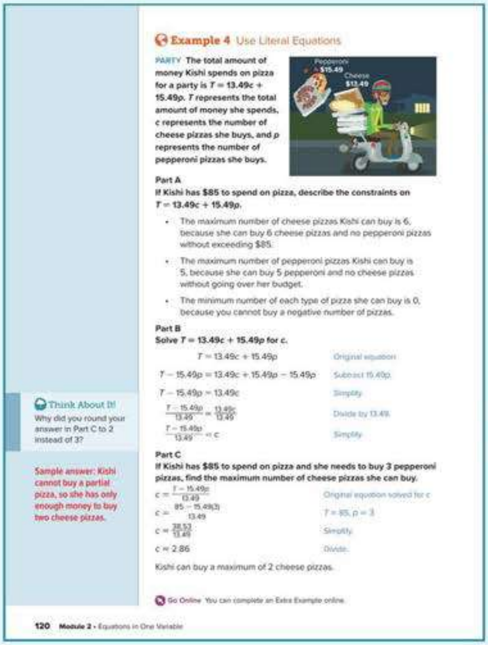
Example 3

TYPE

Students complete the statement to include the height of a trapezoid.

TYPE

Students answer a question to show they understand whether the height of a trapezoid would change if the area were doubled.



Example 4 Use Literal Equations

PARTY **The total amount of money Kishi spends on pizza for a party is $T = 13.49c + 15.49p$. T represents the total amount of money she spends, c represents the number of cheese pizzas she buys, and p represents the number of pepperoni pizzas she buys.**

Part A

If Kishi has $85 to spend on pizza, describe the constraints on $T = 13.49c + 15.49p$.

- The maximum number of cheese pizzas Kishi can buy is 6, because she can buy 6 cheese pizzas and no pepperoni pizzas without exceeding $85.
- The maximum number of pepperoni pizzas Kishi can buy is 5, because she can buy 5 pepperoni and no cheese pizzas without going over her budget.
- The minimum number of each type of pizza she can buy is 0, because you cannot buy a negative number of pizzas.

Part B

Solve $T = 13.49c + 15.49p$ for c.

$T = 13.49c + 15.49p$ Original equation

$T - 15.49p = 13.49c + 15.49p - 15.49p$ Subtract 15.49p.

$T - 15.49p = 13.49c$ Simplify.

$\frac{T - 15.49p}{13.49} = \frac{13.49c}{13.49}$ Divide by 13.49.

$\frac{T - 15.49p}{13.49} = c$ Simplify.

Part C

If Kishi has $85 to spend on pizza and she needs to buy 3 pepperoni pizzas, find the maximum number of cheese pizzas she can buy.

$c = \frac{T - 15.49p}{13.49}$ Original equation solved for c

$c = \frac{85 - 15.49(3)}{13.49}$ $T = 85, p = 3$

$c = \frac{38.53}{13.49}$ Simplify.

$c \approx 2.86$ Divide.

Kishi can buy a maximum of 2 cheese pizzas.

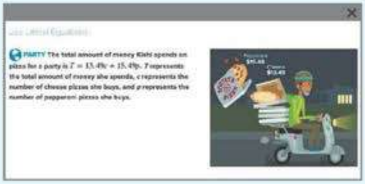
Think About It!
Why did you round your answer in Part C to 2 instead of 3?

Sample answer: Kishi cannot buy a partial pizza, so she has only enough money to buy two cheese pizzas.

Go Online You can complete an Extra Example online.

120 **Module 2** • Equations in One Variable

Interactive Presentation



Use Literal Equations

PARTY The total amount of money Kishi spends on pizza for a party is $T = 13.49c + 15.49p$. T represents the total amount of money she spends, c represents the number of cheese pizzas she buys, and p represents the number of pepperoni pizzas she buys.

Example 4

TYPE

Students answer a question to show they understand rounding answers.

Example 4 Use Literal Equations

Teaching the Mathematical Practices

1 Analyze Givens and Constraints In this example, help students identify the meaning of the problem and look for entry points to its solution.

Questions for Mathematical Discourse

AL How much does each cheese pizza cost? $13.49 Pepperoni pizza? $15.49

OL Explain why the maximum number of cheese pizzas is 6. Sample answer: If Kishi buys 6 cheese pizzas, she will spend 6(13.49) = $80.94. She cannot buy a 7th pizza if she can only spend $85.

BL If Kishi instead bought 2 pepperoni pizzas, how many cheese pizzas could she buy? Show all work.

$$c = \frac{85 - 15.49(2)}{13.49}$$

$$c \approx 4.004$$

She could buy 4 cheese pizzas.

Common Error

Students tend to follow the rules of rounding despite the context of the problem. Because this problem dealt with a budget, rounding up was not appropriate even though the number would ordinarily round up. Reinforce context to students and ensure they understand what their answer represents.

Essential Question Follow-Up

Students have begun solving literal equations for a specified variable.

Ask:

Why is it important to be able to solve literal equations for a specified variable? Sample answer: Many jobs use formulas regularly, but may need a value for one of the variables inside the equation. Being able to solve the general equation for a variable saves time in the long run when that is the needed variable in calculations.

Learn Dimensional Analysis

Objective

Students convert units of measure by applying the properties of equality.

MP Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

Common Misconception

Some students believe they can just solve the problem as is without considering units of measure. During the lesson, point out when unit conversions are necessary and discuss why.

Example 5 Multiply by a Conversion Factor

MP Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL How many liters are in one gallon? 3.785

OL Suppose the problem had given how many gallons of water were in the pool instead of liters. What would the conversion rate have been to convert gallons to liters? $\frac{3.785 \text{ liters}}{1 \text{ gallon}}$

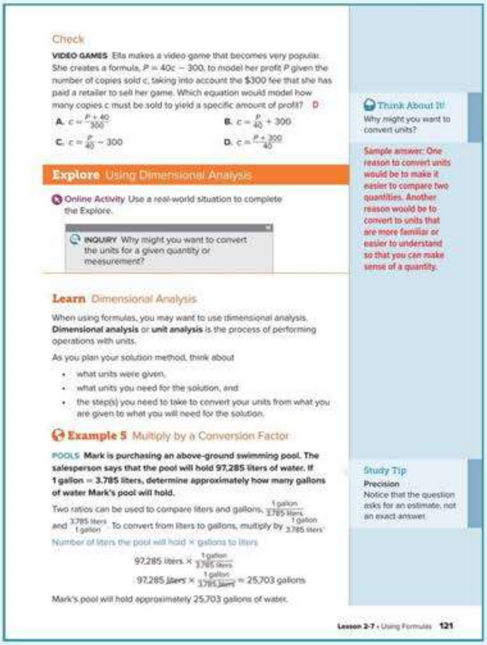
BL About how many quarts of water will the pool hold? Show all work.

$25{,}703 \cancel{\text{ gallons}} \times \frac{4 \text{ quarts}}{1 \cancel{\text{ gallon}}} = 102{,}812$

The pool could hold about 102,812 quarts of water.

Common Error

Many students will write the conversion factor in the way it was presented in the problem rather than consider which unit should be in the numerator and which should be in the denominator. Have students write units with every numerical value to help identify where the units should be in order to be eliminated.



Check

VIDEO GAMES Ella makes a video game that becomes very popular. She creates a formula, $P = 40c - 300$, to model her profit P given the number of copies sold c, taking into account the $300 fee that she has paid a retailer to sell her game. Which equation would model how many copies c must be sold to yield a specific amount of profit? D

A. $c = \frac{P + 40}{300}$

B. $c = \frac{P}{40} + 300$

C. $c = \frac{P}{40} - 300$

D. $c = \frac{P + 300}{40}$

Think About It!

Why might you want to convert units?

Sample answer: One reason to convert units would be to make it easier to compare two quantities. Another reason would be to convert to units that are more familiar or easier to understand so that you can make sense of a quantity.

Explore Using Dimensional Analysis

Online Activity Use a real-world situation to complete the Explore.

INQUIRY Why might you want to convert the units for a given quantity or measurement?

Learn Dimensional Analysis

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** or **unit analysis** is the process of performing operations with units.

As you plan your solution method, think about

- what units were given,
- what units you need for the solution, and
- the step(s) you need to take to convert your units from what you are given to what you will need for the solution.

Example 5 Multiply by a Conversion Factor

POOLS **Mark is purchasing an above-ground swimming pool. The salesperson says that the pool will hold 97,285 liters of water. If 1 gallon = 3.785 liters, determine approximately how many gallons of water Mark's pool will hold.**

Two ratios can be used to compare liters and gallons, $\frac{1 \text{ gallon}}{3.785 \text{ liters}}$ and $\frac{3.785 \text{ liters}}{1 \text{ gallon}}$. To convert from liters to gallons, multiply by $\frac{1 \text{ gallon}}{3.785 \text{ liters}}$.

Number of liters the pool will hold × gallons to liters

$$97{,}285 \text{ liters} \times \frac{1 \text{ gallon}}{3.785 \text{ liters}}$$

$$97{,}285 \cancel{\text{ liters}} \times \frac{1 \text{ gallon}}{3.785 \cancel{\text{ liters}}} \approx 25{,}703 \text{ gallons}$$

Mark's pool will hold approximately 25,703 gallons of water.

Study Tip

Precision Notice that the question asks for an estimate, not an exact answer.

Lesson 2-7 • Using Formulas 121

Interactive Presentation

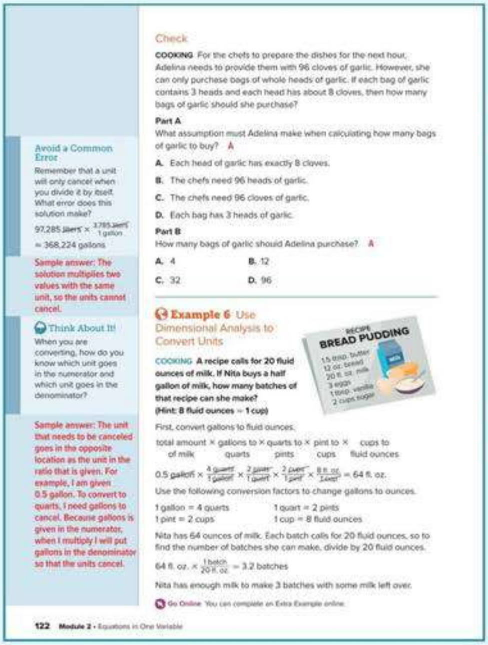
Learn

TYPE

Students answer a question to show they understand why converting units may be important.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Avoid a Common Error

Remember that a unit will only cancel when you divide it by itself. What error does this solution make?

$97{,}285\ \cancel{\text{liters}} \times \frac{3.785\ \cancel{\text{liters}}}{1\text{ gallon}}$

$\approx 368{,}224$ gallons

Sample answer: The solution multiplies two values with the same unit, so the units cannot cancel.

Think About It!

When you are converting, how do you know which unit goes in the numerator and which unit goes in the denominator?

Sample answer: The unit that needs to be canceled goes in the opposite location as the unit in the ratio that is given. For example, I am given 0.5 gallon. To convert to quarts, I need gallons to cancel. Because gallons is given in the numerator, when I multiply I will put gallons in the denominator so that the units cancel.

Check

COOKING For the chefs to prepare the dishes for the next hour, Adelina needs to provide them with 96 cloves of garlic. However, she can only purchase bags of whole heads of garlic. If each bag of garlic contains 3 heads and each head has about 8 cloves, then how many bags of garlic should she purchase?

Part A

What assumption must Adelina make when calculating how many bags of garlic to buy? A

A. Each head of garlic has exactly 8 cloves.

B. The chefs need 96 heads of garlic.

C. The chefs need 96 cloves of garlic.

D. Each bag has 3 heads of garlic.

Part B

How many bags of garlic should Adelina purchase? A

A. 4 **B.** 12

C. 32 **D.** 96

Example 6 Use Dimensional Analysis to Convert Units

COOKING **A recipe calls for 20 fluid ounces of milk. If Nita buys a half gallon of milk, how many batches of that recipe can she make? (Hint: 8 fluid ounces = 1 cup)**

First, convert gallons to fluid ounces.

total amount of milk × gallons to quarts × quarts to pints × pint to cups × cups to fluid ounces

$0.5\ \cancel{\text{gallon}} \times \frac{4\ \cancel{\text{quarts}}}{1\ \cancel{\text{gallon}}} \times \frac{2\ \cancel{\text{pints}}}{1\ \cancel{\text{quart}}} \times \frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{pint}}} \times \frac{8\text{ fl. oz.}}{1\ \cancel{\text{cup}}} = 64\text{ fl. oz.}$

Use the following conversion factors to change gallons to ounces.

1 gallon = 4 quarts
1 quart = 2 pints
1 pint = 2 cups
1 cup = 8 fluid ounces

Nita has 64 ounces of milk. Each batch calls for 20 fluid ounces, so to find the number of batches she can make, divide by 20 fluid ounces.

$64\text{ fl. oz.} \times \frac{1\text{ batch}}{20\text{ fl. oz.}} = 3.2$ batches

Nita has enough milk to make 3 batches with some milk left over.

Go Online You can complete an Extra Example online.

122 **Module 2** • Equations in One Variable

Interactive Presentation

Use Dimensional Analysis to Convert Units

COOKING A recipe calls for 20 fluid ounces of milk. If Nita buys a half gallon of milk, how many batches of that recipe can she make? (Hint: 8 fluid ounces = 1 cup)

Example 6

TAP

Students move through the slides to demonstrate the conversion.

TYPE

Students answer a question to show they understand how to set up a conversion factor.

Example 6 Use Dimensional Analysis to Convert Units

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about dimensional analysis to solve a real-world problem.

Questions for Mathematical Discourse

AL How much milk does Nita buy? a half gallon

OL For this problem, should you convert fluid ounces to gallons or gallons to fluid ounces? Explain. Sample answer: Because the recipe calls for 20 fluid ounces, it would be better to convert gallons to fluid ounces. The solution would be the same if converted to gallons, but we would have to deal with a decimal or fraction value.

BL How many batches of the recipe could Nita have made if she had $\frac{3}{4}$ of a gallon of milk? Show all work.

$0.75\ \cancel{\text{gal}} \times \frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}} \times \frac{2\ \cancel{\text{pts}}}{1\ \cancel{\text{qt}}} \times \frac{2\ \cancel{\text{cups}}}{1\ \cancel{\text{pt}}} \times \frac{8\text{ fl. oz.}}{1\ \cancel{\text{cup}}} = 96\text{ fl. oz.}$

$96\text{ fl. oz.} \div 20\text{ fl. oz.} = 4.8$

Nita could have made 4 batches of the recipe.

Common Error

Many students either unintentionally skip a needed conversion factor or try to combine two factors into one. Either way, this usually produces wrong answers. Encourage students to show all work and not skip conversion factors by going back over the units to ensure the factors eliminate units along the way.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students are struggling to convert units correctly,

THEN show students how a proportion can also be used to convert units rather than multiplying by a conversion factor. In the proportion, the conversion factor is one side and the desired unit with the given unit is the other. Just like in proportions, the units must be in the same place for each ratio; then students can solve.

Example 7 Use Dimensional Analysis to Convert Rates

Teaching the Mathematical Practices

1 Understand the Approaches of Others Work with students to look at the Alternate Method. Ask students to compare and contrast the original method and the alternate method.

Questions for Mathematical Discourse

- **OL** How many meters are there per mile? 1609.344 meters
- **AL** What is the conversion for 1 kanejaku? 1 kanejaku $= \frac{10}{33}$ meters
- **BL** About how many miles could the runner have traveled in 126.5 minutes if he had run 20 kanejaku per second? about 28.6 miles

Common Error

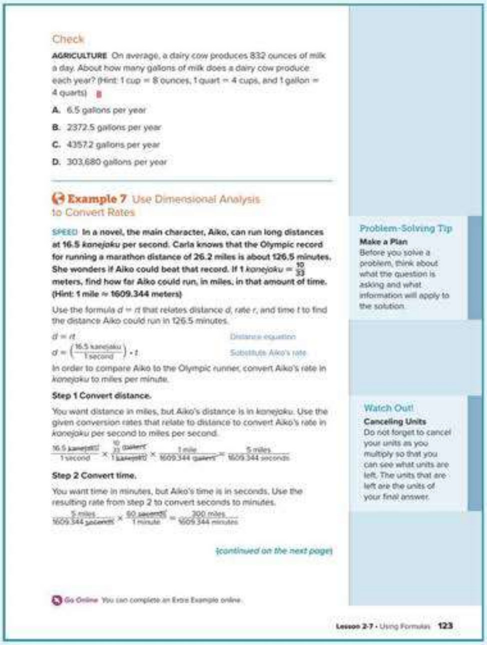
As students set up the conversion factors, encourage them to cancel as they go. Otherwise, they may lose track of which units have canceled and which still remain.

Essential Question Follow-Up

Students have begun solving literal equations for a specified variable.

Ask:

Why is it important in the real world to convert units? Sample answer: Different countries use different units of measure, so to compare distances or speeds we would need to convert to the same unit of measure. Also, many problems need a unit conversion when the desired information is in different units than the information collected.



Check

AGRICULTURE On average, a dairy cow produces 832 ounces of milk a day. About how many gallons of milk does a dairy cow produce each year? (Hint: 1 cup = 8 ounces, 1 quart = 4 cups, and 1 gallon = 4 quarts)

A. 6.5 gallons per year

B. 2372.5 gallons per year

C. 4357.2 gallons per year

D. 303,680 gallons per year

Example 7 Use Dimensional Analysis to Convert Rates

SPEED **In a novel, the main character, Aiko, can run long distances at 16.5 konejoku per second. Carla knows that the Olympic record for running a marathon distance of 26.2 miles is about 126.5 minutes. She wonders if Aiko could beat that record. If 1 konejoku = $\frac{10}{33}$ meters, find how far Aiko could run, in miles, in that amount of time. (Hint: 1 mile ≈ 1609.344 meters)**

Use the formula $d = rt$ that relates distance d, rate r, and time t to find the distance Aiko could run in 126.5 minutes.

$d = rt$ Distance equation

$d = \left(\frac{16.5 \text{ kanejaku}}{1 \text{ second}}\right) \cdot t$ Substitute Aiko's rate

In order to compare Aiko to the Olympic runner, convert Aiko's rate in konejoku to miles per minute.

Step 1 Convert distance.

You want distance in miles, but Aiko's distance is in konejoku. Use the given conversion rates that relate to distance to convert Aiko's rate in konejoku per second to miles per second.

$$\frac{16.5 \text{ kanejaku}}{1 \text{ second}} \times \frac{\frac{10}{33} \text{ meters}}{1 \text{ kanejaku}} \times \frac{1 \text{ mile}}{1609.344 \text{ meters}} = \frac{5 \text{ miles}}{1609.344 \text{ seconds}}$$

Step 2 Convert time.

You want time in minutes, but Aiko's time is in seconds. Use the resulting rate from step 2 to convert seconds to minutes.

$$\frac{5 \text{ miles}}{1609.344 \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{300 \text{ miles}}{1609.344 \text{ minutes}}$$

(continued on the next page)

Go Online You can complete an Extra Example online.

Problem-Solving Tip

Make a Plan

Before you solve a problem, think about what the question is asking and what information will apply to the solution.

Watch Out!

Canceling Units

Do not forget to cancel your units as you multiply so that you can see what units are left. The units that are left are the units of your final answer.

Lesson 2-7 • Using Formulas 123

Interactive Presentation

Example 7

TAP

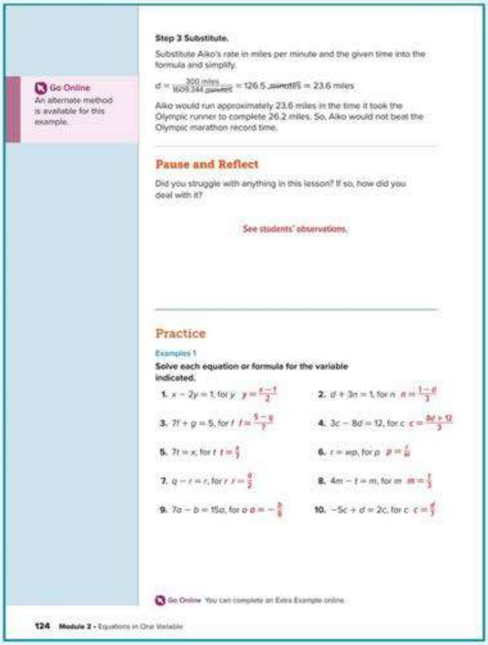
Students move through the steps to convert a rate

TYPE

Students follow directions to show they understand how to compare and contrast two conversion methods.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Step 3 Substitute.

Substitute Aiko's rate in miles per minute and the given time into the formula and simplify.

$d = \frac{300 \text{ miles}}{1609.344 \cancel{\text{minutes}}} \cdot 126.5 \cancel{\text{minutes}} = 23.6 \text{ miles}$

Aiko would run approximately 23.6 miles in the time it took the Olympic runner to complete 26.2 miles. So, Aiko would not beat the Olympic marathon record time.

Go Online An alternate method is available for this example.

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

Practice

Examples 1

Solve each equation or formula for the variable indicated.

1. $x - 2y = 1$, for y $y = \frac{x-1}{2}$
2. $d + 3n = 1$, for n $n = \frac{1-d}{3}$
3. $7f + g = 5$, for f $f = \frac{5-g}{7}$
4. $3c - 8d = 12$, for c $c = \frac{8d+12}{3}$
5. $7t = x$, for t $t = \frac{x}{7}$
6. $r = wp$, for p $p = \frac{r}{w}$
7. $q - r = r$, for r $r = \frac{q}{2}$
8. $4m - t = m$, for m $m = \frac{t}{3}$
9. $7a - b = 15a$, for a $a = -\frac{b}{8}$
10. $-5c + d = 2c$, for c $c = \frac{d}{7}$

Go Online You can complete an Extra Example online.

124 Module 2 • Equations in One Variable

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–33
2	exercises that use a variety of skills from this lesson	34–43
2	exercises that extend concepts learned in this lesson to new contexts	44–47
3	exercises that emphasize higher-order and critical-thinking skills	48–50

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

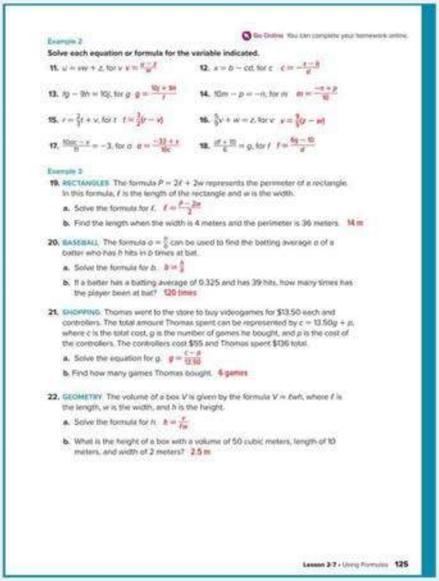
- Practice, Exercises 1–47 odd, 48–50
- Extension: Dimensional Analysis with Area and Volume

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–41 odd
- Remediation, Review Resources: Write and Solve One-Step Equations
- Personal Tutors
- Extra Examples 1–7
- **ALEKS** One–Step Equations; Fractions: Expressions and One–Step Equations; Decimals: Expressions and One–Step Equations

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–33 odd
- Remediation, Review Resources: Write and Solve One-Step Equations
- *Quick Review Math Handbook*: Literal Equations and Dimensional Analysis
- Arrive**MATH** Take Another Look
- **ALEKS** One–Step Equations; Fractions: Expressions and One–Step Equations; Decimals: Expressions and One–Step Equations



Go Online You can complete your homework online.

Example 2

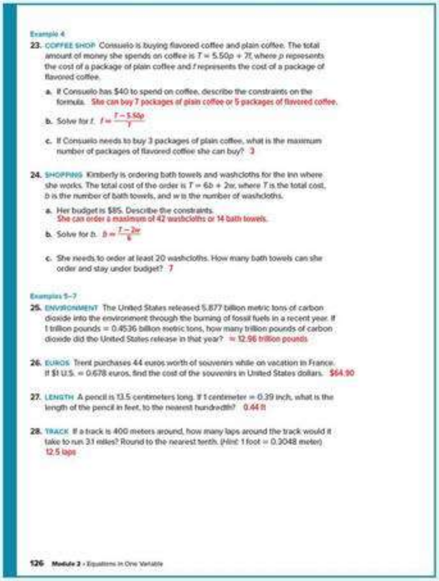
Solve each equation or formula for the variable indicated.

11. $u = vw + z$, for v $v = \frac{u - z}{w}$
12. $x = b - cd$, for c $c = -\frac{x - b}{d}$
13. $fg - 9h = 10j$, for g $g = \frac{10j + 9h}{f}$
14. $10m - p = -n$, for m $m = \frac{-n + p}{10}$
15. $r = \frac{2}{3}t + v$, for t $t = \frac{3}{2}(r - v)$
16. $\frac{5}{9}v + w = z$, for v $v = \frac{9}{5}(z - w)$
17. $\frac{10ac - x}{11} = -3$, for a $a = \frac{-33 + x}{10c}$
18. $\frac{df + 10}{6} = g$, for f $f = \frac{6g - 10}{d}$

Example 3

19. RECTANGLES The formula $P = 2\ell + 2w$ represents the perimeter of a rectangle. In this formula, ℓ is the length of the rectangle and w is the width.
    a. Solve the formula for ℓ. $\ell = \frac{P - 2w}{2}$
    b. Find the length when the width is 4 meters and the perimeter is 36 meters. 14 m
20. BASEBALL The formula $a = \frac{h}{b}$ can be used to find the batting average a of a batter who has h hits in b times at bat.
    a. Solve the formula for b. $b = \frac{h}{a}$
    b. If a batter has a batting average of 0.325 and has 39 hits, how many times has the player been at bat? 120 times
21. SHOPPING Thomas went to the store to buy videogames for \$13.50 each and controllers. The total amount Thomas spent can be represented by $c = 13.50g + p$, where c is the total cost, g is the number of games he bought, and p is the cost of the controllers. The controllers cost \$55 and Thomas spent \$136 total.
    a. Solve the equation for g. $g = \frac{c - p}{13.50}$
    b. Find how many games Thomas bought. 6 games
22. GEOMETRY The volume of a box V is given by the formula $V = \ell wh$, where ℓ is the length, w is the width, and h is the height.
    a. Solve the formula for h. $h = \frac{V}{\ell w}$
    b. What is the height of a box with a volume of 50 cubic meters, length of 10 meters, and width of 2 meters? 2.5 m

Lesson 2-7 • Using Formulas **125**



Example 4

23. COFFEE SHOP Consuelo is buying flavored coffee and plain coffee. The total amount of money she spends on coffee is $T = 5.50p + 7f$, where p represents the cost of a package of plain coffee and f represents the cost of a package of flavored coffee.
    a. If Consuelo has \$40 to spend on coffee, describe the constraints on the formula. She can buy 7 packages of plain coffee or 5 packages of flavored coffee.
    b. Solve for f. $f = \frac{T - 5.50p}{7}$
    c. If Consuelo needs to buy 3 packages of plain coffee, what is the maximum number of packages of flavored coffee she can buy? 3
24. SHOPPING Kimberly is ordering bath towels and washcloths for the inn where she works. The total cost of the order is $T = 6b + 2w$, where T is the total cost, b is the number of bath towels, and w is the number of washcloths.
    a. Her budget is \$85. Describe the constraints. She can order a maximum of 42 washcloths or 14 bath towels.
    b. Solve for b. $b = \frac{T - 2w}{6}$
    c. She needs to order at least 20 washcloths. How many bath towels can she order and stay under budget? 7

Examples 5–7

25. ENVIRONMENT The United States released 5.877 billion metric tons of carbon dioxide into the environment through the burning of fossil fuels in a recent year. If 1 trillion pounds = 0.4536 billion metric tons, how many trillion pounds of carbon dioxide did the United States release in that year? ≈ 12.96 trillion pounds
26. EUROS Trent purchases 44 euros worth of souvenirs while on vacation in France. If \$1 U.S. = 0.678 euros, find the cost of the souvenirs in United States dollars. \$64.90
27. LENGTH A pencil is 13.5 centimeters long. If 1 centimeter = 0.39 inch, what is the length of the pencil in feet, to the nearest hundredth? 0.44 ft
28. TRACK If a track is 400 meters around, how many laps around the track would it take to run 3.1 miles? Round to the nearest tenth. (Hint: 1 foot = 0.3048 meter) 12.5 laps

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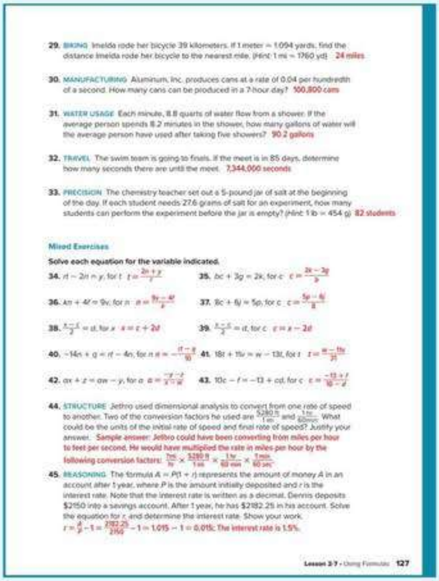


29. BIKING Imelda rode her bicycle 39 kilometers. If 1 meter = 1.094 yards, find the distance Imelda rode her bicycle to the nearest mile. (Hint: 1 mi = 1760 yd) 24 miles

30. MANUFACTURING Aluminum, Inc. produces cans at a rate of 0.04 per hundredth of a second. How many cans can be produced in a 7-hour day? 100,800 cans

31. WATER USAGE Each minute, 8.8 quarts of water flow from a shower. If the average person spends 8.2 minutes in the shower, how many gallons of water will the average person have used after taking five showers? 90.2 gallons

32. TRAVEL The swim team is going to finals. If the meet is in 85 days, determine how many seconds there are until the meet. 7,344,000 seconds

33. PRECISION The chemistry teacher set out a 5-pound jar of salt at the beginning of the day. If each student needs 27.6 grams of salt for an experiment, how many students can perform the experiment before the jar is empty? (Hint: 1 lb = 454 g) 82 students

Mixed Exercises

Solve each equation for the variable indicated.

34. $rt - 2n = y$, for t $t = \frac{2n + y}{r}$

35. $bc + 3g = 2k$, for c $c = \frac{2k - 3g}{b}$

36. $kn + 4f = 9v$, for n $n = \frac{9v - 4f}{k}$

37. $8c + 6j = 5p$, for c $c = \frac{5p - 6j}{8}$

38. $\frac{x - c}{2} = d$, for x $x = c + 2d$

39. $\frac{x - c}{2} = d$, for c $c = x - 2d$

40. $-14n + q = rt - 4n$, for n $n = -\frac{rt - q}{10}$

41. $18t + 11v = w - 13t$, for t $t = \frac{w - 11v}{31}$

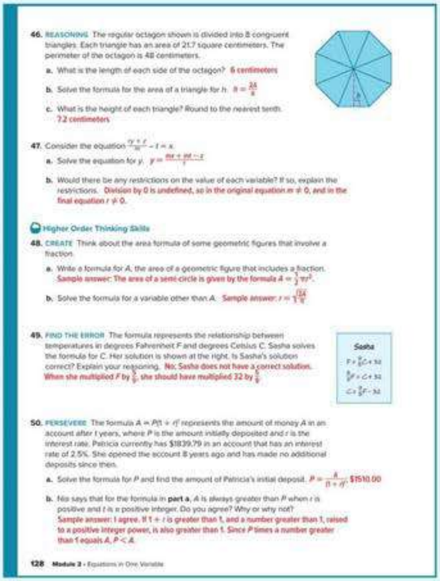
42. $ax + z = aw - y$, for a $a = \frac{-y - z}{x - w}$

43. $10c - f = -13 + cd$, for c $c = \frac{-13 + f}{10 - d}$

44. STRUCTURE Jethro used dimensional analysis to convert from one rate of speed to another. Two of the conversion factors he used are $\frac{5280 \text{ ft}}{1 \text{ mi}}$ and $\frac{1 \text{ hr}}{60 \text{ min}}$. What could be the units of the initial rate of speed and final rate of speed? Justify your answer. Sample answer: Jethro could have been converting from miles per hour to feet per second. He would have multiplied the rate in miles per hour by the following conversion factors: $\frac{\text{mi}}{\text{hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}$

45. REASONING The formula $A = P(1 + r)$ represents the amount of money A in an account after 1 year, where P is the amount initially deposited and r is the interest rate. Note that the interest rate is written as a decimal. Dennis deposits \$2150 into a savings account. After 1 year, he has \$2182.25 in his account. Solve the equation for r, and determine the interest rate. Show your work.
$r = \frac{A}{P} - 1 = \frac{2182.25}{2150} - 1 = 1.015 - 1 = 0.015$; The interest rate is 1.5%.

Lesson 2-7 • Using Formulas 127



46. REASONING The regular octagon shown is divided into 8 congruent triangles. Each triangle has an area of 21.7 square centimeters. The perimeter of the octagon is 48 centimeters.

a. What is the length of each side of the octagon? 6 centimeters

b. Solve the formula for the area of a triangle for h. $h = \frac{2A}{b}$

c. What is the height of each triangle? Round to the nearest tenth. 7.2 centimeters

47. Consider the equation $\frac{ry + t}{m} - t = x$.

a. Solve the equation for y. $y = \frac{mx + mt - t}{r}$

b. Would there be any restrictions on the value of each variable? If so, explain the restrictions. Division by 0 is undefined, so in the original equation $m \neq 0$, and in the final equation $r \neq 0$.

Higher Order Thinking Skills

48. CREATE Think about the area formula of some geometric figures that involve a fraction.

a. Write a formula for A, the area of a geometric figure that includes a fraction. Sample answer: The area of a semi-circle is given by the formula $A = \frac{1}{2}\pi r^2$.

b. Solve the formula for a variable other than A. Sample answer: $r = \sqrt{\frac{2A}{\pi}}$

49. FIND THE ERROR The formula represents the relationship between temperatures in degrees Fahrenheit F and degrees Celsius C. Sasha solves the formula for C. Her solution is shown at the right. Is Sasha's solution correct? Explain your reasoning. No; Sasha does not have a correct solution. When she multiplied F by $\frac{5}{9}$, she should have multiplied 32 by $\frac{5}{9}$.

Sasha
$F = \frac{9}{5}C + 32$
$\frac{5}{9}F = C + 32$
$C = \frac{5}{9}F - 32$

50. PERSEVERE The formula $A = P(1 + r)^t$ represents the amount of money A in an account after t years, where P is the amount initially deposited and r is the interest rate. Patricia currently has \$1839.79 in an account that has an interest rate of 2.5%. She opened the account 8 years ago and has made no additional deposits since then.

a. Solve the formula for P and find the amount of Patricia's initial deposit. $P = \frac{A}{(1 + r)^t}$; \$1510.00

b. Nia says that for the formula in **part a**, A is always greater than P when r is positive and t is a positive integer. Do you agree? Why or why not? Sample answer: I agree. If $1 + r$ is greater than 1, and a number greater than 1, raised to a positive integer power, is also greater than 1. Since P times a number greater than 1 equals A, $P < A$.

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Module 2 • Equations in One Variable

Review

Rate Yourself

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it important for you to be able to write equations to help solve problems in the real world?
- Why would translating equations into verbal sentences be a helpful skill?
- Why is it important to create equivalent equations when solving an equation?
- Why should we be able to set up and solve one-step equations in the real world?
- Why is working backward an important part of the solving process?
- Why might equations have variables on each side in the real-world?
- Why is it important to follow the order of operations even when solving equations?
- Why is absolute value an important concept for real world equations?
- Why is it important to set up and solve proportions in the real world?
- Why is it important to be able to solve literal equations for a specified variable?
- Why is it important in the real world to convert units?

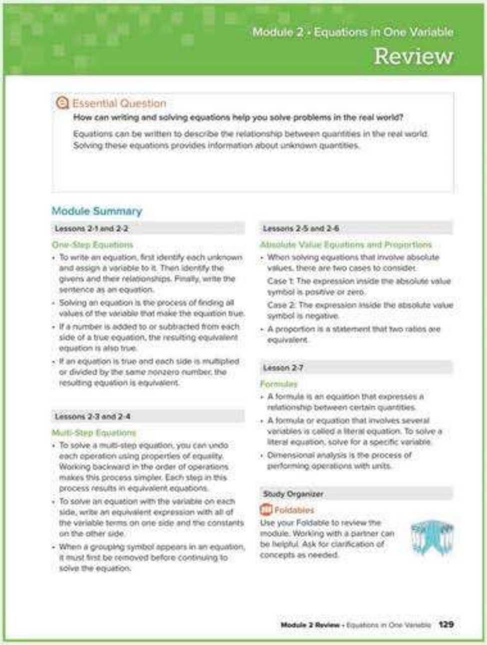
Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the Key Concepts related to writing and interpreting equations.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Relationships Between Quantities and Reasoning with Equations** and **Linear and Exponential Relationships**.

- Reason Quantitatively and Use Units to Solve Problems
- Represent and Solve Equations and Inequalities Graphically



Module 2 • Equations in One Variable

Review

Essential Question

How can writing and solving equations help you solve problems in the real world?

Equations can be written to describe the relationship between quantities in the real world. Solving these equations provides information about unknown quantities.

Module Summary

Lessons 2-1 and 2-2

One-Step Equations

- To write an equation, first identify each unknown and assign a variable to it. Then identify the givens and their relationships. Finally, write the sentence as an equation.
- Solving an equation is the process of finding all values of the variable that make the equation true.
- If a number is added to or subtracted from each side of a true equation, the resulting equivalent equation is also true.
- If an equation is true and each side is multiplied or divided by the same nonzero number, the resulting equation is equivalent.

Lessons 2-3 and 2-4

Multi-Step Equations

- To solve a multi-step equation, you can undo each operation using properties of equality. Working backward in the order of operations makes this process simpler. Each step in this process results in equivalent equations.
- To solve an equation with the variable on each side, write an equivalent expression with all of the variable terms on one side and the constants on the other side.
- When a grouping symbol appears in an equation, it must first be removed before continuing to solve the equation.

Lessons 2-5 and 2-6

Absolute Value Equations and Proportions

- When solving equations that involve absolute values, there are two cases to consider.

  Case 1: The expression inside the absolute value symbol is positive or zero.

  Case 2: The expression inside the absolute value symbol is negative.
- A proportion is a statement that two ratios are equivalent.

Lesson 2-7

Formulas

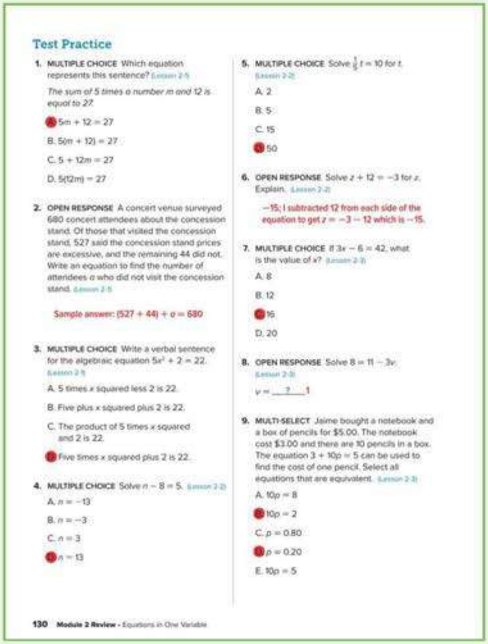
- A formula is an equation that expresses a relationship between certain quantities.
- A formula or equation that involves several variables is called a literal equation. To solve a literal equation, solve for a specific variable.
- Dimensional analysis is the process of performing operations with units.

Study Organizer

Foldables

Use your Foldable to review the module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 2 Review • Equations in One Variable 129



Test Practice

1. **MULTIPLE CHOICE** Which equation represents this sentence? (Lesson 2-1)

   The sum of 5 times a number m and 12 is equal to 27.

   [✓ A.] $5m + 12 = 27$

   B. $5(m + 12) = 27$

   C. $5 + 12m = 27$

   D. $5(12m) = 27$

2. **OPEN RESPONSE** A concert venue surveyed 680 concert attendees about the concession stand. Of those that visited the concession stand, 527 said the concession stand prices are excessive, and the remaining 44 did not. Write an equation to find the number of attendees a who did not visit the concession stand. (Lesson 2-1)

   Sample answer: $(527 + 44) + a = 680$

3. **MULTIPLE CHOICE** Write a verbal sentence for the algebraic equation $5x^2 + 2 = 22$. (Lesson 2-1)

   A. 5 times x squared less 2 is 22.

   B. Five plus x squared plus 2 is 22.

   C. The product of 5 times x squared and 2 is 22.

   [✓ D.] Five times x squared plus 2 is 22.

4. **MULTIPLE CHOICE** Solve $n - 8 = 5$. (Lesson 2-2)

   A. $n = -13$

   B. $n = -3$

   C. $n = 3$

   [✓ D.] $n = 13$

5. **MULTIPLE CHOICE** Solve $\frac{1}{5}t = 10$ for t. (Lesson 2-2)

   A. 2

   B. 5

   C. 15

   [✓ D.] 50

6. **OPEN RESPONSE** Solve $z + 12 = -3$ for z. Explain. (Lesson 2-2)

   -15; I subtracted 12 from each side of the equation to get $z = -3 - 12$ which is -15.

7. **MULTIPLE CHOICE** If $3x - 6 = 42$, what is the value of x? (Lesson 2-3)

   A. 8

   B. 12

   [✓ C.] 16

   D. 20

8. **OPEN RESPONSE** Solve $8 = 11 - 3v$. (Lesson 2-3)

   $v = \underline{\quad ? \quad}$

9. **MULTI-SELECT** Jaime bought a notebook and a box of pencils for $5.00. The notebook cost $3.00 and there are 10 pencils in a box. The equation $3 + 10p = 5$ can be used to find the cost of one pencil. Select all equations that are equivalent. (Lesson 2-3)

   A. $10p = 8$

   [✓ B.] $10p = 2$

   C. $p = 0.80$

   [✓ D.] $p = 0.20$

   E. $10p = 5$

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 2-1 through 2-4
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

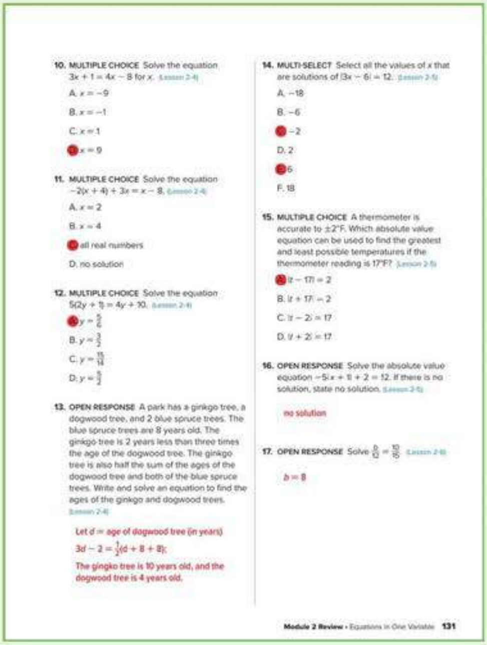
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–23 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 3, 4, 5, 7, 10, 11, 12, 15, 18, 19, 20, 21
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	9, 14
Open Response	Students construct their own response.	2, 6, 8, 13, 16, 17, 22, 23

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
A.CED.1	2-1, 2-4, 2-5, 2-6	1, 3, 13, 15, 19, 20, 21
A.CED.3	2-1	2
A.CED.4	2-7	22, 23
A.REI.1	2-2	6
A.REI.3	2-2 through 2-6	4, 5, 7–12, 14, 16–18



10. MULTIPLE CHOICE Solve the equation $3x + 1 = 4x - 8$ for x. (Lesson 2-4)

A. $x = -9$

B. $x = -1$

C. $x = 1$

[✓ D. $x = 9$]

11. MULTIPLE CHOICE Solve the equation $-2(x + 4) + 3x = x - 8$. (Lesson 2-4)

A. $x = 2$

B. $x = 4$

[✓ C. all real numbers]

D. no solution

12. MULTIPLE CHOICE Solve the equation $5(2y + 1) = 4y + 10$. (Lesson 2-4)

[✓ A. $y = \frac{5}{6}$]

B. $y = \frac{3}{2}$

C. $y = \frac{15}{14}$

D. $y = \frac{5}{2}$

13. OPEN RESPONSE A park has a ginkgo tree, a dogwood tree, and 2 blue spruce trees. The blue spruce trees are 8 years old. The ginkgo tree is 2 years less than three times the age of the dogwood tree. The ginkgo tree is also half the sum of the ages of the dogwood tree and both of the blue spruce trees. Write and solve an equation to find the ages of the ginkgo and dogwood trees. (Lesson 2-4)

Let d = age of dogwood tree (in years)

$3d - 2 = \frac{1}{2}(d + 8 + 8)$;

The gingko tree is 10 years old, and the dogwood tree is 4 years old.

14. MULTI-SELECT Select all the values of x that are solutions of $|3x - 6| = 12$. (Lesson 2-5)

A. -18

B. -6

[✓ C. -2]

D. 2

[✓ E. 6]

F. 18

15. MULTIPLE CHOICE A thermometer is accurate to ±2°F. Which absolute value equation can be used to find the greatest and least possible temperatures if the thermometer reading is 17°F? (Lesson 2-5)

[✓ A. $|t - 17| = 2$]

B. $|t + 17| = 2$

C. $|t - 2| = 17$

D. $|t + 2| = 17$

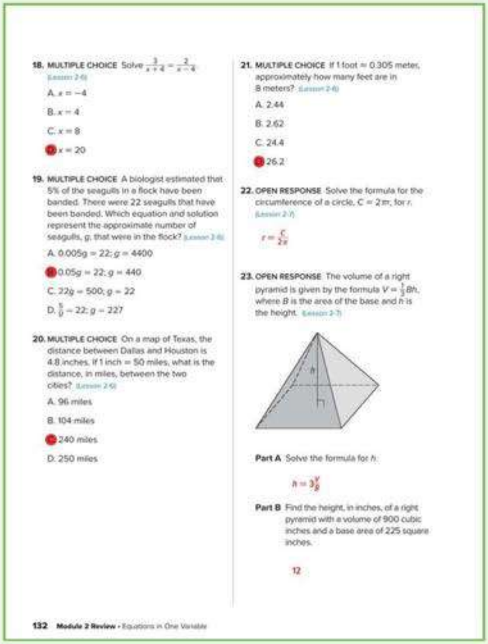
16. OPEN RESPONSE Solve the absolute value equation $-5|x + 1| + 2 = 12$. If there is no solution, state no solution. (Lesson 2-5)

no solution

17. OPEN RESPONSE Solve $\frac{b}{12} = \frac{10}{15}$. (Lesson 2-6)

$b = 8$

Module 2 Review • Equations in One Variable 131



18. **MULTIPLE CHOICE** Solve $\frac{3}{x+4} = \frac{2}{x-4}$. (Lesson 2-6)

A. $x = -4$

B. $x = 4$

C. $x = 8$

[✓ D.] $x = 20$

19. **MULTIPLE CHOICE** A biologist estimated that 5% of the seagulls in a flock have been banded. There were 22 seagulls that have been banded. Which equation and solution represent the approximate number of seagulls, g, that were in the flock? (Lesson 2-6)

A. $0.005g = 22$; $g = 4400$

[✓ B.] $0.05g = 22$; $g = 440$

C. $22g = 500$; $g = 22$

D. $\frac{5}{g} = 22$; $g = 227$

20. **MULTIPLE CHOICE** On a map of Texas, the distance between Dallas and Houston is 4.8 inches. If 1 inch = 50 miles, what is the distance, in miles, between the two cities? (Lesson 2-6)

A. 96 miles

B. 104 miles

[✓ C.] 240 miles

D. 250 miles

21. **MULTIPLE CHOICE** If 1 foot = 0.305 meter, approximately how many feet are in 8 meters? (Lesson 2-6)

A. 2.44

B. 2.62

C. 24.4

[✓ D.] 26.2

22. **OPEN RESPONSE** Solve the formula for the circumference of a circle, $C = 2\pi r$, for r. (Lesson 2-7)

$r = \frac{C}{2\pi}$

23. **OPEN RESPONSE** The volume of a right pyramid is given by the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. (Lesson 2-7)

Part A Solve the formula for h.

$h = 3\frac{V}{B}$

Part B Find the height, in inches, of a right pyramid with a volume of 900 cubic inches and a base area of 225 square inches.

12

132 **Module 2 Review** • Equations in One Variable

Module 3

Relations and Functions

Module Goals

- Students represent relations, and determine whether a relation is a function.
- Students use function notation, and find function values.
- Students graph linear and nonlinear functions, and identify their attributes.

Focus

Domains: Number and Quantity, Algebra, Functions

Standards for Mathematical Content:

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Also addresses N.Q.1, F.IF.5, A.REI.10, and F.IF.9

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Coherence

Vertical Alignment

Previous

Students understood the concept of a function.

8.F.1

Now

Students graph functions and interpret key features in graphs of functions.

F.IF.1, F.IF.4

Next

Students will construct linear and nonlinear functions to model and solve real-world problems.

F.BF.1, F.LE.2

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

EXPLORE → **LEARN** → **EXAMPLE & PRACTICE**

Suggested Pacing

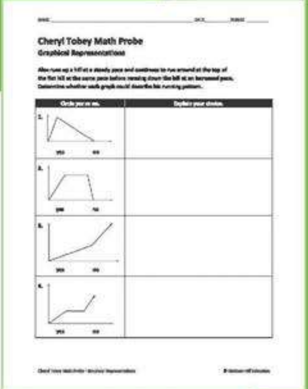
Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
3-1 Representing Relations	N.Q.1, F.IF.1	2	1
3-2 Functions	F.IF.1, F.IF.2	1	0.5
3-3 Linearity and Continuity of Graphs	F.IF.4, F.IF.5	1	0.5
3-4 Intercepts of Graphs	A.REI.10, F.IF.4	2	1
3-5 Shapes of Graphs	F.IF.4	2	1
Put It All Together: Lessons 3-1 through 3-5		1	0.5
3-6 Sketching Graphs and Comparing Functions	F.IF.4, F.IF.9	2	1
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	**14**	**7**

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Graphical Representations

Module Resource


Cheryl Tobey Math Probe
Graphical Representations

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which graph correctly represents the situation described and explain their choices.

Targeted Concepts Understand the graphical representation of a verbal description that describes a relationship between two quantities.

Targeted Misconceptions

- Students may interpret a graph as a literal picture rather than a representation of the relationship between time and distance.
- Students may interpret a rate of change (slope) in the graph as part of the hill in the verbal description.
- Students may interpret a constant function as running on a flat surface (the top of the hill).

Use the Probe after Lesson 3-1.

Correct Answers:

1. no **2.** no **3.** yes **4.** no

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
1. yes	is interpreting the graph as a literal picture of the verbal description (going uphill, then going downhill) instead of a representation of time versus distance.
2. yes	is interpreting the graph as a literal picture of the verbal description, but considers running at the top of the hill as a horizontal line.
3. no	doesn't recognize a time versus distance graph and expects it to look like a literal picture of a hill.
4. yes	is beginning to recognize the relationship between time and distance but needs more experience with connecting pace to rate of change (slope).

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Sets, Relations, and Functions
- Lesson 3-1, Learn, Example 2

Revisit the probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

Why are representations of relations and functions useful? Sample answer: Relations and functions can help you visualize relationships between quantities. They can also be used to display data, identify trends, and make predictions.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

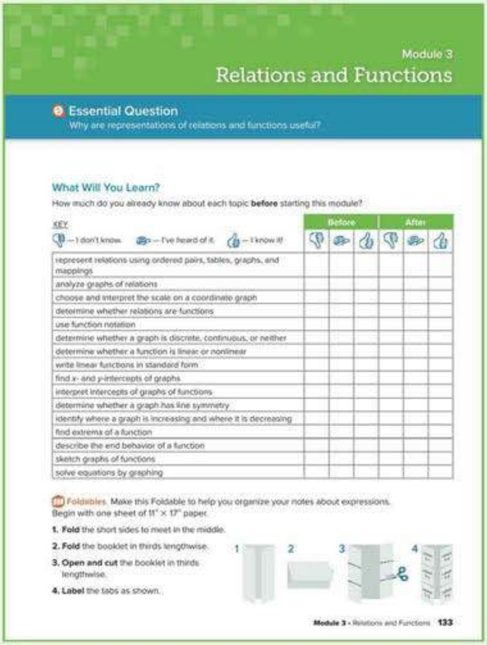
Focus Students write notes about new terms and concepts as they are presented in each lesson of this module.

Teach Have students construct their Foldable as illustrated. Have students write an explanation of each term or concept on the appropriate section of their Foldable while working through each lesson. Encourage students to record examples of each term or concept on the back of each flap.

When to Use It Encourage students to add to their Foldable as they work through the module, and to use it to review for the module test.

Launch the Module

For this module, the Launch the Module video uses data analysis to describe relations and functions. Students learn about using relations and functions in computer networks and weather.



Module 3

Relations and Functions

Essential Question

Why are representations of relations and functions useful?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. ✋ — I've heard of it. 👍 — I know it!

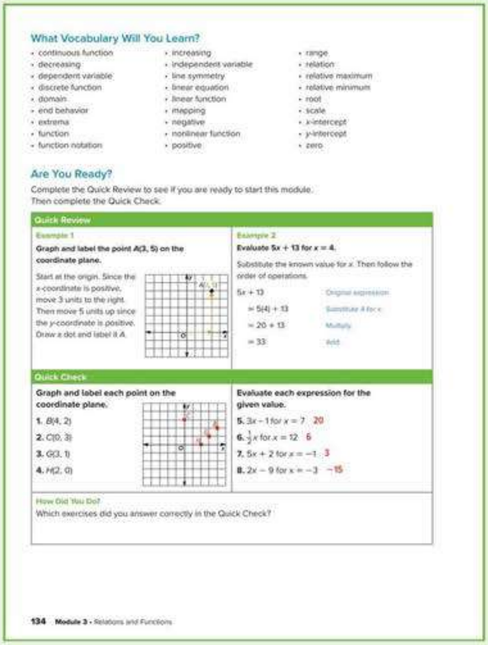
	Before			After		
	👎	✋	👍	👎	✋	👍
represent relations using ordered pairs, tables, graphs, and mappings						
analyze graphs of relations						
choose and interpret the scale on a coordinate graph						
determine whether relations are functions						
use function notation						
determine whether a graph is discrete, continuous, or neither						
determine whether a function is linear or nonlinear						
write linear functions in standard form						
find x- and y-intercepts of graphs						
interpret intercepts of graphs of functions						
determine whether a graph has line symmetry						
identify where a graph is increasing and where it is decreasing						
find extrema of a function						
describe the end behavior of a function						
sketch graphs of functions						
solve equations by graphing						

Foldables Make this Foldable to help you organize your notes about expressions. Begin with one sheet of 11" × 17" paper.

1. **Fold** the short sides to meet in the middle.
2. **Fold** the booklet in thirds lengthwise.
3. **Open and cut** the booklet in thirds lengthwise.
4. **Label** the tabs as shown.

Module 3 • Relations and Functions **133**

Interactive Presentation



What Vocabulary Will You Learn?

- continuous function
- decreasing
- dependent variable
- discrete function
- domain
- end behavior
- extrema
- function
- function notation
- increasing
- independent variable
- line symmetry
- linear equation
- linear function
- mapping
- negative
- nonlinear function
- positive
- range
- relation
- relative maximum
- relative minimum
- root
- scale
- x-intercept
- y-intercept
- zero

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Graph and label the point $A(3, 5)$ on the coordinate plane.

Start at the origin. Since the x-coordinate is positive, move 3 units to the right. Then move 5 units up since the y-coordinate is positive. Draw a dot and label it A.

Example 2

Evaluate $5x + 13$ for $x = 4$.

Substitute the known value for x. Then follow the order of operations.

$5x + 13$ Original expression

$= 5(4) + 13$ Substitute 4 for x.

$= 20 + 13$ Multiply.

$= 33$ Add.

Quick Check

Graph and label each point on the coordinate plane.

1. $B(4, 2)$
2. $C(0, 3)$
3. $G(3, 1)$
4. $H(2, 0)$

Evaluate each expression for the given value.

5. $3x - 1$ for $x = 7$ 20
6. $\frac{1}{2}x$ for $x = 12$ 6
7. $5x + 2$ for $x = -1$ 3
8. $2x - 9$ for $x = -3$ −15

How Did You Do?

Which exercises did you answer correctly in the Quick Check?

134 Module 3 • Relations and Functions

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define A function is a relation in which each element of the domain is paired with *exactly* one element from the range.

Example {(1, 3), (2, 4), (2, 6)}

Ask Is the relation a function? Explain. No. Because 2, an element of the domain, is paired with both 4 and 6, the relation is not a function.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- naming quadrants
- identifying ordered pairs
- graphing ordered pairs
- analyzing qualitative graphs

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the topics in the **Functions and Lines** module—who is ready to learn these topics and who isn't quite ready to learn them yet—in order to adjust your instruction as appropriate.

Mindset Matters

Foster Grit

Grit is defined as a student's perseverance and passion for long-term goals. A student's ability to work hard, endure struggle, remain committed to their goals, make mistakes, and try again are important factors in learning.

How Can I Apply It?

Assign students the **Put It All Together** activity for each module and allow them an opportunity to work through the problems, make mistakes, share their strategies and receive feedback, and then work on the problems again to try new strategies.

Lesson 3-1

Representing Relations

N.Q.1, F.IF.1

LESSON GOAL

Students represent relations with graphs, ordered pairs, tables, and mappings.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Relations
- Representations of a Relation

Analyzing Graphs of Relations
- Analyze Graphs

Explore: Choosing Scales

Develop:

The Coordinate System
- Use Appropriate Scales
- Choose an Appropriate Origin
- Interpret Scales and Origins

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: The Coordinate Plane	●	●		●
Extension: Misleading Scales		●	●	●

Language Development Handbook

Assign page 14 of the *Language Development Handbook* to help your students build mathematical language related to representing relations.

ELL You can use the tips and suggestions on page T14 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Number and Quantity, Functions

Standards for Mathematical Content:

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

6 Attend to precision.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous

Students used multiple representations (tables, graphs, ordered pairs) to represent relationships between two quantities.

6.EE.9, 7.RP.3a, 8.EE.5

Now

Students represent relations.

N.Q.1, F.IF.1

Next

Students will determine whether a relation is a function and find function values.

F.IF.1, F.IF.2

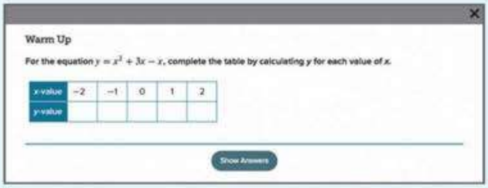
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of relations as a prelude to understanding functions. They apply their understanding by solving real-world problems involving relations.

Interactive Presentation

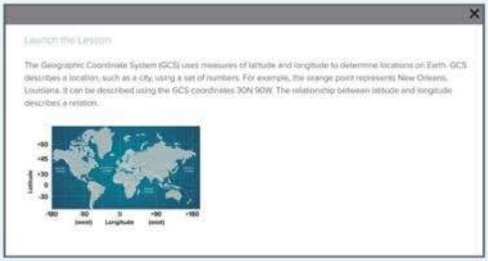


Warm Up

For the equation $y = x^2 + 3x - x$, complete the table by calculating y for each value of x.

x-value	−2	−1	0	1	2
y-value					

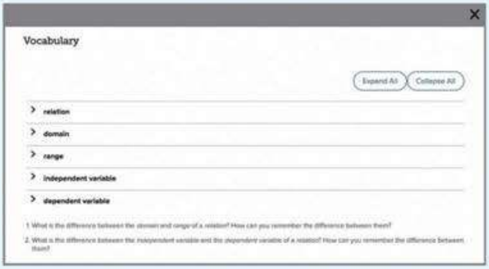
Warm Up



Launch the Lesson

The Geographic Coordinate System (GCS) uses measures of latitude and longitude to determine locations on Earth. GCS describes a location, such as a city, using a set of numbers. For example, the orange point represents New Orleans, Louisiana. It can be described using the GCS coordinates 30N 90W. The relationship between latitude and longitude describes a relation.

Launch the Lesson



Vocabulary

- relation
- domain
- range
- independent variable
- dependent variable

1. What is the difference between the domain and range of a relation? How can you remember the difference between them?
2. What is the difference between the independent variable and the dependent variable of a relation? How can you remember the difference between them?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- naming quadrants

Answers:

x-value	−2	−1	0	1	2
y-value	0	−1	0	3	8

Launch the Lesson

MP Teaching the Mathematical Practices

1 Explain correspondences Encourage students to explain the relationships between the GCS coordinates and the actual location of a city.

6 Use quantities Have students examine the scales and labels of the axes.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer to read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

A relation can be represented as a set of ordered pairs, as an equation, a table, a mapping, or a graph. A mapping lists the x-values in the domain (independent variable) and the y-values in the range (dependent variable) with arrows drawn from the x-values to the corresponding y-values. A table lists the set of x-coordinates in the first column and their corresponding y-coordinates in the second column. A graph consists of a horizontal axis (x-axis), a vertical axis (y-axis), and the intersection of the axes (origin).

The scale of graph is the distance between tick marks on the x- and y-axes. Using a scale other than 1 can make graphing a relation easier.

Explore Choosing Scales

Objective

Students use a sketch to explore how to choose appropriate scales for situations.

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This Explore asks students to justify their conclusions.

1 Seek Information Students may need to change the viewing windows on their graphing calculators to complete this Explore.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

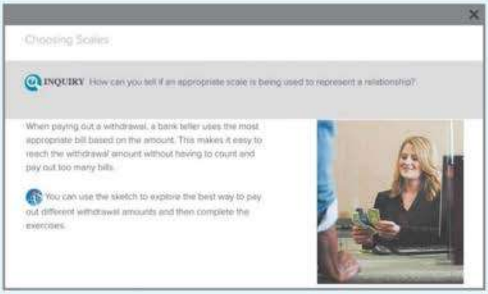
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will explore how choosing an appropriate scale is similar to choosing appropriate bills for a withdrawal amount from a bank. They will then use a graphing calculator to explore how the choice of scale affects the graph of a set of data. Then, students will complete the Inquiry Question.

(continued on the next page)

Interactive Presentation



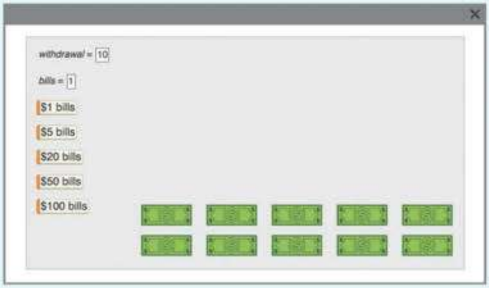
Choosing Scales

INQUIRY How can you tell if an appropriate scale is being used to represent a relationship?

When paying out a withdrawal, a bank teller uses the most appropriate bill based on the amount. This makes it easy to reach the withdrawal amount without having to count and pay out too many bills.

You can use the sketch to explore the best way to pay out different withdrawal amounts and then complete the exercises.

Explore



withdrawal = 10

bills = 1

Explore

WEB SKETCHPAD

Students use the sketch to complete an activity in which they explore the best way to pay out different withdrawal amounts.

Interactive Presentation

Explore

TYPE

a|

Students will respond to the Inquiry Question and can view a sample answer.

Explore Choosing Scales (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why would it be harder for the teller to use $1 bills? If they still existed, would it be reasonable to use a $1000 bill? Sample answer: The teller would have to count out one thousand $1 bills, which would take more time. It's not reasonable to use a $1000 bill, even though there would only be one bill. There are not many (or any) places where you could use a $1000 bill.
- Would a scale of 5 be appropriate to use for the given data? Why or why not? Sample answer: While a scale of 5 seems to fit with the given values because almost all are multiples of 5, this would mean close to 20 tick marks in each quadrant. It makes more sense to use a scale of 10.

Inquiry

How can you tell if an appropriate scale is being used to represent a relationship? Sample answer: The scale is appropriate if it allows you to view all of the data in a reasonably-sized graph, and allows you to read or estimate data values.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Relations

Objective

Students represent relations by matching sets of ordered pairs to tables, graphs, and mappings.

MP Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of relations in this Learn.

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students Are Learning

Students are learning that a relation is a pairing of elements from one set (the domain) with elements from a second set (the range). They learn that the pairing can be shown using different representations.

Common Misconception

A common misconception some students may have is that the domain and the range of a relation must have the same number of elements. Explain that this is not true, as an element of either the domain or the range can be paired with more than one element from the other set.

Essential Question Follow-Up

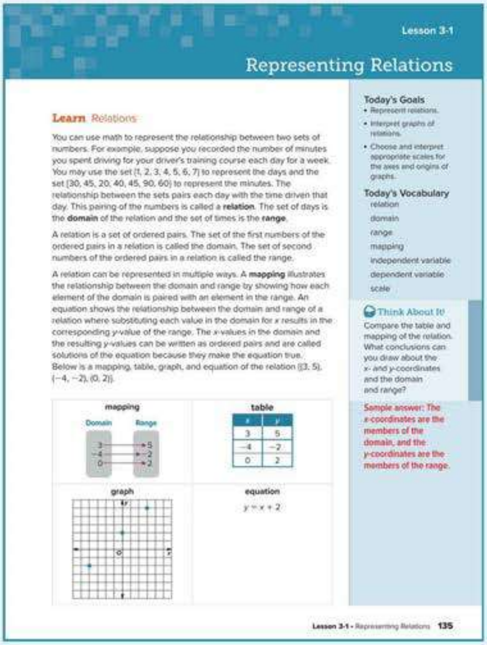
Students learn that there are different ways of representing a relation.

Ask:

Why is it helpful to have several different representations of the same relation? Sample answer: Different representations of the same relation can show different aspects of the relationship. For example, a mapping is helpful because it allows you to quickly visualize how many times an element in the domain is paired with an element in the range, or vice versa.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-1

Representing Relations

Learn Relations

You can use math to represent the relationship between two sets of numbers. For example, suppose you recorded the number of minutes you spent driving for your driver's training course each day for a week. You may use the set {1, 2, 3, 4, 5, 6, 7} to represent the days and the set {30, 45, 20, 40, 45, 90, 60} to represent the minutes. The relationship between the sets pairs each day with the time driven that day. This pairing of the numbers is called a **relation**. The set of days is the **domain** of the relation and the set of times is the **range**.

A relation is a set of ordered pairs. The set of the first numbers of the ordered pairs in a relation is called the domain. The set of second numbers of the ordered pairs in a relation is called the range.

A relation can be represented in multiple ways. A **mapping** illustrates the relationship between the domain and range by showing how each element of the domain is paired with an element in the range. An equation shows the relationship between the domain and range of a relation where substituting each value in the domain for x results in the corresponding y-value of the range. The x-values in the domain and the resulting y-values can be written as ordered pairs and are called solutions of the equation because they make the equation true. Below is a mapping, table, graph, and equation of the relation {(3, 5), (−4, −2), (0, 2)}.

mapping

table

x	y
3	5
−4	−2
0	2

graph

equation

$y = x + 2$

Today's Goals

- Represent relations.
- Interpret graphs of relations.
- Choose and interpret appropriate scales for the axes and origins of graphs.

Today's Vocabulary

relation
domain
range
mapping
independent variable
dependent variable
scale

Think About It!

Compare the table and mapping of the relation. What conclusions can you draw about the x- and y-coordinates and the domain and range?

Sample answer: The x-coordinates are the members of the domain, and the y-coordinates are the members of the range.

Lesson 3-1 • Representing Relations 135

Interactive Presentation

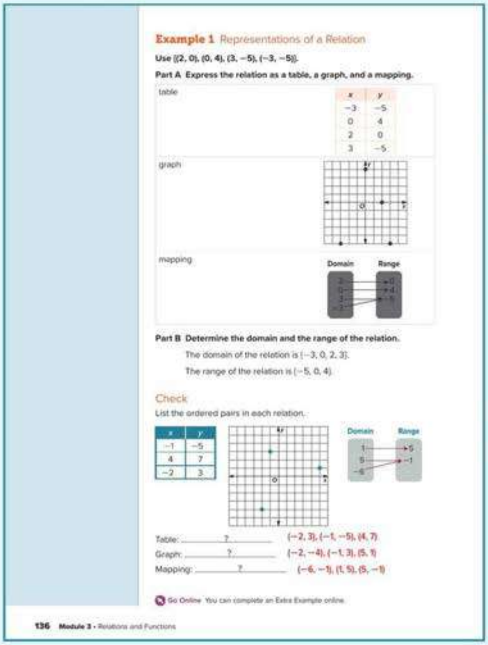
Learn

TAP

Students will tap through different terms to learn more about the vocabulary.

TYPE

Students will compare a table and mapping of a relation to the x-coordinates and y-coordinates of a graph.



Example 1 Representations of a Relation

Use {(2, 0), (0, 4), (3, −5), (−3, −5)}.

Part A Express the relation as a table, a graph, and a mapping.

table

x	y
−3	−5
0	4
2	0
3	−5

graph

mapping

Part B Determine the domain and the range of the relation.

The domain of the relation is {−3, 0, 2, 3}.

The range of the relation is {−5, 0, 4}.

Check

List the ordered pairs in each relation.

x	y
−1	−5
4	7
−2	3

Table: ___?___ {(−2, 3), (−1, −5), (4, 7)}

Graph: ___?___ {(−2, −4), (−1, 3), (5, 1)}

Mapping: ___?___ {(−6, −1), (1, 5), (5, −1)}

Go Online You can complete an Extra Example online.

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Interactive Presentation

Representations of a Relation

Example 1

DRAG & DROP

Students drag objects to create different representations of a relation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 1 Representations of a Relation

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain how the table, graph, and mapping are related.

Questions for Mathematical Discourse

AL Which value in each given ordered pair goes in the x-column of the table? the first value in each pair

OL Which representation makes it easier for you to see the relationship between the x- and y-coordinates? Explain. Sample answer: The mapping because I can see how many unique values are in the domain and range.

BL How can you tell if the relationship is linear? Explain. Sample answer: By looking at the graph, I can see that it is not linear because the plotted points do not lie on a line.

Common Error

Students may think they need to write the −5 in the range of the mapping diagram twice. Explain that this is not the case, and that more than one arrow can be drawn from or to any of the elements.

Learn Analyzing Graphs of Relations

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of graphs of relations in this Learn.

Objective

Students interpret graphs of relations by analyzing their shapes and selecting the independent and dependent variables.

What Students Are Learning

The dependent variable *depends on* the independent variable. For example, if, each y-value in a relation is 2 more than its corresponding x-value, then the values of y, the dependent variable, *depend on* the values of x, the independent variable.

DIFFERENTIATE

Language Development Activity AL ELL

Discuss with students how the shape of a graph changes based on what it represents. The graph of Days and Driving Time represents the time the student has spent driving over several days of training. Ask students what they think the graph might look like if the vertical axis represented *Distance Driven* or *Distance from Starting Point*. If the driver starts and ends each driving session at home, would the graphs be the same or different? If they are different, how are they different? This would be an excellent time to have a student use a stopwatch to record the time x it takes another student to walk or run a specific distance from a starting point. Make a table and graph the data to represent the situation in different ways.

Example 2 Analyze Graphs

Teaching the Mathematical Practices

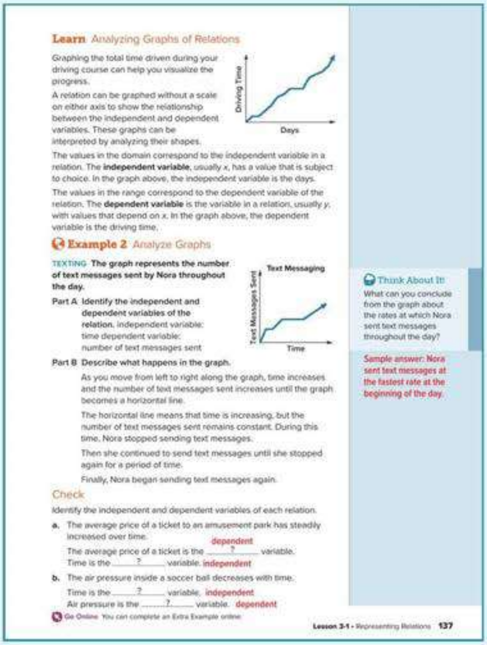
4 Make Assumptions Discuss with students the reasonableness of the assumptions they make while solving problems. Ask them to explain why they made the assumptions that they did.

Questions for Mathematical Discourse

AL Describe the graph. The graph increases, then stays the same, then increases again, then stays the same, and then increases again.

OL Which variable continues to increase regardless of the change in the other variable? time

BL Does it make sense for the graph to continue to increase? Why? Sample answer: The graph will not continue to increase forever because it would not make sense for Nora to send an infinite number of texts in one day.



Learn Analyzing Graphs of Relations

Graphing the total time driven during your driving course can help you visualize the progress.

A relation can be graphed without a scale on either axis to show the relationship between the independent and dependent variables. These graphs can be interpreted by analyzing their shapes.

The values in the domain correspond to the independent variable in a relation. The **independent variable**, usually x, has a value that is subject to choice. In the graph above, the independent variable is the days.

The values in the range correspond to the dependent variable of the relation. The **dependent variable** is the variable in a relation, usually y, with values that depend on x. In the graph above, the dependent variable is the driving time.

Example 2 Analyze Graphs

TEXTING **The graph represents the number of text messages sent by Nora throughout the day.**

Part A Identify the independent and dependent variables of the relation. independent variable: time dependent variable: number of text messages sent

Part B Describe what happens in the graph.

As you move from left to right along the graph, time increases and the number of text messages sent increases until the graph becomes a horizontal line.

The horizontal line means that time is increasing, but the number of text messages sent remains constant. During this time, Nora stopped sending text messages.

Then she continued to send text messages until she stopped again for a period of time.

Finally, Nora began sending text messages again.

Think About It!
What can you conclude from the graph about the rates at which Nora sent text messages throughout the day?

Sample answer: Nora sent text messages at the fastest rate at the beginning of the day.

Check

Identify the independent and dependent variables of each relation.

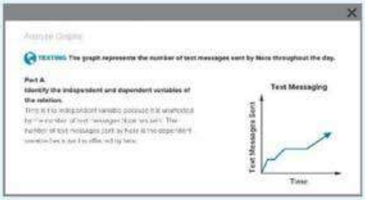
a. The average price of a ticket to an amusement park has steadily increased over time.
The average price of a ticket is the ___?___ variable. dependent
Time is the ___?___ variable. independent

b. The air pressure inside a soccer ball decreases with time.
Time is the ___?___ variable. independent
Air pressure is the ___?___ variable. dependent

Go Online You can complete an Extra Example online.

Lesson 3-1 • Representing Relations 137

Interactive Presentation



Analyze Graphs

TEXTING The graph represents the number of text messages sent by Nora throughout the day.

Part A

Identify the independent and dependent variables of the relation.

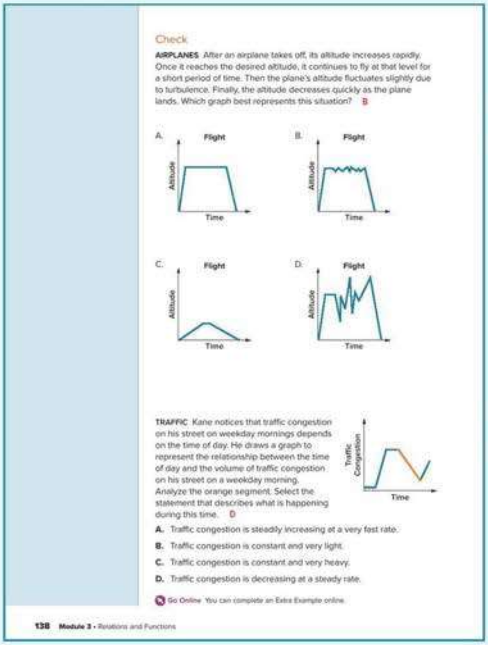
Example 2

TAP

Students move through the slides to analyze the graph.

TYPE

Students discuss conclusions from the graph about the rates at which Nora sent text messages.



Check

AIRPLANES After an airplane takes off, its altitude increases rapidly. Once it reaches the desired altitude, it continues to fly at that level for a short period of time. Then the plane's altitude fluctuates slightly due to turbulence. Finally, the altitude decreases quickly as the plane lands. Which graph best represents this situation? B

TRAFFIC Kane notices that traffic congestion on his street on weekday mornings depends on the time of day. He draws a graph to represent the relationship between the time of day and the volume of traffic congestion on his street on a weekday morning. Analyze the orange segment. Select the statement that describes what is happening during this time. D

A. Traffic congestion is steadily increasing at a very fast rate.

B. Traffic congestion is constant and very light.

C. Traffic congestion is constant and very heavy.

D. Traffic congestion is decreasing at a steady rate.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Check

MULTIPLE CHOICE

Students decide which graph best represents the situation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Common Error

Students may interpret a horizontal segment as increasing, as the *x*-values of the points along such a segment are increasing. Explain that it is the *y*-values that determine whether a relation is increasing, decreasing, or constant.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty interpreting qualitative graphs,
THEN have them think about the *y*-axis as a "thermometer," and use the idea of the temperature reading rising, falling, and remaining constant to determine whether the parts of the graph indicate that the values are increasing, decreasing, or remaining constant.

Enrichment Activity BL

Have students draw a graph that shows the height of water in a tub as it is filling, when the water is turned off, when a person gets into the water, when they get out, and as the tub is draining. Remind students to label the axes of their graphs. Have students exchange and discuss their graphs. See students' graphs.

Learn The Coordinate System

Objective

Students choose and interpret appropriate scales and origins of graphs.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Things to Remember

When different scales are used for the *x*- and *y*-axes, distances on a graph will be distorted. For example, a point that appears to be equidistant from the two axes with different scales will actually be fewer units from one axis than from the other.

Common Misconception

A common misconception some students may have is that the scales on the *x*- and *y*-axes must be the same. Point out that the scale for each axis should be chosen based on the values of the related coordinate in the ordered pairs.

Example 3 Use Appropriate Scales

Teaching the Mathematical Practices

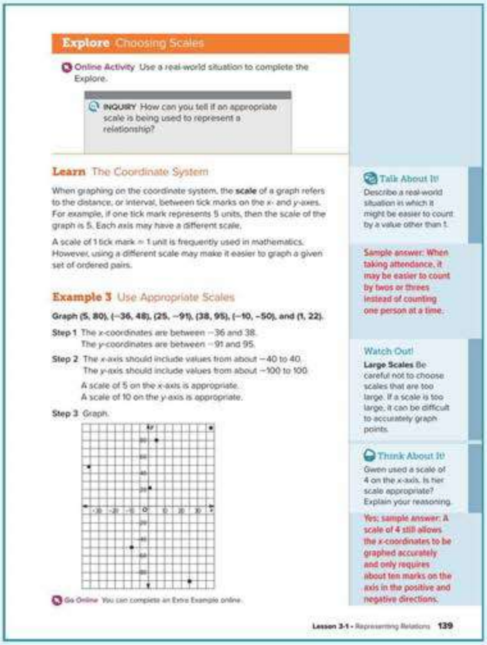
3 Compare Arguments Mathematically proficient students can compare arguments, determine which one is flawed, and explain the flaw. In this Think About It!, students have to evaluate and correct a solution.

Questions for Mathematical Discourse

AL If you used a scale of 1, about how many tick marks would be required along the *x*-axis to accommodate all the ordered pairs? about 80 along the *y*-axis? about 200

OL Why is it important to choose a scale other than 1 for this situation? Sample answer: The number of tick marks needed for the axes (about 80 on the *x*-axis and about 200 on the *y*-axis) is unreasonable, and would make the graph difficult to use and read.

BL If you knew you wanted 16 tick marks along the *x*-axis and 20 tick marks along the *y*-axis, how could you determine the appropriate scale for each axis? Sample answer: Divide 80 by 16 to determine a scale of 5 for the *x*-axis, and divide 200 by 20 to determine a scale of 10 for the *y*-axis.



Explore Choosing Scales

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you tell if an appropriate scale is being used to represent a relationship?

Learn The Coordinate System

When graphing on the coordinate system, the **scale** of a graph refers to the distance, or interval, between tick marks on the *x*- and *y*-axes. For example, if one tick mark represents 5 units, then the scale of the graph is 5. Each axis may have a different scale.

A scale of 1 tick mark = 1 unit is frequently used in mathematics. However, using a different scale may make it easier to graph a given set of ordered pairs.

Example 3 Use Appropriate Scales

Graph (5, 80), (−36, 48), (25, −91), (38, 95), (−10, −50), and (1, 22).

Step 1 The *x*-coordinates are between −36 and 38.
The *y*-coordinates are between −91 and 95.

Step 2 The *x*-axis should include values from about −40 to 40.
The *y*-axis should include values from about −100 to 100.

A scale of 5 on the *x*-axis is appropriate.
A scale of 10 on the *y*-axis is appropriate.

Step 3 Graph.

Go Online You can complete an Extra Example online.

Talk About It! Describe a real-world situation in which it might be easier to count by a value other than 1.

Sample answer: When taking attendance, it may be easier to count by twos or threes instead of counting one person at a time.

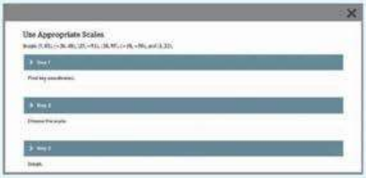
Watch Out! **Large Scales** Be careful not to choose scales that are too large. If a scale is too large, it can be difficult to accurately graph points.

Think About It! Gwen used a scale of 4 on the *x*-axis. Is her scale appropriate? Explain your reasoning.

Yes; sample answer: A scale of 4 still allows the *x*-coordinates to be graphed accurately and only requires about ten marks on the axis in the positive and negative directions.

Lesson 3-1 • Representing Relations 139

Interactive Presentation



Use Appropriate Scales

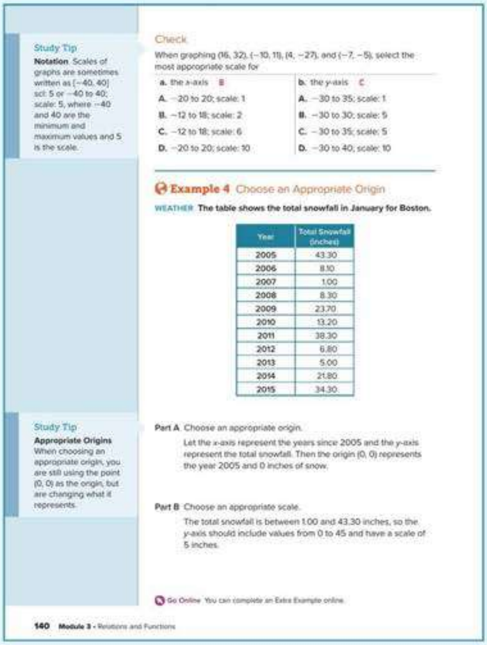
Example 3

TAP

Students move through the steps to graphing a relation using appropriate scales.

TYPE

Students determine if a given scale is appropriate.



Study Tip

Notation Scales of graphs are sometimes written as [−40, 40] scl: 5 or −40 to 40; scale: 5, where −40 and 40 are the minimum and maximum values and 5 is the scale.

Check

When graphing (16, 32), (−10, 11), (4, −27), and (−7, −5), select the most appropriate scale for

a. the x-axis B	b. the y-axis C
A. −20 to 20; scale: 1	A. −30 to 35; scale: 1
B. −12 to 18; scale: 2	B. −30 to 30; scale: 5
C. −12 to 18; scale: 6	C. −30 to 35; scale: 5
D. −20 to 20; scale: 10	D. −30 to 40; scale: 10

Example 4 Choose an Appropriate Origin

WEATHER **The table shows the total snowfall in January for Boston.**

Year	Total Snowfall (inches)
2005	43.30
2006	8.10
2007	1.00
2008	8.30
2009	23.70
2010	13.20
2011	38.30
2012	6.80
2013	5.00
2014	21.80
2015	34.30

Study Tip

Appropriate Origins When choosing an appropriate origin, you are still using the point (0, 0) as the origin, but are changing what it represents.

Part A Choose an appropriate origin.

Let the x-axis represent the years since 2005 and the y-axis represent the total snowfall. Then the origin (0, 0) represents the year 2005 and 0 inches of snow.

Part B Choose an appropriate scale.

The total snowfall is between 1.00 and 43.30 inches, so the y-axis should include values from 0 to 45 and have a scale of 5 inches.

Go Online You can complete an Extra Example online.

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Interactive Presentation

When solving real-world problems, you will want to choose what the origin represents in terms of the context of the situation.

WEATHER The table provides the total snowfall in January for Boston.

Example 4

TAP

Students move through the steps to choosing an appropriate origin and graphing the data.

TYPE

Students describe a situation where choosing a different meaning for the axes and origin may be necessary.

Example 4 Choose an Appropriate Origin

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about choosing an appropriate origin to solving a real-world problem.

Questions for Mathematical Discourse

AL How does letting the x-axis represent the number of years since 2005 make it easier to graph the data? Sample answer: It makes the x-values 0, 1, 2, 3, ... instead of 2005, 2006, 2007, 2008..., and it is easier to create a scale for the smaller numbers.

OL Why is it important to change the meaning of the origin in this situation? Sample answer: If the meaning of the origin was not changed, then the point (0, 0) would represent the year 0 and 0 inches of snow. The next smallest x-value on the graph would be 2005, which would make for an unreasonable scale on the x-axis.

BL Why is it unnecessary to include any negative y-values for this graph? Sample answer: The number of inches of snow cannot be negative, so it is unnecessary to include negative numbers in this context.

Common Error

Students may think they can use the years for the tick marks on the x-axis. Explain that since the value of x at the origin is 0, making the next tick mark 2005 would make the scale on the x-axis 2005 years. This means that the second tick mark would be 4010.

Example 5 Interpret Scales and Origins

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

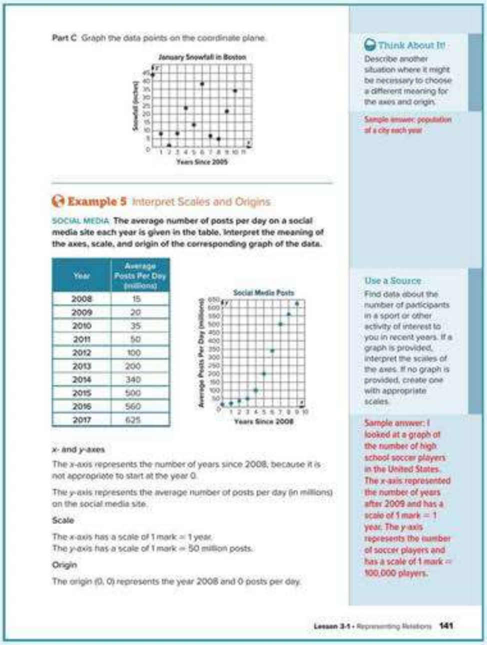
AL Are the average number of social media posts per day increasing or decreasing each year? increasing How does the graph show this? Sample answer: As the *x*-values increase, the *y*-values increase.

OL What does the ordered pair (6, 340) represent on the graph? Six years after 2008, or in 2014, the average number of posts on social media per day was 340 million.

BL What would the ordered pair (—2, 5) represent if it was on the graph? Two years before 2008, or in 2006, the average number of posts on social media per day was 5 million.

Common Error

Students may think that the scale on the *y*-axis represents 50 posts. Remind them to always read the label on an axis in order to correctly interpret the scale. In this example, the scale is 50 million posts, not 50 posts.



Part C Graph the data points on the coordinate plane.

Think About It!
Describe another situation where it might be necessary to choose a different meaning for the axes and origin.

Sample answer: population of a city each year

Example 5 Interpret Scales and Origins

SOCIAL MEDIA **The average number of posts per day on a social media site each year is given in the table. Interpret the meaning of the axes, scale, and origin of the corresponding graph of the data.**

Year	Average Posts Per Day (millions)
2008	15
2009	20
2010	35
2011	50
2012	100
2013	200
2014	340
2015	500
2016	560
2017	625

Use a Source
Find data about the number of participants in a sport or other activity of interest to you in recent years. If a graph is provided, interpret the scales of the axes. If no graph is provided, create one with appropriate scales.

Sample answer: I looked at a graph of the number of high school soccer players in the United States. The *x*-axis represented the number of years after 2009 and has a scale of 1 mark = 1 year. The *y*-axis represents the number of soccer players and has a scale of 1 mark = 100,000 players.

x- and y-axes

The *x*-axis represents the number of years since 2008, because it is not appropriate to start at the year 0.

The *y*-axis represents the average number of posts per day (in millions) on the social media site.

Scale

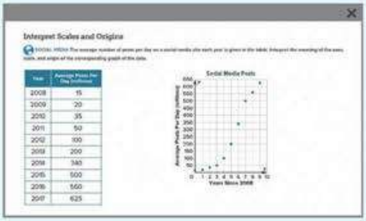
The *x*-axis has a scale of 1 mark = 1 year.
The *y*-axis has a scale of 1 mark = 50 million posts.

Origin

The origin (0, 0) represents the year 2008 and 0 posts per day.

Lesson 3-1 • Representing Relations 141

Interactive Presentation



Interpret Scales and Origins

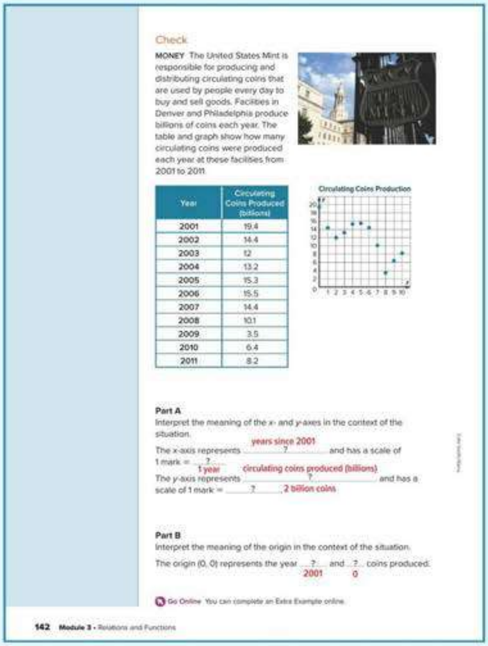
Year	Average Posts Per Day (millions)
2008	15
2009	20
2010	35
2011	50
2012	100
2013	200
2014	340
2015	500
2016	560
2017	625

Example 5

TYPE

a|

Students enter the meaning of the origin, axes, and scales of a graph they find.



Check

MONEY The United States Mint is responsible for producing and distributing circulating coins that are used by people every day to buy and sell goods. Facilities in Denver and Philadelphia produce billions of coins each year. The table and graph show how many circulating coins were produced each year at these facilities from 2001 to 2011.

Year	Circulating Coins Produced (billions)
2001	19.4
2002	14.4
2003	12
2004	13.2
2005	15.3
2006	15.5
2007	14.4
2008	10.1
2009	3.5
2010	6.4
2011	8.2

Part A

Interpret the meaning of the *x*- and *y*-axes in the context of the situation.

The *x*-axis represents ? years since 2001 and has a scale of 1 mark = ? 1 year.

The *y*-axis represents ? circulating coins produced (billions) and has a scale of 1 mark = ? 2 billion coins.

Part B

Interpret the meaning of the origin in the context of the situation.

The origin (0, 0) represents the year ? 2001 and ? 0 coins produced.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Question 2

This question has two parts. First, answer Part A. Then, answer Part B.

Part A

Check

CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–10
2	exercises that use a variety of skills from this lesson	11–24
3	exercises that emphasize higher-order and critical-thinking skills	25–29

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–23 odd, 25–29
- Extension: Misleading Scales
- **ALEKS** Introduction to Functions; Ordered Pairs

OL

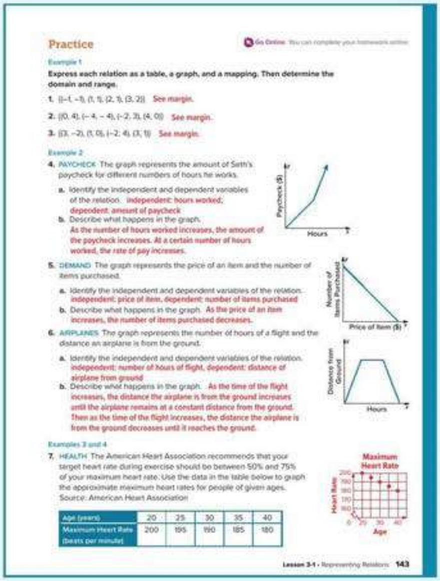
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–29 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- Personal Tutors
- Extra Examples 1–5
- **ALEKS** Ordered Pairs; Converting Between Fractions and Decimals

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–9 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- *Quick Review Math Handbook*: Relations and Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Ordered Pairs; Converting Between Fractions and Decimals



Practice

Go Online You can complete your homework online.

Example 1

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. {(−1, −1), (1, 1), (2, 1), (3, 2)} See margin.
2. {(0, 4), (−4, −4), (−2, 3), (4, 0)} See margin.
3. {(3, −2), (1, 0), (−2, 4), (3, 1)} See margin.

Example 2

4. PAYCHECK The graph represents the amount of Seth's paycheck for different numbers of hours he works.
   a. Identify the independent and dependent variables of the relation. independent: hours worked; dependent: amount of paycheck
   b. Describe what happens in the graph. As the number of hours worked increases, the amount of the paycheck increases. At a certain number of hours worked, the rate of pay increases.

5. DEMAND The graph represents the price of an item and the number of items purchased.
   a. Identify the independent and dependent variables of the relation. independent: price of item, dependent: number of items purchased
   b. Describe what happens in the graph. As the price of an item increases, the number of items purchased decreases.

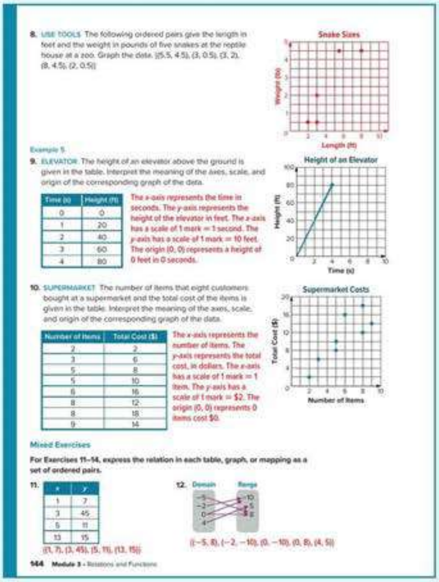
6. AIRPLANES The graph represents the number of hours of a flight and the distance an airplane is from the ground.
   a. Identify the independent and dependent variables of the relation. independent: number of hours of flight, dependent: distance of airplane from ground
   b. Describe what happens in the graph. As the time of the flight increases, the distance the airplane is from the ground increases until the airplane remains at a constant distance from the ground. Then as the time of the flight increases, the distance the airplane is from the ground decreases until it reaches the ground.

Examples 3 and 4

7. HEALTH The American Heart Association recommends that your target heart rate during exercise should be between 50% and 75% of your maximum heart rate. Use the data in the table below to graph the approximate maximum heart rates for people of given ages.
Source: American Heart Association

Age (years)	20	25	30	35	40
Maximum Heart Rate (beats per minute)	200	195	190	185	180

Lesson 3-1 · Representing Relations 143



8. USE TOOLS The following ordered pairs give the length in feet and the weight in pounds of five snakes at the reptile house at a zoo. Graph the data. {(5.5, 4.5), (3, 0.5), (3, 2), (8, 4.5), (2, 0.5)}

Example 5

9. ELEVATOR The height of an elevator above the ground is given in the table. Interpret the meaning of the axes, scale, and origin of the corresponding graph of the data.

Time (s)	Height (ft)
0	0
1	20
2	40
3	60
4	80

The x-axis represents the time in seconds. The y-axis represents the height of the elevator in feet. The x-axis has a scale of 1 mark = 1 second. The y-axis has a scale of 1 mark = 10 feet. The origin (0, 0) represents a height of 0 feet in 0 seconds.

10. SUPERMARKET The number of items that eight customers bought at a supermarket and the total cost of the items is given in the table. Interpret the meaning of the axes, scale, and origin of the corresponding graph of the data.

Number of Items	Total Cost ($)
2	2
3	6
5	8
5	10
6	16
8	12
8	18
9	14

The x-axis represents the number of items. The y-axis represents the total cost, in dollars. The x-axis has a scale of 1 mark = 1 item. The y-axis has a scale of 1 mark = $2. The origin (0, 0) represents 0 items cost $0.

Mixed Exercises

For Exercises 11–14, express the relation in each table, graph, or mapping as a set of ordered pairs.

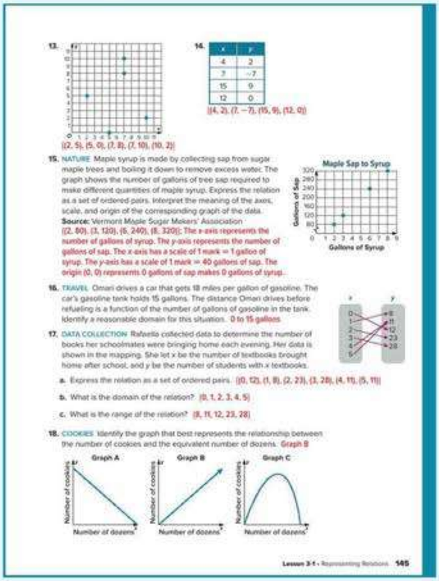
11.

x	y
1	7
3	45
5	11
13	15

{(1, 7), (3, 45), (5, 11), (13, 15)}

12. {(−5, 8), (−2, −10), (0, −10), (0, 8), (4, 5)}

144 Module 3 · Relations and Functions



13. {(2, 5), (5, 0), (7, 8), (7, 10), (10, 2)}

14.

x	y
4	2
7	−7
15	9
12	0

{(4, 2), (7, −7), (15, 9), (12, 0)}

15. NATURE Maple syrup is made by collecting sap from sugar maple trees and boiling it down to remove excess water. The graph shows the number of gallons of tree sap required to make different quantities of maple syrup. Express the relation as a set of ordered pairs. Interpret the meaning of the axes, scale, and origin of the corresponding graph of the data.
Source: Vermont Maple Sugar Makers' Association
{(2, 80), (3, 120), (6, 240), (8, 320)}; The x-axis represents the number of gallons of syrup. The y-axis represents the number of gallons of sap. The x-axis has a scale of 1 mark = 1 gallon of syrup. The y-axis has a scale of 1 mark = 40 gallons of sap. The origin (0, 0) represents 0 gallons of sap makes 0 gallons of syrup.

16. TRAVEL Omari drives a car that gets 18 miles per gallon of gasoline. The car's gasoline tank holds 15 gallons. The distance Omari drives before refueling is a function of the number of gallons of gasoline in the tank. Identify a reasonable domain for this situation. 0 to 15 gallons

17. DATA COLLECTION Rafaella collected data to determine the number of books her schoolmates were bringing home each evening. Her data is shown in the mapping. She let x be the number of textbooks brought home after school, and y be the number of students with x textbooks.

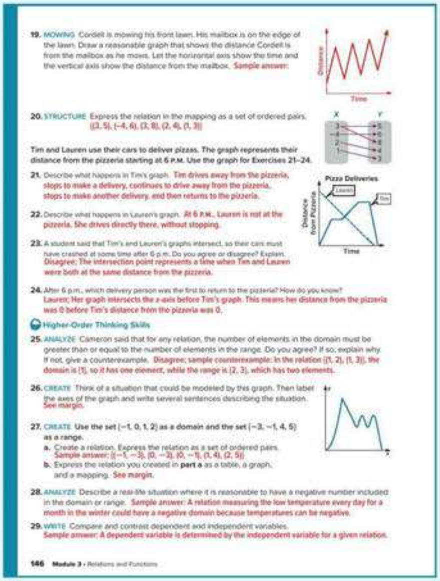
a. Express the relation as a set of ordered pairs. {(0, 12), (1, 8), (2, 23), (3, 28), (4, 11), (5, 11)}

b. What is the domain of the relation? {0, 1, 2, 3, 4, 5}

c. What is the range of the relation? {8, 11, 12, 23, 28}

18. COOKIES Identify the graph that best represents the relationship between the number of cookies and the equivalent number of dozens. Graph B

Lesson 3-1 • Representing Relations 145



19. MOWING Cordell is mowing his front lawn. His mailbox is on the edge of the lawn. Draw a reasonable graph that shows the distance Cordell is from the mailbox as he moves. Let the horizontal axis show the time and the vertical axis show the distance from the mailbox. Sample answer:

20. STRUCTURE Express the relation in the mapping as a set of ordered pairs. {(3, 5), (−4, 6), (3, 8), (2, 4), (1, 3)}

Tim and Lauren use their cars to deliver pizzas. The graph represents their distance from the pizzeria starting at 6 P.M. Use the graph for Exercises 21–24.

21. Describe what happens in Tim's graph. Tim drives away from the pizzeria, stops to make a delivery, continues to drive away from the pizzeria, stops to make another delivery, and then returns to the pizzeria.

22. Describe what happens in Lauren's graph. At 6 P.M., Lauren is not at the pizzeria. She drives directly there, without stopping.

23. A student said that Tim's and Lauren's graphs intersect, so their cars must have crashed at some time after 6 p.m. Do you agree or disagree? Explain. Disagree; The intersection point represents a time when Tim and Lauren were both at the same distance from the pizzeria.

24. After 6 p.m., which delivery person was the first to return to the pizzeria? How do you know? Lauren; Her graph intersects the x-axis before Tim's graph. This means her distance from the pizzeria was 0 before Tim's distance from the pizzeria was 0.

Higher-Order Thinking Skills

25. ANALYZE Cameron said that for any relation, the number of elements in the domain must be greater than or equal to the number of elements in the range. Do you agree? If so, explain why. If not, give a counterexample. Disagree; sample counterexample: In the relation {(1, 2), (1, 3)}, the domain is {1}, so it has one element, while the range is {2, 3}, which has two elements.

26. CREATE Think of a situation that could be modeled by this graph. Then label the axes of the graph and write several sentences describing the situation. See margin.

27. CREATE Use the set {−1, 0, 1, 2} as a domain and the set {−3, −1, 4, 5} as a range.

a. Create a relation. Express the relation as a set of ordered pairs. Sample answer: {(−1, −3), (0, −3), (0, −1), (1, 4), (2, 5)}

b. Express the relation you created in **part a** as a table, a graph, and a mapping. See margin.

28. ANALYZE Describe a real-life situation where it is reasonable to have a negative number included in the domain or range. Sample answer: A relation measuring the low temperature every day for a month in the winter could have a negative domain because temperatures can be negative.

29. WRITE Compare and contrast dependent and independent variables. Sample answer: A dependent variable is determined by the independent variable for a given relation.

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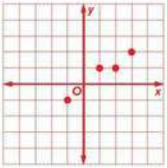
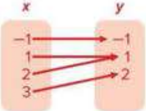
Answers

1.

x	y
−1	−1
1	1
2	1
3	2

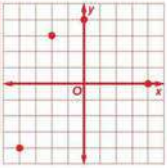


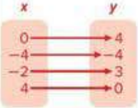


$D = \{-1, 1, 2, 3\}$; $R = \{-1, 1, 2\}$

2.

x	y
0	4
−4	−4
−2	3
4	0

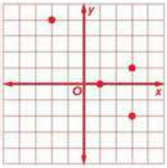


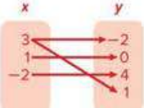


$D = \{-4, -2, 0, 4\}$; $R = \{-4, 0, 3, 4\}$

3.

x	y
3	−2
1	0
−2	4
3	1



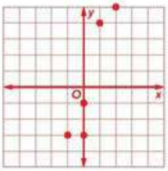


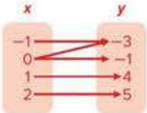
$D = \{-2, 1, 3\}$; $R = \{-2, 0, 1, 4\}$

26. Sample answer: The distance a boy is from his home as a function of time. Label the vertical axis as distance and the horizontal axis as time. The boy rides his bike to the post office to drop off a letter. He rides to his high school, which is a bit closer to his house. He jogs twice around the track, then rides his bike straight home.

27b.

x	y
−1	−3
0	−3
0	−1
1	4
2	5





Lesson 3-2

Functions

F.IF.1, F.IF.2

LESSON GOAL

Students determine whether a relation is a function and find function values.

1 LAUNCH

Launch the lesson with a Warm Up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Vertical Line Test

Develop:

Functions
- Identify Functions
- Analyze Data
- Equations as Functions

Function Values
- Find Function Values
- Evaluate Functions
- Interpret Function Values

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Graph Reflections of Points	●	●		●
Extension: Composite Functions		●	●	●

Language Development Handbook

Assign page 15 of the *Language Development Handbook* to help your students build mathematical language related to determining whether a relation is a function.

ELL You can use the tips and suggestions on page T15 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Standards for Mathematical Practice:

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students understood the concept of a function.
8.F.1

Now
Students determine whether a relation is a function, and find function values.
F.IF.1, F.IF.2

Next
Students will identify linear and nonlinear functions and continuous and discrete functions.
F.IF.4

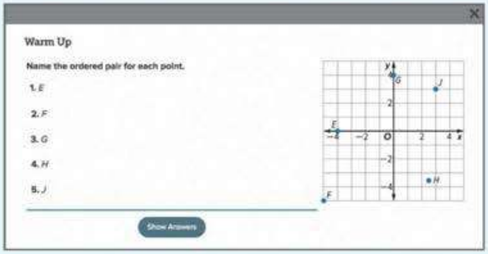
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of relations to include functions and function notation. They build fluency by determining which relations are functions and apply their understanding by solving real-world problems involving functions.

Interactive Presentation

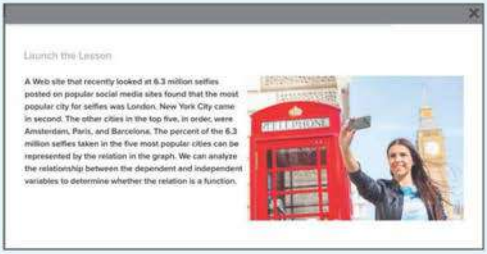


Warm Up

Name the ordered pair for each point.

1. *E*
2. *F*
3. *G*
4. *H*
5. *J*

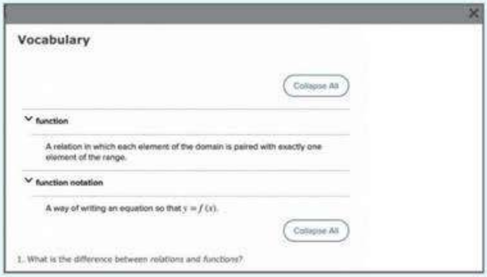
Warm Up



Launch the Lesson

A Web site that recently looked at 6.3 million selfies posted on popular social media sites found that the most popular city for selfies was London. New York City came in second. The other cities in the top five, in order, were Amsterdam, Paris, and Barcelona. The percent of the 6.3 million selfies taken in the five most popular cities can be represented by the relation in the graph. We can analyze the relationship between the dependent and independent variables to determine whether the relation is a function.

Launch the Lesson



Vocabulary

function

A relation in which each element of the domain is paired with exactly one element of the range.

function notation

A way of writing an equation so that $y = f(x)$.

1. What is the difference between *relations* and *functions*?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying ordered pairs

Answers:

1. (−4, 0)
2. (−5, −5)
3. (0, 4)
4. $\left(2\frac{1}{2}, -3\frac{1}{2}\right)$
5. (3, 3)

Launch the Lesson

Teaching the Mathematical Practices

4 Analyze Relationships Mathematically Encourage students to describe the relationship between the dependent and independent variables to determine whether the relation in the graph is a function.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

A function is a relationship between input and output in which each input value has exactly one output. The set of input values is the domain of the function, and the set of output values is the range. The vertical line test can be used to determine whether a graph represents a function. If the graph does not intersect any drawn vertical line more than once, it is a function. Functions can be written using function notation. In a function, if *x* represents the independent quantity (elements of the domain), *f*(*x*) represents the dependent quantity (elements of the range).

william87/iStock/Getty Images

Explore Vertical Line Test

Objective

Students use a sketch to determine whether relations are functions.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use dynamic geometry software. Work with students to explore and deepen their understanding of relations and functions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

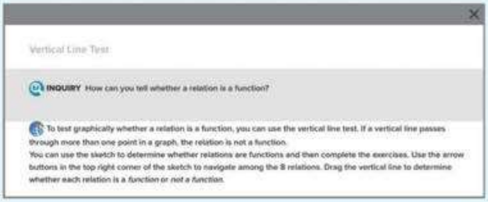
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will learn that they can use a vertical line to test the graph of a relation to see if the relation is a function. They will be presented with different types of graphs, and will use a movable vertical line to test whether each graph represents a function. Then, students will complete the Inquiry Question.

(continued on the next page)

Interactive Presentation



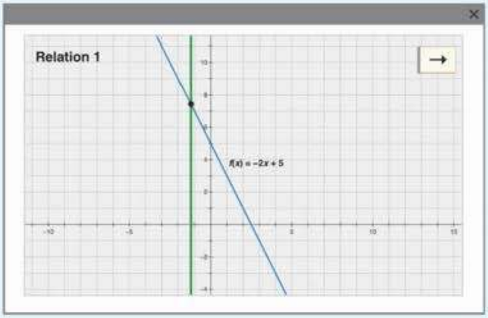
Vertical Line Test

INQUIRY How can you tell whether a relation is a function?

To test graphically whether a relation is a function, you can use the vertical line test. If a vertical line passes through more than one point in a graph, the relation is not a function.

You can use the sketch to determine whether relations are functions and then complete the exercises. Use the arrow buttons in the top right corner of the sketch to navigate among the 8 relations. Drag the vertical line to determine whether each relation is a *function* or *not a function*.

Explore



Explore

WEB SKETCHPAD

Students will use a sketch to determine whether relations are functions.

MULTIPLE SELECT

Students will select all of the relations that are functions.

TYPE

Students describe their observations from the graphing activity and explain how to tell if a relation is a function.

Interactive Presentation

INQUIRY How can you tell whether a relation is a function?

Explore

TYPE

Students will respond to the Inquiry Question and can view a sample answer.

Explore Vertical Line Test (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is a circle not a function? Sample answer: A vertical line drawn would intersect a circle at more than one point.

Inquiry

How can you tell whether a relation is a function? Sample answer: You can look at its graph and use the vertical line test. If a vertical line intersects the graph in more than one point, the relation is not a function.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Functions

Objective

Students determine whether relations are functions by analyzing mappings, tables, or graphs.

MP Teaching the Mathematical Practices

1 Different Methods Mathematically proficient students look for different ways to solve problems. Encourage them to work through different ways to solve the problem and to choose the method that works best for them.

7 Use Structure Help students to explore the structure of functions in this Learn.

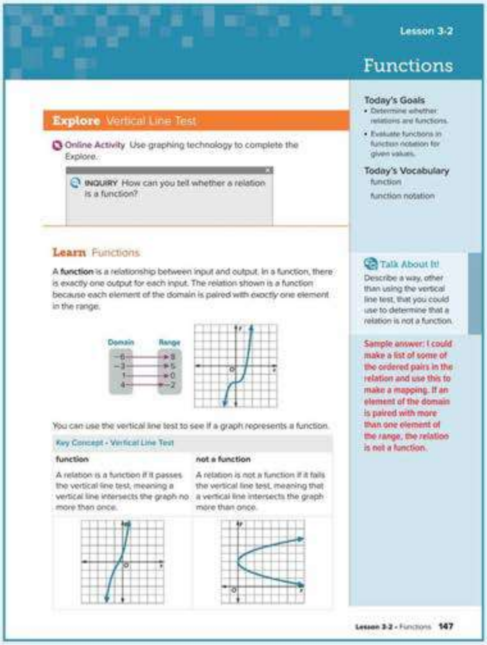
About the Key Concept It is important that students understand *why* the vertical line test works. Lead them to see that if a vertical line passes through more than one point of the graph, those points will have the same x-coordinate and different y-coordinates. This means that an element of the domain is paired with more than one element of the range, and therefore the relation is not a function. A vertical line has an equation of $x = a$, where a is a constant. If a vertical line intersects with a graph twice, then two range values would be paired with a.

Common Misconception

A common misconception some students may have is that if more than one x-value is mapped to the same y-value, then the relation is not a function. Point out that as long as each input value is mapped with only one output value, the relation is a function.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-2

Functions

Explore Vertical Line Test

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you tell whether a relation is a function?

Learn Functions

A **function** is a relationship between input and output. In a function, there is exactly one output for each input. The relation shown is a function because each element of the domain is paired with exactly one element in the range.

You can use the vertical line test to see if a graph represents a function.

Key Concept • Vertical Line Test

function	not a function
A relation is a function if it passes the vertical line test, meaning a vertical line intersects the graph no more than once.	A relation is not a function if it fails the vertical line test, meaning that a vertical line intersects the graph more than once.

Today's Goals
- Determine whether relations are functions.
- Evaluate functions in function notation for given values.

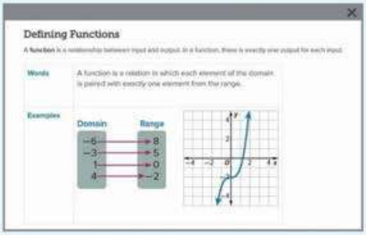
Today's Vocabulary
function
function notation

Talk About It!
Describe a way, other than using the vertical line test, that you could use to determine that a relation is not a function.

Sample answer: I could make a list of some of the ordered pairs in the relation and use this to make a mapping. If an element of the domain is paired with more than one element of the range, the relation is not a function.

Lesson 3-2 • Functions 147

Interactive Presentation



Defining Functions

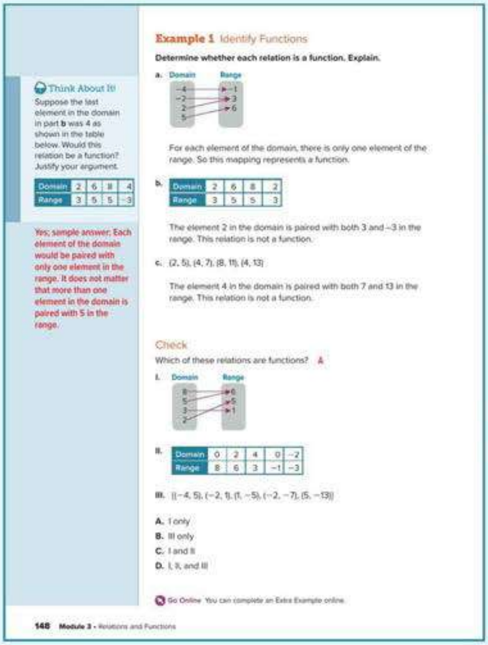
A **function** is a relationship between input and output. In a function, there is exactly one output for each input.

Words	A function is a relation in which each element of the domain is paired with exactly one element from the range.
Examples	

Learn

TYPE

Students describe another way to determine whether a relation is a function.



Think About It!

Suppose the last element in the domain in part **b** was 4 as shown in the table below. Would this relation be a function? Justify your argument.

Domain	2	6	8	4
Range	3	5	5	−3

Yes; sample answer: Each element of the domain would be paired with only one element in the range. It does not matter that more than one element in the domain is paired with 5 in the range.

Example 1 Identify Functions

Determine whether each relation is a function. Explain.

a. Domain Range

−4, −2, 2, 5 → 1, 3, 6

For each element of the domain, there is only one element of the range. So this mapping represents a function.

b.

Domain	2	6	8	2
Range	3	5	5	3

The element 2 in the domain is paired with both 3 and −3 in the range. This relation is not a function.

c. (2, 5), (4, 7), (8, 11), (4, 13)

The element 4 in the domain is paired with both 7 and 13 in the range. This relation is not a function.

Check

Which of these relations are functions? A

I. Domain Range

8, 5, 3, 2 → 6, 5, 1

II.

Domain	0	2	4	0	−2
Range	8	6	3	−1	−3

III. {(−4, 5), (−2, 1), (1, −5), (−2, −7), (5, −13)}

A. I only
B. III only
C. I and II
D. I, II, and III

Go Online You can complete an Extra Example online.

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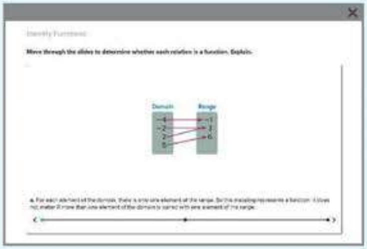
Interactive Presentation



Identify Functions

Move through the slides to determine whether each relation is a function. Explain.

Example 1

TAP

Students move through the slides to determine whether each relation is a function.

TYPE

Students determine if a relation would be a function if the domain in part **b** was changed.

Example 1 Identify Functions

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to justify their conclusions.

Questions for Mathematical Discourse

AL In each part, are there any values of the domain that are used more than once? a: no; b: yes; c: yes

OL In part **a**, how can you use the arrows in the diagram to determine whether the relation is a function? Sample answer: I can see if any of the numbers in the domain have more than one arrow.

BL In part **c**, if the order of the coordinates in each ordered pair were reversed, would the resulting relation represent a function? Explain. Yes; sample answer: Because no two ordered pairs would have the same x-coordinate, the relation would be a function.

Common Error

Some students may think that they can conclude that a set of ordered pairs is not a function if any of the ordered pairs have the same x-coordinate or the same y-coordinate. Help them to see that in a function, it is possible that more than one ordered pair may have the same y-coordinate, but not the same x-coordinate, since each element of the domain must be paired only once.

Example 2 Analyze Data

Teaching the Mathematical Practices

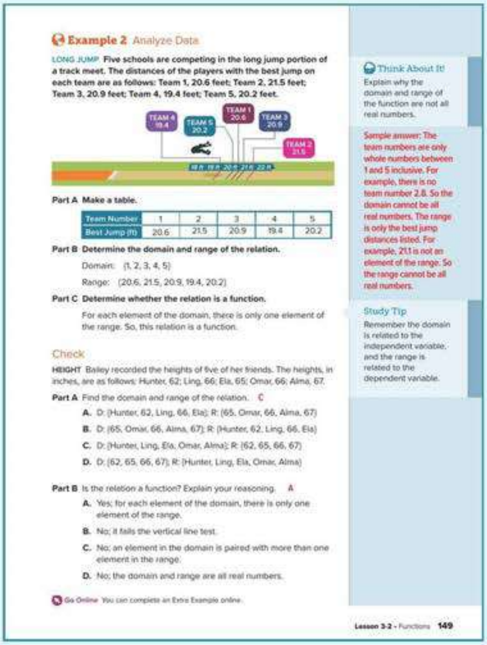
2 Consider Units Point out that it is important to note the units involved in this problem.

Questions for Mathematical Discourse

AL What are the independent and dependent variables? The independent variable is the team number and the dependent variable is the distance of the best jump.

OL Would the relation be a function if the best jump for Team 5 was 21.5 feet instead of 20.2 feet? Explain. Yes; sample answer: Each element of the domain would still only be paired with only one element in the range.

BL In the context of the situation, is it possible for one *x*-value to be mapped to more than one *y*-value? Explain. No; sample answer: It is not possible for one team to have more than one best jump.



Example 2 Analyze Data

LONG JUMP **Five schools are competing in the long jump portion of a track meet. The distances of the players with the best jump on each team are as follows: Team 1, 20.6 feet; Team 2, 21.5 feet; Team 3, 20.9 feet; Team 4, 19.4 feet; Team 5, 20.2 feet.**

Part A Make a table.

Team Number	1	2	3	4	5
Best Jump (ft)	20.6	21.5	20.9	19.4	20.2

Part B Determine the domain and range of the relation.

Domain: {1, 2, 3, 4, 5}

Range: {20.6, 21.5, 20.9, 19.4, 20.2}

Part C Determine whether the relation is a function.

For each element of the domain, there is only one element of the range. So, this relation is a function.

Check

HEIGHT Bailey recorded the heights of five of her friends. The heights, in inches, are as follows: Hunter, 62; Ling, 66; Ela, 65; Omar, 66; Alma, 67.

Part A Find the domain and range of the relation. C

- **A.** D: {Hunter, 62, Ling, 66, Ela}; R: {65, Omar, 66, Alma, 67}
- **B.** D: {65, Omar, 66, Alma, 67}; R: {Hunter, 62, Ling, 66, Ela}
- **C.** D: {Hunter, Ling, Ela, Omar, Alma}; R: {62, 65, 66, 67}
- **D.** D: {62, 65, 66, 67}; R: {Hunter, Ling, Ela, Omar, Alma}

Part B Is the relation a function? Explain your reasoning. A

- **A.** Yes; for each element of the domain, there is only one element of the range.
- **B.** No; it fails the vertical line test.
- **C.** No; an element in the domain is paired with more than one element in the range.
- **D.** No; the domain and range are all real numbers.

Go Online You can complete an Extra Example online.

Think About It!

Explain why the domain and range of the function are not all real numbers.

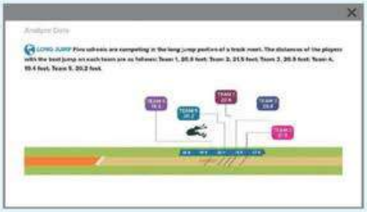
Sample answer: The team numbers are only whole numbers between 1 and 5 inclusive. For example, there is no team number 2.8. So the domain cannot be all real numbers. The range is only the best jump distances listed. For example, 21.1 is not an element of the range. So the range cannot be all real numbers.

Study Tip

Remember the domain is related to the independent variable, and the range is related to the dependent variable.

Lesson 3-2 • Functions 149

Interactive Presentation



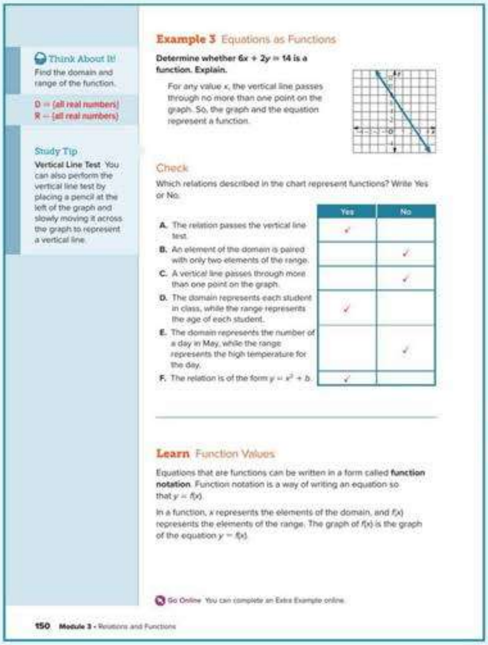
Example 2

TYPE

Students make a table of values showing the relation between the team and the best jump.

TYPE

Students explain why the domain and range are not all real numbers.



Think About It!
Find the domain and range of the function.

D = {all real numbers}
R = {all real numbers}

Study Tip
Vertical Line Test You can also perform the vertical line test by placing a pencil at the left of the graph and slowly moving it across the graph to represent a vertical line.

Example 3 Equations as Functions

Determine whether $6x + 2y = 14$ is a function. Explain.

For any value x, the vertical line passes through no more than one point on the graph. So, the graph and the equation represent a function.

Check

Which relations described in the chart represent functions? Write Yes or No.

	Yes	No
A. The relation passes the vertical line test.	✓	
B. An element of the domain is paired with only two elements of the range.		✓
C. A vertical line passes through more than one point on the graph.		✓
D. The domain represents each student in class, while the range represents the age of each student.	✓	
E. The domain represents the number of a day in May, while the range represents the high temperature for the day.		✓
F. The relation is of the form $y = x^2 + b$.	✓	

Learn Function Values

Equations that are functions can be written in a form called **function notation**. Function notation is a way of writing an equation so that $y = f(x)$.

In a function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. The graph of $f(x)$ is the graph of the equation $y = f(x)$.

Go Online You can complete an Extra Example online.

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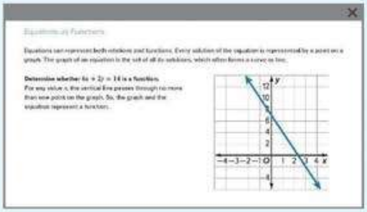
Interactive Presentation



Example 3

TYPE

Students find the domain and range for the given function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 3 Equations as Functions

Teaching the Mathematical Practices

3 Explain Correspondences Encourage students to explain the relationships between the equation of a function and its graph used in this example.

Questions for Mathematical Discourse

AL What is the independent variable? x the dependent variable? y

OL Are there any x-values that are paired with more than one y-value? Explain. No; sample answer: If there were, there would be points vertically aligned with each other.

BL Give an example of an equation that has a graph that would not pass the vertical line test. Sample answer: $x = 3$

Common Error

Some students reverse the concept of the vertical line test, thinking if a vertical line passes through more than one point on the graph, the graph represents a function. Help them to understand why this reasoning does not make sense.

Learn Function Values

Objective

Students evaluate functions in function notation for given values by writing or selecting the correct solution.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Important to Know

It is important to recognize that the ordered pairs that belong to the function are of the form $(x, f(x))$, where x represents the elements of the domain.

Common Misconception

Some students may not understand the notation, mistaking $f(x)$ to mean f times x. Compare an equation such as $f(x) = x + 1$ to $f(5) = 5 + 1$ in order to illustrate that the x acts as a placeholder for the numbers that will be evaluated in the function.

Example 4 Find Function Values

Teaching the Mathematical Practices

3 Construct Arguments In the Think About It! feature, students use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL What does the notation $f(4)$ mean? the value of the function when evaluated for $x = 4$

OL In part **b**, why do we not combine 4 and 3 to make $f(7)$ in the first step? Sample answer: The 4 is the x value that needs to be evaluated in the functions, so it cannot be combined with the 3.

BL What value of x would make $f(x) = 9$? Explain. 0; because $-2(0) + 9 = 9$

Example 5 Evaluate Functions

Teaching the Mathematical Practices

7 Use Structure Help students use the structure of functions in this example to evaluate them for the given values.

Questions for Mathematical Discourse

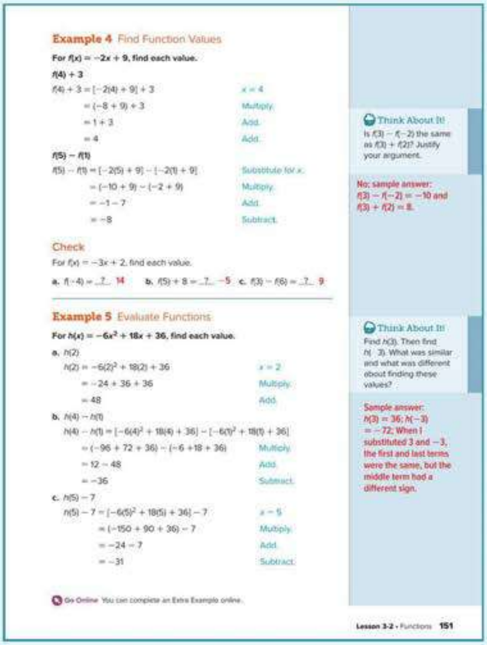
AL How do you read $h(4) - h(1)$? h of 4 minus h of 1

OL Explain the difference between the steps needed to solve part **b** and the steps needed to solve part **c**. Sample answer: In part **b**, you need to evaluate the function for two different numbers and then subtract the answers. In part **c**, you need to evaluate the function only for 5 and then subtract 7 from the result.

BL If the ordered pair $(0, y)$ belongs to function h, what is the value of y? Explain. 36; the value of y is the value of $f(0)$, which is equal to 36.

Common Error

When evaluating quadratic functions, such as the function in this example, some students may forget to follow the order of operations. Remind them to simplify exponents before multiplying, and to multiply before adding.



Example 4 Find Function Values

For $f(x) = -2x + 9$, find each value.

$f(4) + 3$

$f(4) + 3 = [-2(4) + 9] + 3$ $x = 4$

$= (-8 + 9) + 3$ Multiply.

$= 1 + 3$ Add.

$= 4$ Add.

$f(5) - f(1)$

$f(5) - f(1) = [-2(5) + 9] - [-2(1) + 9]$ Substitute for x.

$= (-10 + 9) - (-2 + 9)$ Multiply.

$= -1 - 7$ Add.

$= -8$ Subtract.

Check

For $f(x) = -3x + 2$, find each value.

a. $f(-4) =$? 14 b. $f(5) + 8 =$? -5 c. $f(3) - f(6) =$? 9

Example 5 Evaluate Functions

For $h(x) = -6x^2 + 18x + 36$, find each value.

a. $h(2)$

$h(2) = -6(2)^2 + 18(2) + 36$ $x = 2$

$= -24 + 36 + 36$ Multiply.

$= 48$ Add.

b. $h(4) - h(1)$

$h(4) - h(1) = [-6(4)^2 + 18(4) + 36] - [-6(1)^2 + 18(1) + 36]$

$= (-96 + 72 + 36) - (-6 + 18 + 36)$ Multiply.

$= 12 - 48$ Add.

$= -36$ Subtract.

c. $h(5) - 7$

$h(5) - 7 = [-6(5)^2 + 18(5) + 36] - 7$ $x = 5$

$= (-150 + 90 + 36) - 7$ Multiply.

$= -24 - 7$ Add.

$= -31$ Subtract.

Go Online You can complete an Extra Example online.

Think About It!
Is $f(3) - f(-2)$ the same as $f(3) + f(2)$? Justify your argument.

No; sample answer: $f(3) - f(-2) = -10$ and $f(3) + f(2) = 8$.

Think About It!
Find $h(3)$. Then find $h(-3)$. What was similar and what was different about finding these values?

Sample answer: $h(3) = 36$; $h(-3) = -72$; When I substituted 3 and -3, the first and last terms were the same, but the middle term had a different sign.

Lesson 3-2 • Functions 151

Interactive Presentation

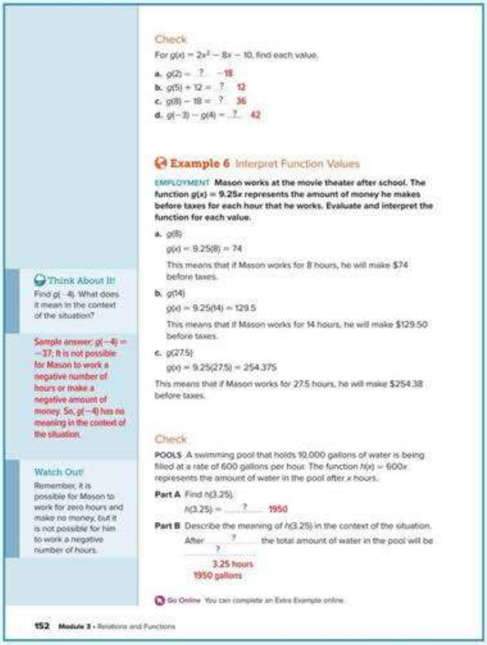
Example 5

TAP

Students move through the slides to complete the exercises.

TYPE

Students determine if two expressions are equivalent.



Check

For $g(x) = 2x^2 - 8x - 10$, find each value.

a. $g(2) =$? -18
b. $g(5) + 12 =$? 12
c. $g(8) - 18 =$? 36
d. $g(-3) - g(4) =$? 42

Example 6 Interpret Function Values

EMPLOYMENT **Mason works at the movie theater after school. The function $g(x) = 9.25x$ represents the amount of money he makes before taxes for each hour that he works. Evaluate and interpret the function for each value.**

a. $g(8)$
$g(x) = 9.25(8) = 74$
This means that if Mason works for 8 hours, he will make $74 before taxes.

b. $g(14)$
$g(x) = 9.25(14) = 129.5$
This means that if Mason works for 14 hours, he will make $129.50 before taxes.

c. $g(27.5)$
$g(x) = 9.25(27.5) = 254.375$
This means that if Mason works for 27.5 hours, he will make $254.38 before taxes.

Think About It!
Find $g(-4)$. What does it mean in the context of the situation?

Sample answer: $g(-4) = -37$; It is not possible for Mason to work a negative number of hours or make a negative amount of money. So, $g(-4)$ has no meaning in the context of the situation.

Watch Out!
Remember, it is possible for Mason to work for zero hours and make no money, but it is not possible for him to work a negative number of hours.

Check

POOLS A swimming pool that holds 10,000 gallons of water is being filled at a rate of 600 gallons per hour. The function $h(x) = 600x$ represents the amount of water in the pool after x hours.

Part A Find $h(3.25)$.
$h(3.25) =$? 1950

Part B Describe the meaning of $h(3.25)$ in the context of the situation.
After ? the total amount of water in the pool will be ?.
3.25 hours
1950 gallons

Go Online You can complete an Extra Example online.

152 Module 3 • Relations and Functions

Interactive Presentation

Interpret Function Values

EMPLOYMENT Mason works at the movie theater after school. The function $g(x) = 9.25x$ represents the amount of money he makes before taxes for each hour that he works. Evaluate and interpret the function for each value.

Example 6

TYPE

a|

Students fill in expressions as they evaluate the given function.

TYPE

a|

Students find a function value and explain its meaning in context.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 6 Interpret Function Values

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL What does 9.25 represent in the context of the situation? the amount of money Mason makes for each hour that he works

OL How can Mason determine the total amount of money he will make before taxes if he knows the number of hours that he worked? He can substitute the number of hours he worked for x in the function and then evaluate the expression.

BL Find the domain and range in the context of the situation. Justify your answer. The domain and range are all real numbers greater than or equal to 0. Mason cannot work negative hours or make a negative amount of money.

DIFFERENTIATE

Language Development Activity ELL

Beginning/Intermediate Have students work in small groups. This strategy allows every student to have an opportunity to speak several times. Ask a question or give a prompt about functions such as “Name a real-world situation that can be modeled by a function.” Then pass a stick or other object to the student. The student speaks, everyone listens, and then the student passes the object to the next person. The next student speaks, everyone listens, and then the student passes the object on until everyone has had one or two turns.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. You will also find these questions online in the Practice Bank for customization, digital assignment, and auto-scoring.

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–31
2	exercises that use a variety of skills from this lesson	32–44
2	exercises that extend concepts learned in this lesson to new contexts	45–47
3	exercises that emphasize higher-order and critical-thinking skills	48–53

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: OL BL

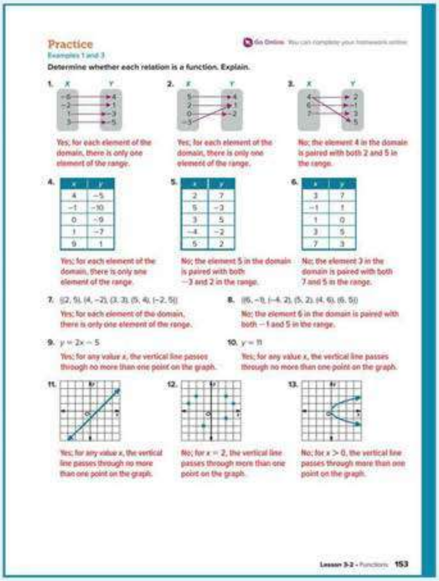
- Practice, Exercises 1–47 odd, 48–53
- Extension: Composite Functions
- **ALEKS** Introduction to Functions

IF students score 66%–89% on the Checks, **THEN** assign: AL OL

- Independent Practice, Exercises 1–53 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- Personal Tutors
- Extra Examples 1–6
- **ALEKS** Converting Between Fractions and Decimals; Ordered Pairs

IF students score 65% or less on the Checks, **THEN** assign: AL

- Independent Practice, Exercises 1–31 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- *Quick Review Math Handbook*: Relations and Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Converting Between Fractions and Decimals; Ordered Pairs



Practice

Go Online You can complete your homework online.

Examples 1 and 3

Determine whether each relation is a function. Explain.

1. Yes; for each element of the domain, there is only one element of the range.

2. Yes; for each element of the domain, there is only one element of the range.

3. No; the element 4 in the domain is paired with both 2 and 5 in the range.

4.

x	y
4	−5
−1	−10
0	−9
1	−7
9	1

Yes; for each element of the domain, there is only one element of the range.

5.

x	y
2	7
5	−3
3	5
−4	−2
5	2

No; the element 5 in the domain is paired with both −3 and 2 in the range.

6.

x	y
3	7
−1	1
1	0
3	5
7	3

No; the element 3 in the domain is paired with both 7 and 5 in the range.

7. {(2, 5), (4, −2), (3, 3), (5, 4), (−2, 5)} Yes; for each element of the domain, there is only one element of the range.

8. {(6, −1), (−4, 2), (5, 2), (4, 6), (6, 5)} No; the element 6 in the domain is paired with both −1 and 5 in the range.

9. $y = 2x - 5$ Yes; for any value x, the vertical line passes through no more than one point on the graph.

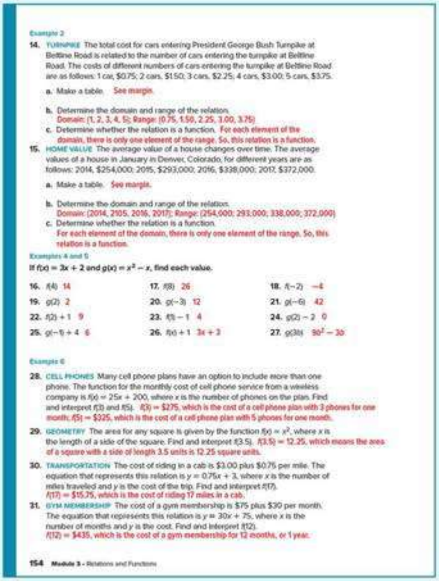
10. $y = 11$ Yes; for any value x, the vertical line passes through no more than one point on the graph.

11. Yes; for any value x, the vertical line passes through no more than one point on the graph.

12. No; for $x = 2$, the vertical line passes through more than one point on the graph.

13. No; for $x > 0$, the vertical line passes through more than one point on the graph.

Lesson 3-2 • Functions 153



Example 2

14. TURNPIKE The total cost for cars entering President George Bush Turnpike at Beltline Road is related to the number of cars entering the turnpike at Beltline Road. The costs of different numbers of cars entering the turnpike at Beltline Road are as follows: 1 car, \$0.75; 2 cars, \$1.50; 3 cars, \$2.25; 4 cars, \$3.00; 5 cars, \$3.75.
    a. Make a table. See margin.
    b. Determine the domain and range of the relation. Domain: {1, 2, 3, 4, 5}; Range: {0.75, 1.50, 2.25, 3.00, 3.75}
    c. Determine whether the relation is a function. For each element of the domain, there is only one element of the range. So, this relation is a function.

15. HOME VALUE The average value of a house changes over time. The average values of a house in January in Denver, Colorado, for different years are as follows: 2014, \$254,000; 2015, \$293,000; 2016, \$338,000; 2017, \$372,000.
    a. Make a table. See margin.
    b. Determine the domain and range of the relation. Domain: {2014, 2105, 2016, 2017}; Range: {254,000; 293,000; 338,000; 372,000}
    c. Determine whether the relation is a function. For each element of the domain, there is only one element of the range. So, this relation is a function.

Examples 4 and 5

If $f(x) = 3x + 2$ and $g(x) = x^2 - x$, find each value.

16. $f(4)$ 14
17. $f(8)$ 26
18. $f(-2)$ −4
19. $g(2)$ 2
20. $g(-3)$ 12
21. $g(-6)$ 42
22. $f(2) + 1$ 9
23. $f(1) - 1$ 4
24. $g(2) - 2$ 0
25. $g(-1) + 4$ 6
26. $f(x) + 1$ $3x + 3$
27. $g(3b)$ $9b^2 - 3b$

Example 6

28. CELL PHONES Many cell phone plans have an option to include more than one phone. The function for the monthly cost of cell phone service from a wireless company is $f(x) = 25x + 200$, where x is the number of phones on the plan. Find and interpret $f(3)$ and $f(5)$. $f(3) = \$275$, which is the cost of a cell phone plan with 3 phones for one month; $f(5) = \$325$, which is the cost of a cell phone plan with 5 phones for one month.

29. GEOMETRY The area for any square is given by the function $f(x) = x^2$, where x is the length of a side of the square. Find and interpret $f(3.5)$. $f(3.5) = 12.25$, which means the area of a square with a side of length 3.5 units is 12.25 square units.

30. TRANSPORTATION The cost of riding in a cab is \$3.00 plus \$0.75 per mile. The equation that represents this relation is $y = 0.75x + 3$, where x is the number of miles traveled and y is the cost of the trip. Find and interpret $f(17)$. $f(17) = \$15.75$, which is the cost of riding 17 miles in a cab.

31. GYM MEMBERSHIP The cost of a gym membership is \$75 plus \$30 per month. The equation that represents this relation is $y = 30x + 75$, where x is the number of months and y is the cost. Find and interpret $f(12)$. $f(12) = \$435$, which is the cost of a gym membership for 12 months, or 1 year.

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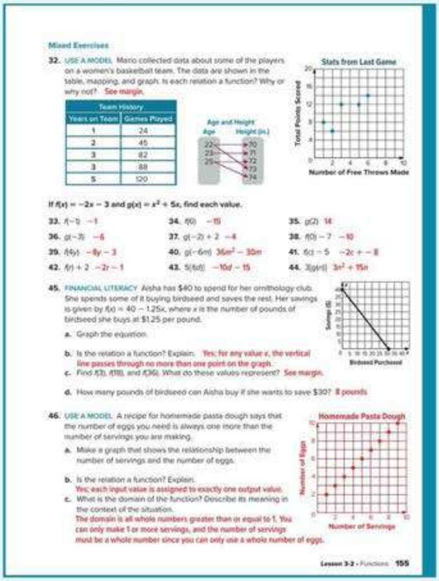


Mixed Exercises

32. USE A MODEL Mario collected data about some of the players on a women's basketball team. The data are shown in the table, mapping, and graph. Is each relation a function? Why or why not? See margin.

Team History	
Years on Team	Games Played
1	24
2	45
3	82
3	88
5	120

If $f(x) = -2x - 3$ and $g(x) = x^2 + 5x$, find each value.

33. $f(-1)$ -1
34. $f(6)$ -15
35. $g(2)$ 14
36. $g(-3)$ -6
37. $g(-2) + 2$ -4
38. $f(0) - 7$ -10
39. $f(4y)$ $-8y - 3$
40. $g(-6m)$ $36m^2 - 30m$
41. $f(c) - 5$ $-2c + -8$
42. $f(r) + 2$ $-2r - 1$
43. $5[f(d)]$ $-10d - 15$
44. $3[g(n)]$ $3n^2 + 15n$

45. FINANCIAL LITERACY Aisha has $40 to spend for her ornithology club. She spends some of it buying birdseed and saves the rest. Her savings is given by $f(x) = 40 - 1.25x$, where x is the number of pounds of birdseed she buys at $1.25 per pound.

a. Graph the equation.

b. Is the relation a function? Explain. Yes; for any value x, the vertical line passes through no more than one point on the graph.

c. Find $f(3)$, $f(18)$, and $f(36)$. What do these values represent? See margin.

d. How many pounds of birdseed can Aisha buy if she wants to save $30? 8 pounds

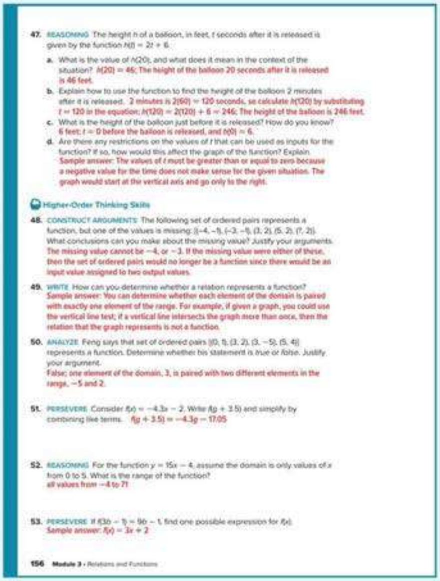
46. USE A MODEL A recipe for homemade pasta dough says that the number of eggs you need is always one more than the number of servings you are making.

a. Make a graph that shows the relationship between the number of servings and the number of eggs.

b. Is the relation a function? Explain.
Yes; each input value is assigned to exactly one output value.

c. What is the domain of the function? Describe its meaning in the context of the situation.
The domain is all whole numbers greater than or equal to 1. You can only make 1 or more servings, and the number of servings must be a whole number since you can only use a whole number of eggs.

Lesson 3-2 • Functions 155



47. REASONING The height h of a balloon, in feet, t seconds after it is released is given by the function $h(t) = 2t + 6$.

a. What is the value of $h(20)$, and what does it mean in the context of the situation? $h(20) = 46$; The height of the balloon 20 seconds after it is released is 46 feet.

b. Explain how to use the function to find the height of the balloon 2 minutes after it is released. 2 minutes is $2(60) = 120$ seconds, so calculate $h(120)$ by substituting $t = 120$ in the equation: $h(120) = 2(120) + 6 = 246$; The height of the balloon is 246 feet.

c. What is the height of the balloon just before it is released? How do you know? 6 feet; $t = 0$ before the balloon is released, and $h(0) = 6$.

d. Are there any restrictions on the values of t that can be used as inputs for the function? If so, how would this affect the graph of the function? Explain. Sample answer: The values of t must be greater than or equal to zero because a negative value for the time does not make sense for the given situation. The graph would start at the vertical axis and go only to the right.

Higher-Order Thinking Skills

48. CONSTRUCT ARGUMENTS The following set of ordered pairs represents a function, but one of the values is missing: $\{(-4, -1), (-3, -1), (3, 2), (5, 2), (7, 2)\}$. What conclusions can you make about the missing value? Justify your arguments. The missing value cannot be −4, or −3. If the missing value were either of these, then the set of ordered pairs would no longer be a function since there would be an input value assigned to two output values.

49. WRITE How can you determine whether a relation represents a function? Sample answer: You can determine whether each element of the domain is paired with exactly one element of the range. For example, if given a graph, you could use the vertical line test; if a vertical line intersects the graph more than once, then the relation that the graph represents is not a function.

50. ANALYZE Feng says that set of ordered pairs $\{(0, 1), (3, 2), (3, -5), (5, 4)\}$ represents a function. Determine whether his statement is *true* or *false*. Justify your argument.
False; one element of the domain, 3, is paired with two different elements in the range, −5 and 2.

51. PERSEVERE Consider $f(x) = -4.3x - 2$. Write $f(g + 3.5)$ and simplify by combining like terms. $f(g + 3.5) = -4.3g - 17.05$

52. REASONING For the function $y = 15x - 4$, assume the domain is only values of x from 0 to 5. What is the range of the function?
all values from −4 to 71

53. PERSEVERE If $f(3b - 1) = 9b - 1$, find one possible expression for $f(x)$.
Sample answer: $f(x) = 3x + 2$

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Answers

14a.

Number of Cars	1	2	3	4	5
Cost ($)	0.75	1.50	2.25	3.00	3.75

15a.

Year	2014	2015	2016	2017
Value ($)	254,000	293,000	338,000	372,000

32. The relation in the table is not a function because 3 maps to both 82 and 88; the relation in the mapping is not a function because 22 maps to both 70 and 73; the relation in the graph is a function because each *x* only maps to one *y*.

45c. $f(3) = 36.25$, which means if Aisha buys 3 pounds of birdseed, she saves $36.25; $f(18) = 17.50$, which means if Aisha buys 18 pounds of birdseed, she saves $17.50; $f(36) = -5$, which means if Aisha wants to buy 36 pounds of birdseed, she needs $5 extra.

Lesson 3-3

Linearity and Continuity of Graphs

LESSON GOAL

Students identify linear and nonlinear functions and continuous and discrete functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Representing Discrete and Continuous Functions

Develop:

Discrete and Continuous Functions

- Determine Continuity
- Determine Continuity by Using Graphs
- Apply Discrete and Continuous Functions

Linear and Nonlinear Functions

- Linear and Nonlinear Functions
- Identify Linear and Nonlinear Functions
- Functions in Table Form
- Identify Linear Functions by Graphing

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Nonlinear Functions	●	●		●
Extension: Point Discontinuity		●	●	●

Language Development Handbook

Assign page 16 of the *Language Development Handbook* to help your students build mathematical language related to linear and nonlinear functions.

ELL You can use the tips and suggestions on page T16 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

Coherence

Vertical Alignment

Previous
Students determined whether a relation was a function and found function values. **8.F.1, F.IF.1, F.IF.2, F.IF.5**

Now
Students identify linear and nonlinear functions and continuous and discrete functions. **F.IF.4, F.IF.5**

Next
Students will find intercepts of graphs.
F.IF.4; A.REI.10

Rigor

The Three Pillars of Rigor

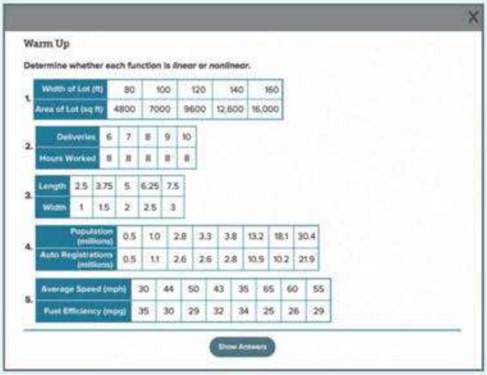
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of functions to include graphs of functions. They apply their understanding by solving real-world problems involving discrete, continuous, linear, and nonlinear functions.

Mathematical Background

The graph of a discrete function consists of points that are not connected. The domain and range of a discrete function are described by sets of individual values. The graph of a continuous function forms a line or smooth curve. The domain and range of a continuous function consist of infinitely many values. A linear function is a function in which no independent variable is raised to a power other than 1. A linear function can be described by a linear equation.

Interactive Presentation



Warm Up

Determine whether each function is *linear* or *nonlinear*.

1.

Width of Lot (ft)	80	100	120	140	160
Area of Lot (sq ft)	4800	7000	9600	12,600	16,000

2.

Deliveries	6	7	8	9	10
Hours Worked	8	8	8	8	8

3.

Length	2.5	3.75	5	6.25	7.5
Width	1	1.5	2	2.5	3

4.

Population (millions)	0.5	1.0	2.8	3.3	3.8	13.2	18.1	30.4
Auto Registrations (millions)	0.5	1.1	2.6	2.6	2.8	10.9	10.2	21.9

5.

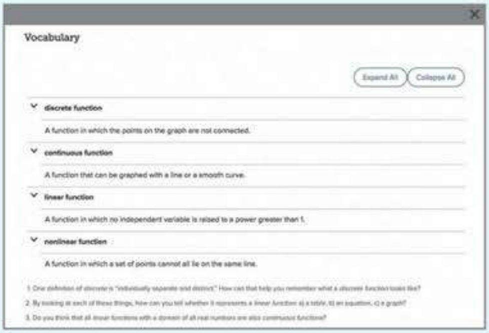
Average Speed (mph)	30	44	50	43	35	65	60	55
Fuel Efficiency (mpg)	35	30	29	32	34	25	26	29

Warm Up

Launch the Lesson

Watch this video to learn about the difference between linear and nonlinear functions.

Launch the Lesson



Vocabulary

discrete function	A function in which the points on the graph are not connected.
continuous function	A function that can be graphed with a line or a smooth curve.
linear function	A function in which no independent variable is raised to a power greater than 1.
nonlinear function	A function in which a set of points cannot all lie on the same line.

1. One definition of discrete is "individually separate and distinct." How can that help you remember what a discrete function looks like?
2. By looking at each of these things, how can you tell whether it represents a linear function a) a table, b) an equation, c) a graph?
3. Do you think that all linear functions with a domain of all real numbers are also continuous functions?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying linear functions

Answers:

1. nonlinear
2. linear
3. linear
4. nonlinear
5. nonlinear

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal descriptions and the graphs in the video.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Representing Discrete and Continuous Functions

Objective

Students use a sketch to explore whether a situation can be represented by a continuous or discrete function.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

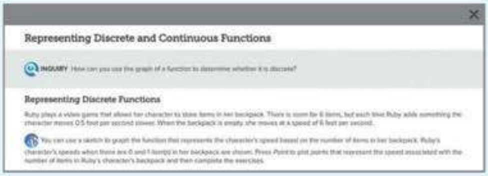
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will use the sketch to create and explore a graph that represents data related to a video game. They will then answer questions about the nature of the graph, requiring them to think about how the graph models the given situation. Then students will complete the Inquiry Question.

(continued on the next page)

Interactive Presentation

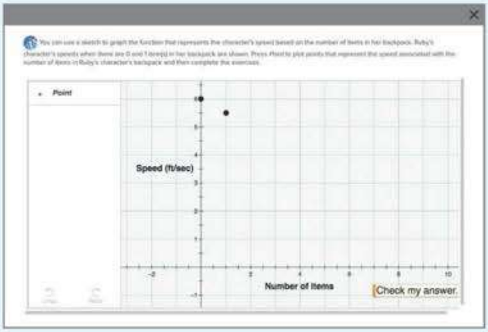


Representing Discrete and Continuous Functions

INQUIRY

Representing Discrete Functions

Explore



Explore

WEB SKETCHPAD

Students use a sketch to determine if a function is discrete.

TYPE

Students answer questions regarding discrete functions.

Interactive Presentation

Explore

TYPE

Students will respond to the Inquiry Question and can view a sample answer.

Explore Representing Discrete and Continuous Functions (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How is measuring the number of items in Ruby's backpack different than measuring time? Sample answer: For items, you can only use whole numbers. When you are measuring time, you can use decimals or fractions for seconds or even milliseconds.
- If a café were counting the bottles of water sold in a day, is this discrete or continuous? What if the café counted the liters of water sold? Sample answer: Counting numbers of water bottles sold would be discrete because you cannot sell part of a water bottle. Counting liters could be continuous because you can measure partial liters.

Inquiry

How can you use the graph of a function to determine whether it is discrete? Sample answer: You can look to see if the graph consists of individual points. If it does, then the function is discrete.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Discrete and Continuous Functions

Objective

Students determine whether functions are continuous, discrete, or neither continuous nor discrete by classifying given functions.

MP Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

Common Misconception

A common misconception some students may have is that if a graph is not a continuous line or curve, it represents a discrete function. Use examples to show them that this is not the case, and reinforce that in order for a graph to represent a discrete function, it must consist only of individual points.

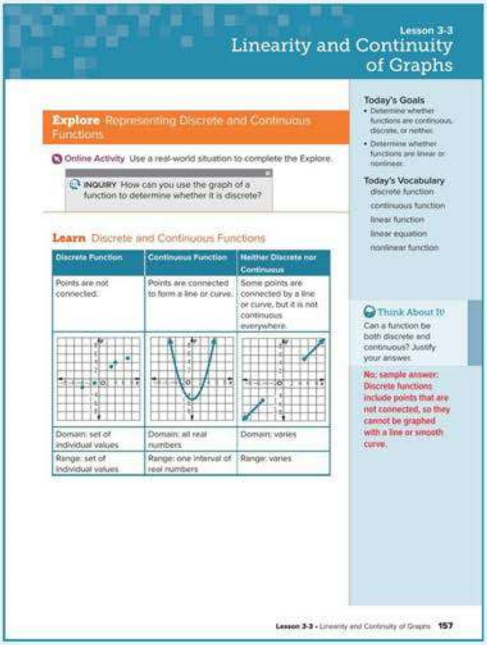
DIFFERENTIATE

Enrichment Activity AL BL ELL

Have students describe real-world situations that can be described by discrete functions, and real-world situations that can be described by continuous functions. Encourage students to explain what it is about each situation that makes the related function discrete or continuous.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-3

Linearity and Continuity of Graphs

Explore Representing Discrete and Continuous Functions

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you use the graph of a function to determine whether it is discrete?

Learn Discrete and Continuous Functions

Discrete Function	Continuous Function	Neither Discrete nor Continuous
Points are not connected.	Points are connected to form a line or curve.	Some points are connected by a line or curve, but it is not continuous everywhere.
Domain: set of individual values	Domain: all real numbers	Domain: varies
Range: set of individual values	Range: one interval of real numbers	Range: varies

Today's Goals
- Determine whether functions are continuous, discrete, or neither.
- Determine whether functions are linear or nonlinear.

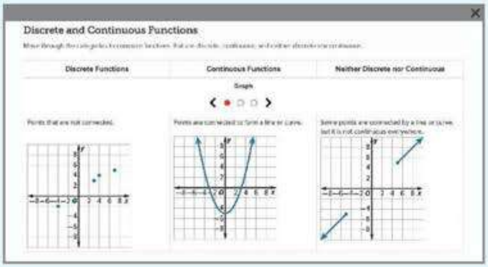
Today's Vocabulary
discrete function
continuous function
linear function
linear equation
nonlinear function

Think About It!
Can a function be both discrete and continuous? Justify your answer.

No; sample answer: Discrete functions include points that are not connected, so they cannot be graphed with a line or smooth curve.

Lesson 3-3 • Linearity and Continuity of Graphs 157

Interactive Presentation



Discrete and Continuous Functions

Discrete Functions | Continuous Functions | Neither Discrete nor Continuous

Graph

Points that are not connected.

Points are connected to form a line or curve.

Some points are connected by a line or curve, but it is not continuous everywhere.

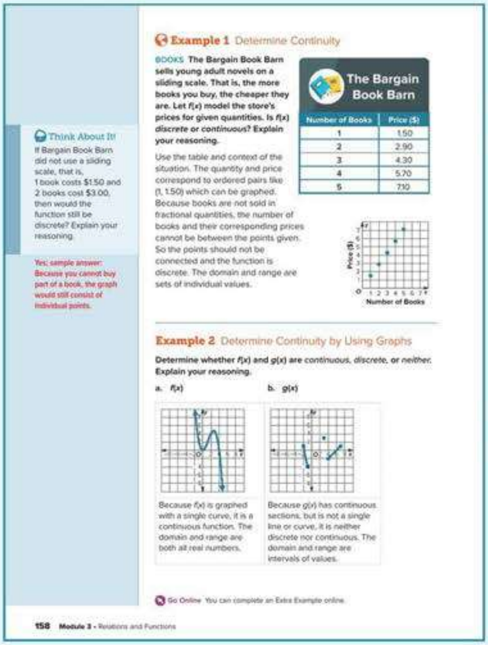
Learn

TAP

Students move through the categories to compare functions that are discrete, continuous, or neither.

TYPE

Students determine if a function can be both discrete and continuous.



Think About It!

If Bargain Book Barn did not use a sliding scale, that is, 1 book costs $1.50 and 2 books cost $3.00, then would the function still be discrete? Explain your reasoning.

Yes; sample answer: Because you cannot buy part of a book, the graph would still consist of individual points.

Example 1 Determine Continuity

BOOKS **The Bargain Book Barn sells young adult novels on a sliding scale. That is, the more books you buy, the cheaper they are. Let *f*(*x*) model the store's prices for given quantities. Is *f*(*x*) discrete or continuous? Explain your reasoning.**

The Bargain Book Barn

Number of Books	Price ($)
1	1.50
2	2.90
3	4.30
4	5.70
5	7.10

Use the table and context of the situation. The quantity and price correspond to ordered pairs like (1, 1.50) which can be graphed. Because books are not sold in fractional quantities, the number of books and their corresponding prices cannot be between the points given. So the points should not be connected and the function is discrete. The domain and range are sets of individual values.

Example 2 Determine Continuity by Using Graphs

Determine whether *f*(*x*) and *g*(*x*) are *continuous, discrete,* or *neither.* Explain your reasoning.

a. *f*(*x*)

Because *f*(*x*) is graphed with a single curve, it is a continuous function. The domain and range are both all real numbers.

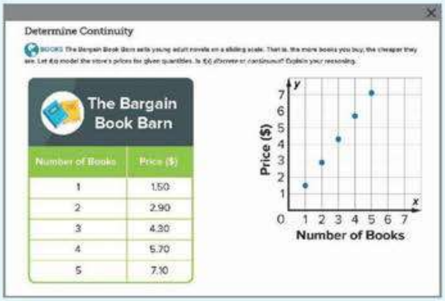
b. *g*(*x*)

Because *g*(*x*) has continuous sections, but is not a single line or curve, it is neither discrete nor continuous. The domain and range are intervals of values.

Go Online You can complete an Extra Example online.

158 Module 3 • Relations and Functions

Interactive Presentation



Determine Continuity

BOOKS The Bargain Book Barn sells young adult novels on a sliding scale. That is, the more books you buy, the cheaper they are. Let *f*(*x*) model the store's prices for given quantities. Is *f*(*x*) discrete or continuous? Explain your reasoning.

The Bargain Book Barn

Number of Books	Price ($)
1	1.50
2	2.90
3	4.30
4	5.70
5	7.10

Example 1

TYPE

Students determine if the function will still be discrete if they did not use a sliding scale.

TAP

Students tap to see how to determine continuity.

Example 1 Determine Continuity

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about continuity to solving a real-world problem.

Questions for Mathematical Discourse

AL Is the relation a function? Explain. Yes; each *x*-value is mapped to only one *y*-value.

OL Why is the number of books the independent variable and the price the dependent variable? Sample answer: The price depends on how many books someone buys, so it has to be the dependent variable.

BL Could the store adjust its pricing so that the function representing price was continuous? Explain. Sample answer: No; because you cannot have a part or a piece of a book.

Example 2 Determine Continuity by Using Graphs

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to justify their conclusions.

Questions for Mathematical Discourse

AL Do either of these graphs represent a function? Explain. Yes, they are both functions. No *x*-value is mapped to more than one *y*-value.

OL How do discrete functions and functions that are neither continuous nor discrete differ? Sample answer: Discrete functions are composed of single, unconnected points. Functions that are neither continuous nor discrete may have some points that are connected, but the graph is not one continuous line or curve.

BL Which parts of *g*(*x*) are continuous? Sample answer: for *x* values from −2 to −1 and from 4 to 7

Example 3 Apply Discrete and Continuous Functions

Teaching the Mathematical Practices

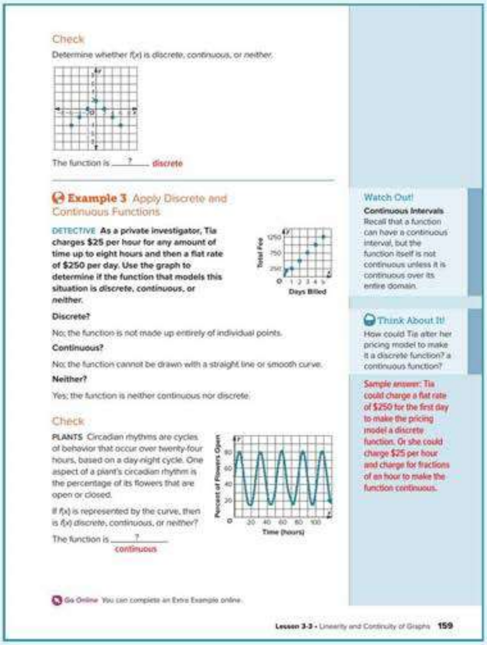
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others. Have them use the terms discrete and continuous instead of descriptive words like connected or broken.

Questions for Mathematical Discourse

AL Are there any portions of this graph that are continuous? If so, where? yes; from 0 to 8 hours

OL Tia finishes an investigation after 4 days of work. How much will she charge her client? $1000

BL Why is this function not continuous? Sample answer: The graph begins as a continuous line, but then changes to have points at specific *x*-values. Because of the change, it cannot be considered continuous.



Check

Determine whether *f*(*x*) is discrete, continuous, or neither.

The function is ___?___ discrete

Example 3 Apply Discrete and Continuous Functions

DETECTIVE **As a private investigator, Tia charges $25 per hour for any amount of time up to eight hours and then a flat rate of $250 per day. Use the graph to determine if the function that models this situation is *discrete*, *continuous*, or *neither*.**

Discrete?

No; the function is not made up entirely of individual points.

Continuous?

No; the function cannot be drawn with a straight line or smooth curve.

Neither?

Yes; the function is neither continuous nor discrete.

Check

PLANTS Circadian rhythms are cycles of behavior that occur over twenty-four hours, based on a day-night cycle. One aspect of a plant's circadian rhythm is the percentage of its flowers that are open or closed.

If *f*(*x*) is represented by the curve, then is *f*(*x*) discrete, continuous, or neither?

The function is ___?___ continuous

Go Online You can complete an Extra Example online.

Watch Out!

Continuous Intervals Recall that a function can have a continuous interval, but the function itself is not continuous unless it is continuous over its entire domain.

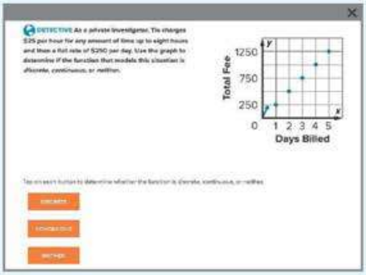
Think About It!

How could Tia alter her pricing model to make it a discrete function? a continuous function?

Sample answer: Tia could charge a flat rate of $250 for the first day to make the pricing model a discrete function. Or she could charge $25 per hour and charge for fractions of an hour to make the function continuous.

Lesson 3-3 • Linearity and Continuity of Graphs 159

Interactive Presentation



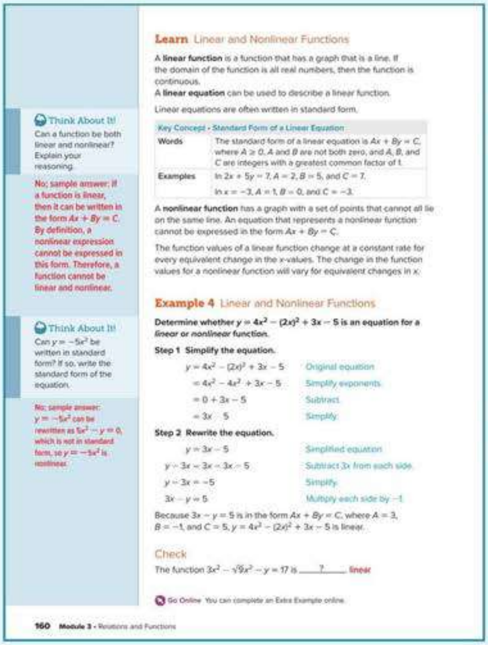
Example 3

TYPE

Students explain how they could alter the model to make the function discrete or continuous.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Think About It! Can a function be both linear and nonlinear? Explain your reasoning.

No; sample answer: If a function is linear, then it can be written in the form $Ax + By = C$. By definition, a nonlinear expression cannot be expressed in this form. Therefore, a function cannot be linear and nonlinear.

Think About It! Can $y = -5x^2$ be written in standard form? If so, write the standard form of the equation.

No; sample answer: $y = -5x^2$ can be rewritten as $5x^2 - y = 0$, which is not in standard form, so $y = -5x^2$ is nonlinear.

Learn Linear and Nonlinear Functions

A **linear function** is a function that has a graph that is a line. If the domain of the function is all real numbers, then the function is continuous.

A **linear equation** can be used to describe a linear function.

Linear equations are often written in standard form.

Key Concept • Standard Form of a Linear Equation

Words	The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1.
Examples	In $2x + 5y = 7$, $A = 2$, $B = 5$, and $C = 7$. In $x = -3$, $A = 1$, $B = 0$, and $C = -3$.

A **nonlinear function** has a graph with a set of points that cannot all lie on the same line. An equation that represents a nonlinear function cannot be expressed in the form $Ax + By = C$.

The function values of a linear function change at a constant rate for every equivalent change in the x-values. The change in the function values for a nonlinear function will vary for equivalent changes in x.

Example 4 Linear and Nonlinear Functions

Determine whether $y = 4x^2 - (2x)^2 + 3x - 5$ is an equation for a *linear or nonlinear* function.

Step 1 Simplify the equation.

$y = 4x^2 - (2x)^2 + 3x - 5$ Original equation
$= 4x^2 - 4x^2 + 3x - 5$ Simplify exponents.
$= 0 + 3x - 5$ Subtract.
$= 3x - 5$ Simplify.

Step 2 Rewrite the equation.

$y = 3x - 5$ Simplified equation
$y - 3x = 3x - 3x - 5$ Subtract 3x from each side.
$y - 3x = -5$ Simplify.
$3x - y = 5$ Multiply each side by −1.

Because $3x - y = 5$ is in the form $Ax + By = C$, where $A = 3$, $B = -1$, and $C = 5$, $y = 4x^2 - (2x)^2 + 3x - 5$ is linear.

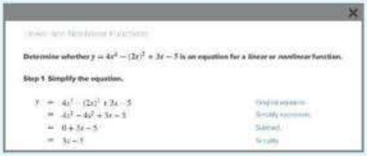
Check

The function $3x^2 - \sqrt{9x^2} - y = 17$ is ___?___ linear

Go Online You can complete an Extra Example online.

160 Module 3 • Relations and Functions

Interactive Presentation



Determine whether $y = 4x^2 - (2x)^2 + 3x - 5$ is an equation for a linear or nonlinear function.

Step 1 Simplify the equation.

$y = 4x^2 - (2x)^2 + 3x - 5$
$= 4x^2 - 4x^2 + 3x - 5$
$= 0 + 3x - 5$
$= 3x - 5$

Example 4

TYPE

Students explain if a function can be written in standard form.

Learn Linear and Nonlinear Functions

Objective

Students determine whether functions are linear or nonlinear by rewriting the related equations in standard form.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

A common misconception some students may have is that a linear equation must contain both *x* and *y*. Help them to see what happens to the equation and the resulting graph if $A = 0$ (produces an equation and graph of a horizontal line), and then if $B = 0$ (produces an equation and graph of a vertical line).

Example 4 Linear and Nonlinear Functions

Teaching the Mathematical Practices

1 Seek Information Mathematically proficient students must be able to transform algebraic expressions to reach solutions. Point out that gaining fluency in this skill is as important as learning their math facts was in the elementary grades.

Questions for Mathematical Discourse

AL What is the standard form of a linear equation? $Ax + By = C$

OL Why do you think that $y = 4x^2 - (2x)^2 + 3x - 5$ is sometimes mistakenly thought to be nonlinear? Sample answer: There are variables raised to a power greater than 1.

BL Can the equation $y = 2$ be written in standard form? If so, write the equation. yes; $0x + y = 2$

Common Error

Students may write the answer to Example 4 as $-3x + y = -5$. Remind them that *A*, the coefficient of *x*, must be greater than or equal to 0.

Example 5 Identify Linear and Nonlinear Functions

Teaching the Mathematical Practices

3 Construct Arguments In the Think About It! feature, students use stated assumptions, definitions, and previously established results to construct an argument about whether a function is linear or nonlinear.

Questions for Mathematical Discourse

AL In a linear function, what can be the values of the exponents on the variables? 0 or 1

OL How can you tell that the function $y = 3x^3 - x^3 + 3x + 6$ is nonlinear? Sample answer: There are variables raised to a power greater than 1 in the equation of the function after it is simplified.

BL Write an equation that simplifies to a linear function, but has variables raised to a power greater than 1. Sample answer: $y = \overset{5}{5}x + 3x \overset{3}{-} 5x + 10$

Example 6 Functions in Table Form

Teaching the Mathematical Practices

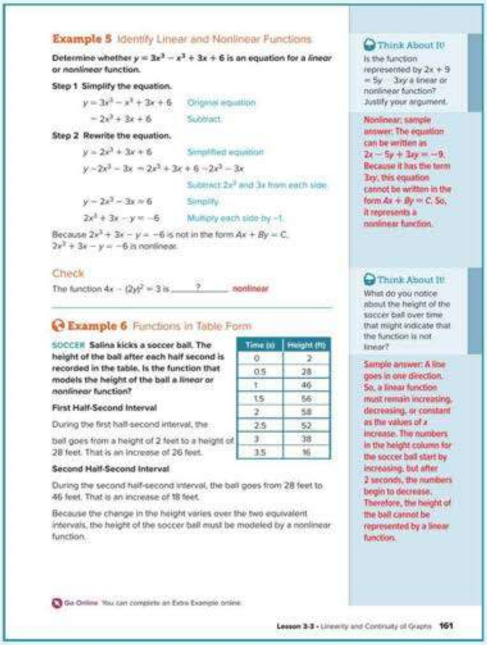
7 Use Structure Help students explore the structure of the table in this example to determine if the function is linear.

Questions for Mathematical Discourse

AL What is another way that you could use the information in the table to determine whether the function is linear or nonlinear? Sample answer: Plot the data on a coordinate plane.

OL Why can you use the table to determine whether the function is linear or nonlinear? Sample answer: Linear functions must show a constant increase or decrease.

BL In the context of this situation, would it make sense for the data to be linear? Explain. Sample answer: No; because a ball must come back down once it is kicked, the data will show heights that are first increasing and then heights that are decreasing. There will not be a constant rate of change.



Example 5 Identify Linear and Nonlinear Functions

Determine whether $y = 3x^3 - x^3 + 3x + 6$ is an equation for a *linear* or *nonlinear* function.

Step 1 Simplify the equation.

$y = 3x^3 - x^3 + 3x + 6$ Original equation

$= 2x^3 + 3x + 6$ Subtract.

Step 2 Rewrite the equation.

$y = 2x^3 + 3x + 6$ Simplified equation

$y - 2x^3 - 3x = 2x^3 + 3x + 6 - 2x^3 - 3x$ Subtract $2x^3$ and $3x$ from each side.

$y - 2x^3 - 3x = 6$ Simplify.

$2x^3 + 3x - y = -6$ Multiply each side by −1.

Because $2x^3 + 3x - y = -6$ is not in the form $Ax + By = C$, $2x^3 + 3x - y = -6$ is nonlinear.

Check

The function $4x - (2y)^2 = 3$ is ___?___. nonlinear

Example 6 Functions in Table Form

SOCCER **Salina kicks a soccer ball. The height of the ball after each half second is recorded in the table. Is the function that models the height of the ball a linear or nonlinear function?**

Time (s)	Height (ft)
0	2
0.5	28
1	46
1.5	56
2	58
2.5	52
3	38
3.5	16

First Half-Second Interval

During the first half-second interval, the ball goes from a height of 2 feet to a height of 28 feet. That is an increase of 26 feet.

Second Half-Second Interval

During the second half-second interval, the ball goes from 28 feet to 46 feet. That is an increase of 18 feet.

Because the change in the height varies over the two equivalent intervals, the height of the soccer ball must be modeled by a nonlinear function.

Go Online You can complete an Extra Example online.

Think About It!
Is the function represented by $2x + 9 = 5y - 3xy$ a linear or nonlinear function? Justify your argument.

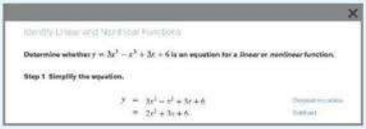
Nonlinear; sample answer: The equation can be written as $2x - 5y + 3xy = -9$. Because it has the term $3xy$, this equation cannot be written in the form $Ax + By = C$. So, it represents a nonlinear function.

Think About It!
What do you notice about the height of the soccer ball over time that might indicate that the function is not linear?

Sample answer: A line goes in one direction. So, a linear function must remain increasing, decreasing, or constant as the values of x increase. The numbers in the height column for the soccer ball start by increasing, but after 2 seconds, the numbers begin to decrease. Therefore, the height of the ball cannot be represented by a linear function.

Lesson 3-3 • Linearity and Continuity of Graphs 161

Interactive Presentation



Identify Linear and Nonlinear Functions

Determine whether $y = 3x^3 - x^3 + 3x + 6$ is an equation for a *linear* or *nonlinear* function.

Step 1 Simplify the equation.

$y = 3x^3 - x^3 + 3x + 6$ Original equation

$= 2x^3 + 3x + 6$ Subtract

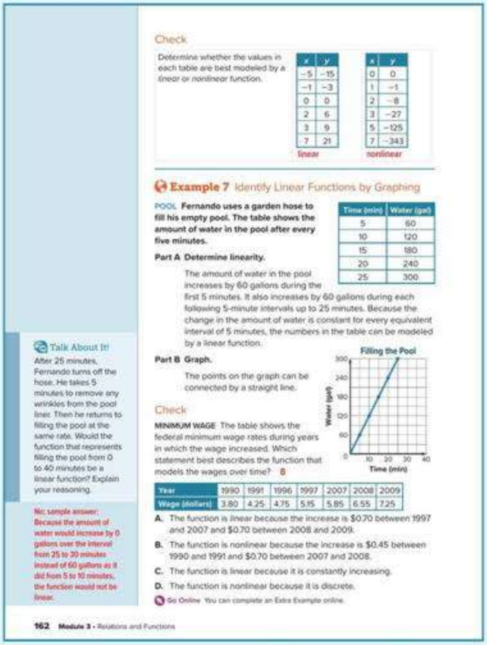
Example 5

SELECT

Students select the correct operation used when simplifying expressions.

TYPE

Students determine if a given equation represents a linear or nonlinear function.



Check

Determine whether the values in each table are best modeled by a linear or nonlinear function.

x	y
−5	−15
−1	−3
0	0
2	6
3	9
7	21

linear

x	y
0	0
1	−1
2	−8
3	−27
5	−125
7	−343

nonlinear

Example 7 Identify Linear Functions by Graphing

POOL **Fernando uses a garden hose to fill his empty pool. The table shows the amount of water in the pool after every five minutes.**

Time (min)	Water (gal)
5	60
10	120
15	180
20	240
25	300

Part A Determine linearity.

The amount of water in the pool increases by 60 gallons during the first 5 minutes. It also increases by 60 gallons during each following 5-minute intervals up to 25 minutes. Because the change in the amount of water is constant for every equivalent interval of 5 minutes, the numbers in the table can be modeled by a linear function.

Part B Graph.

The points on the graph can be connected by a straight line.

Talk About It!

After 25 minutes, Fernando turns off the hose. He takes 5 minutes to remove any wrinkles from the pool liner. Then he returns to filling the pool at the same rate. Would the function that represents filling the pool from 0 to 40 minutes be a linear function? Explain your reasoning.

No; sample answer: Because the amount of water would increase by 0 gallons over the interval from 25 to 30 minutes instead of 60 gallons as it did from 5 to 10 minutes, the function would not be linear.

Check

MINIMUM WAGE The table shows the federal minimum wage rates during years in which the wage increased. Which statement best describes the function that models the wages over time? B

Year	1990	1991	1996	1997	2007	2008	2009
Wage (dollars)	3.80	4.25	4.75	5.15	5.85	6.55	7.25

A. The function is linear because the increase is $0.70 between 1997 and 2007 and $0.70 between 2008 and 2009.

B. The function is nonlinear because the increase is $0.45 between 1990 and 1991 and $0.70 between 2007 and 2008.

C. The function is linear because it is constantly increasing.

D. The function is nonlinear because it is discrete.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Identify Linear Functions by Graphing

POOL Fernando uses a garden hose to fill his empty pool. The table shows the amount of water in the pool after every five minutes.

Time (min)	Water (gal)
5	60
10	120
15	180
20	240
25	300

Example 7

TAP

Students move through the two parts to determine if the table represents a linear function.

TYPE

Students explain if a new scenario represents a linear function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 7 Identify Linear Functions by Graphing

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL Why is it reasonable to expect that this situation would be represented by a function that is linear? Sample answer: because water from a garden hose flows at a constant rate

OL How can you test that the plotted points model a linear function? Sample answer: Check that the points form a straight line by using a ruler or straightedge.

BL Could this function be represented by a continuous line? Explain. Sample answer: Yes; although the table and the graph show only 5 points, each point on the line that connects those points represents another number of minutes and number of gallons as the pool is filling.

Essential Question Follow-Up

Students analyze a table of values and the related graph of a function for a real-world situation.

Ask:

Why is it useful to have a graph of a function for a real-world situation? Sample answer: You can use the graph to obtain data values for the situation, and to get a picture of how those data values are related. This can give you the information you need to understand the situation.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–25
2	exercises that use a variety of skills from this lesson	26–33
3	exercises that emphasize higher-order and critical-thinking skills	34–36

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–33 odd, 34–36
- Extension: Point Discontinuity
- **ALEKS** Other Topics Available: Functions and Lines

OL

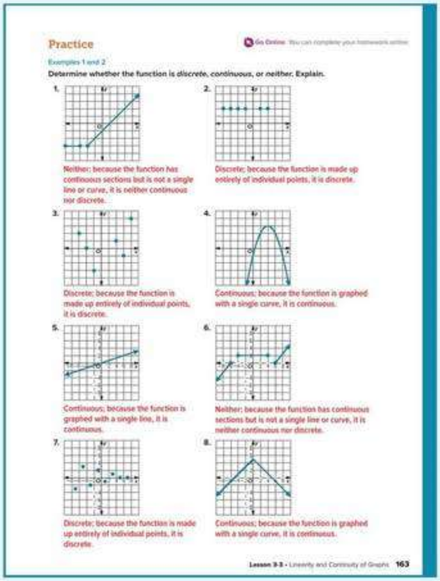
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–35 odd
- Remediation, Review Resources: Nonlinear Functions
- Personal Tutors
- Extra Examples 1–7
- **ALEKS** Scatter Plots and Lines of Best Fit

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–25 odd
- Remediation, Review Resources: Nonlinear Functions
- *Quick Review Math Handbook*: Interpreting Graphs of Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Scatter Plots and Lines of Best Fit



Practice

Go Online You can complete your homework online.

Examples 1 and 2

Determine whether the function is discrete, continuous, or neither. Explain.

1. Neither; because the function has continuous sections but is not a single line or curve, it is neither continuous nor discrete.

2. Discrete; because the function is made up entirely of individual points, it is discrete.

3. Discrete; because the function is made up entirely of individual points, it is discrete.

4. Continuous; because the function is graphed with a single curve, it is continuous.

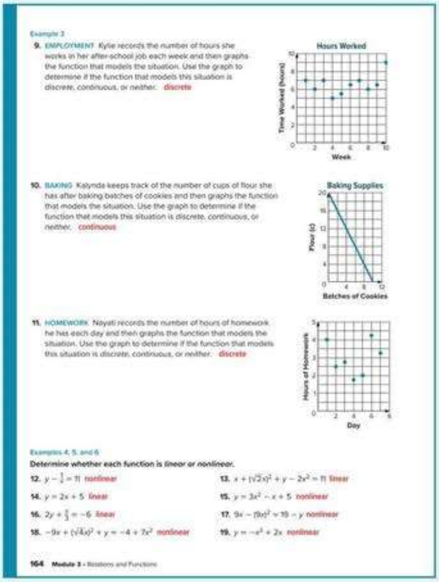
5. Continuous; because the function is graphed with a single line, it is continuous.

6. Neither; because the function has continuous sections but is not a single line or curve, it is neither continuous nor discrete.

7. Discrete; because the function is made up entirely of individual points, it is discrete.

8. Continuous; because the function is graphed with a single curve, it is continuous.

Lesson 3-3 • Linearity and Continuity of Graphs 163



Example 3

9. EMPLOYMENT Kylie records the number of hours she works in her after-school job each week and then graphs the function that models the situation. Use the graph to determine if the function that models this situation is discrete, continuous, or neither. discrete

10. BAKING Kalynda keeps track of the number of cups of flour she has after baking batches of cookies and then graphs the function that models the situation. Use the graph to determine if the function that models this situation is discrete, continuous, or neither. continuous

11. HOMEWORK Nayati records the number of hours of homework he has each day and then graphs the function that models the situation. Use the graph to determine if the function that models this situation is discrete, continuous, or neither. discrete

Examples 4, 5, and 6

Determine whether each function is *linear* or *nonlinear*.

12. $y-\frac{1}{x}=11$ nonlinear

13. $x+(\sqrt{2}x)^2+y-2x^2=11$ linear

14. $y=2x+5$ linear

15. $y=3x^2-x+5$ nonlinear

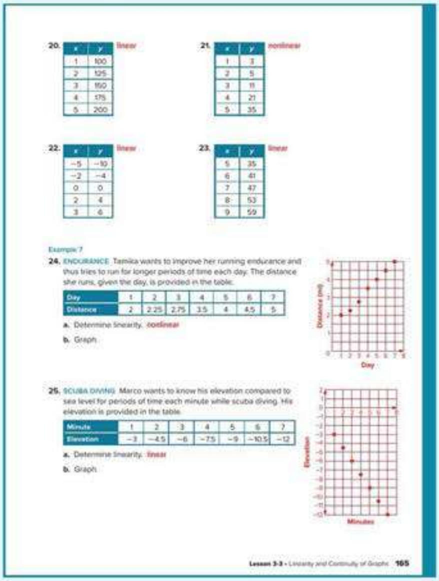
16. $2y+\frac{x}{3}=-6$ linear

17. $9x-(9x)^2=19-y$ nonlinear

18. $-9x+(\sqrt{4}x)^2+y=-4+7x^2$ nonlinear

19. $y=-x^3+2x$ nonlinear

164 Module 3 • Relations and Functions



20. linear

x	y
1	100
2	125
3	150
4	175
5	200

21. nonlinear

x	y
1	3
2	5
3	11
4	21
5	35

22. linear

x	y
−5	−10
−2	−4
0	0
2	4
3	6

23. linear

x	y
5	35
6	41
7	47
8	53
9	59

Example 7

24. ENDURANCE Tamika wants to improve her running endurance and thus tries to run for longer periods of time each day. The distance she runs, given the day, is provided in the table.

Day	1	2	3	4	5	6	7
Distance	2	2.25	2.75	3.5	4	4.5	5

a. Determine linearity. nonlinear

b. Graph.

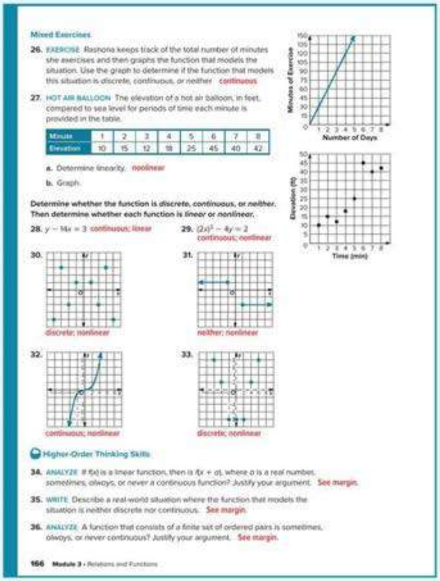
25. SCUBA DIVING Marco wants to know his elevation compared to sea level for periods of time each minute while scuba diving. His elevation is provided in the table.

Minute	1	2	3	4	5	6	7
Elevation	−3	−4.5	−6	−7.5	−9	−10.5	−12

a. Determine linearity. linear

b. Graph.

Lesson 3-3 • Linearity and Continuity of Graphs 165



Mixed Exercises

26. EXERCISE Rashona keeps track of the total number of minutes she exercises and then graphs the function that models the situation. Use the graph to determine if the function that models this situation is discrete, continuous, or neither. continuous

27. HOT AIR BALLOON The elevation of a hot air balloon, in feet, compared to sea level for periods of time each minute is provided in the table.

Minute	1	2	3	4	5	6	7	8
Elevation	10	15	12	18	25	45	40	42

a. Determine linearity. nonlinear

b. Graph.

Determine whether the function is discrete, continuous, or *neither*. Then determine whether each function is *linear* or *nonlinear*.

28. $y - 14x = 3$ continuous; linear

29. $(2x)^2 - 4y = 2$ continous; nonlinear

30. discrete; nonlinear

31. neither; nonlinear

32. continuous; nonlinear

33. discrete; nonlinear

Higher-Order Thinking Skills

34. ANALYZE If $f(x)$ is a linear function, then is $f(x + a)$, where a is a real number, sometimes, always, or never a continuous function? Justify your argument. See margin.

35. WRITE Describe a real-world situation where the function that models the situation is neither discrete nor continuous. See margin.

36. ANALYZE A function that consists of a finite set of ordered pairs is sometimes, always, or never continuous? Justify your argument. See margin.

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Answers

34. Sample answer: Always; if a function is linear, then it can be written in the form $Ax + By = C$. If x is increased by a real number a, then the new function could be written as $A(x + a) + By = C$. This can be rewritten as $Ax + Aa + By = C$, or $Ax + By = C - Aa$. Because C, A, and a are all real numbers, the expression $C - Aa$ is a real number, which means $Ax + By = C - Aa$ is also a linear function.

35. Sample answer: A studio charges musicians to use the space and recording equipment by the hour, rounding a fraction of an hour up. So, for up to 1 hour, the studio charges $100, but for up to 2 hours, the studio charges $200, and so on. The function that models this situation is neither discrete nor continuous.

36. Never; if a function consists of a finite set of ordered pairs, then the function is made up of a set of individual points. Thus, itis by definition discrete.

Lesson 3-4

Intercepts of Graphs

A.REI.10, F.IF.4

LESSON GOAL

Students identify intercepts of functions and solve equations by graphing.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Intercepts of Graphs of Functions

- Intercepts of the Graph of a Linear Function
- Intercepts of the Graph of a Nonlinear Function
- Find Intercepts from a Graph
- Find Intercepts from a Table

Solving Equations by Graphing

- Solve a Linear Equation by Graphing
- Solve a Nonlinear Equation by Graphing
- Solve an Equation of a Horizontal Line by Graphing
- Estimate Solutions by Graphing

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: The Coordinate Plane	●	●		●
Extension: Even and Odd Functions		●	●	●

Language Development Handbook

Assign page 17 of the *Language Development Handbook* to help your students build mathematical language related to solving equations by graphing.

ELL You can use the tips and suggestions on page T17 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous

Students interpreted the *y*-intercept of a linear function, including reading from a graph.

8.F.4

Now

Students identify intercepts of functions and solve equations by graphing.

A.REI.10, F.IF.4

Next

Students will identify characteristics of functions.

F.IF.4

Rigor

The Three Pillars of Rigor

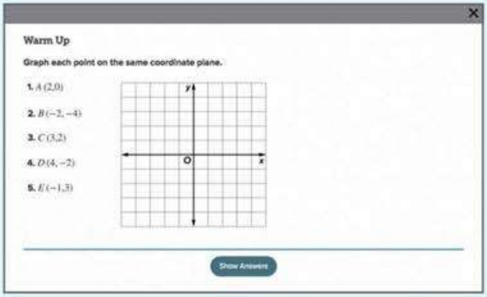
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of functions by finding the intercepts of graphs of functions. They apply their understanding by solving real-world problems that require them to interpret the intercepts of graphs.

Mathematical Background

The intercepts of the graph of a function are the points where the graph intersects the *x*-axis and the *y*-axis.

Interactive Presentation

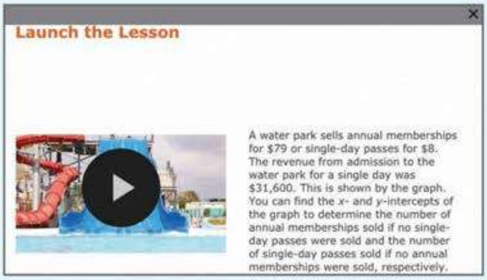


Warm Up

Graph each point on the same coordinate plane.

1. $A(2,0)$
2. $B(-2,-4)$
3. $C(3,2)$
4. $D(4,-2)$
5. $E(-1,3)$

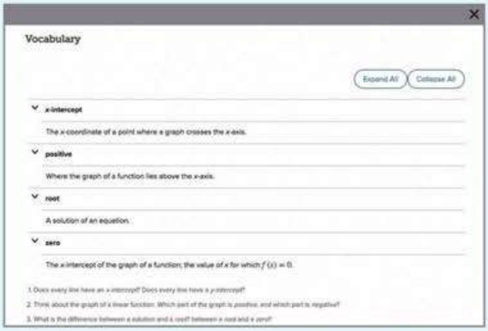
Warm Up



Launch the Lesson

A water park sells annual memberships for \$79 or single-day passes for \$8. The revenue from admission to the water park for a single day was \$31,600. This is shown by the graph. You can find the x- and y-intercepts of the graph to determine the number of annual memberships sold if no single-day passes were sold and the number of single-day passes sold if no annual memberships were sold, respectively.

Launch the Lesson



Vocabulary

x-intercept

The x-coordinate of a point where a graph crosses the x-axis.

positive

Where the graph of a function lies above the x-axis.

root

A solution of an equation.

zero

The x-intercept of the graph of a function; the value of x for which $f(x) = 0$.

1. Does every line have an x-intercept? Does every line have a y-intercept?
2. Think about the graph of a linear function. Which part of the graph is positive, and which part is negative?
3. What is the difference between a solution and a root? between a root and a zero?

Today's Vocabulary

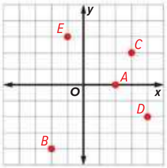
Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- graphing ordered pairs

Answers:



Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the verbal description about water park admission and the corresponding graph.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Learn Intercepts of Graphs of Functions

Objective

Students analyze the graphs or tables of functions to identify the intercepts of the functions and determine their meaning in real-world contexts.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

A common misconception some students may have is that a function is negative where its graph lies to the left of the *y*-axis, and positive where its graph lies to the right of the *y*-axis. Explain that the references to “positive” and “negative” are to the *y*-values, so these terms refer to where the *y*-values are positive or negative, which are, respectively, above the *x*-axis and below the *x*-axis.

Example 1 Intercepts of the Graph of a Linear Function

Teaching the Mathematical Practices

3 Construct Arguments In the Think About It! feature, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL How many times does the graph cross the *x*-axis? 1 the *y*-axis? 1

OL Does the graph have parts that lie above the *x*-axis? yes below the *x*-axis? yes What does this mean about the function? It is positive over one interval and negative over another.

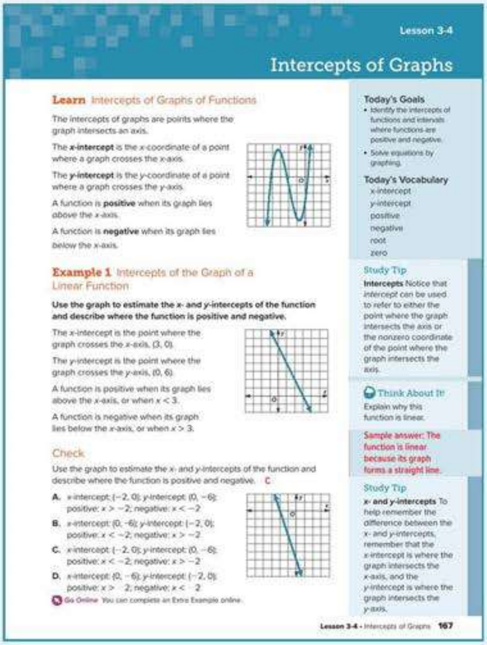
BL Can a linear function be positive or negative over more than one interval of *x*-values? Explain your reasoning. Sample answer: No; the graph of a linear function is a straight line, so it will not cross the *x*-axis more than once.

Common Error

Some students may think that a function is negative when $x < 0$. Help them to see that although the *x*-values for that piece of the graph are negative, the *y*-values are positive. Emphasize that it is the sign of the *y*-values that determines where the function is positive or negative.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-4

Intercepts of Graphs

Learn Intercepts of Graphs of Functions

The intercepts of graphs are points where the graph intersects an axis.

The ***x*-intercept** is the *x*-coordinate of a point where a graph crosses the *x*-axis.

The ***y*-intercept** is the *y*-coordinate of a point where a graph crosses the *y*-axis.

A function is **positive** when its graph lies above the *x*-axis.

A function is **negative** when its graph lies below the *x*-axis.

Example 1 Intercepts of the Graph of a Linear Function

Use the graph to estimate the *x*- and *y*-intercepts of the function and describe where the function is positive and negative.

The *x*-intercept is the point where the graph crosses the *x*-axis, (3, 0).

The *y*-intercept is the point where the graph crosses the *y*-axis, (0, 6).

A function is positive when its graph lies above the *x*-axis, or when $x < 3$.

A function is negative when its graph lies below the *x*-axis, or when $x > 3$.

Check

Use the graph to estimate the *x*- and *y*-intercepts of the function and describe where the function is positive and negative. C

A. *x*-intercept: (−2, 0); *y*-intercept: (0, −6); positive: $x > -2$; negative: $x < -2$

B. *x*-intercept: (0, −6); *y*-intercept: (−2, 0); positive: $x < -2$; negative: $x > -2$

C. *x*-intercept: (−2, 0); *y*-intercept: (0, −6); positive: $x < -2$; negative: $x > -2$

D. *x*-intercept: (0, −6); *y*-intercept: (−2, 0); positive: $x > -2$; negative: $x < -2$

Go Online You can complete an Extra Example online.

Today's Goals
- Identify the intercepts of functions and intervals where functions are positive and negative.
- Solve equations by graphing.

Today's Vocabulary
x-intercept
y-intercept
positive
negative
root
zero

Study Tip

Intercepts Notice that *intercept* can be used to refer to either the point where the graph intersects the axis or the nonzero coordinate of the point where the graph intersects the axis.

Think About It!

Explain why this function is linear.

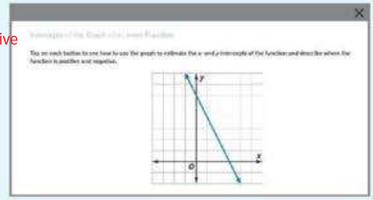
Sample answer: The function is linear because its graph forms a straight line.

Study Tip

***x*- and *y*-intercepts** To help remember the difference between the *x*- and *y*-intercepts, remember that the *x*-intercept is where the graph intersects the *x*-axis, and the *y*-intercept is where the graph intersects the *y*-axis.

Lesson 3-4 • Intercepts of Graphs 167

Interactive Presentation



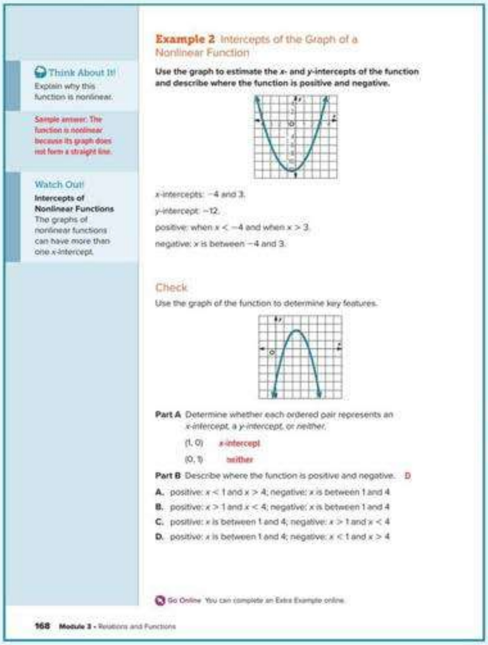
Example 1

TAP

Students tap on each button to see what portions of the graph of a function represent the intercepts and intervals where the function is positive or negative.

TYPE

Students explain why the given function is linear.



Think About It!

Explain why this function is nonlinear.

Sample answer: The function is nonlinear because its graph does not form a straight line.

Watch Out!

Intercepts of Nonlinear Functions The graphs of nonlinear functions can have more than one x-intercept.

Example 2 Intercepts of the Graph of a Nonlinear Function

Use the graph to estimate the x- and y-intercepts of the function and describe where the function is positive and negative.

x-intercepts: -4 and 3.

y-intercept: -12.

positive: when $x < -4$ and when $x > 3$.

negative: x is between -4 and 3.

Check

Use the graph of the function to determine key features.

Part A Determine whether each ordered pair represents an x-intercept, a y-intercept, or *neither*.

(1, 0) x-intercept

(0, 1) neither

Part B Describe where the function is positive and negative. D

A. positive: $x < 1$ and $x > 4$; negative: x is between 1 and 4

B. positive: $x > 1$ and $x < 4$; negative: x is between 1 and 4

C. positive: x is between 1 and 4; negative: $x > 1$ and $x < 4$

D. positive: x is between 1 and 4; negative: $x < 1$ and $x > 4$

Go Online You can complete an Extra Example online.

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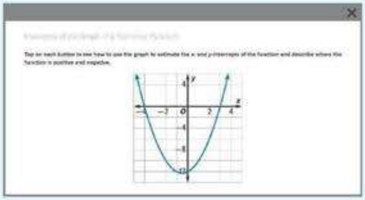
Interactive Presentation



Example 2

TAP

Students tap on each button to see how to use the graph to estimate intercepts of the function and where the function is positive and negative.

TYPE

Students explain why the function is nonlinear.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Intercepts of the Graph of a Nonlinear Function

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How many times does the graph cross the x-axis? 2 the y-axis? 1

OL Does the graph have parts that lie above the x-axis? yes below the x-axis? yes What does this mean about the function? It is positive over some intervals and negative over others.

BL Can a nonlinear function be positive or negative over more than one interval of x-values? Explain your reasoning. Sample answer: Yes; the graph of a nonlinear function often curves, so it could cross the x-axis more than once.

Common Error

Some students may reference the negative parts of the function where $x < 3$. Help them to recognize that the points on the piece of the graph that lies below the x-axis have x-values that lie between -4 and 3, and that this is the correct way to describe this piece of the graph.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have trouble identifying the intervals over which a function is positive and over which it is negative,

THEN have them circle the numbers on the x-axis that correspond to the x-intercepts, and have them trace the pieces of the graph with their finger, stopping at each circled intercept. Students can then record the interval over which they traced each piece, and state if the function is negative or positive in that interval.

Example 3 Find Intercepts from a Graph

Teaching the Mathematical Practices

2 Consider Units Point out that it is important to note the units involved in this problem.

Questions for Mathematical Discourse

AL Why does the entire graph lie above the *x*-axis? because the height of the ball is always a positive number or 0

OL What are the coordinates of the *y*-intercept? (0, 4) the *x*-intercept? (9, 0)

BL What is the domain of the function? real numbers from 0 to 9, inclusive the range? real numbers from 0 to 9, inclusive

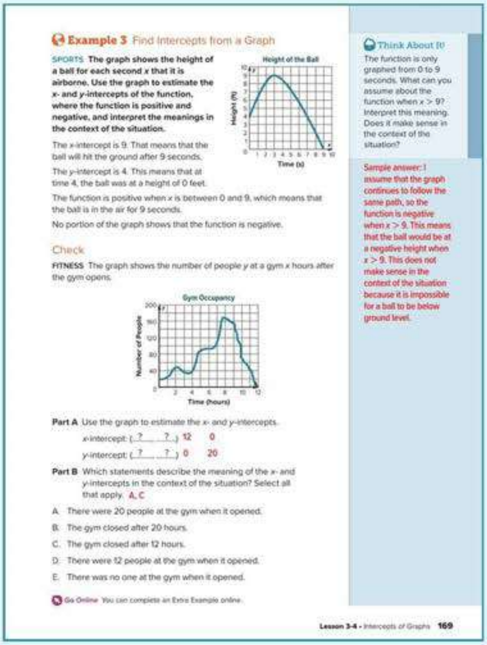
Common Error

Some students may misinterpret the meaning of the *y*-intercept. Explain that this point represents the value of the function when $x = 0$, which is the start time ("time 0"). The point (0, 4) indicates that the starting height of the ball was 4 feet.

DIFFERENTIATE

Enrichment Activity AL BL

Challenge students to write a description about what the shape of the Gym Occupancy graph indicates about the number of people at the gym on the day the data were recorded. Have the students reference the different parts of the graph, and interpret them in the context of the situation.



Example 3 Find Intercepts from a Graph

SPORTS **The graph shows the height of a ball for each second *x* that it is airborne. Use the graph to estimate the *x*- and *y*-intercepts of the function, where the function is positive and negative, and interpret the meanings in the context of the situation.**

The *x*-intercept is 9. That means that the ball will hit the ground after 9 seconds.

The *y*-intercept is 4. This means that at time 4, the ball was at a height of 0 feet.

The function is positive when *x* is between 0 and 9, which means that the ball is in the air for 9 seconds.

No portion of the graph shows that the function is negative.

Check

FITNESS The graph shows the number of people *y* at a gym *x* hours after the gym opens.

Part A Use the graph to estimate the *x*- and *y*-intercepts.

x-intercept: (? , ?) 12 0

y-intercept: (? , ?) 0 20

Part B Which statements describe the meaning of the *x*- and *y*-intercepts in the context of the situation? Select all that apply. A, C

A. There were 20 people at the gym when it opened.

B. The gym closed after 20 hours.

C. The gym closed after 12 hours.

D. There were 12 people at the gym when it opened.

E. There was no one at the gym when it opened.

Go Online You can complete an Extra Example online.

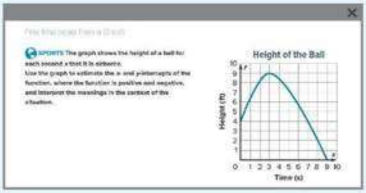
Think About It!

The function is only graphed from 0 to 9 seconds. What can you assume about the function when $x > 9$? Interpret this meaning. Does it make sense in the context of the situation?

Sample answer: I assume that the graph continues to follow the same path, so the function is negative when $x > 9$. This means that the ball would be at a negative height when $x > 9$. This does not make sense in the context of the situation because it is impossible for a ball to be below ground level.

Lesson 3-4 • Intercepts of Graphs 169

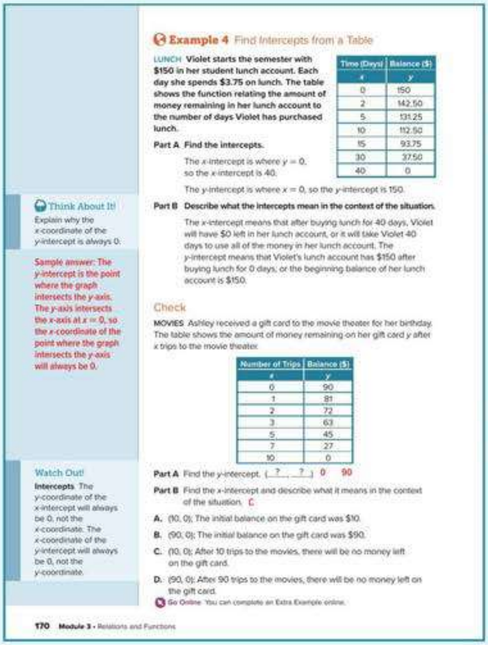
Interactive Presentation



Example 3

TYPE

Students explain what the function represents after 9 seconds.



Example 4 Find Intercepts from a Table

LUNCH **Violet starts the semester with $150 in her student lunch account. Each day she spends $3.75 on lunch. The table shows the function relating the amount of money remaining in her lunch account to the number of days Violet has purchased lunch.**

Time (Days)	Balance ($)
x	y
0	150
2	142.50
5	131.25
10	112.50
15	93.75
30	37.50
40	0

Part A Find the intercepts.

The x-intercept is where $y = 0$, so the x-intercept is 40.

The y-intercept is where $x = 0$, so the y-intercept is 150.

Part B Describe what the intercepts mean in the context of the situation.

The x-intercept means that after buying lunch for 40 days, Violet will have $0 left in her lunch account, or it will take Violet 40 days to use all of the money in her lunch account. The y-intercept means that Violet's lunch account has $150 after buying lunch for 0 days, or the beginning balance of her lunch account is $150.

Think About It!

Explain why the x-coordinate of the y-intercept is always 0.

Sample answer: The y-intercept is the point where the graph intersects the y-axis. The y-axis intersects the x-axis at $x = 0$, so the x-coordinate of the point where the graph intersects the y-axis will always be 0.

Check

MOVIES Ashley received a gift card to the movie theater for her birthday. The table shows the amount of money remaining on her gift card y after x trips to the movie theater.

Number of Trips	Balance ($)
x	y
0	90
1	81
2	72
3	63
5	45
7	27
10	0

Part A Find the y-intercept. (_?_, _?_) 0 90

Part B Find the x-intercept and describe what it means in the context of the situation. C

A. (10, 0); The initial balance on the gift card was $10.

B. (90, 0); The initial balance on the gift card was $90.

C. (10, 0); After 10 trips to the movies, there will be no money left on the gift card.

D. (90, 0); After 90 trips to the movies, there will be no money left on the gift card.

Go Online You can complete an Extra Example online.

Watch Out!

Intercepts The y-coordinate of the x-intercept will always be 0, not the x-coordinate. The x-coordinate of the y-intercept will always be 0, not the y-coordinate.

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Interactive Presentation

Example 4

TAP

Students tap on each button to find the intercepts of the graph.

TYPE

Students explain why the x-coordinate of the y-intercept is always zero.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Find Intercepts from a Table

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about finding intercepts to solving a real-world problem.

Questions for Mathematical Discourse

AL Are either of the intercepts given in the table? How can you find them? Yes; both are given. To find the y-intercept, look for the row where the x-value is 0. To find the x-intercept, look for the row where the y-value is 0.

OL What value does each intercept represent in the context of the situation? Sample answer: The x-intercept means that at 40 days the balance will be zero, and the y-intercept means that at the beginning of the semester the balance will be $150.

BL Does the function have a second x-intercept? Explain. Sample answer: No; there is only one time when the balance is $0, and that is at 40 days.

Common Error

Some students might think the x-intercept appears in the table where $x = 0$. Remind them that the y-coordinate of any point on the x-axis is 0, so the x-intercept is the entry in the table where y is 0.

Learn Solving Equations by Graphing

Objective

Students solve equations by graphing and identifying where the given graphs intersect the *x*-axis.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Important to Know

The distinction between roots of an equation and zeros of a function is an important one. An equation can be solved, producing solutions, or roots. Functions are not "solved"; they are evaluated. If a function produces a value of 0 when evaluated for a given number, that number is a zero of the function. Graphically, the zeros are the *x*-intercepts of the graph of the function.

Example 5 Solve a Linear Equation by Graphing

MP Teaching the Mathematical Practices

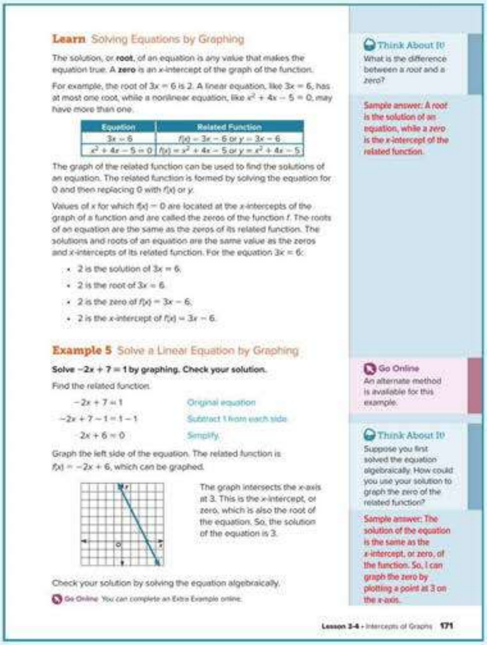
1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students need to check their answer. Point out that they should ask themselves whether their answer makes sense and whether they have answered the problem question.

Questions for Mathematical Discourse

AL What does knowing the zero of a function tell you about the graph of the function? It tells me the *x*-intercept.

OL After graphing the equation, how can you verify that the line crosses the *x*-axis at 3? Substitute 3 into the function for *x* and make sure that $f(3) = 0$.

BL What would be true about the graph of a function that has no zero? Sample answer: The graph will never cross the *x*-axis.



Learn Solving Equations by Graphing

The solution, or **root**, of an equation is any value that makes the equation true. A **zero** is an *x*-intercept of the graph of the function.

For example, the root of $3x = 6$ is 2. A linear equation, like $3x = 6$, has at most one root, while a nonlinear equation, like $x^2 + 4x - 5 = 0$, may have more than one.

Equation	Related Function
$3x = 6$	$f(x) = 3x - 6$ or $y = 3x - 6$
$x^2 + 4x - 5 = 0$	$f(x) = x^2 + 4x - 5$ or $y = x^2 + 4x - 5$

The graph of the related function can be used to find the solutions of an equation. The related function is formed by solving the equation for 0 and then replacing 0 with $f(x)$ or y.

Values of x for which $f(x) = 0$ are located at the x-intercepts of the graph of a function and are called the zeros of the function f. The roots of an equation are the same as the zeros of its related function. The solutions and roots of an equation are the same value as the zeros and x-intercepts of its related function. For the equation $3x = 6$:

- 2 is the solution of $3x = 6$.
- 2 is the root of $3x = 6$.
- 2 is the zero of $f(x) = 3x - 6$.
- 2 is the x-intercept of $f(x) = 3x - 6$.

Example 5 Solve a Linear Equation by Graphing

Solve $-2x + 7 = 1$ by graphing. Check your solution.

Find the related function.

$-2x + 7 = 1$ Original equation

$-2x + 7 - 1 = 1 - 1$ Subtract 1 from each side.

$-2x + 6 = 0$ Simplify.

Graph the left side of the equation. The related function is $f(x) = -2x + 6$, which can be graphed.

The graph intersects the x-axis at 3. This is the x-intercept, or zero, which is also the root of the equation. So, the solution of the equation is 3.

Check your solution by solving the equation algebraically.

Go Online You can complete an Extra Example online.

Think About It! What is the difference between a *root* and a *zero*?

Sample answer: A root is the solution of an equation, while a zero is the x-intercept of the related function.

Go Online An alternate method is available for this example.

Think About It! Suppose you first solved the equation algebraically. How could you use your solution to graph the zero of the related function?

Sample answer: The solution of the equation is the same as the x-intercept, or zero, of the function. So, I can graph the zero by plotting a point at 3 on the x-axis.

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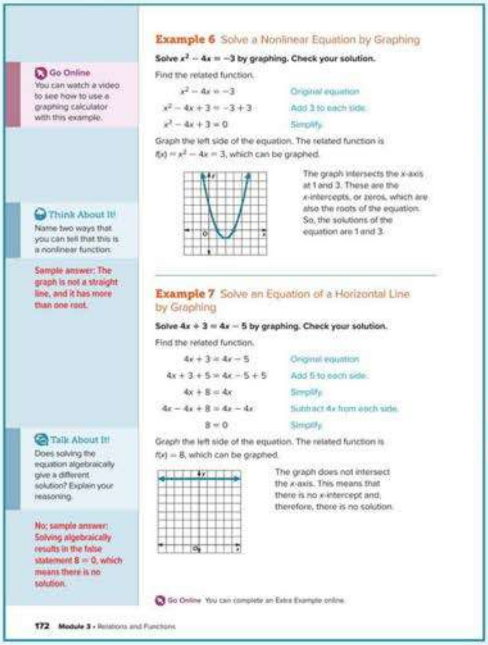
Interactive Presentation

Learn

TYPE

Students describe the difference between a root and a zero.



Go Online You can watch a video to see how to use a graphing calculator with this example.

Think About It! Name two ways that you can tell that this is a nonlinear function.

Sample answer: The graph is not a straight line, and it has more than one root.

Talk About It! Does solving the equation algebraically give a different solution? Explain your reasoning.

No; sample answer: Solving algebraically results in the false statement 8 = 0, which means there is no solution.

Example 6 Solve a Nonlinear Equation by Graphing

Solve $x^2 - 4x = -3$ by graphing. Check your solution.

Find the related function.

$x^2 - 4x = -3$ Original equation

$x^2 - 4x + 3 = -3 + 3$ Add 3 to each side.

$x^2 - 4x + 3 = 0$ Simplify.

Graph the left side of the equation. The related function is $f(x) = x^2 - 4x + 3$, which can be graphed.

The graph intersects the x-axis at 1 and 3. These are the x-intercepts, or zeros, which are also the roots of the equation. So, the solutions of the equation are 1 and 3.

Example 7 Solve an Equation of a Horizontal Line by Graphing

Solve $4x + 3 = 4x - 5$ by graphing. Check your solution.

Find the related function.

$4x + 3 = 4x - 5$ Original equation

$4x + 3 + 5 = 4x - 5 + 5$ Add 5 to each side.

$4x + 8 = 4x$ Simplify.

$4x - 4x + 8 = 4x - 4x$ Subtract 4x from each side.

$8 = 0$ Simplify.

Graph the left side of the equation. The related function is $f(x) = 8$, which can be graphed.

The graph does not intersect the x-axis. This means that there is no x-intercept and, therefore, there is no solution.

Go Online You can complete an Extra Example online.

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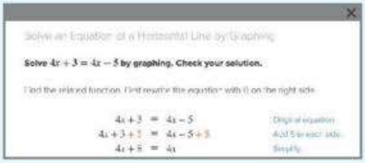
Interactive Presentation



Solve an Equation of a Horizontal Line by Graphing

Solve $4x + 3 = 4x - 5$ by graphing. Check your solution.

Find the related function. First rewrite the equation with 0 on the right side.

$4x + 3 = 4x - 5$ Original equation

$4x + 3 + 5 = 4x - 5 + 5$ Add 5 to each side.

$4x + 8 = 4x$ Simplify.

Example 7

TYPE

Students explain how the solution found algebraically gives the same solution as solving the equation graphically.

Example 6 Solve a Nonlinear Equation by Graphing

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the equation and the graph used in this example.

Questions for Mathematical Discourse

AL How do you find the zeros by graphing? Graph and find the *x*-intercepts.

OL How many zeros will this function have? 2 Explain your reasoning. The graph crosses the *x*-axis in 2 places.

BL Describe the shape of the graph of a function that has three zeros. Sample answer: It could be increasing, then decreasing, then increasing again, crossing the *x*-axis three times.

Example 7 Solve an Equation of a Horizontal Line by Graphing

Teaching the Mathematical Practices

7 Interpret Complicated Expressions Mathematically proficient students can see complicated expressions as single objects or as being composed of several objects. In Example 7, guide students to see what information they can gather about the equation just from looking at it.

Questions for Mathematical Discourse

AL How do you use the equation to get the related function? You add and subtract terms so that one side of the equation equals zero.

OL What type of line is the graph of the resulting function? horizontal

BL Why does the function not have any zeros? The function does not have any zeros because there are no numbers that make the function equal to 0. If there were, the graph would have an *x*-intercept.

Common Error

Some students may have difficulty identifying a related function for an equation such as the one in Example 7, where the resulting equation does not contain a variable. Help them to make the connection between this type of equation and the related constant function, with a graph that is always a horizontal line.

Apply Example 8 Estimate Solutions by Graphing

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

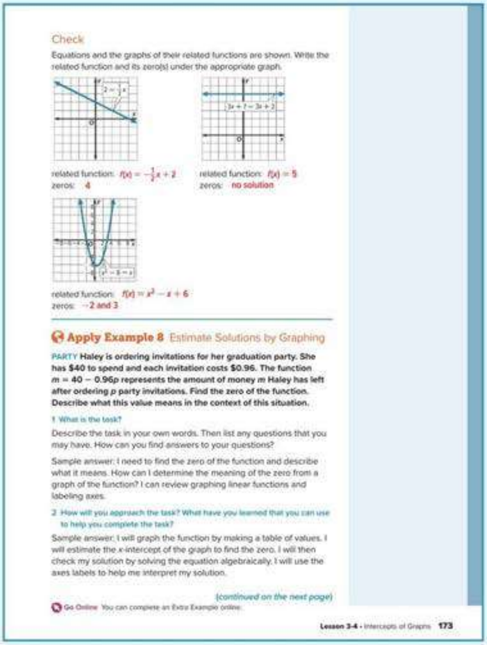
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you use the equation to determine the zero?
- What does the y-intercept represent in the context of the situation?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Check

Equations and the graphs of their related functions are shown. Write the related function and its zero(s) under the appropriate graph.

related function: $f(x) = -\frac{1}{2}x + 2$
zeros: 4

related function: $f(x) = 5$
zeros: no solution

related function: $f(x) = x^2 - x + 6$
zeros: −2 and 3

Apply Example 8 Estimate Solutions by Graphing

PARTY **Haley is ordering invitations for her graduation party. She has $40 to spend and each invitation costs $0.96. The function $m = 40 - 0.96p$ represents the amount of money m Haley has left after ordering p party invitations. Find the zero of the function. Describe what this value means in the context of this situation.**

1 What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to find the zero of the function and describe what it means. How can I determine the meaning of the zero from a graph of the function? I can review graphing linear functions and labeling axes.

2 How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will graph the function by making a table of values. I will estimate the x-intercept of the graph to find the zero. I will then check my solution by solving the equation algebraically. I will use the axes labels to help me interpret my solution.

(continued on the next page)

Go Online You can complete an Extra Example online.

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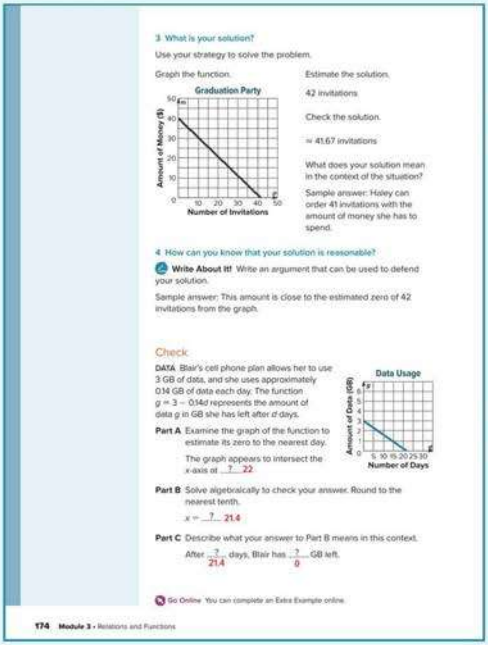
Interactive Presentation

PARTY **Haley is ordering invitations for her graduation party. She has $40 to spend and each invitation costs $0.96. The function $m = 40 - 0.96p$ represents the amount of money m Haley has left after ordering p party invitations. Find the zero of the function. Describe what this value means in the context of this situation.**

Apply Example 8

TAP

Students tap on each step to solve a real-world problem.



3 What is your solution?

Use your strategy to solve the problem.

Graph the function.

Estimate the solution.

42 invitations

Check the solution.

≈ 41.67 invitations

What does your solution mean in the context of the situation?

Sample answer: Haley can order 41 invitations with the amount of money she has to spend.

4 How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: This amount is close to the estimated zero of 42 invitations from the graph.

Check

DATA Blair's cell phone plan allows her to use 3 GB of data, and she uses approximately 0.14 GB of data each day. The function $g = 3 - 0.14d$ represents the amount of data g in GB she has left after d days.

Part A Examine the graph of the function to estimate its zero to the nearest day.

The graph appears to intersect the x-axis at ? 22

Part B Solve algebraically to check your answer. Round to the nearest tenth.

$x =$? 21.4

Part C Describe what your answer to Part B means in this context.

After ? 21.4 days, Blair has ? 0 GB left.

Go Online You can complete an Extra Example online.

174 **Module 3** • Relations and Functions

Interactive Presentation

Check

TYPE

Students solve a real-world problem graphically and algebraically.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–21
2	exercises that use a variety of skills from this lesson	22–27
3	exercises that emphasize higher-order and critical-thinking skills	28–31

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–27 odd, 28–31
- Extension: Even and Odd Functions
- **ALEKS** Tables and Graphs of Lines

OL

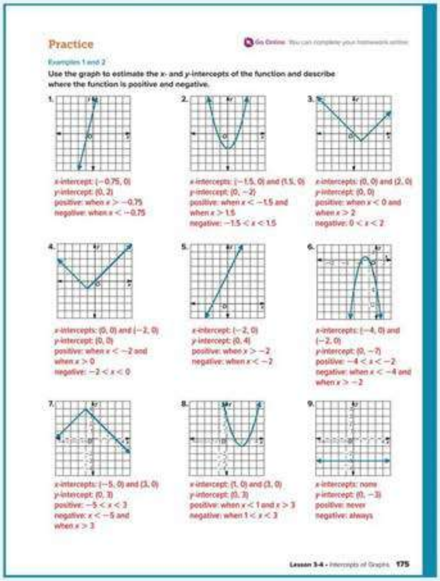
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–31 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- Personal Tutors
- Extra Examples 1–8
- **ALEKS** Converting Between Fractions and Decimals; Ordered Pairs

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–21 odd
- Remediation, Review Resources: Compare and Order Rational Numbers
- *Quick Review Math Handbook*: Interpreting Graphs of Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Converting Between Fractions and Decimals; Ordered Pairs



Practice

Go Online You can complete your homework online.

Examples 1 and 2

Use the graph to estimate the x- and y-intercepts of the function and describe where the function is positive and negative.

1. x-intercept: (−0.75, 0)
y-intercept: (0, 2)
positive: when $x > -0.75$
negative: when $x < -0.75$

2. x-intercepts: (−1.5, 0) and (1.5, 0)
y-intercept: (0, −2)
positive: when $x < -1.5$ and when $x > 1.5$
negative: $-1.5 < x < 1.5$

3. x-intercepts: (0, 0) and (2, 0)
y-intercept: (0, 0)
positive: when $x < 0$ and when $x > 2$
negative: $0 < x < 2$

4. x-intercepts: (0, 0) and (−2, 0)
y-intercept: (0, 0)
positive: when $x < -2$ and when $x > 0$
negative: $-2 < x < 0$

5. x-intercept: (−2, 0)
y-intercept: (0, 4)
positive: when $x > -2$
negative: when $x < -2$

6. x-intercepts: (−4, 0) and (−2, 0)
y-intercept: (0, −7)
positive: $-4 < x < -2$
negative: when $x < -4$ and when $x > -2$

7. x-intercepts: (−5, 0) and (3, 0)
y-intercept: (0, 3)
positive: $-5 < x < 3$
negative: $x < -5$ and when $x > 3$

8. x-intercept: (1, 0) and (3, 0)
y-intercept: (0, 3)
positive: when $x < 1$ and $x > 3$
negative: when $1 < x < 3$

9. x-intercepts: none
y-intercept: (0, −3)
positive: never
negative: always

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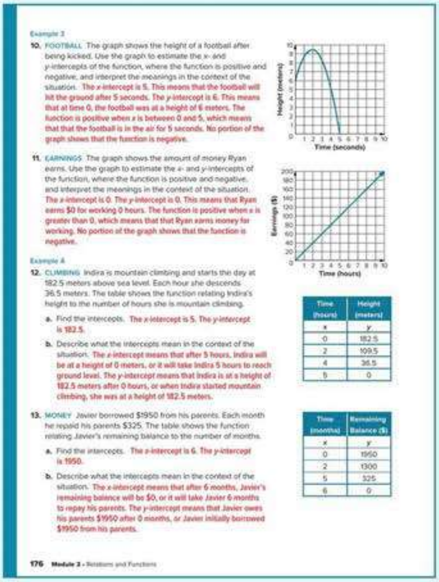

Example 3

10. FOOTBALL The graph shows the height of a football after being kicked. Use the graph to estimate the x- and y-intercepts of the function, where the function is positive and negative, and interpret the meanings in the context of the situation. The x-intercept is 5. This means that the football will hit the ground after 5 seconds. The y-intercept is 6. This means that at time 0, the football was at a height of 6 meters. The function is positive when x is between 0 and 5, which means that that the football is in the air for 5 seconds. No portion of the graph shows that the function is negative.

11. EARNINGS The graph shows the amount of money Ryan earns. Use the graph to estimate the x- and y-intercepts of the function, where the function is positive and negative, and interpret the meanings in the context of the situation. The x-intercept is 0. The y-intercept is 0. This means that Ryan earns $0 for working 0 hours. The function is positive when x is greater than 0, which means that that Ryan earns money for working. No portion of the graph shows that the function is negative.

Example 4

12. CLIMBING Indira is mountain climbing and starts the day at 182.5 meters above sea level. Each hour she descends 36.5 meters. The table shows the function relating Indira's height to the number of hours she is mountain climbing.

Time (hours)	Height (meters)
x	y
0	182.5
2	109.5
4	36.5
5	0

a. Find the intercepts. The x-intercept is 5. The y-intercept is 182.5.

b. Describe what the intercepts mean in the context of the situation. The x-intercept means that after 5 hours, Indira will be at a height of 0 meters, or it will take Indira 5 hours to reach ground level. The y-intercept means that Indira is at a height of 182.5 meters after 0 hours, or when Indira started mountain climbing, she was at a height of 182.5 meters.

13. MONEY Javier borrowed $1950 from his parents. Each month he repaid his parents $325. The table shows the function relating Javier's remaining balance to the number of months.

Time (months)	Remaining Balance ($)
x	y
0	1950
2	1300
5	325
6	0

a. Find the intercepts. The x-intercept is 6. The y-intercept is 1950.

b. Describe what the intercepts mean in the context of the situation. The x-intercept means that after 6 months, Javier's remaining balance will be $0, or it will take Javier 6 months to repay his parents. The y-intercept means that Javier owes his parents $1950 after 0 months, or Javier initially borrowed $1950 from his parents.

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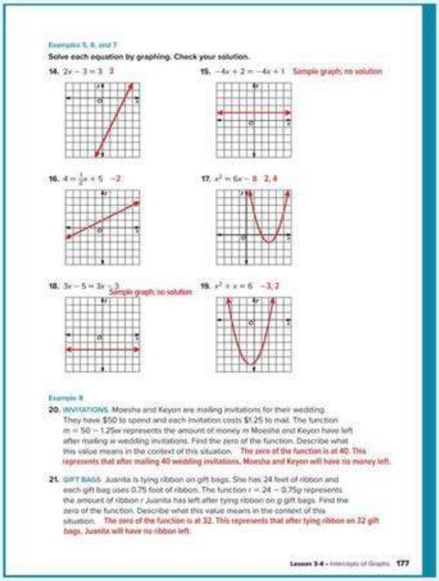


Examples 5, 6, and 7

Solve each equation by graphing. Check your solution.

14. $2x - 3 = 3$ 3

15. $-4x + 2 = -4x + 1$ Sample graph; no solution

16. $4 = \frac{1}{2}x + 5$ -2

17. $x^2 = 6x - 8$ 2, 4

18. $3x - 5 = 3x - 3$ Sample graph; no solution

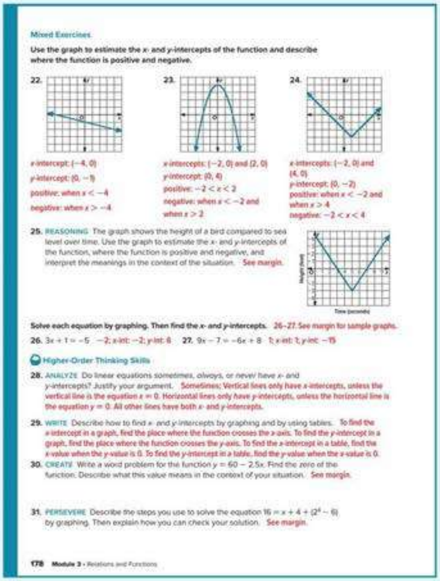
19. $x^2 + x = 6$ -3, 2

Example 8

20. INVITATIONS Moesha and Keyon are mailing invitations for their wedding. They have $50 to spend and each invitation costs $1.25 to mail. The function $m = 50 - 1.25w$ represents the amount of money m Moesha and Keyon have left after mailing w wedding invitations. Find the zero of the function. Describe what this value means in the context of this situation. The zero of the function is at 40. This represents that after mailing 40 wedding invitations, Moesha and Keyon will have no money left.

21. GIFT BAGS Juanita is tying ribbon on gift bags. She has 24 feet of ribbon and each gift bag uses 0.75 foot of ribbon. The function $r = 24 - 0.75g$ represents the amount of ribbon r Juanita has left after tying ribbon on g gift bags. Find the zero of the function. Describe what this value means in the context of this situation. The zero of the function is at 32. This represents that after tying ribbon on 32 gift bags, Juanita will have no ribbon left.

Lesson 3-4 • Intercepts of Graphs 177



Mixed Exercises

Use the graph to estimate the *x*- and *y*-intercepts of the function and describe where the function is positive and negative.

22.
x-intercept: $(-4, 0)$
y-intercept: $(0, -1)$
positive: when $x < -4$
negative: when $x > -4$

23.
x-intercepts: $(-2, 0)$ and $(2, 0)$
y-intercept: $(0, 4)$
positive: $-2 < x < 2$
negative: when $x < -2$ and when $x > 2$

24.
x-intercepts: $(-2, 0)$ and $(4, 0)$
y-intercept: $(0, -2)$
positive: when $x < -2$ and when $x > 4$
negative: $-2 < x < 4$

25. REASONING The graph shows the height of a bird compared to sea level over time. Use the graph to estimate the *x*- and *y*-intercepts of the function, where the function is positive and negative, and interpret the meanings in the context of the situation. See margin.

Solve each equation by graphing. Then find the *x*- and *y*-intercepts. 26–27. See margin for sample graphs.

26. $3x + 1 = -5$ -2; x-int: -2; y-int: 6 **27.** $9x - 7 = -6x + 8$ 1; x-int: 1; y-int: -15

Higher-Order Thinking Skills

28. ANALYZE Do linear equations sometimes, always, or never have *x*- and *y*-intercepts? Justify your argument. Sometimes; Vertical lines only have x-intercepts, unless the vertical line is the equation $x = 0$. Horizontal lines only have y-intercepts, unless the horizontal line is the equation $y = 0$. All other lines have both x- and y-intercepts.

29. WRITE Describe how to find *x*- and *y*-intercepts by graphing and by using tables. To find the x-intercept in a graph, find the place where the function crosses the x-axis. To find the y-intercept in a graph, find the place where the function crosses the y-axis. To find the x-intercept in a table, find the x-value when the y-value is 0. To find the y-intercept in a table, find the y-value when the x-value is 0.

30. CREATE Write a word problem for the function $y = 60 - 2.5x$. Find the zero of the function. Describe what this value means in the context of your situation. See margin.

31. PERSEVERE Describe the steps you use to solve the equation $16 = x + 4 + (2^4 - 6)$ by graphing. Then explain how you can check your solution. See margin.

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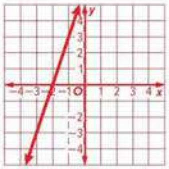
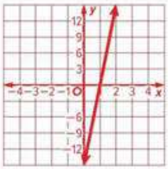
Answers

25. The x-intercepts are 3 and 7. That means that the bird will be at sea level at 3 seconds and at 7 seconds. The y-intercept is 4.5. This means that at time 0, the bird was at a height of 4.5 feet. The function is positive when x is less than 3 and when x is greater than 7, which means that the bird is above sea level from 0 to 3 seconds and after 7 seconds. The function is negative when x is between 3 and 7, which means that that the bird is below sea level, or under water, for 4 seconds.

26.



27.



30. Sample answer: Mr. Devono is purchasing notebooks. He has $60 to spend and each notebook costs $2.50. The function $y = 60 - 2.5x$ represents the amount of money y Mr. Devono has left after purchasing x notebooks. The zero of the function is at 24. This represents that after purchasing 24 notebooks, Mr. Devono will have no money left.

31. Find the related function. Subtract 16 from each side: $0 = x + 4 + (2^4 - 6) - 16$. Evaluate the exponent: $0 = x + 4 + (16 - 6) - 16$. Evaluate the expression in parentheses: $0 = x + 4 + 10 - 16$. Add and subtract: $0 = x - 2$. Replace 0 for $f(x)$. The related function is $f(x) = x - 2$. The graph of the related function intersects the x-axis at 2. This is the x-intercept, or zero. So the solution of the equation is 2. Check the solution by solving the equation algebraically. Evaluate the exponent: $16 = x + 4 + (16 - 6)$. Evaluate the expression in parentheses: $16 = x + 4 + 10$. Add: $16 = x + 14$. Subtract 14 from each side: $2 = x$.

Lesson 3-5

Shapes of Graphs

F.IF.4

LESSON GOAL

Students identify symmetry, extrema, and end behavior of functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore:
- Line Symmetry

Develop:
Symmetry and Graphs of Functions
- Line Symmetry
- Interpret Symmetry

Explore:
- Relative High and Low Points

Develop:
Extrema of Graphs of Functions
- Determine Increasing and Decreasing Parts of the Graph of a Function
- Determine Extrema of the Graph of a Function
- Interpret Extrema of the Graph of a Function

End Behavior of Graphs of Functions
- Determine End Behavior of the Graph of a Linear Function
- Determine End Behavior of the Graph of a Nonlinear Function

You may want your students to complete the **Checks online**.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Qualitative Graphs	●	●		●
Extension: Optimization with Graphs		●	●	●

Language Development Handbook

Assign page 18 of the *Language Development Handbook* to help your students build mathematical language related to symmetry, extrema, and end behavior of functions.

ELL You can use the tips and suggestions on page T18 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students identified intercepts of functions and solved equations by graphing.
A.REI.10, F.IF.4

Now
Students identify symmetry, extrema, and end behavior of functions.
F.IF.4

Next
Students will sketch graphs of functions and compare two or more functions.
F.IF.4, F.IF.5, F.IF.9

Rigor

The Three Pillars of Rigor

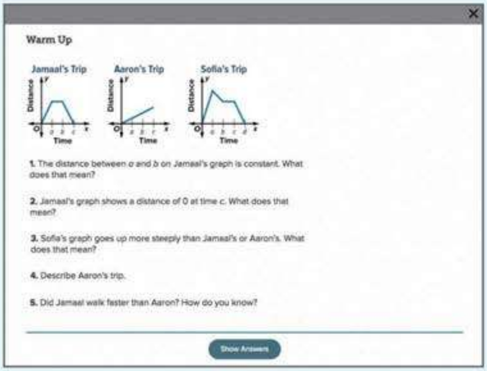
1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Conceptual Bridge In this lesson, students expand on their understanding of graphs of functions by exploring the shapes of graphs. They apply their understanding by solving real-world problems that require them to interpret the symmetry, extrema, and end behavior of graphs.

Mathematical Background

A function is increasing over an interval if its graph goes up when viewed from left to right and is decreasing over the interval if the graph goes down. A function may have extrema, in the form of relative maxima and minima. End behavior of a function is the behavior of the values of a function at the positive and negative extremes of its domain.

Interactive Presentation



Warm Up

1. The distance between *a* and *b* on Jamaal's graph is constant. What does that mean?

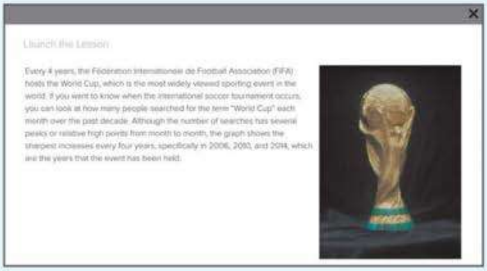
2. Jamaal's graph shows a distance of 0 at time *c*. What does that mean?

3. Sofia's graph goes up more steeply than Jamaal's or Aaron's. What does that mean?

4. Describe Aaron's trip.

5. Did Jamaal walk faster than Aaron? How do you know?

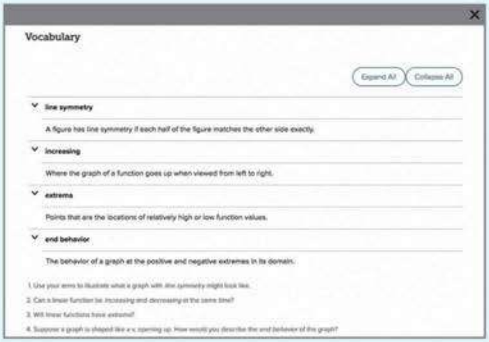
Warm Up



Launch the Lesson

Every 4 years, the Fédération Internationale de Football Association (FIFA) hosts the World Cup, which is the most widely viewed sporting event in the world. If you want to know when the international soccer tournament occurs, you can look at how many people searched for the term "World Cup" each month over the past decade. Although the number of searches has several peaks or relative high points from month to month, the graph shows the sharpest increases every four years, specifically in 2006, 2010, and 2014, which are the years that the event has been held.

Launch the Lesson



Vocabulary

line symmetry
A figure has line symmetry if each half of the figure matches the other side exactly.

increasing
Where the graph of a function goes up when viewed from left to right.

extrema
Points that are the locations of relatively high or low function values.

end behavior
The behavior of a graph at the positive and negative extremes in its domain.

1. Use your arms to illustrate what a graph with line symmetry might look like.
2. Can a linear function be increasing and decreasing at the same time?
3. Will linear functions have extrema?
4. Suppose a graph is shaped like a ∪, opening up. How would you describe the end behavior of the graph?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- analyzing qualitative graphs

Answers:

1. Jamaal did not move forward during this time.
2. Jamaal came back to where he started.
3. Sofia walked faster.
4. Aaron walked steadily forward.
5. Yes; Jamaal's graph is steeper.

Launch the Lesson

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the verbal description of the number of searches and the graph representing the situation.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Line Symmetry

Objective

Students use a sketch to determine whether functions have line symmetry.

MP Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use dynamic geometry software. Work with students to explore and deepen their understanding of symmetry of functions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

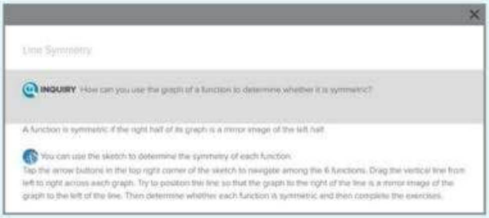
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will explore the concept of line symmetry as it relates to the graphs of functions. Students will use a movable vertical line to analyze the graphs of different types of functions. If they can position the vertical line so that the left side of the graph is a mirror image of the right, then the function is symmetric. Then, students will complete the Inquiry Question.

(continued on the next page)

Interactive Presentation



Line Symmetry

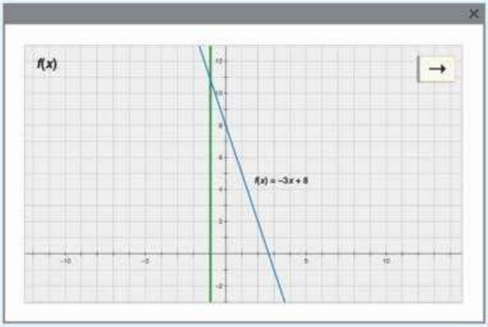
INQUIRY How can you use the graph of a function to determine whether it is symmetric?

A function is symmetric if the right half of its graph is a mirror image of the left half.

You can use the sketch to determine the symmetry of each function.

Tap the arrow buttons in the top right corner of the sketch to navigate among the 6 functions. Drag the vertical line from left to right across each graph. Try to position the line so that the graph to the right of the line is a mirror image of the graph to the left of the line. Then determine whether each function is symmetric and then complete the exercises.

Explore



Explore

WEB SKETCHPAD

Students use the sketch to determine line symmetry.

Interactive Presentation

Explore

TYPE

Students will respond to the Inquiry Question and can view a sample answer.

Explore Line Symmetry (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What is another way to describe a mirror image? Sample answer: a reflection
- If a function is symmetric about the *y*-axis, and you know the coordinates for a positive *x*-value, what else do you also know? Sample answer: If you know the function is symmetric about the *y*-axis, it means that the *y*-values will be the same at equal distances from the *x*-axis. For example, if you know there is a point on the graph at (3, 5), then you also know that there is a point on the graph at (−3, 5).

Inquiry

How can you use the graph of a function to determine whether it is symmetric? Sample answer: If you can find a line such that the left and right halves of the graph are mirror images of one another, then the graph of the function is symmetric.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Explore Relative High and Low Points

Objective

Students use a sketch to explore the relative high and low points of the graph of a function.

MP Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use dynamic geometry software. Work with students to explore and deepen their understanding of relative high and low points of functions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will trace the curve of a given graph using a movable point that allows them to see the coordinates of the points on the graph. They will explore how the *y*-values of the relative high points and the relative low points compare to the *y*-values of nearby points. Then, students will complete the Inquiry Question.

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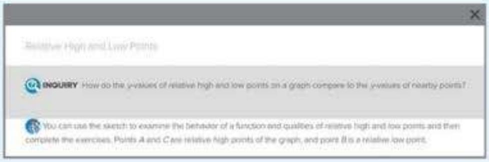
Interactive Presentation

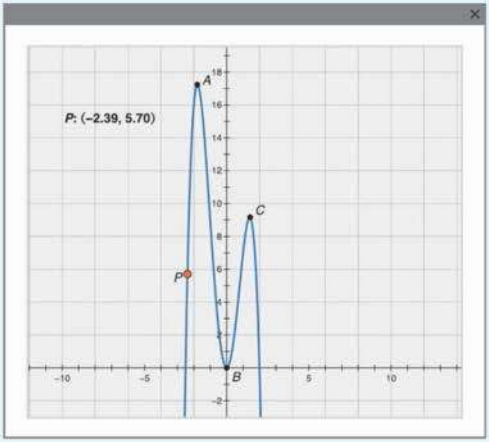


Relative High and Low Points

INQUIRY How do the *y*-values of relative high and low points on a graph compare to the *y*-values of nearby points?

You can use the sketch to examine the behavior of a function and qualities of relative high and low points and then complete the exercises. Points *A* and *C* are relative high points of the graph, and point *B* is a relative low point.

Explore



Explore

WEB SKETCHPAD

Students will use the sketch to determine relative high and low points.

TYPE

a|

Students will compare locations of the relative high and low points.

Interactive Presentation

Explore

TYPE

Students will respond to the Inquiry Question and can view a sample answer.

Explore Relative High and Low Points *(continued)*

Questions

Have students complete the Explore activity.

Ask:

- What does the word “relative” mean in other uses? How is that similar to the use here? Sample answer: *Relative* often means related to, like a family member or a species. This is similar to how *relative* is used in this Explore because it discusses how a point may be high or low, compared to the values that surround it.
- Are there other relative low points in the given function? Why or why not? Sample answer: There are no other relative low points for this function. It appears to go down to negative infinity for both positive and negative x-values.

Inquiry

How do the y-values of relative high and low points on a graph compare to the y-values of nearby points? Sample answer: The y-value of a relative high point is greater than the y-values of nearby points. The y-value of a relative low point is less than the y-values of nearby points.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Symmetry and Graphs of Functions

Objective

Students determine whether functions have line symmetry and, if so, find the line of symmetry by analyzing graphs.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

A common misconception some students may have is thinking that a function is symmetric only if its graph is symmetric about the *y*-axis. Correct this misconception by using the second graph on this page and other similar graphs. Help students see that any vertical line can serve as the line that divides the graph in half.

Go Online

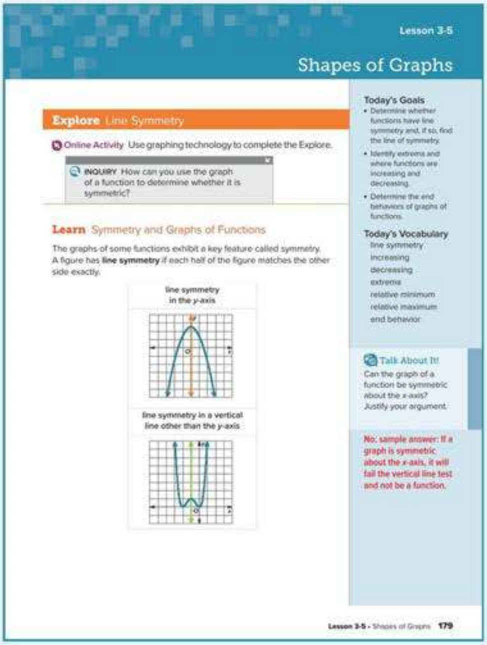
- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty determining whether the graph of a function exhibits line symmetry,

THEN have them use tracing paper to trace the shape of the graph, and see if they can fold the paper in such a way so that one half of the figure folds exactly on top of the other half.



Lesson 3-5

Shapes of Graphs

Explore Line Symmetry

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use the graph of a function to determine whether it is symmetric?

Learn Symmetry and Graphs of Functions

The graphs of some functions exhibit a key feature called symmetry. A figure has **line symmetry** if each half of the figure matches the other side exactly.

line symmetry in the *y*-axis

line symmetry in a vertical line other than the *y*-axis

Today's Goals

- Determine whether functions have line symmetry and, if so, find the line of symmetry.
- Identify extrema and where functions are increasing and decreasing.
- Determine the end behaviors of graphs of functions.

Today's Vocabulary

line symmetry
increasing
decreasing
extrema
relative minimum
relative maximum
end behavior

Talk About It!

Can the graph of a function be symmetric about the *x*-axis? Justify your argument.

No; sample answer: If a graph is symmetric about the *x*-axis, it will fail the vertical line test and not be a function.

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Interactive Presentation

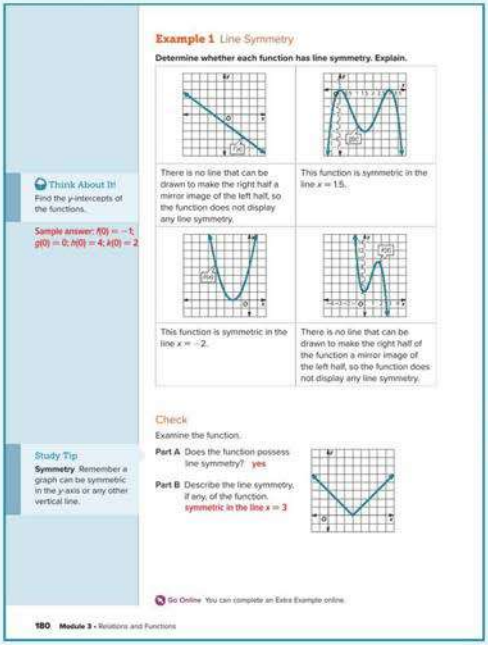
Learn

TAP

Students tap on each card to learn more about function symmetry.

TYPE

Students explain if the graph of a function can be symmetric about the *x*-axis.



Example 1 Line Symmetry

Determine whether each function has line symmetry. Explain.

There is no line that can be drawn to make the right half a mirror image of the left half, so the function does not display any line symmetry.

This function is symmetric in the line $x = 1.5$.

This function is symmetric in the line $x = -2$.

There is no line that can be drawn to make the right half of the function a mirror image of the left half, so the function does not display any line symmetry.

Think About It!
Find the y-intercepts of the functions.
Sample answer: $f(0) = -1$; $g(0) = 0$; $h(0) = 4$; $k(0) = 2$

Check
Examine the function.

Part A Does the function possess line symmetry? yes

Part B Describe the line symmetry, if any, of the function.
symmetric in the line $x = 3$

Study Tip
Symmetry Remember a graph can be symmetric in the y-axis or any other vertical line.

Go Online You can complete an Extra Example online.

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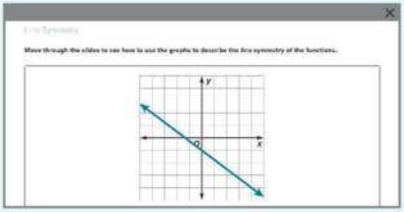
Interactive Presentation



Line Symmetry

Move through the slides to see how to use the graphs to describe the line symmetry of the functions.

Example 1

TAP

Students move through the slides to see how to use graphs to describe the line symmetry of the function.

TYPE

Students find the y-intercepts of the functions in Example 1.

Example 1 Line Symmetry

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between line symmetry and graphs used in this example.

Questions for Mathematical Discourse

AL Does the right side of each graph appear to be a mirror image of the left? a. no; b. yes; c. yes; d. no

OL In the top right graph, how far are the x-intercepts from the line of symmetry? Both x-intercepts are 1.5 units from the line of symmetry. Does this make sense? Explain. Sample answer: Yes; because the graph of one side is a mirror image of the other side, the x-intercepts should be the same distance from that line.

BL In the graphs with line symmetry, what do you notice about where the line of symmetry crosses the graph? Sample answer: It crosses at a point where the graph changes from increasing to decreasing or from decreasing to increasing.

Common Error

Some students may think that all linear functions have line symmetry because their graphs are lines. Use a tracing of the first graph in the example to correct the error in this reasoning by showing students that this graph cannot be folded on top of itself around a vertical line.

DIFFERENTIATE

Enrichment Activity BL ELL

Have pairs of students challenge each other by each drawing graphs of three functions: one that has line symmetry and two that do not. Have the students trade graphs and they must determine which of the three graphs drawn by their partner represents a function that has line symmetry. Have them share and discuss their observations and conclusions.

Example 2 Interpret Symmetry

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

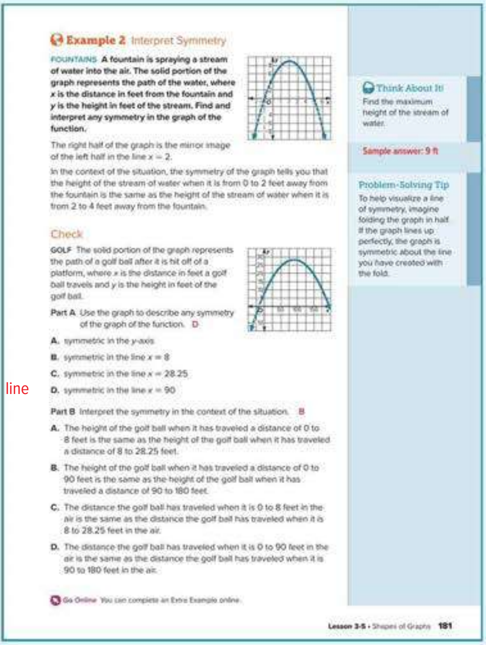
AL Does the right side of the graph appear to be a mirror image of the left? yes

OL Does the function have a second *x*-intercept? Explain. Sample answer: Yes; because the graph is symmetric, the graph will cross the *x*-axis again at some negative value of *x*.

BL Without extending the graph, how can you determine the other *x*-intercept? Sample answer: The line of symmetry is $x = 2$. The *x*-intercept we know is $x = 5$, which is 3 units away from the line of symmetry. The other *x*-intercept should be 3 units in the other direction at $x = -1$.

Common Error

Students may state that the function represented by the graph in this example does not have line symmetry because the piece of the graph that is shown is not symmetric about a vertical line. Explain that although a real-world situation may place restrictions on the domain of a function, the determination of whether the function has line symmetry must be made by considering the function graphed over all real numbers.



Example 2 Interpret Symmetry

FOUNTAINS A fountain is spraying a stream of water into the air. The solid portion of the graph represents the path of the water, where *x* is the distance in feet from the fountain and *y* is the height in feet of the stream. Find and interpret any symmetry in the graph of the function.

The right half of the graph is the mirror image of the left half in the line $x = 2$.

In the context of the situation, the symmetry of the graph tells you that the height of the stream of water when it is from 0 to 2 feet away from the fountain is the same as the height of the stream of water when it is from 2 to 4 feet away from the fountain.

Check

GOLF The solid portion of the graph represents the path of a golf ball after it is hit off of a platform, where *x* is the distance in feet a golf ball travels and *y* is the height in feet of the golf ball.

Part A Use the graph to describe any symmetry of the graph of the function. D

A. symmetric in the *y*-axis

B. symmetric in the line $x = 8$

C. symmetric in the line $x = 28.25$

D. symmetric in the line $x = 90$

Part B Interpret the symmetry in the context of the situation. B

A. The height of the golf ball when it has traveled a distance of 0 to 8 feet is the same as the height of the golf ball when it has traveled a distance of 8 to 28.25 feet.

B. The height of the golf ball when it has traveled a distance of 0 to 90 feet is the same as the height of the golf ball when it has traveled a distance of 90 to 180 feet.

C. The distance the golf ball has traveled when it is 0 to 8 feet in the air is the same as the distance the golf ball has traveled when it is 8 to 28.25 feet in the air.

D. The distance the golf ball has traveled when it is 0 to 90 feet in the air is the same as the distance the golf ball has traveled when it is 90 to 180 feet in the air.

Go Online You can complete an Extra Example online.

Think About It! Find the maximum height of the stream of water.

Sample answer: 9 ft

Problem-Solving Tip To help visualize a line of symmetry, imagine folding the graph in half. If the graph lines up perfectly, the graph is symmetric about the line you have created with the fold.

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Interactive Presentation

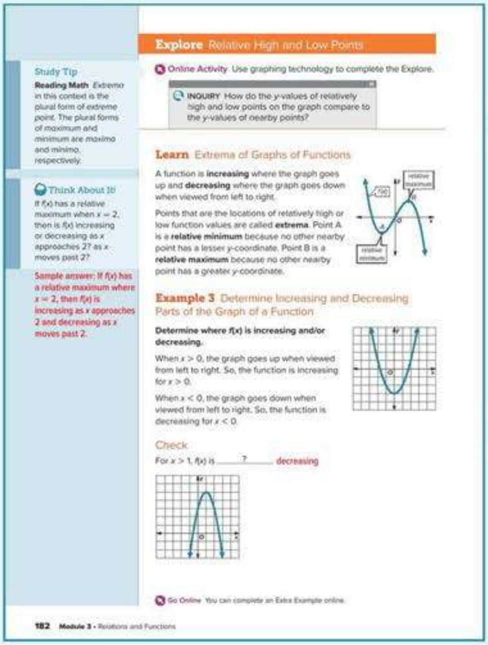
Example 2

TYPE

Students find the maximum height of the stream of water.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Explore Relative High and Low Points

Online Activity Use graphing technology to complete the Explore.

INQUIRY How do the *y*-values of relatively high and low points on the graph compare to the *y*-values of nearby points?

Study Tip

Reading Math *Extrema* in this context is the plural form of extreme point. The plural forms of *maximum* and *minimum* are *maxima* and *minima*, respectively.

Think About It!

If $f(x)$ has a relative maximum when $x = 2$, then is $f(x)$ increasing or decreasing as x approaches 2? as x moves past 2?

Sample answer: If $f(x)$ has a relative maximum where $x = 2$, then $f(x)$ is increasing as x approaches 2 and decreasing as x moves past 2.

Learn Extrema of Graphs of Functions

A function is **increasing** where the graph goes up and **decreasing** where the graph goes down when viewed from left to right.

Points that are the locations of relatively high or low function values are called **extrema**. Point A is a **relative minimum** because no other nearby point has a lesser *y*-coordinate. Point B is a **relative maximum** because no other nearby point has a greater *y*-coordinate.

Example 3 Determine Increasing and Decreasing Parts of the Graph of a Function

Determine where $f(x)$ is increasing and/or decreasing.

When $x > 0$, the graph goes up when viewed from left to right. So, the function is increasing for $x > 0$.

When $x < 0$, the graph goes down when viewed from left to right. So, the function is decreasing for $x < 0$.

Check

For $x > 1$, $f(x)$ is ___?___. decreasing

Go Online You can complete an Extra Example online.

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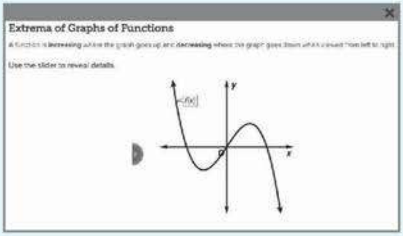
Interactive Presentation



Extrema of Graphs of Functions

A function is **increasing** where the graph goes up and **decreasing** where the graph goes down when viewed from left to right.

Use the slider to reveal details.

Learn

SWIPE

Students drag the slider to reveal details about the increasing and decreasing intervals of the function.

TYPE

Students determine if a function is increasing or decreasing given different information.

Learn Extrema of Graphs of Functions

Objective

Students identify extrema and where functions are increasing and decreasing by analyzing graphs.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of a function to determine where it is increasing or decreasing.

Common Misconception

A common misconception some students may have is thinking that the arrows at the ends of the graph are the indicators of where the graph is increasing and where it is decreasing. Correct this misconception, and reinforce the need to read the graph from left to right in order to determine where the graph is increasing and where it is decreasing.

Example 3 Determine Increasing and Decreasing Parts of the Graph of a Function

Teaching the Mathematical Practices

5 Analyze Graphs Help students analyze the graph in this example to determine where the function is increasing and decreasing.

Questions for Mathematical Discourse

AL The given graph of $f(x)$ curves. Should we expect $f(x)$ to be always increasing or decreasing? Because of the curve in the graph, we should expect $f(x)$ to increase over some interval and decrease over some interval.

OL Starting at the leftmost point of the graph and moving from the left to the right, is the graph increasing or decreasing? decreasing At what *x*-value does the graph change to be increasing? at $x = 0$

BL Do the arrows on the ends of the graph indicate whether the function is increasing or decreasing? Explain. Sample answer: No; the arrows on the ends of the graph indicate only that the graph continues in each direction. To tell whether the graph is increasing or decreasing, you trace the graph from its leftmost point.

Common Error

Some students may state that the graph is increasing on both sides of the *y*-axis. It is likely that these students are looking at the left side of the graph from right to left instead of from left to right. Correct this error and show these students why this part of the graph is decreasing.

Example 4 Determine Extrema of the Graph of a Function

MP Teaching the Mathematical Practices

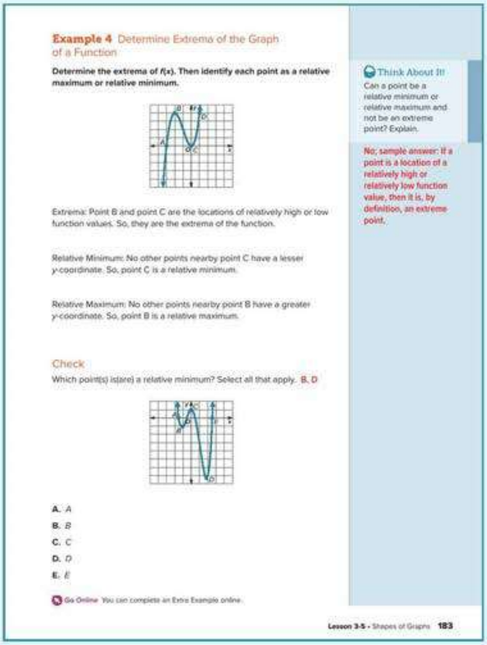
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How many extrema does the graph have? 2

OL What must be true for a point to be a relative minimum? There must be no points nearby the point that have a lesser *y*-coordinate. Which point of the graph satisfies this condition? point *C*

BL Point *A* has the same *y*-coordinate as point *C*, and point *D* has the same *y*-coordinate as point *B*. Why are points *A* and *D* not extrema? Sample answer: There are points to the left of point *A* with lesser *y*-coordinates and to the right of point *A* with greater *y*-coordinates, so it is neither a relative minimum nor a relative maximum. The same is true for point *D*.



Example 4 Determine Extrema of the Graph of a Function

Determine the extrema of *f*(*x*). Then identify each point as a relative maximum or relative minimum.

Extrema: Point B and point C are the locations of relatively high or low function values. So, they are the extrema of the function.

Relative Minimum: No other points nearby point C have a lesser *y*-coordinate. So, point C is a relative minimum.

Relative Maximum: No other points nearby point B have a greater *y*-coordinate. So, point B is a relative maximum.

Check

Which point(s) is(are) a relative minimum? Select all that apply. B, D

A. *A*
B. *B*
C. *C*
D. *D*
E. *E*

Go Online You can complete an Extra Example online.

Think About It! Can a point be a relative minimum or relative maximum and not be an extreme point? Explain.

No; sample answer: If a point is a location of a relatively high or relatively low function value, then it is, by definition, an extreme point.

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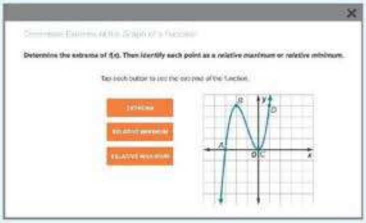
Interactive Presentation



Determine the extrema of f(x). Then identify each point as a relative maximum or relative minimum.

Tap each button to see the extrema of the function.

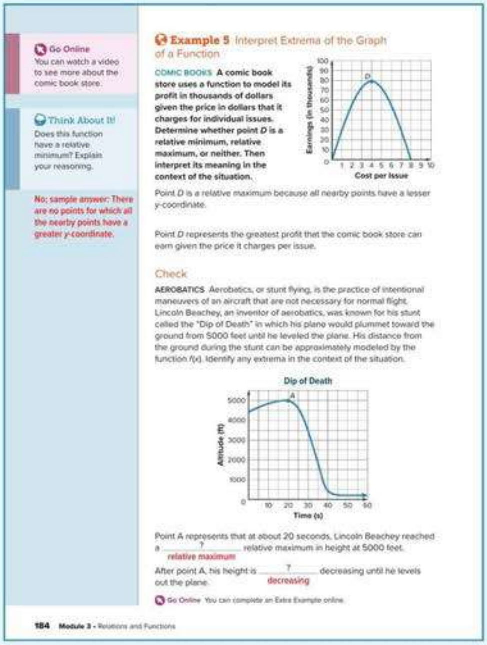
Example 4

TAP

Students tap on each button to see where the extrema of the function occur.

TYPE

Students define an extreme point as a maximum or minimum.



Go Online You can watch a video to see more about the comic book store.

Think About It! Does this function have a relative minimum? Explain your reasoning.

No; sample answer: There are no points for which all the nearby points have a greater *y*-coordinate.

Example 5 Interpret Extrema of the Graph of a Function

COMIC BOOKS **A comic book store uses a function to model its profit in thousands of dollars given the price in dollars that it charges for individual issues. Determine whether point *D* is a relative minimum, relative maximum, or neither. Then interpret its meaning in the context of the situation.**

Point *D* is a relative maximum because all nearby points have a lesser *y*-coordinate.

Point *D* represents the greatest profit that the comic book store can earn given the price it charges per issue.

Check

AEROBATICS Aerobatics, or stunt flying, is the practice of intentional maneuvers of an aircraft that are not necessary for normal flight. Lincoln Beachey, an inventor of aerobatics, was known for his stunt called the "Dip of Death" in which his plane would plummet toward the ground from 5000 feet until he leveled the plane. His distance from the ground during the stunt can be approximately modeled by the function $f(x)$. Identify any extrema in the context of the situation.

Point A represents that at about 20 seconds, Lincoln Beachey reached a ___?___ relative maximum in height at 5000 feet.
relative maximum

After point A, his height is ___?___ decreasing until he levels out the plane.
decreasing

Go Online You can complete an Extra Example online.

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Interactive Presentation

COMIC BOOKS Watch this animation to see how a comic book store uses a function to model their profit in thousands of dollars given the price in dollars that they charge for individual issues.

Example 5

WATCH

Students watch the animation to see how a comic book store uses a function to model their profit.

TYPE

Students determine if the function has a relative minimum.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 5 Interpret Extrema of the Graph of a Function

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about extrema to solving a real-world problem.

Questions for Mathematical Discourse

AL What do the *x*- and *y*-axes represent? The *x*-axis represents the price of one comic book in dollars; the *y*-axis represents the publisher's profit in thousands of dollars.

OL Is the function increasing or decreasing as the graph approaches point *D* from the left? increasing As the graph moves away from point *D* toward the right, is the function increasing or decreasing? decreasing

BL Describe the behavior of a function, in terms of increasing and decreasing, at an extreme. Sample answer: At an extreme, the function changes from increasing to decreasing or vice versa.

Common Error

Some students may misinterpret the meaning of the maximum in the context of the situation. Make sure they understand what the scale on each axis represents. This is an important first step in interpreting the meaning of any point on the graph.

Learn End Behavior of Graphs of Functions

Objective

Students determine the end behaviors of graphs of functions by analyzing graphs.

MP Teaching the Mathematical Practices

5 Analyze Graphs Help students to analyze the graph of a function in order to determine the function's end behavior in this Learn.

Example 6 Determine End Behavior of the Graph of a Linear Function

MP Teaching the Mathematical Practices

3 Make Conjectures In the Think About It! feature, students will make conjectures and then build a logical progression of statements to validate the conjectures. Once students have made their conjectures, guide the students to validate them.

Questions for Mathematical Discourse

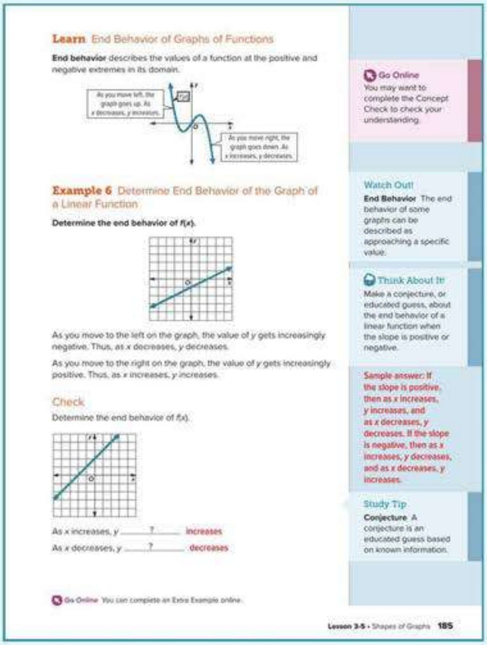
AL Is the slope of the graph positive, negative, zero, or undefined? positive

OL Start at the *y*-intercept. Follow the graph to the left. Do the *y*-values increase or decrease? decrease Start at the *y*-intercept again. Follow the graph to the right. Do the *y*-values increase or decrease? increase

BL Can a linear function ever increase both as *x* increases and as *x* decreases? Explain. Sample answer: No; for a function to increase as *x* becomes both increasingly positive and increasingly negative, the graph would need to turn and change direction. Because the graph of a linear function is a straight line, this is not possible.

Common Error

Some students may mistakenly analyze the graph from left to right, as they do when determining whether a graph is increasing or decreasing. Reinforce that when determining the end behavior as *x* is decreasing, students must read the graph from right to left.



Learn End Behavior of Graphs of Functions

End behavior describes the values of a function at the positive and negative extremes in its domain.

As you move left, the graph goes up. As *x* decreases, *y* increases.

As you move right, the graph goes down. As *x* increases, *y* decreases.

Example 6 Determine End Behavior of the Graph of a Linear Function

Determine the end behavior of $f(x)$.

As you move to the left on the graph, the value of *y* gets increasingly negative. Thus, as *x* decreases, *y* decreases.

As you move to the right on the graph, the value of *y* gets increasingly positive. Thus, as *x* increases, *y* increases.

Check

Determine the end behavior of $f(x)$.

As *x* increases, *y* ___?___ increases

As *x* decreases, *y* ___?___ decreases

Go Online You can complete an Extra Example online.

Go Online You may want to complete the Concept Check to check your understanding.

Watch Out!

End Behavior The end behavior of some graphs can be described as approaching a specific value.

Think About It!

Make a conjecture, or educated guess, about the end behavior of a linear function when the slope is positive or negative.

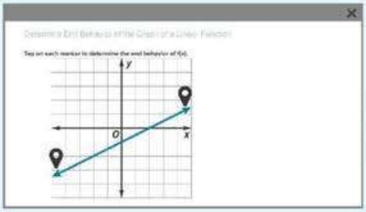
Sample answer: If the slope is positive, then as *x* increases, *y* increases, and as *x* decreases, *y* decreases. If the slope is negative, then as *x* increases, *y* decreases, and as *x* decreases, *y* increases.

Study Tip

Conjecture A conjecture is an educated guess based on known information.

Lesson 3-5 • Shapes of Graphs 185

Interactive Presentation



Determine End Behavior of the Graph of a Linear Function

Tap on each marker to determine the end behavior of $f(x)$.

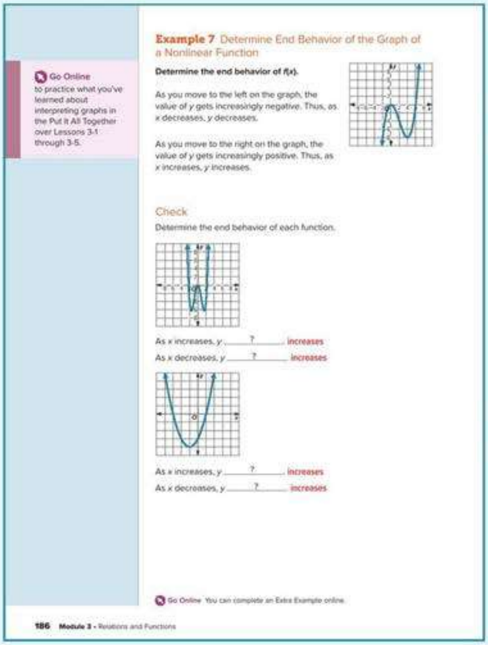
Example 6

TAP

Students tap on each marker to determine the end behavior of the function.

TYPE

Students make a conjecture about the end behavior of a linear function when the slope is positive or negative.



Example 7 Determine End Behavior of the Graph of a Nonlinear Function

Determine the end behavior of $f(x)$.

As you move to the left on the graph, the value of y gets increasingly negative. Thus, as x decreases, y decreases.

As you move to the right on the graph, the value of y gets increasingly positive. Thus, as x increases, y increases.

Go Online to practice what you've learned about interpreting graphs in the Put It All Together over Lessons 3-1 through 3-5.

Check

Determine the end behavior of each function.

As x increases, y ___?___ increases

As x decreases, y ___?___ increases

As x increases, y ___?___ increases

As x decreases, y ___?___ increases

Go Online You can complete an Extra Example online.

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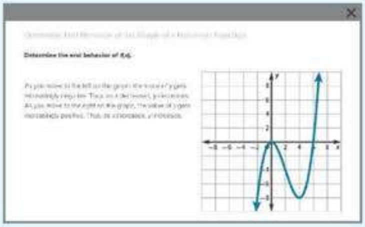
Interactive Presentation



Determine the end behavior of $f(x)$.

Example 7

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 7 Determine End Behavior of the Graph of a Nonlinear Function

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and the end behavior of the function used in this example.

Questions for Mathematical Discourse

AL Start at the y-intercept. Follow the graph to the left. Does the graph go up or down? down

OL Is the end behavior at each end of this graph the same or different? different How do you know? Sample answer: The arrows at each of the two ends are pointing in different directions. One arrow is pointing up and the other is pointing down.

BL Can a nonlinear function ever increase both as x increases and as x decreases? yes What must be true for this to happen? Sample answer: For a function to increase as x becomes both increasingly positive and increasingly negative, the graph would need to turn once, three times, or any odd number of times. The same is true for decreasing end behavior.

Essential Question Follow-Up

Students have explored interpreting graphs of functions.

Ask:

Why are graphs useful representations of functions? Sample answer: You can use the graph of a function to find where the function is increasing and decreasing and the end behavior and extrema of the function.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–20
2	exercises that use a variety of skills from this lesson	21–23
3	exercises that emphasize higher-order and critical-thinking skills	24, 25

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–23 odd, 24, 25
- Extension: Optimization with Graphs
- **ALEKS** Graphs of Functions

OL

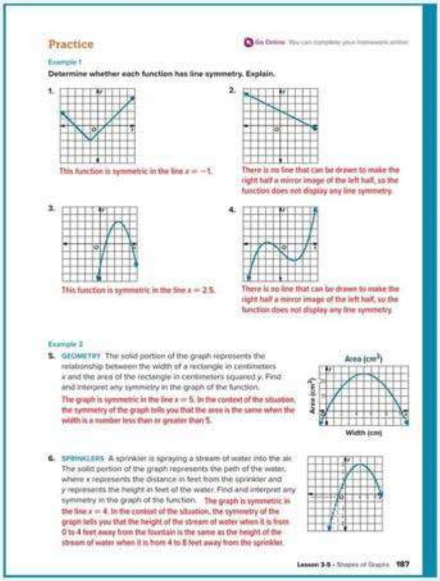
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–25 odd
- Remediation, Review Resources: Qualitative Graphs
- Personal Tutors
- Extra Examples 1–7
- **ALEKS** Graphs of Functions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–19 odd
- Remediation, Review Resources: Qualitative Graphs
- *Quick Review Math Handbook*: Interpreting Graphs of Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Graphs of Functions



Practice

Go Online You can complete your homework online.

Example 1

Determine whether each function has line symmetry. Explain.

1. This function is symmetric in the line $x = -1$.

2. There is no line that can be drawn to make the right half a mirror image of the left half, so the function does not display any line symmetry.

3. This function is symmetric in the line $x = 2.5$.

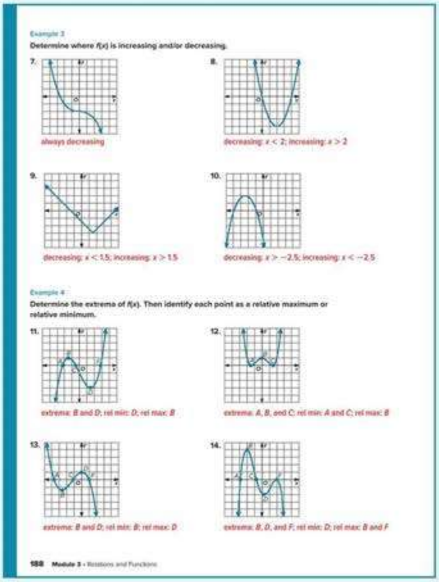
4. There is no line that can be drawn to make the right half a mirror image of the left half, so the function does not display any line symmetry.

Example 2

5. GEOMETRY The solid portion of the graph represents the relationship between the width of a rectangle in centimeters x and the area of the rectangle in centimeters squared y. Find and interpret any symmetry in the graph of the function. The graph is symmetric in the line $x = 5$. In the context of the situation, the symmetry of the graph tells you that the area is the same when the width is a number less than or greater than 5.

6. SPRINKLERS A sprinkler is spraying a stream of water into the air. The solid portion of the graph represents the path of the water, where x represents the distance in feet from the sprinkler and y represents the height in feet of the water. Find and interpret any symmetry in the graph of the function. The graph is symmetric in the line $x = 4$. In the context of the situation, the symmetry of the graph tells you that the height of the stream of water when it is from 0 to 4 feet away from the fountain is the same as the height of the stream of water when it is from 4 to 8 feet away from the sprinkler.

Lesson 3-5 • Shapes of Graphs 187



Example 3

Determine where $f(x)$ is increasing and/or decreasing.

7. always decreasing

8. decreasing: $x < 2$; increasing: $x > 2$

9. decreasing: $x < 1.5$; increasing: $x > 1.5$

10. decreasing: $x > -2.5$; increasing: $x < -2.5$

Example 4

Determine the extrema of $f(x)$. Then identify each point as a relative maximum or relative minimum.

11. extrema: B and D; rel min: D; rel max: B

12. extrema: A, B, and C; rel min: A and C; rel max: B

13. extrema: B and D; rel min: B; rel max: D

14. extrema: B, D, and F; rel min: D; rel max: B and F

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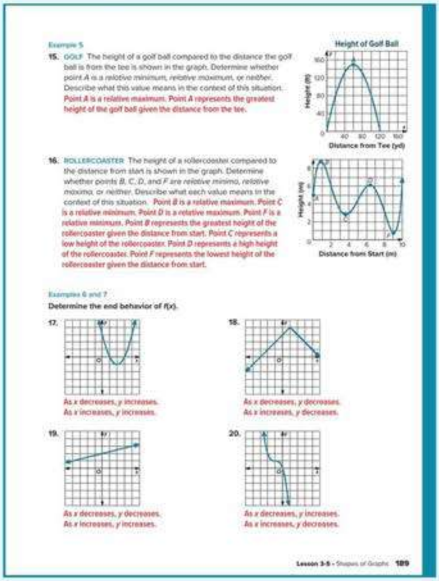
3 REFLECT AND PRACTICE

F.IF.4

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 5

15. GOLF The height of a golf ball compared to the distance the golf ball is from the tee is shown in the graph. Determine whether point *A* is a *relative minimum*, *relative maximum*, or *neither*. Describe what this value means in the context of this situation. Point *A* is a relative maximum. Point *A* represents the greatest height of the golf ball given the distance from the tee.

16. ROLLERCOASTER The height of a rollercoaster compared to the distance from start is shown in the graph. Determine whether points *B*, *C*, *D*, and *F* are *relative minima*, *relative maxima*, or *neither*. Describe what each value means in the context of this situation. Point *B* is a relative maximum. Point *C* is a relative minimum. Point *D* is a relative maximum. Point *F* is a relative minimum. Point *B* represents the greatest height of the rollercoaster given the distance from start. Point *C* represents a low height of the rollercoaster. Point *D* represents a high height of the rollercoaster. Point *F* represents the lowest height of the rollercoaster given the distance from start.

Examples 6 and 7

Determine the end behavior of *f*(*x*).

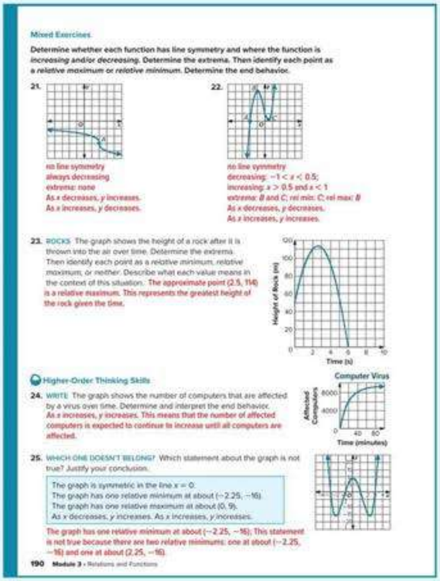
17. As *x* decreases, *y* increases. As *x* increases, *y* increases.

18. As *x* decreases, *y* decreases. As *x* increases, *y* decreases.

19. As *x* decreases, *y* decreases. As *x* increases, *y* increases.

20. As *x* decreases, *y* increases. As *x* increases, *y* decreases.

Lesson 3-5 • Shapes of Graphs **189**



Mixed Exercises

Determine whether each function has line symmetry and where the function is *increasing* and/or *decreasing*. Determine the extrema. Then identify each point as a *relative maximum* or *relative minimum*. Determine the end behavior.

21.
no line symmetry
always decreasing
extrema: none
As *x* decreases, *y* increases.
As *x* increases, *y* decreases.

22.
no line symmetry
decreasing: $-1 < x < 0.5$;
increasing: $x > 0.5$ and $x < 1$
extrema: *B* and *C*; rel min: *C*; rel max: *B*
As *x* decreases, *y* decreases.
As *x* increases, *y* increases.

23. ROCKS The graph shows the height of a rock after it is thrown into the air over time. Determine the extrema. Then identify each point as a *relative minimum*, *relative maximum*, or *neither*. Describe what each value means in the context of this situation. The approximate point (2.5, 114) is a relative maximum. This represents the greatest height of the rock given the time.

Higher-Order Thinking Skills

24. WRITE The graph shows the number of computers that are affected by a virus over time. Determine and interpret the end behavior. As *x* increases, *y* increases. This means that the number of affected computers is expected to continue to increase until all computers are affected.

25. WHICH ONE DOESN'T BELONG? Which statement about the graph is not true? Justify your conclusion.

> The graph is symmetric in the line $x = 0$.
> The graph has one relative minimum at about $(-2.25, -16)$.
> The graph has one relative maximum at about $(0, 9)$.
> As *x* decreases, *y* increases. As *x* increases, *y* increases.

The graph has one relative minimum at about $(-2.25, -16)$; This statement is not true because there are two relative minimums: one at about $(-2.25, -16)$ and one at about $(2.25, -16)$.

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Lesson 3-6

F.IF.4, F.IF.9

Sketching Graphs and Comparing Functions

LESSON GOAL

Students sketch graphs of functions and compare two or more functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Modeling Relationships by Using Functions

Develop:

Sketching Graphs of Functions

- Sketch the Graph of a Linear Function
- Sketch the Graph of a Symmetric Function
- Sketch the Graph of a Nonlinear Function
- Compare Properties of Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Identify Functions	●	●		●
Extension: Solving Equations: $f(x) = g(x)$		●	●	●

Language Development Handbook

Assign page 19 of the *Language Development Handbook* to help your students build mathematical language related to graphs of functions.

ELL You can use the tips and suggestions on page T19 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, sketch graphs showing key features given a verbal description of the relationship.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by ver descriptions).

Standards for Mathematical Practice:

6 Attend to precision.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students sketched functions to qualitatively describe the relationship (increasing, decreasing, linear, nonlinear).

8.F.5

Now

Students sketch graphs of functions and compare two or more functions.

F.IF.4, F.IF.9

Next

Students interpret key features of graphs that represent functions, and compare two or more functions.

F.IF.4, F.IF.5, F.IF.9 (Course 1, Course 2, Course 3)

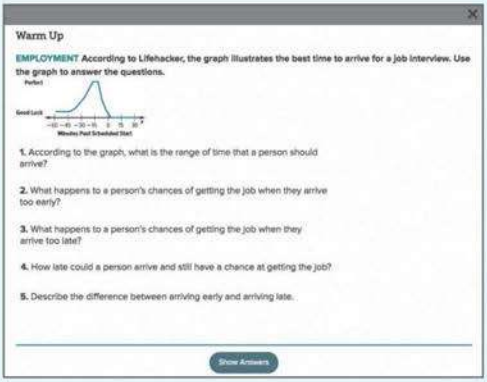
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Conceptual Bridge In this lesson, students expand on their understanding of graphs of functions and build fluency by using key features to sketch graphs. They apply their understanding by solving real-world problems that require them to compare and interpret the key features of graphs.

Interactive Presentation

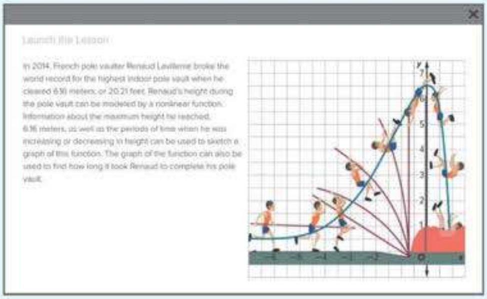


Warm Up

EMPLOYMENT According to Lifehacker, the graph illustrates the best time to arrive for a job interview. Use the graph to answer the questions.

1. According to the graph, what is the range of time that a person should arrive?
2. What happens to a person's chances of getting the job when they arrive too early?
3. What happens to a person's chances of getting the job when they arrive too late?
4. How late could a person arrive and still have a chance at getting the job?
5. Describe the difference between arriving early and arriving late.

Warm Up



Launch the Lesson

In 2014, French pole vaulter Renaud Lavillenie broke the world record for the highest indoor pole vault when he cleared 6.16 meters, or 20.21 feet. Renaud's height during the pole vault can be modeled by a nonlinear function. Information about the maximum height he reached, 6.16 meters, as well as the periods of time when he was increasing or decreasing in height can be used to sketch a graph of this function. The graph of the function can also be used to find how long it took Renaud to complete his pole vault.

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skills for this lesson:

- sketching graphs of relations
- analyzing qualitative graphs

Answers:

1. between 14 and 16 minutes before the scheduled start
2. The chances decrease the earlier they arrive.
3. Being late appears to guarantee that the person will not get the job.
4. about 1 minute late
5. When a person arrives early, there is still a small chance that they will get the job. But when they arrive late, there is almost no chance.

Launch the Lesson

Teaching the Mathematical Practices

6 Use Quantities Have students read the verbal description about the world record for indoor pole vaulting and then determine how the axes should be labeled on the graph.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards*? and *How can I use these practices*?, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Mathematical Background

You can sketch the graph of a function if you are given information about its key features. Key features include intercepts, relative maxima and minima, intervals over which the graph is increasing or decreasing, and the end behavior of the function. Also helpful is information about any symmetry the graph may possess, and the real-world constraints of the situation modeled by the function.

Explore Modeling Relationships by Using Functions

Objective

Students use data and a sketch to predict key features of the graph of a function.

Teaching the Mathematical Practices

3 Construct Arguments In this Explore, students will use stated assumptions, definitions, and previously established results to construct an argument.

5 Compare Predictions with Data Point out that in this Explore, students should use a graphing calculator to compare their predictions with the data.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

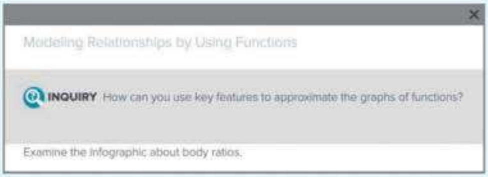
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. They will examine an infographic about body ratios. They will collect data from their classmates about wrist and neck circumferences and graph the data. They will use the resulting graph and their knowledge of the key features of graphs to answer questions and make a prediction about the function that models this relationship. Then, students will complete the Inquiry Question.

(continued on the next page)

Interactive Presentation

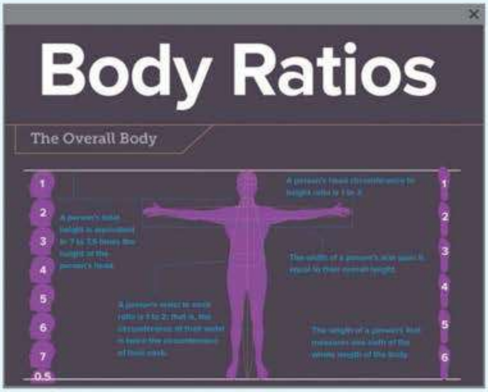


Modeling Relationships by Using Functions

INQUIRY How can you use key features to approximate the graphs of functions?

Examine the infographic about body ratios.

Explore



Explore

TYPE

Students measure their wrist and neck and enter the class data into a table. Then they answer questions about the data.

WEB SKETCHPAD

Students use a sketch to graph the collected data.

Interactive Presentation

INQUIRY How can you use key features to approximate the graphs of functions?

Explore

TYPE

Students will respond to the Inquiry Question and can view a sample answer.

Explore Modeling Relationships by Using Functions (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does it make sense for the graph to pass through the origin? Explain in context. Sample answer: The graph represents the relationship between the wrist circumference and neck circumference. If one circumference is 0 inches, then it does not exist, and the other would not exist either.
- For the relationship between the length of a person's foot and the length of the body, what key features of the function do you know without experimentation? Sample answer: We're told that a foot is about one-sixth of the whole length of the body, so we know that the graph will pass through the origin. If a person is 0 ft tall, they will have feet that are 0 ft long. We also know that the graph will be increasing because as a person's foot grows, usually their height is growing too. Also, it is not possible for these lengths to be negative.

Inquiry

How can you use key features to approximate the graphs of functions? Sample answer: Since key features tell you about the general shape and behavior of a function, you can use them to predict what the graph of a function will look like.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Sketching Graphs of Functions

Objective

Students sketch graphs of functions by using key features.

Teaching the Mathematical Practices

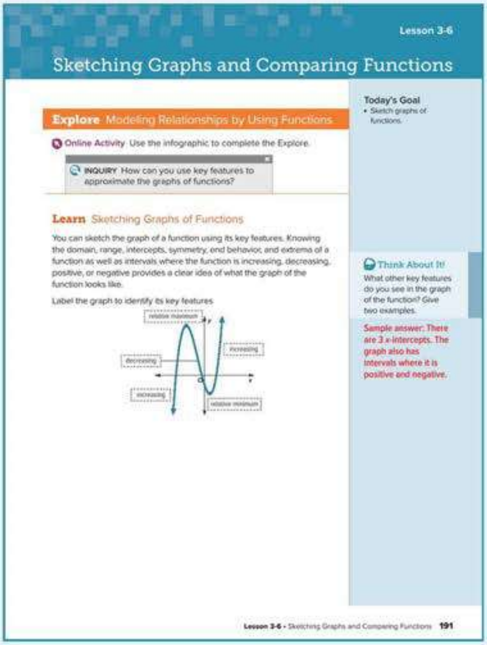
1 Explain Correspondences Encourage students to explain the relationships between the verbal descriptions and the graphs they create.

Common Misconception

A common misconception some students may have is that increasing and decreasing describes what the *x*-values of the function are doing, rather than what the *y*-values are doing. Reinforce to students that for a function to be increasing, the *y*-values are increasing as the *x*-values increase, but a function is decreasing if the *y*-values are decreasing as the *x*-values increase.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-6

Sketching Graphs and Comparing Functions

Explore Modeling Relationships by Using Functions

Online Activity Use the infographic to complete the Explore.

INQUIRY How can you use key features to approximate the graphs of functions?

Learn Sketching Graphs of Functions

You can sketch the graph of a function using its key features. Knowing the domain, range, intercepts, symmetry, end behavior, and extrema of a function as well as intervals where the function is increasing, decreasing, positive, or negative provides a clear idea of what the graph of the function looks like.

Label the graph to identify its key features.

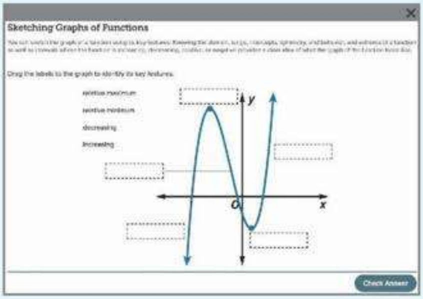
Today's Goal
- Sketch graphs of functions.

Think About It!
What other key features do you see in the graph of the function? Give two examples.

Sample answer: There are 3 *x*-intercepts. The graph also has intervals where it is positive and negative.

Lesson 3-6 • Sketching Graphs and Comparing Functions 191

Interactive Presentation



Sketching Graphs of Functions

Drag the labels to the graph to identify its key features.

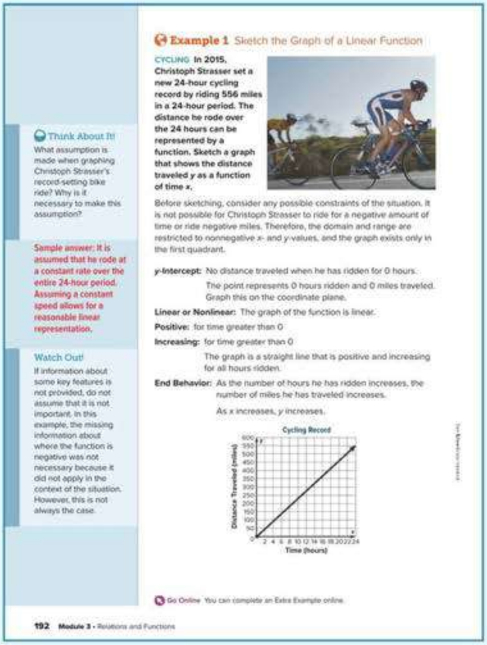
Learn

DRAG & DROP

Students drag and drop labels to describe key features of the graph.

TYPE

Students describe other key features present in the graph of the function.



Example 1 Sketch the Graph of a Linear Function

CYCLING **In 2015, Christoph Strasser set a new 24-hour cycling record by riding 556 miles in a 24-hour period. The distance he rode over the 24 hours can be represented by a function. Sketch a graph that shows the distance traveled *y* as a function of time *x*.**

Before sketching, consider any possible constraints of the situation. It is not possible for Christoph Strasser to ride for a negative amount of time or ride negative miles. Therefore, the domain and range are restricted to nonnegative *x*- and *y*-values, and the graph exists only in the first quadrant.

y-Intercept: No distance traveled when he has ridden for 0 hours.

The point represents 0 hours ridden and 0 miles traveled. Graph this on the coordinate plane.

Linear or Nonlinear: The graph of the function is linear.

Positive: for time greater than 0

Increasing: for time greater than 0

The graph is a straight line that is positive and increasing for all hours ridden.

End Behavior: As the number of hours he has ridden increases, the number of miles he has traveled increases.

As *x* increases, *y* increases.

Go Online You can complete an Extra Example online.

Think About It!
What assumption is made when graphing Christoph Strasser's record-setting bike ride? Why is it necessary to make this assumption?

Sample answer: It is assumed that he rode at a constant rate over the entire 24-hour period. Assuming a constant speed allows for a reasonable linear representation.

Watch Out!
If information about some key features is not provided, do not assume that it is not important. In this example, the missing information about where the function is negative was not necessary because it did not apply in the context of the situation. However, this is not always the case.

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Interactive Presentation

Example 1

TAP

Students move through the slides to see how to use key features to sketch a graph of the function.

TYPE

Students discuss the assumption made when graphing the bike ride.

Example 1 Sketch the Graph of a Linear Function

Teaching the Mathematical Practices

4 Make Assumptions In the Think About It!, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL Does the graph have an *x*-intercept? yes What are the coordinates of the intercept? (0, 0)

OL What does the *x*-intercept mean in this context? Sample answer: When no time has passed, he has ridden 0 miles.

BL Given this situation, would it make sense for the function to decrease over any interval? Explain. Sample answer: No; distances cannot be negative. Even if he had turned around at some point, the total distance he had ridden would still increase. The slope of the line may vary over the 24 hours, but it will never be negative.

Example 2 Sketch the Graph of a Symmetric Function

Teaching the Mathematical Practices

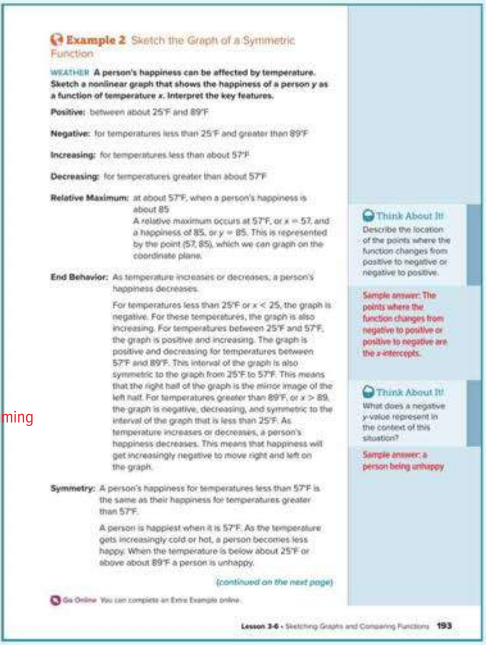
1 Explain Correspondences Guide students as they use each piece of information in Example 2 to sketch a graph to represent the situation.

Questions for Mathematical Discourse

AL How many x-intercepts does the function have? 2

OL What is happening in the graph when it changes from increasing to decreasing? Sample answer: This is the point where the graph curves. In this case, it is the maximum.

BL How many quadrants are needed to show the whole graph? Explain. Sample answer: 3; The positive part of the graph appears in Quadrant I, but because we know the function is negative for $x < 25$ and $x > 89$, the graph extends into Quadrant IV. Assuming that the left half of the graph continues as shown, it will eventually cross into Quadrant III, which makes sense because temperatures can be negative.



Example 2 Sketch the Graph of a Symmetric Function

WEATHER **A person's happiness can be affected by temperature. Sketch a nonlinear graph that shows the happiness of a person y as a function of temperature x. Interpret the key features.**

Positive: between about 25°F and 89°F

Negative: for temperatures less than 25°F and greater than 89°F

Increasing: for temperatures less than about 57°F

Decreasing: for temperatures greater than about 57°F

Relative Maximum: at about 57°F, when a person's happiness is about 85
A relative maximum occurs at 57°F, or $x = 57$, and a happiness of 85, or $y = 85$. This is represented by the point (57, 85), which we can graph on the coordinate plane.

End Behavior: As temperature increases or decreases, a person's happiness decreases.

For temperatures less than 25°F or $x < 25$, the graph is negative. For these temperatures, the graph is also increasing. For temperatures between 25°F and 57°F, the graph is positive and increasing. The graph is positive and decreasing for temperatures between 57°F and 89°F. This interval of the graph is also symmetric to the graph from 25°F to 57°F. This means that the right half of the graph is the mirror image of the left half. For temperatures greater than 89°F, or $x > 89$, the graph is negative, decreasing, and symmetric to the interval of the graph that is less than 25°F. As temperature increases or decreases, a person's happiness decreases. This means that happiness will get increasingly negative to move right and left on the graph.

Symmetry: A person's happiness for temperatures less than 57°F is the same as their happiness for temperatures greater than 57°F.

A person is happiest when it is 57°F. As the temperature gets increasingly cold or hot, a person becomes less happy. When the temperature is below about 25°F or above about 89°F a person is unhappy.

(continued on the next page)

Go Online You can complete an Extra Example online.

Think About It! Describe the location of the points where the function changes from positive to negative or negative to positive.

Sample answer: The points where the function changes from negative to positive or positive to negative are the x-intercepts.

Think About It! What does a negative y-value represent in the context of this situation?

Sample answer: a person being unhappy

Lesson 3-6 • Sketching Graphs and Comparing Functions 193

Interactive Presentation

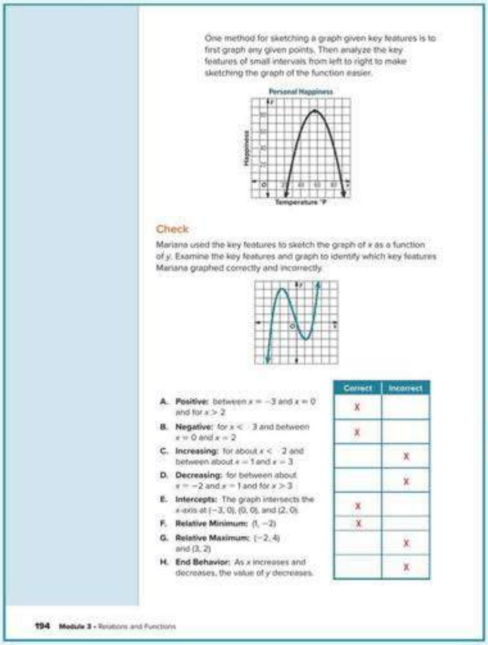
Example 2

TAP

Students move through the steps to graph a real-world function.

TYPE

Students describe the location of the points where the function changes signs.



One method for sketching a graph given key features is to first graph any given points. Then analyze the key features of small intervals from left to right to make sketching the graph of the function easier.

Check

Mariana used the key features to sketch the graph of x as a function of y. Examine the key features and graph to identify which key features Mariana graphed correctly and incorrectly.

	Correct	Incorrect
A. **Positive:** between $x = -3$ and $x = 0$ and for $x > 2$	X	
B. **Negative:** for $x < -3$ and between $x = 0$ and $x = 2$	X	
C. **Increasing:** for about $x < -2$ and between about $x = 1$ and $x = 3$		X
D. **Decreasing:** for between about $x = -2$ and $x = 1$ and for $x > 3$		X
E. **Intercepts:** The graph intersects the x-axis at $(-3, 0)$, $(0, 0)$, and $(2, 0)$.	X	
F. **Relative Minimum:** $(1, -2)$	X	
G. **Relative Maximum:** $(-2, 4)$ and $(3, 2)$		X
H. **End Behavior:** As x increases and decreases, the value of y decreases.		X

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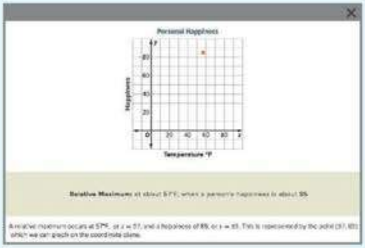
Interactive Presentation



Example 2

SWYPE

Students move through the slides to analyze the key features of a function.

Common Error

Some students may confuse "positive" with "increasing" and "negative" with "decreasing." Review these key features and help students reinforce their understanding of their differences

DIFFERENTIATE

Language Development Activity AL BL ELL

IF students are having trouble sketching the graph of a nonlinear function,

THEN have students work together to discuss the examples in this lesson. You may also want them to complete some of the Check exercises cooperatively.

DIFFERENTIATE

Enrichment Activity BL

Have pairs of students challenge each other to draw graphs with given key features. One student draws a graph without showing it to the other and describes its key features. The second student should draw a graph that fits the description. The students should discuss similarities and differences in the graphs and whether both graphs fit the description. Then have them switch roles and repeat.

Example 3 Sketch the Graph of a Nonlinear Function

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about sketching graphs of functions to solving a real-world problem.

Questions for Mathematical Discourse

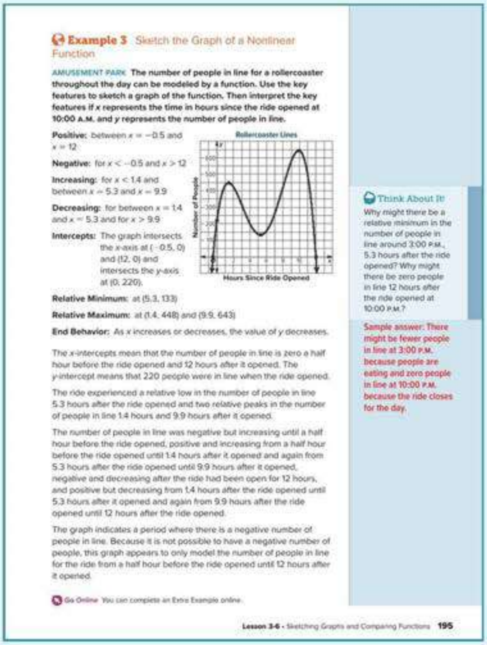
AL Which points are the easiest to graph first? the intercepts and the relative extrema

OL How many quadrants are needed to show the whole graph? Explain. Sample answer: 2; Most of the graph appears in Quadrant I, but because people lined up before the ride opened, we need Quadrant II. We do not need Quadrants III or IV because it is not possible for a negative number of people to be in line.

BL Is this the only possible graph for this situation and the given key features? Explain. No; sample answer: The example shows smooth curves between the known points, but we could draw straight lines or more wiggly lines connecting the points instead, as long as we do not introduce more local extrema in the process.

Common Error

Some students may be confused about the fact that the graph does not contain the origin. Discuss what the *y*-intercept represents in this context, and why the *y*-intercept is not 0.



Example 3 Sketch the Graph of a Nonlinear Function

AMUSEMENT PARK **The number of people in line for a rollercoaster throughout the day can be modeled by a function. Use the key features to sketch a graph of the function. Then interpret the key features if x represents the time in hours since the ride opened at 10:00 A.M. and y represents the number of people in line.**

Positive: between $x = -0.5$ and $x = 12$

Negative: for $x < -0.5$ and $x > 12$

Increasing: for $x < 1.4$ and between $x = 5.3$ and $x = 9.9$

Decreasing: for between $x = 1.4$ and $x = 5.3$ and for $x > 9.9$

Intercepts: The graph intersects the x-axis at $(-0.5, 0)$ and $(12, 0)$ and intersects the y-axis at $(0, 220)$.

Relative Minimum: at $(5.3, 133)$

Relative Maximum: at $(1.4, 448)$ and $(9.9, 643)$

End Behavior: As x increases or decreases, the value of y decreases.

The x-intercepts mean that the number of people in line is zero a half hour before the ride opened and 12 hours after it opened. The y-intercept means that 220 people were in line when the ride opened.

The ride experienced a relative low in the number of people in line 5.3 hours after the ride opened and two relative peaks in the number of people in line 1.4 hours and 9.9 hours after it opened.

The number of people in line was negative but increasing until a half hour before the ride opened, positive and increasing from a half hour before the ride opened until 1.4 hours after it opened and again from 5.3 hours after the ride opened until 9.9 hours after it opened, negative and decreasing after the ride had been open for 12 hours, and positive but decreasing from 1.4 hours after the ride opened until 5.3 hours after it opened and again from 9.9 hours after the ride opened until 12 hours after the ride opened.

The graph indicates a period where there is a negative number of people in line. Because it is not possible to have a negative number of people, this graph appears to only model the number of people in line for the ride from a half hour before the ride opened until 12 hours after it opened.

Go Online You can complete an Extra Example online.

Think About It! Why might there be a relative minimum in the number of people in line around 3:00 P.M., 5.3 hours after the ride opened? Why might there be zero people in line 12 hours after the ride opened at 10:00 P.M.?

Sample answer: There might be fewer people in line at 3:00 P.M. because people are eating and zero people in line at 10:00 P.M. because the ride closes for the day.

Lesson 3-6 • Sketching Graphs and Comparing Functions 195

Interactive Presentation

Sketch the Graph of a Nonlinear Function

AMUSEMENT PARK The number of people in line for a roller coaster throughout the day can be modeled by a function. Move through the slides to see how to use the key features to sketch a graph that shows the number of people in line y as a function of time x in hours since the ride opened at 10:00 A.M. Interpret the key features.

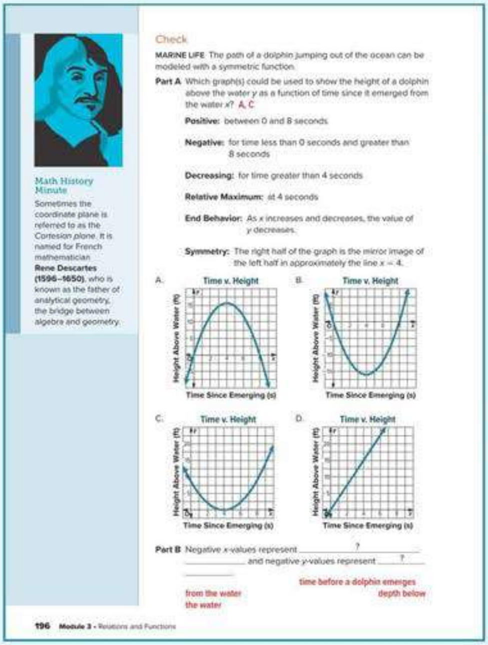
Example 3

TAP

Students move through the steps to graph a nonlinear function.

TYPE

Students explain why there may be a relative minimum at a certain time and why there might be zero people in line at a certain time.



Math History Minute

Sometimes the coordinate plane is referred to as the *Cartesian plane*. It is named for French mathematician **Rene Descartes (1596–1650)**, who is known as the father of analytical geometry, the bridge between algebra and geometry.

Check

MARINE LIFE The path of a dolphin jumping out of the ocean can be modeled with a symmetric function.

Part A Which graph(s) could be used to show the height of a dolphin above the water y as a function of time since it emerged from the water x? A, C

Positive: between 0 and 8 seconds

Negative: for time less than 0 seconds and greater than 8 seconds

Decreasing: for time greater than 4 seconds

Relative Maximum: at 4 seconds

End Behavior: As x increases and decreases, the value of y decreases.

Symmetry: The right half of the graph is the mirror image of the left half in approximately the line $x = 4$.

Part B Negative x-values represent ___?___ and negative y-values represent ___?___

time before a dolphin emerges from the water

depth below the water

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Interactive Presentation

Check

TYPE

Students interpret the key features in the context of the situation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Essential Question Follow-Up

Students have explored sketching graphs of functions.

Ask:

How does knowing about the key features of graphs of functions help you to sketch the graph of a function that represents a real-world situation? Sample answer: You can use the information about the situation to determine where to plot the intercepts and the extrema, and where the graph is increasing or decreasing and positive or negative. You can also get information about the end behavior to help you sketch the graph.

Example 4 Compare Properties of Functions

Teaching the Mathematical Practices

4 Model with Mathematics In Example 4, students apply what they have learned about sketching graphs to solving a real-world problem.

Questions for Mathematical Discourse

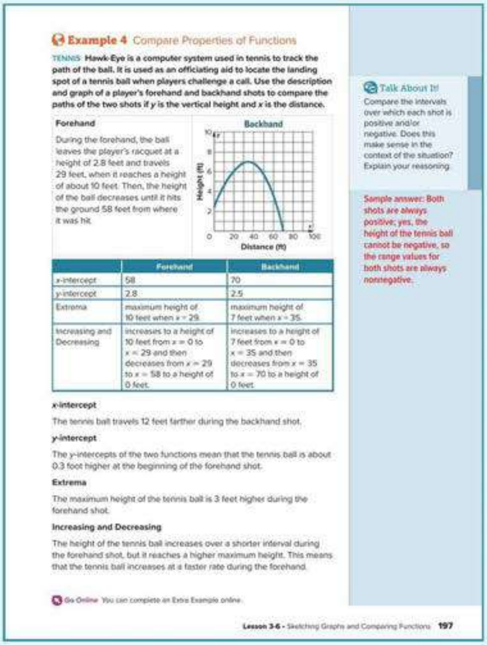
AL At what height does the ball leave the player's racquet on a forehand shot? 2.8 ft What key feature is this on the graph? the *y*-intercept

OL How far from the player does the ball hit the ground on a forehand shot? 58 ft on a backhand shot? 70 ft What key feature is this on the graph? the *x*-intercept

BL Why do you think the ball reaches a higher point on a forehand shot? Sample answer: The ball is hit from a slightly higher point, and perhaps the ball is at more of an upward angle.

Common Error

Some students may not recognize that the description of the forehand is a description of a continuous function. Help them to see that just as the graph represents the backhand function, the verbal description of the forehand contains information about the key features of the function that models the forehand.



Example 4 Compare Properties of Functions

TENNIS **Hawk-Eye is a computer system used in tennis to track the path of the ball. It is used as an officiating aid to locate the landing spot of a tennis ball when players challenge a call. Use the description and graph of a player's forehand and backhand shots to compare the paths of the two shots if *y* is the vertical height and *x* is the distance.**

Forehand

During the forehand, the ball leaves the player's racquet at a height of 2.8 feet and travels 29 feet, when it reaches a height of about 10 feet. Then, the height of the ball decreases until it hits the ground 58 feet from where it was hit.

	Forehand	Backhand
x-intercept	58	70
y-intercept	2.8	2.5
Extrema	maximum height of 10 feet when $x = 29$.	maximum height of 7 feet when $x = 35$.
Increasing and Decreasing	increases to a height of 10 feet from $x = 0$ to $x = 29$ and then decreases from $x = 29$ to $x = 58$ to a height of 0 feet.	increases to a height of 7 feet from $x = 0$ to $x = 35$ and then decreases from $x = 35$ to $x = 70$ to a height of 0 feet.

x-intercept

The tennis ball travels 12 feet farther during the backhand shot.

y-intercept

The *y*-intercepts of the two functions mean that the tennis ball is about 0.3 foot higher at the beginning of the forehand shot.

Extrema

The maximum height of the tennis ball is 3 feet higher during the forehand shot.

Increasing and Decreasing

The height of the tennis ball increases over a shorter interval during the forehand shot, but it reaches a higher maximum height. This means that the tennis ball increases at a faster rate during the forehand.

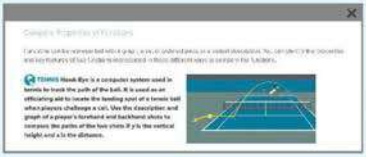
Go Online You can complete an Extra Example online.

Talk About It! Compare the intervals over which each shot is positive and/or negative. Does this make sense in the context of the situation? Explain your reasoning.

Sample answer: Both shots are always positive; yes, the height of the tennis ball cannot be negative, so the range values for both shots are always nonnegative.

Lesson 3-6 • Sketching Graphs and Comparing Functions 197

Interactive Presentation



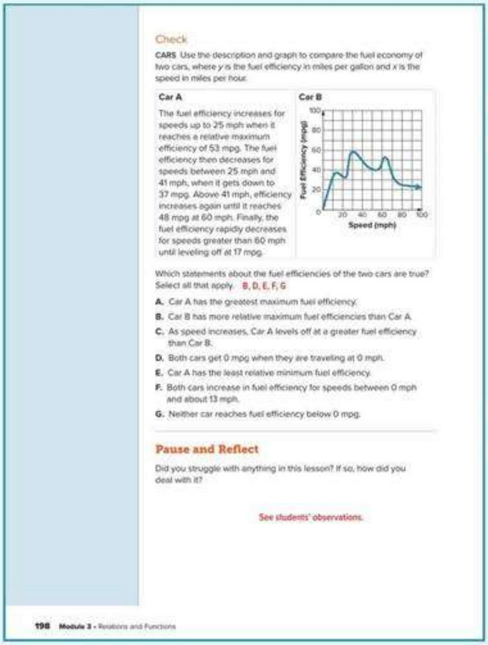
Example 4

TAP

Students move through the categories to compare the key features for each shot.

TYPE

Students compare the intervals over which each shot is positive and/or negative.



Check

CARS Use the description and graph to compare the fuel economy of two cars, where y is the fuel efficiency in miles per gallon and x is the speed in miles per hour.

Car A

The fuel efficiency increases for speeds up to 25 mph when it reaches a relative maximum efficiency of 53 mpg. The fuel efficiency then decreases for speeds between 25 mph and 41 mph, when it gets down to 37 mpg. Above 41 mph, efficiency increases again until it reaches 48 mpg at 60 mph. Finally, the fuel efficiency rapidly decreases for speeds greater than 60 mph until leveling off at 17 mpg.

Car B

Which statements about the fuel efficiencies of the two cars are true? Select all that apply. B, D, E, F, G

A. Car A has the greatest maximum fuel efficiency.

B. Car B has more relative maximum fuel efficiencies than Car A.

C. As speed increases, Car A levels off at a greater fuel efficiency than Car B.

D. Both cars get 0 mpg when they are traveling at 0 mph.

E. Car A has the least relative minimum fuel efficiency.

F. Both cars increase in fuel efficiency for speeds between 0 mph and about 13 mph.

G. Neither car reaches fuel efficiency below 0 mpg.

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

See students' observations.

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Interactive Presentation

Check

MULTIPLE CHOICE

Students decide which statements best represent a graph.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–8
2	exercises that use a variety of skills from this lesson	9–11
3	exercises that emphasize higher-order and critical-thinking skills	12–16

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–11 odd, 12–16
- Extension: Solving Equations: $f(x) = g(x)$
- **ALEKS** Graphs of Functions

OL

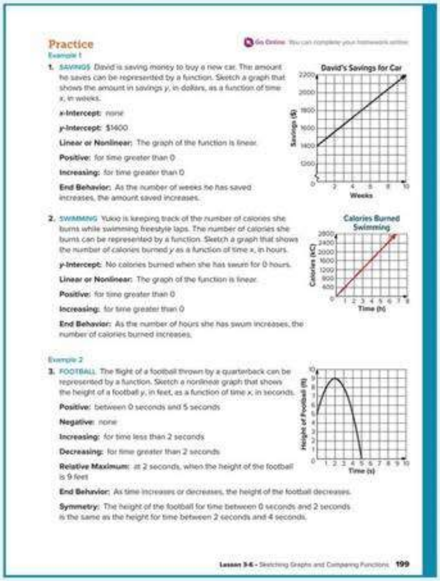
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–15 odd
- Remediation, Review Resources: Identify Functions
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** Introduction to Functions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–7 odd
- Remediation, Review Resources: Identify Functions
- *Quick Review Math Handbook*: Interpreting Graphs of Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Introduction to Functions



Practice

Go Online You can complete your homework online.

Example 1

1. SAVINGS David is saving money to buy a new car. The amount he saves can be represented by a function. Sketch a graph that shows the amount in savings y, in dollars, as a function of time x, in weeks.

x-Intercept: none

y-Intercept: $1400

Linear or Nonlinear: The graph of the function is linear.

Positive: for time greater than 0

Increasing: for time greater than 0

End Behavior: As the number of weeks he has saved increases, the amount saved increases.

2. SWIMMING Yuko is keeping track of the number of calories she burns while swimming freestyle laps. The number of calories she burns can be represented by a function. Sketch a graph that shows the number of calories burned y as a function of time x, in hours.

y-Intercept: No calories burned when she has swum for 0 hours.

Linear or Nonlinear: The graph of the function is linear.

Positive: for time greater than 0

Increasing: for time greater than 0

End Behavior: As the number of hours she has swum increases, the number of calories burned increases.

Example 2

3. FOOTBALL The flight of a football thrown by a quarterback can be represented by a function. Sketch a nonlinear graph that shows the height of a football y, in feet, as a function of time x, in seconds.

Positive: between 0 seconds and 5 seconds

Negative: none

Increasing: for time less than 2 seconds

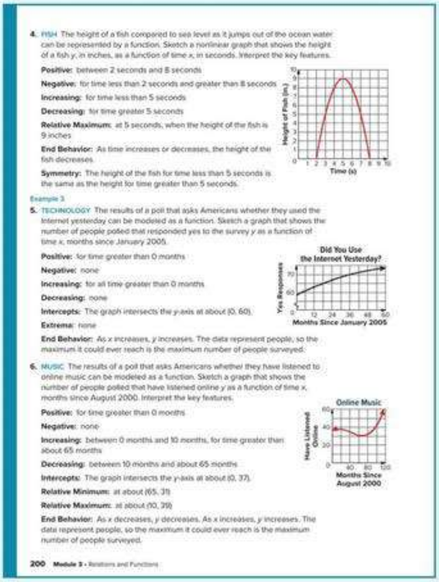
Decreasing: for time greater than 2 seconds

Relative Maximum: at 2 seconds, when the height of the football is 9 feet

End Behavior: As time increases or decreases, the height of the football decreases.

Symmetry: The height of the football for time between 0 seconds and 2 seconds is the same as the height for time between 2 seconds and 4 seconds.

Lesson 3-6 • Sketching Graphs and Comparing Functions **199**



4. FISH The height of a fish compared to sea level as it jumps out of the ocean water can be represented by a function. Sketch a nonlinear graph that shows the height of a fish y, in inches, as a function of time x, in seconds. Interpret the key features.

Positive: between 2 seconds and 8 seconds

Negative: for time less than 2 seconds and greater than 8 seconds

Increasing: for time less than 5 seconds

Decreasing: for time greater 5 seconds

Relative Maximum: at 5 seconds, when the height of the fish is 9 inches

End Behavior: As time increases or decreases, the height of the fish decreases.

Symmetry: The height of the fish for time less than 5 seconds is the same as the height for time greater than 5 seconds.

Example 3

5. TECHNOLOGY The results of a poll that asks Americans whether they used the Internet yesterday can be modeled as a function. Sketch a graph that shows the number of people polled that responded yes to the survey y as a function of time x, months since January 2005.

Positive: for time greater than 0 months

Negative: none

Increasing: for all time greater than 0 months

Decreasing: none

Intercepts: The graph intersects the y-axis at about (0, 60).

Extrema: none

End Behavior: As x increases, y increases. The data represent people, so the maximum it could ever reach is the maximum number of people surveyed.

Did You Use the Internet Yesterday?
Yes Responses
0
60
70
Months Since January 2005
12
24
36
48
60

6. MUSIC The results of a poll that asks Americans whether they have listened to online music can be modeled as a function. Sketch a graph that shows the number of people polled that have listened online y as a function of time x, months since August 2000. Interpret the key features.

Positive: for time greater than 0 months

Negative: none

Increasing: between 0 months and 10 months, for time greater than about 65 months

Decreasing: between 10 months and about 65 months

Intercepts: The graph intersects the y-axis at about (0, 37).

Relative Minimum: at about (65, 31)

Relative Maximum: at about (10, 39)

End Behavior: As x decreases, y decreases. As x increases, y increases. The data represent people, so the maximum it could ever reach is the maximum number of people surveyed.

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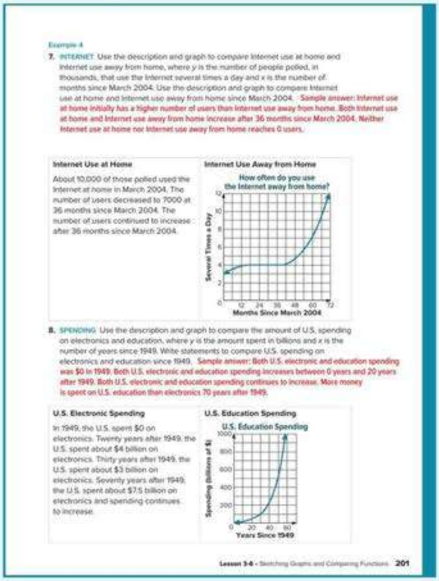


Example 4

7. INTERNET Use the description and graph to compare Internet use at home and Internet use away from home, where *y* is the number of people polled, in thousands, that use the Internet several times a day and *x* is the number of months since March 2004. Use the description and graph to compare Internet use at home and Internet use away from home since March 2004. Sample answer: Internet use at home initially has a higher number of users than Internet use away from home. Both Internet use at home and Internet use away from home increase after 36 months since March 2004. Neither Internet use at home nor Internet use away from home reaches 0 users.

Internet Use at Home

About 10,000 of those polled used the Internet at home in March 2004. The number of users decreased to 7000 at 36 months since March 2004. The number of users continued to increase after 36 months since March 2004.

Internet Use Away from Home

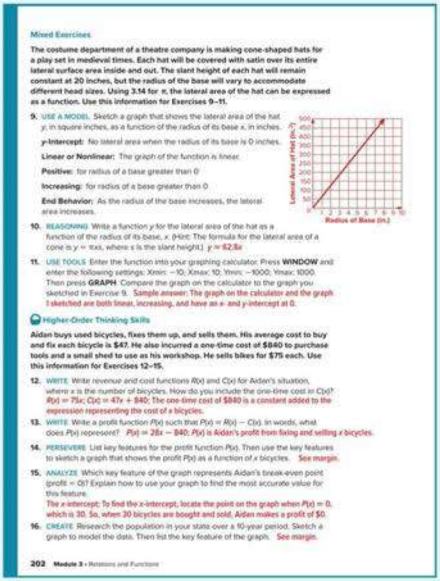
8. SPENDING Use the description and graph to compare the amount of U.S. spending on electronics and education, where *y* is the amount spent in billions and *x* is the number of years since 1949. Write statements to compare U.S. spending on electronics and education since 1949. Sample answer: Both U.S. electronic and education spending was \$0 in 1949. Both U.S. electronic and education spending increases between 0 years and 20 years after 1949. Both U.S. electronic and education spending continues to increase. More money is spent on U.S. education than electronics 70 years after 1949.

U.S. Electronic Spending

In 1949, the U.S. spent \$0 on electronics. Twenty years after 1949, the U.S. spent about \$4 billion on electronics. Thirty years after 1949, the U.S. spent about \$3 billion on electronics. Seventy years after 1949, the U.S. spent about \$7.5 billion on electronics and spending continues to increase.

U.S. Education Spending

Lesson 3-6 • Sketching Graphs and Comparing Functions 201



Mixed Exercises

The costume department of a theatre company is making cone-shaped hats for a play set in medieval times. Each hat will be covered with satin over its entire lateral surface area inside and out. The slant height of each hat will remain constant at 20 inches, but the radius of the base will vary to accommodate different head sizes. Using 3.14 for π, the lateral area of the hat can be expressed as a function. Use this information for Exercises 9–11.

9. USE A MODEL Sketch a graph that shows the lateral area of the hat *y*, in square inches, as a function of the radius of its base *x*, in inches.

y-Intercept: No lateral area when the radius of its base is 0 inches.

Linear or Nonlinear: The graph of the function is linear.

Positive: for radius of a base greater than 0

Increasing: for radius of a base greater than 0

End Behavior: As the radius of the base increases, the lateral area increases.

10. REASONING Write a function *y* for the lateral area of the hat as a function of the radius of its base, *x*. (Hint: The formula for the lateral area of a cone is $y = \pi x s$, where *s* is the slant height.) $y = 62.8x$

11. USE TOOLS Enter the function into your graphing calculator. Press **WINDOW** and enter the following settings: Xmin: −10; Xmax: 10; Ymin: −1000; Ymax: 1000. Then press **GRAPH**. Compare the graph on the calculator to the graph you sketched in Exercise 9. Sample answer: The graph on the calculator and the graph I sketched are both linear, increasing, and have an *x*- and *y*-intercept at 0.

Higher-Order Thinking Skills

Aidan buys used bicycles, fixes them up, and sells them. His average cost to buy and fix each bicycle is \$47. He also incurred a one-time cost of \$840 to purchase tools and a small shed to use as his workshop. He sells bikes for \$75 each. Use this information for Exercises 12–15.

12. WRITE Write revenue and cost functions $R(x)$ and $C(x)$ for Aidan's situation, where *x* is the number of bicycles. How do you include the one-time cost in $C(x)$? $R(x) = 75x$; $C(x) = 47x + 840$; The one-time cost of \$840 is a constant added to the expression representing the cost of *x* bicycles.

13. WRITE Write a profit function $P(x)$ such that $P(x) = R(x) - C(x)$. In words, what does $P(x)$ represent? $P(x) = 28x - 840$; $P(x)$ is Aidan's profit from fixing and selling *x* bicycles.

14. PERSEVERE List key features for the profit function $P(x)$. Then use the key features to sketch a graph that shows the profit $P(x)$ as a function of *x* bicycles. See margin.

15. ANALYZE Which key feature of the graph represents Aidan's break-even point (profit = 0)? Explain how to use your graph to find the most accurate value for this feature.
The *x*-intercept; To find the *x*-intercept, locate the point on the graph when $P(x) = 0$, which is 30. So, when 30 bicycles are bought and sold, Aidan makes a profit of \$0.

16. CREATE Research the population in your state over a 10-year period. Sketch a graph to model the data. Then list the key feature of the graph. See margin.

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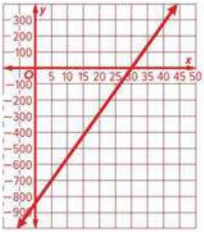
Answers

14. *y*-intercept: The profit for buying and selling 0 bicycles is −\$840. Linear or Nonlinear: The function is linear. Positive: for greater than 30 bicycles. Increasing: for greater than 0 bicycles. End behavior: As the number of bicycles increases, the profit increases.



16. Sample answer: I researched the population of Pennsylvania from 2007 to 2017. The graph is a nonlinear function that is always positive. The *x*-axis represent years since 2007 and the *y*-axis represents the population in millions. The *y*-intercept is (0, 12.52) meaning that 0 years since 2007, or in 2007, the population of Pennsylvania was 12.52 million. As time increases, the population increases.

Module 3 • Relations and Functions

Review

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it helpful to have several different representations of the same relation?
- Why is it useful to have a graph of a function for a real-world situation?
- Why are graphs useful representations of functions?
- How does knowing about the key features of graphs of functions help you to sketch the graph of a function that represents a real-world situation?

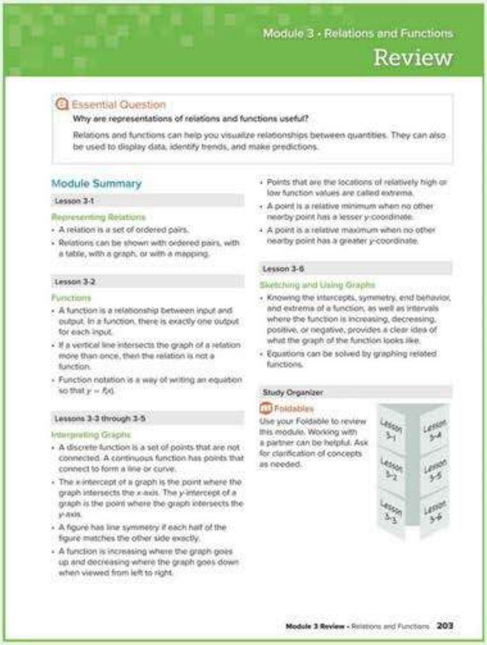
Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to relations and functions.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Linear and Exponential Relationships**.

- Understand the Concept of a Function and Use Function Notation
- Analyze Functions Using Different Representations
- Interpret Functions that Arise in Applications in Terms of the Context



Module 3 • Relations and Functions

Review

Essential Question

Why are representations of relations and functions useful?

Relations and functions can help you visualize relationships between quantities. They can also be used to display data, identify trends, and make predictions.

Module Summary

Lesson 3-1

Representing Relations

- A relation is a set of ordered pairs.
- Relations can be shown with ordered pairs, with a table, with a graph, or with a mapping.

Lesson 3-2

Functions

- A function is a relationship between input and output. In a function, there is exactly one output for each input.
- If a vertical line intersects the graph of a relation more than once, then the relation is not a function.
- Function notation is a way of writing an equation so that $y = f(x)$.

Lessons 3-3 through 3-5

Interpreting Graphs

- A discrete function is a set of points that are not connected. A continuous function has points that connect to form a line or curve.
- The x-intercept of a graph is the point where the graph intersects the x-axis. The y-intercept of a graph is the point where the graph intersects the y-axis.
- A figure has line symmetry if each half of the figure matches the other side exactly.
- A function is increasing where the graph goes up and decreasing where the graph goes down when viewed from left to right.
- Points that are the locations of relatively high or low function values are called extrema.
- A point is a relative minimum when no other nearby point has a lesser y-coordinate.
- A point is a relative maximum when no other nearby point has a greater y-coordinate.

Lesson 3-6

Sketching and Using Graphs

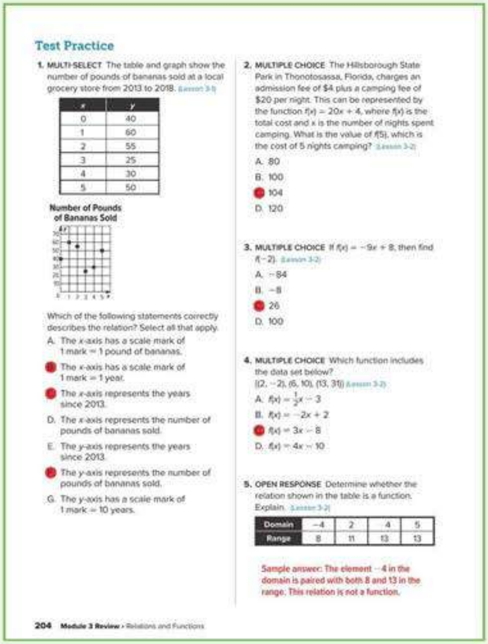
- Knowing the intercepts, symmetry, end behavior, and extrema of a function, as well as intervals where the function is increasing, decreasing, positive, or negative, provides a clear idea of what the graph of the function looks like.
- Equations can be solved by graphing related functions.

Study Organizer

Foldables

Use your Foldable to review this module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 3 Review • Relations and Functions 203



Test Practice

1. MULTI-SELECT The table and graph show the number of pounds of bananas sold at a local grocery store from 2013 to 2018. (Lesson 3-1)

x	y
0	40
1	60
2	55
3	25
4	30
5	50

Number of Pounds of Bananas Sold

Which of the following statements correctly describes the relation? Select all that apply.

A. The x-axis has a scale mark of 1 mark = 1 pound of bananas.

● The x-axis has a scale mark of 1 mark = 1 year.

● The x-axis represents the years since 2013.

D. The x-axis represents the number of pounds of bananas sold.

E. The y-axis represents the years since 2013.

● The y-axis represents the number of pounds of bananas sold.

G. The y-axis has a scale mark of 1 mark = 10 years.

2. MULTIPLE CHOICE The Hillsborough State Park in Thonotosassa, Florida, charges an admission fee of $4 plus a camping fee of $20 per night. This can be represented by the function $f(x) = 20x + 4$, where $f(x)$ is the total cost and x is the number of nights spent camping. What is the value of $f(5)$, which is the cost of 5 nights camping? (Lesson 3-2)

A. 80

B. 100

● 104

D. 120

3. MULTIPLE CHOICE If $f(x) = -9x + 8$, then find $f(-2)$. (Lesson 3-2)

A. −84

B. −8

● 26

D. 100

4. MULTIPLE CHOICE Which function includes the data set below?
$\{(2, -2), (6, 10), (13, 31)\}$ (Lesson 3-2)

A. $f(x) = \frac{1}{2}x - 3$

B. $f(x) = -2x + 2$

● $f(x) = 3x - 8$

D. $f(x) = 4x - 10$

5. OPEN RESPONSE Determine whether the relation shown in the table is a function. Explain. (Lesson 3-2)

Domain	−4	2	4	5
Range	8	11	13	13

Sample answer: The element −4 in the domain is paired with both 8 and 13 in the range. This relation is not a function.

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 3-1 through 3-5
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

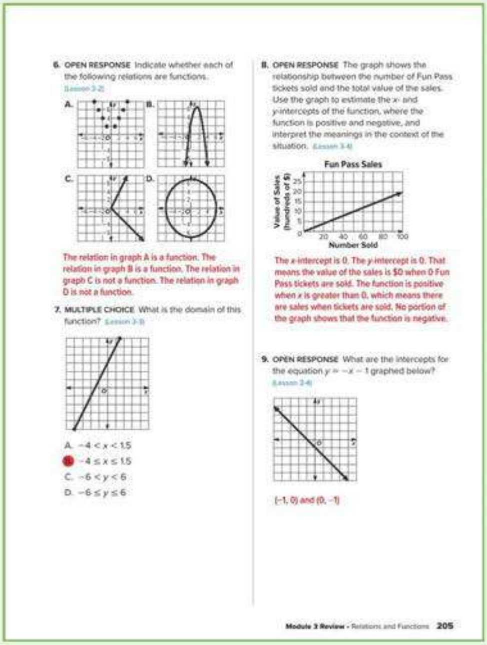
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	2–4, 7, 12
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	1
Table Item	Students complete a table by entering in the correct values.	6
Open Response	Students construct their own response.	5, 8–11, 13

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
N.Q.1	3-1	1
F.IF.1	3-2	4–6
F.IF.2	3-2	2, 3
F.IF.4	3-4, 3-5	8, 10–12
F.IF.5	3-3	7
F.IF.9	3-6	13
A.REI.10	3-4	9



6. **OPEN RESPONSE** Indicate whether each of the following relations are functions. (Lesson 3-2)

A. B. C. D.

The relation in graph A is a function. The relation in graph B is a function. The relation in graph C is not a function. The relation in graph D is not a function.

7. **MULTIPLE CHOICE** What is the domain of this function? (Lesson 3-3)

A. $-4 < x < 1.5$
● $-4 \leq x \leq 1.5$
C. $-6 < y < 6$
D. $-6 \leq y \leq 6$

8. **OPEN RESPONSE** The graph shows the relationship between the number of Fun Pass tickets sold and the total value of the sales. Use the graph to estimate the x- and y-intercepts of the function, where the function is positive and negative, and interpret the meanings in the context of the situation. (Lesson 3-4)

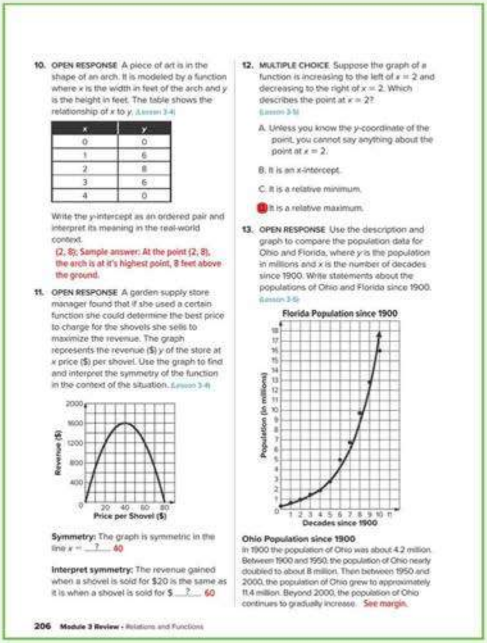
Fun Pass Sales

The x-intercept is 0. The y-intercept is 0. That means the value of the sales is \$0 when 0 Fun Pass tickets are sold. The function is positive when x is greater than 0, which means there are sales when tickets are sold. No portion of the graph shows that the function is negative.

9. **OPEN RESPONSE** What are the intercepts for the equation $y = -x - 1$ graphed below? (Lesson 3-4)

(−1, 0) and (0, −1)

Module 3 Review • Relations and Functions 205



10. **OPEN RESPONSE** A piece of art is in the shape of an arch. It is modeled by a function where x is the width in feet of the arch and y is the height in feet. The table shows the relationship of x to y. (Lesson 3-4)

x	y
0	0
1	6
2	8
3	6
4	0

Write the y-intercept as an ordered pair and interpret its meaning in the real-world context.

(2, 8); Sample answer: At the point (2, 8), the arch is at it's highest point, 8 feet above the ground.

11. **OPEN RESPONSE** A garden supply store manager found that if she used a certain function she could determine the best price to charge for the shovels she sells to maximize the revenue. The graph represents the revenue ($) y of the store at x price ($) per shovel. Use the graph to find and interpret the symmetry of the function in the context of the situation. (Lesson 3-4)

Symmetry: The graph is symmetric in the line $x =$ __?__ 40

Interpret symmetry: The revenue gained when a shovel is sold for $20 is the same as it is when a shovel is sold for $ __?__ 60

12. **MULTIPLE CHOICE** Suppose the graph of a function is increasing to the left of $x = 2$ and decreasing to the right of $x = 2$. Which describes the point at $x = 2$? (Lesson 3-5)

A. Unless you know the y-coordinate of the point, you cannot say anything about the point at $x = 2$.

B. It is an x-intercept.

C. It is a relative minimum.

[✓ D.] It is a relative maximum.

13. **OPEN RESPONSE** Use the description and graph to compare the population data for Ohio and Florida, where y is the population in millions and x is the number of decades since 1900. Write statements about the populations of Ohio and Florida since 1900. (Lesson 3-5)

Florida Population since 1900

Ohio Population since 1900
In 1900 the population of Ohio was about 4.2 million. Between 1900 and 1950, the population of Ohio nearly doubled to about 8 million. Then between 1950 and 2000, the population of Ohio grew to approximately 11.4 million. Beyond 2000, the population of Ohio continues to gradually increase. See margin.

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Answers

13. Sample answer: In 1900 the population of Ohio was nearly 4 million more than the population of Florida. Both populations grew between 1900 and 1950. At this point, the population of Ohio exceeded that of Florida by approximately 5 million, indicating a greater growth rate for Ohio than Florida during those decades. Then from 1950 to 2000, the population of Ohio grew by about 3.4 million, whereas the population of Florida grew by about 13 million, indicating a significantly greater growth rate for Florida during those decades. In fact, by 2000, the population of Florida surpassed Ohio by more than 4 million people.

Module 4

Linear and Nonlinear Functions

Module Goals

- Students graph linear, piecewise-defined, step, and absolute value functions.
- Students find and interpret the rate of change and slope of lines.
- Students identify the effects of transformations on the graphs of linear and absolute value functions.

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Also addresses A.CED.2, A.REI.10, F.IF.4, F.IF.6, F.BF.1a, F.BF.2, F.LE.1a, F.LE.2, and F.LE.5.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Be Sure to Cover

To completely cover F.LE.1a, go online to assign the following activity:

- Linear Growth Patterns (Expand 4-3)

Coherence

Vertical Alignment

Previous

Students interpreted the equation $y = mx + b$ as defining a linear function and gave examples of functions that are not linear.

8.F.3

Now

Students write and graph linear and nonlinear equations.

F.IF.7a, F.IF.7b, F.BF.3

Next

Students will build linear and nonlinear functions to model real-world data and relationships.

F.BF.1 (Course 1, Course 2, Course 3)

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

EXPLORE → LEARN → EXAMPLE & PRACTICE

Suggested Pacing

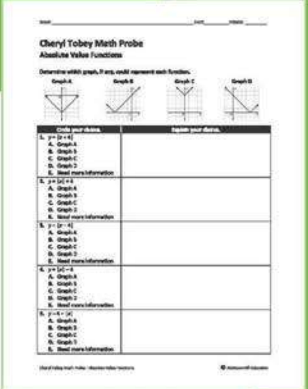
Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
4-1 Graphing Linear Functions	A.REI.10, F.IF.7a, F.LE.5	1	0.5
4-2 Rate of Change and Slope	F.IF.6, F.LE.5	1	0.5
4-3 Slope-Intercept Form	A.CED.2, F.IF.7a, F.LE.5	2	1
4-3 Expand Linear Growth Patterns	F.LE.1a	1	0.5
4-4 Transformations of Linear Functions	F.IF.7a, F.BF.3	2	1
4-5 Arithmetic Sequences	F.BF.1a, F.BF.2, F.LE.2	1	0.5
4-6 Piecewise and Step Functions	F.IF.4, F.IF.7b	1	0.5
4-7 Absolute Value Functions	F.IF.7b, F.BF.3	2	1
Put It All Together: Lessons 4-6 through 4-7		1	0.5
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	**15**	**7.5**

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Absolute Value Functions

Module Resource



Correct Answers: 1. B 2. C 3. D 4. A 5. C

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which graph matches the correct function and explain their choices.

Targeted Concepts Certain modifications to the parent function of an absolute value function will result in predictable transformations of the graph.

Targeted Misconceptions

- Students may not recognize a horizontal transformation and/or predict an incorrect direction of a horizontal transformation.
- Students may not recognize a vertical transformation and/or predict an incorrect direction of a vertical transformation.

Use the Probe after Lesson 4-7.

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
1. D **3.** B	recognizes the horizontal shift but fails to use the opposite value of the number associated with *x* to determine the direction of the shift. **Example:** For Item 1, the student recognizes that positive 4 is associated with the *x*-value (horizontal shift) but moves the graph to the right.
2. A **4.** C	recognizes the vertical shift but fails to use the same value of the number associated with *y* to determine the direction of the shift. **Example:** For Item 2, the student recognizes that positive 4 is associated with the *y*-value (vertical shift) but moves the graph down.
5. A	recognizes the vertical shift but is confused with the direction of the shift when the number is placed on the same side as *y*. **Example:** For Item 5, the student recognizes that negative 4 is associated with the *y*-value (vertical shift) but does not solve for *y* before using the “rules” of transformation and moves the graph down.
1. A **2.** D **3.** C **4.** B	confuses a horizontal shift with a vertical shift **Example:** For Item 3, the student incorrectly moves the graph up 4 units instead of to the right 4 units.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Absolute Value Functions
- Lesson 4-7, all Learns, all Examples

Revisit the probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

What can a function tell you about the relationship that it represents? Sample answer: It can tell you about the rate of change, whether the relationship is positive or negative, the locations of the *x*- and *y*-intercepts, and what points fall on the graph.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

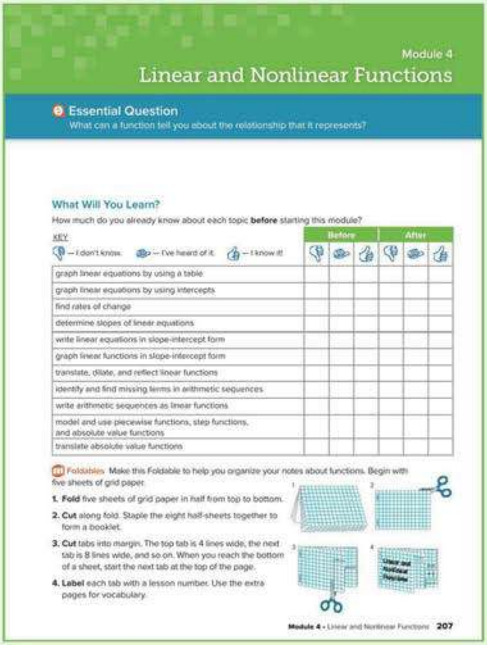
Focus As students read and study this module, they should show examples and write notes about linear functions and relations.

Teach Have students make and label their Foldables as illustrated. Students should label the front of each half page with the lesson title. On the back of each of these pages, they can record concepts and notes from that particular lesson.

When to Use It Encourage students to add to their Foldables as they work through the module and to use them to review for the module test.

Launch the Module

For this module, the Launch the Module video uses real-world scenarios to illustrate how functions and their graphs can be used to model both linear and nonlinear relationships. Students learn about using graphs to model the change in altitude of an airplane and the change in strength of a Wi-Fi signal.



Module 4

Linear and Nonlinear Functions

Essential Question

What can a function tell you about the relationship that it represents?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. ✋ — I've heard of it. 👍 — I know it!

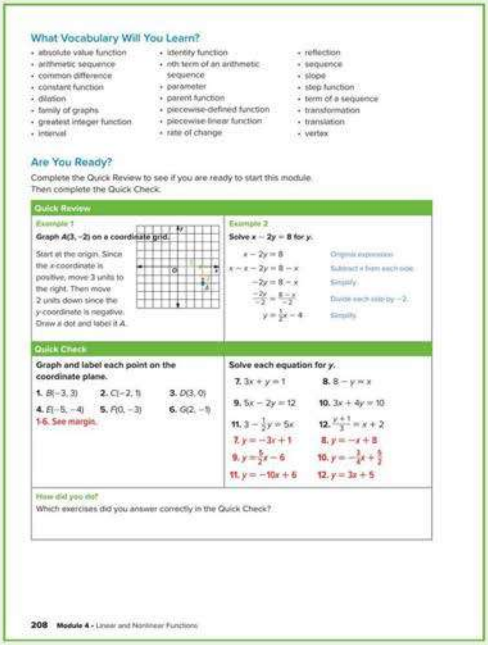
	Before			After		
	👎	✋	👍	👎	✋	👍
graph linear equations by using a table						
graph linear equations by using intercepts						
find rates of change						
determine slopes of linear equations						
write linear equations in slope-intercept form						
graph linear functions in slope-intercept form						
translate, dilate, and reflect linear functions						
identify and find missing terms in arithmetic sequences						
write arithmetic sequences as linear functions						
model and use piecewise functions, step functions, and absolute value functions						
translate absolute value functions						

Foldables Make this Foldable to help you organize your notes about functions. Begin with five sheets of grid paper.

1. **Fold** five sheets of grid paper in half from top to bottom.
2. **Cut** along fold. Staple the eight half-sheets together to form a booklet.
3. **Cut** tabs into margin. The top tab is 4 lines wide, the next tab is 8 lines wide, and so on. When you reach the bottom of a sheet, start the next tab at the top of the page.
4. **Label** each tab with a lesson number. Use the extra pages for vocabulary.

Module 4 • Linear and Nonlinear Functions 207

Interactive Presentation



What Vocabulary Will You Learn?

- absolute value function
- arithmetic sequence
- common difference
- constant function
- dilation
- family of graphs
- greatest integer function
- interval
- identity function
- nth term of an arithmetic sequence
- parameter
- parent function
- piecewise-defined function
- piecewise-linear function
- rate of change
- reflection
- sequence
- slope
- step function
- term of a sequence
- transformation
- translation
- vertex

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Graph $A(3, -2)$ on a coordinate grid.

Start at the origin. Since the x-coordinate is positive, move 3 units to the right. Then move 2 units down since the y-coordinate is negative. Draw a dot and label it A.

Example 2

Solve $x - 2y = 8$ for y.

$x - 2y = 8$	Original expression
$x - x - 2y = 8 - x$	Subtract x from each side.
$-2y = 8 - x$	Simplify.
$\frac{-2y}{-2} = \frac{8 - x}{-2}$	Divide each side by -2.
$y = \frac{1}{2}x - 4$	Simplify.

Quick Check

Graph and label each point on the coordinate plane.

1. $B(-3, 3)$
2. $C(-2, 1)$
3. $D(3, 0)$
4. $E(-5, -4)$
5. $F(0, -3)$
6. $G(2, -1)$

1-6. See margin.

Solve each equation for y.

7. $3x + y = 1$
8. $8 - y = x$
9. $5x - 2y = 12$
10. $3x + 4y = 10$
11. $3 - \frac{1}{2}y = 5x$
12. $\frac{y + 1}{3} = x + 2$

7. $y = -3x + 1$
8. $y = -x + 8$
9. $y = \frac{5}{2}x - 6$
10. $y = -\frac{3}{4}x + \frac{5}{2}$
11. $y = -10x + 6$
12. $y = 3x + 5$

How did you do?

Which exercises did you answer correctly in the Quick Check?

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define The **slope** of a line is the rate of change in the y-coordinates (rise) for the corresponding change in the x-coordinates (run) for points on the line.

Example A line passes through the points (1, 4) and (3, 8).

Ask What is the slope of the line? 2

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- identifying domain and range
- identifying slopes
- translating and reflecting geometric figures
- finding the next terms in patterns
- graphing linear functions
- evaluating absolute value expressions

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the topics in the **Functions and Lines** module—who is ready to learn these topics and who isn't quite ready to learn them yet—in order to adjust your instruction as appropriate.

Mindset Matters

Collaborative Risk Taking

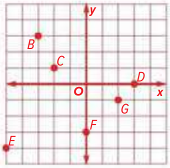
Some students may be averse to taking risks during math class, like sharing an idea, strategy, or solution. They may worry about their grades or scores on tests, or some might feel less confident solving math problems, especially in front of their peers.

How Can I Apply It?

Assign the **Practice** problems of each lesson and encourage students to take risks as they solve problems, try new paths, and discuss their strategies with their partner or group.

Answer

1-6.



Lesson 4-1

Graphing Linear Functions

A.REI.10, F.IF.7a, F.LE.5

LESSON GOAL

Students graph linear functions by using tables and intercepts.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Points on a Line

Develop:

Graphing Linear Functions by Using Tables
- Graph by Making a Table
- Choose Appropriate Domain Values
- Graph $y = a$
- Graph $x = a$

Explore: Lines Through Two Points

Develop:

Graphing Linear Functions by Using the Intercepts
- Graph by Using Intercepts
- Use Intercepts

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Proportional Relationships and Slope	●	●		●
Extension: Graphing Equations in Three Dimensions		●	●	●

Language Development Handbook

Assign page 20 of the *Language Development Handbook* to help your students build mathematical language related to graphing linear functions.

ELL You can use the tips and suggestions on page T20 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students sketched graphs and compared graphs of functions.
8.F.2, 8.F.4, F.IF.4, F.IF.9

Now
Students graph linear functions using tables and intercepts.
A.REI.10, F.IF.7a, F.LE.5

Next
Students will investigate rate of change and slope.
F.IF.6, F.LE.5

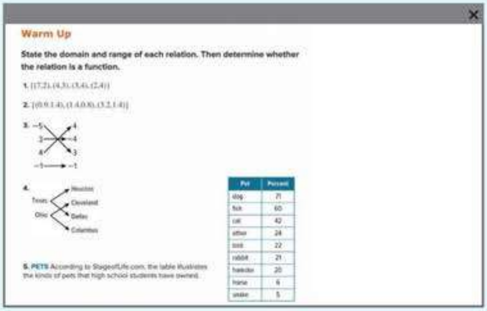
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and fluency with linear functions (first studied in Grade 8) to graphing linear functions by using a table and by using intercepts. They apply their understanding of linear functions by solving real-world problems.

Interactive Presentation



Warm Up

State the domain and range of each relation. Then determine whether the relation is a function.

1. {(7,2), (4,3), (3,4), (2,4)}

2. {(0.9,1.4), (1.4,0.8), (3.2,1.4)}

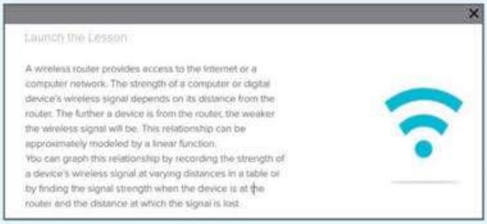
3.

4.

5. **PETS** According to StageofLife.com, the table illustrates the kinds of pets that high school students have owned.

Pet	Percent
dog	71
fish	60
cat	42
other	24
bird	22
rabbit	21
hamster	20
horse	6
snake	5

Warm Up



Launch the Lesson

A wireless router provides access to the Internet or a computer network. The strength of a computer or digital device's wireless signal depends on its distance from the router. The further a device is from the router, the weaker the wireless signal will be. This relationship can be approximately modeled by a linear function.
You can graph this relationship by recording the strength of a device's wireless signal at varying distances in a table or by finding the signal strength when the device is at the router and the distance at which the signal is lost.

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying domain and range

Answers:

1. D: {2, 3, 4, 7}, R: {2, 3, 4}; yes
2. D: {0.9, 1.4, 3.2}, R: {0.8, 1.4}; yes
3. D: {–5, –1, 3, 4}, R: {–4, –1, 3, 4}; yes
4. D: {Ohio, Texas}, R: {Cleveland, Columbus, Dallas, Houston}; no
5. D: {dog, fish, cat, other, bird, rabbit, hamster, horse, snake}, R: {5, 6, 20, 21, 22, 24, 42, 60, 71}; yes

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain how the verbal description of the relationship between the device's strength and the distance from the router can be modeled by a function, which can be used to create a table of values and a graph.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Mathematical Background

The graph of a linear function is a line. The coordinates of the points on the line are the solutions of the related linear equation. If you know at least two solutions of the equation, you can use them to graph the line. You can also use the x- and y-intercepts to graph the line. The intercepts can be found by alternately replacing x and y with 0. The line that connects the intercepts is the graph of the linear equation.

Explore Points on a Line

Objective

Students explore the relationship between graphs of linear equations and their solutions.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in this Explore.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

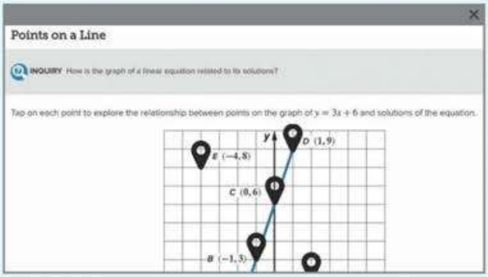
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will be presented with a linear equation and its graph. Several points on the coordinate plane are marked and labeled, some on the graph, and some not on the graph. Students will record the coordinates of the marked points, and determine whether each pair of coordinates makes the equation true. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

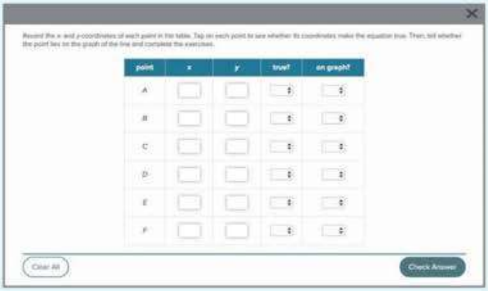


Points on a Line

INQUIRY How is the graph of a linear equation related to its solutions?

Tap on each point to explore the relationship between points on the graph of $y = 3x + 6$ and solutions of the equation.

Explore



Record the x- and y-coordinates of each point in the table. Tap on each point to see whether its coordinates make the equation true. Then, tell whether the point lies on the graph of the line and complete the exercises.

point	x	y	true?	on graph?
A				
B				
C				
D				
E				
F				

Explore

TAP

Students tap on each point to explore the relationship between points on a graph and solutions of an equation.

TYPE

Students complete a table and answer questions about the points that make an equation true.

Interactive Presentation

INQUIRY How is the graph of a linear equation related to its solutions?

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Points on a Line (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is it important to know if coordinates make an equation true? Sample answer: It is important to know when the substituted values make both sides of the equation equal. The coordinates that make the equation true are solutions of the equation.
- Given a graph of a linear function, how could you find a solution of the related equation? Sample answer: I could look for coordinates on the line because any point on the line is a solution of the related equation.

Inquiry

How is the graph of a linear equation related to its solutions?
Sample answer: The graph of a line is all of the solutions of its equation plotted on a coordinate plane.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Explore Lines Through Two Points

Objective

Students use a sketch to explore the number of lines that pass through two points.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use a sketch. Work with students to explore and deepen their understanding of lines through two points.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to explore the number of lines that can be drawn through a single point. They will then explore the number of lines that can be drawn through two points. Then, students will answer the Inquiry Question.

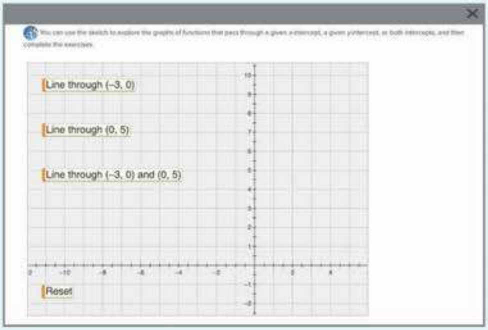
(continued on the next page)

Interactive Presentation

Lines Through Two Points

INQUIRY How many lines can be formed with two given points?

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore the graphs of linear functions.

TYPE

Students answer questions about the graphed functions.

Interactive Presentation

INQUIRY How many lines can be formed with two given points?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Lines Through Two Points (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Can you graph a function from a table that has only two points? Sample answer: As long as you know that the function is linear, it is okay for the table to only list two points.
- When graphing, do you think it would be better to use two points close together or farther apart? Sample answer: Farther apart would help you get a better idea of where the line should be drawn. If the points are too close together, you might not have your ruler or line tool lined up correctly.

Inquiry

How many lines can be formed with two given points? Sample answer: There is only one line that can be formed with two given points.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphing Linear Functions by Using Tables

Objective

Students graph linear functions by making a table of values.

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the table, coordinates, equation, and graph of a linear function.

What Students Are Learning

Students come to understand that although a table of values can be used to construct the graph of a linear function, the graph represents all of the solutions of the equation. They learn that every point on the graph represents a pair of coordinates that is a solution of the equation.

Example 1 Graph by Making a Table

MP Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

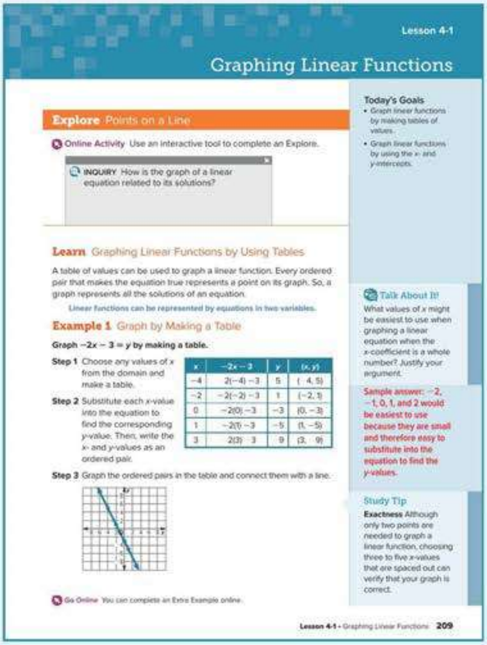
AL What values are in the domain of the function? all real numbers

OL Why is it helpful to choose both positive and negative values? Sample answer: Choosing positive and negative values gives you a better idea of what the graph will look like and will show you where the graph crosses the *y*-axis.

BL What should you do if one of the points you graph is not on the same line as the others? Sample answer: Check your work to see if you miscalculated the *y*-value.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-1

Graphing Linear Functions

Explore Points on a Line

Online Activity Use an interactive tool to complete an Explore.

INQUIRY How is the graph of a linear equation related to its solutions?

Learn Graphing Linear Functions by Using Tables

A table of values can be used to graph a linear function. Every ordered pair that makes the equation true represents a point on its graph. So, a graph represents all the solutions of an equation.

Linear functions can be represented by equations in two variables.

Example 1 Graph by Making a Table

Graph $-2x - 3 = y$ by making a table.

Step 1 Choose any values of x from the domain and make a table.

Step 2 Substitute each x-value into the equation to find the corresponding y-value. Then, write the x- and y-values as an ordered pair.

x	$-2x - 3$	y	(x, y)
-4	$-2(-4) - 3$	5	$(-4, 5)$
-2	$-2(-2) - 3$	1	$(-2, 1)$
0	$-2(0) - 3$	-3	$(0, -3)$
1	$-2(1) - 3$	-5	$(1, -5)$
3	$-2(3) - 3$	-9	$(3, -9)$

Step 3 Graph the ordered pairs in the table and connect them with a line.

Go Online You can complete an Extra Example online.

Today's Goals
- Graph linear functions by making tables of values.
- Graph linear functions by using the x- and y-intercepts.

Talk About It!

What values of x might be easiest to use when graphing a linear equation when the x-coefficient is a whole number? Justify your argument.

Sample answer: −2, −1, 0, 1, and 2 would be easiest to use because they are small and therefore easy to substitute into the equation to find the y-values.

Study Tip

Exactness Although only two points are needed to graph a linear function, choosing three to five x-values that are spaced out can verify that your graph is correct.

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Interactive Presentation

Graph $-2x - 3 = y$ by making a table.

x	$-2x - 3$	y	(x, y)
-4			
-2			
0			
1			
3			

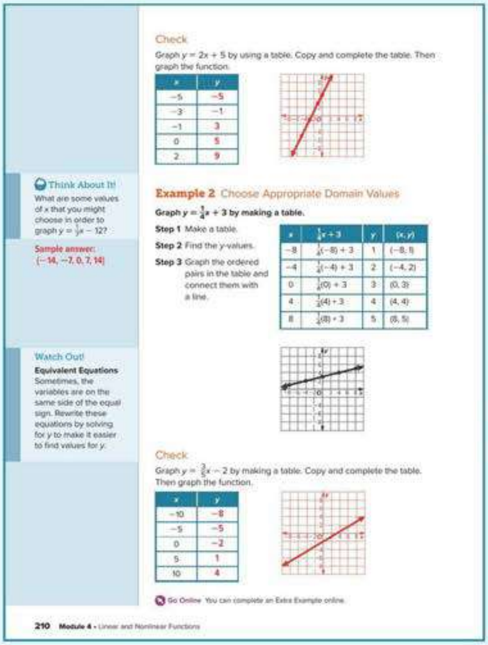
Example 1

TAP

Students move through the steps to see how to make a table of values for a line.

TYPE

Students discuss the *x*-values that would be easiest to use when graphing a linear equation if the coefficient of *x* is an integer.



Think About It!

What are some values of x that you might choose in order to graph $y = \frac{1}{7}x - 12$?

Sample answer: {−14, −7, 0, 7, 14}

Watch Out!

Equivalent Equations Sometimes, the variables are on the same side of the equal sign. Rewrite these equations by solving for y to make it easier to find values for y.

Check

Graph $y = 2x + 5$ by using a table. Copy and complete the table. Then graph the function.

x	y
−5	−5
−3	−1
−1	3
0	5
2	9

Example 2 Choose Appropriate Domain Values

Graph $y = \frac{1}{4}x + 3$ by making a table.

Step 1 Make a table.

Step 2 Find the y-values.

Step 3 Graph the ordered pairs in the table and connect them with a line.

x	$\frac{1}{4}x + 3$	y	(x, y)
−8	$\frac{1}{4}(-8) + 3$	1	(−8, 1)
−4	$\frac{1}{4}(-4) + 3$	2	(−4, 2)
0	$\frac{1}{4}(0) + 3$	3	(0, 3)
4	$\frac{1}{4}(4) + 3$	4	(4, 4)
8	$\frac{1}{4}(8) + 3$	5	(8, 5)

Check

Graph $y = \frac{3}{5}x - 2$ by making a table. Copy and complete the table. Then graph the function.

x	y
−10	−8
−5	−5
0	−2
5	1
10	4

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 2

WEB SKETCHPAD

Students use a sketch to graph the ordered pairs from the table of values.

TYPE

Students give the possible domain values for a given linear equation with a rational slope.

Example 2 Choose Appropriate Domain Values

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the equation, table, and graph in this example.

Questions for Mathematical Discourse

AL What values are in the domain? all real numbers

OL Why were the values selected for x in the table −8, −4, 0, 4, and 8? Sample answer: They were all multiples of 4 and since the coefficient is $\frac{1}{4}$ this makes multiplication easier.

BL What would happen if you used multiples of 2 for x in the table? Sample answer: Multiples of 2 that are also multiples of 4 would cancel out the denominator, but others would reduce to have a denominator of 2.

Common Error

Some students may make calculation errors when working with a coefficient that is a fraction. Help them avoid this by suggesting that they write the integer that they are substituting for x as a fraction with a denominator of 1.

Example 3 Graph $y = a$

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How is this equation different from other linear equations that you have worked with? Sample answer: The coefficient of x is zero.

OL What would the table look like for other values of x? Sample answer: The y-values would all be 5.

BL Is the graph a function? Explain Y es; sample answer: This is a function because it passes the vertical line test.

Common Error

Some students may interpret an equation such as $y = 5$ as a point, not a line. Help them to see that although the equation specifies that $y = 5$, x could be infinitely many values. Use a table to show how this leads to the graph of $y = 5$ consisting of more than one point.

Example 4 Graph $x = a$

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this example, students will need to use a sketch. Work with students to explore and deepen their understanding of graphs of horizontal lines.

Questions for Mathematical Discourse

AL How is this equation different from other linear equations that you have worked with? Sample answer: There is only one variable, x.

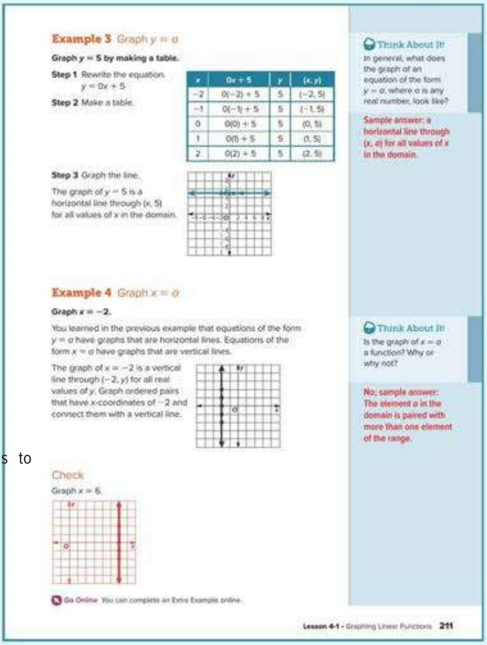
OL What is the x-intercept for the graph of an equation of the form $x = a$? $(a, 0)$

BL Why does every point of the form $(-2, y)$ satisfy the equation? Sample answer: Because the equation has no y-variable, substituting any point $(-2, y)$ into the equation will result in the true statement $-2 = -2$.

DIFFERENTIATE

Language Development Activity AL BL ELL

IF students are having difficulty remembering which equations represent horizontal lines and which represent vertical lines, **THEN** have them use the acronyms HOY and VUX to remember which is which. HOY stands for "Horizontal, 0 slope, $y =$," and VUX stands for "Vertical, Undefined slope, $x =$."



Example 3 Graph $y = a$

Graph $y = 5$ by making a table.

Step 1 Rewrite the equation.
$y = 0x + 5$

Step 2 Make a table.

x	$0x + 5$	y	(x, y)
-2	$0(-2) + 5$	5	$(-2, 5)$
-1	$0(-1) + 5$	5	$(-1, 5)$
0	$0(0) + 5$	5	$(0, 5)$
1	$0(1) + 5$	5	$(1, 5)$
2	$0(2) + 5$	5	$(2, 5)$

Step 3 Graph the line.

The graph of $y = 5$ is a horizontal line through $(x, 5)$ for all values of x in the domain.

Think About It! In general, what does the graph of an equation of the form $y = a$, where a is any real number, look like?

Sample answer: a horizontal line through (x, a) for all values of x in the domain.

Example 4 Graph $x = a$

Graph $x = -2$.

You learned in the previous example that equations of the form $y = a$ have graphs that are horizontal lines. Equations of the form $x = a$ have graphs that are vertical lines.

The graph of $x = -2$ is a vertical line through $(-2, y)$ for all real values of y. Graph ordered pairs that have x-coordinates of -2 and connect them with a vertical line.

Think About It! Is the graph of $x = a$ a function? Why or why not?

No; sample answer: The element a in the domain is paired with more than one element of the range.

Check

Graph $x = 6$.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Graph $y = a$

Graph $y = 5$ by making a table.

Rewrite the equation.

Make a table.

Graph the line.

Example 3

TAP

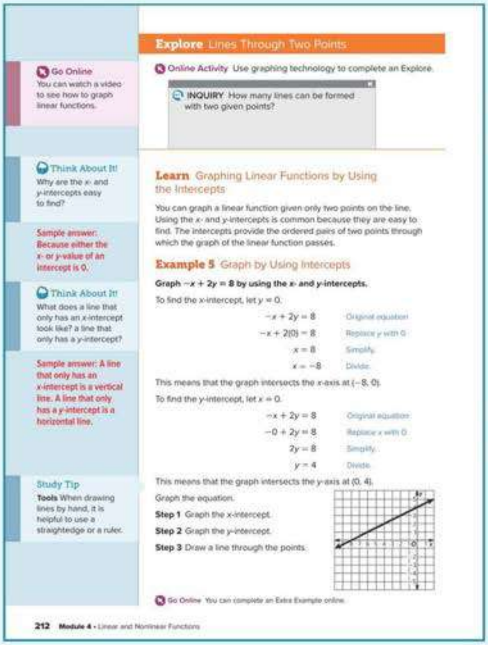
Students move through the steps to graph a line in theform $y = a$.

TYPE

Students explain what the graph of an equation in the form $y = a$ will look like.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Explore Lines Through Two Points

Go Online You can watch a video to see how to graph linear functions.

Online Activity Use graphing technology to complete an Explore.

INQUIRY How many lines can be formed with two given points?

Think About It! Why are the x- and y-intercepts easy to find?

Sample answer: Because either the x- or y-value of an intercept is 0.

Learn Graphing Linear Functions by Using the Intercepts

You can graph a linear function given only two points on the line. Using the x- and y-intercepts is common because they are easy to find. The intercepts provide the ordered pairs of two points through which the graph of the linear function passes.

Example 5 Graph by Using Intercepts

Graph $-x + 2y = 8$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$-x + 2y = 8$ Original equation
$-x + 2(0) = 8$ Replace y with 0.
$x = 8$ Simplify.
$x = -8$ Divide.

This means that the graph intersects the x-axis at $(-8, 0)$.

To find the y-intercept, let $x = 0$.

$-x + 2y = 8$ Original equation
$-0 + 2y = 8$ Replace x with 0.
$2y = 8$ Simplify.
$y = 4$ Divide.

This means that the graph intersects the y-axis at $(0, 4)$.

Graph the equation.

Step 1 Graph the x-intercept.

Step 2 Graph the y-intercept.

Step 3 Draw a line through the points.

Think About It! What does a line that only has an x-intercept look like? a line that only has a y-intercept?

Sample answer: A line that only has an x-intercept is a vertical line. A line that only has a y-intercept is a horizontal line.

Study Tip
Tools When drawing lines by hand, it is helpful to use a straightedge or a ruler.

Go Online You can complete an Extra Example online.

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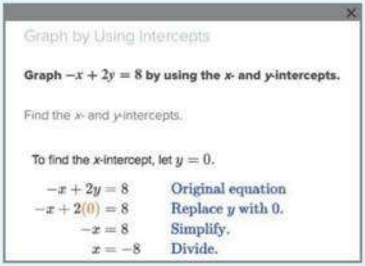
Interactive Presentation



Graph by Using Intercepts

Graph $-x + 2y = 8$ by using the x- and y-intercepts.

Find the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$-x + 2y = 8$ Original equation
$-x + 2(0) = 8$ Replace y with 0.
$-x = 8$ Simplify.
$x = -8$ Divide.

Example 5

WEB SKETCHPAD

Students use a sketch to graph a linear function.

TYPE

a

Students describe what a line looks like that only has an x– or y– intercept.

Learn Graphing Linear Functions by Using the Intercepts

Objective

Students graph linear functions by using the x- and y-intercepts.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

Some students may think that the x- and y-intercepts are the coefficients of x and y. Use an example such as $3x + 2y = 12$ to review the process of finding intercepts and show that neither coefficient is an intercept.

Example 5 Graph by Using Intercepts

MP Teaching the Mathematical Practices

5 Decide When to Use Tools Mathematically proficient students can make sound decisions about when to use mathematical tools such as a straightedge. Help them see why using these tools will help to solve problems and what the limitations are of using the tool.

Questions for Mathematical Discourse

AL What are the intercepts of the graph of a linear function? the points where the line crosses the x- and y-axes

OL How does finding the x- and y-intercepts help you to graph the function? Sample answer: Two points make a line, so a line can be drawn using the intercepts as the two points.

BL When finding the x-intercept, why do you substitute 0 for y in the equation? Sample answer: The y-coordinate of any point on the x-axis is 0, so substituting 0 for y in the equation tells you the value of x when $y = 0$, which is the x-intercept of the graph of the function.

Example 6 Use Intercepts

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about graphing linear functions to solving a real-world problem.

Questions for Mathematical Discourse

AL What information is given in the problem? Angelina starts with 60 cups of dog food and feeds her dog $\frac{5}{2}$ cups per day.

OL What does each variable represent, and what does this tell you about the intercepts? Sample answer: x represents days, and y represents cups of food. So the x-intercept represents the number of days when there are 0 cups of food left, and the y-intercept represents the amount of food when 0 days have passed.

BL Explain what the intercepts mean in the context of the problem. Sample answer: At 24 days, there is no food left. The bag started with 60 cups of food and after 24 days, the bag was empty.

Common Error

Some students may interchange the intercepts, thinking that when they let $x = 0$, they are finding the x-intercept or vice versa. Help students avoid this error by having them write the ordered pairs with the zeros in place before they solve algebraically. Then have them fill in the values they find, and plot the points from the ordered pairs.

Essential Question Follow-Up

Students have used a variety of methods to graph linear equations.

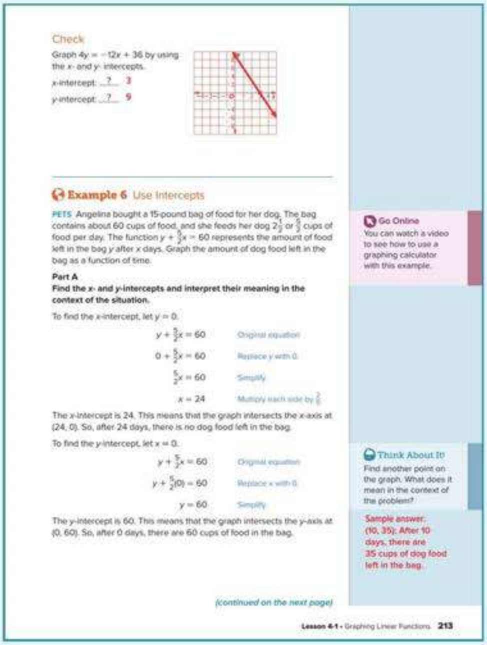
Ask:

Why is it helpful to have different ways to graph linear functions? Sample answer: Some methods of graphing are easier in different contexts. For instance, graphing by finding the x- and y-intercepts might be obvious from inspecting the particular equation. For a function that represents a real-world situation, it might be easier to create a table of values for the situation.

DIFFERENTIATE

Enrichment Activity **BL**

Have students work in pairs to create a poster about graphing linear equations. Have them include information about tables of values, intercepts, and the solutions of the equations in their display.



Check

Graph $4y = -12x + 36$ by using the x- and y-intercepts.

x-intercept: ? 3

y-intercept: ? 9

Example 6 Use Intercepts

PETS Angelina bought a 15-pound bag of food for her dog. The bag contains about 60 cups of food, and she feeds her dog $2\frac{1}{2}$ or $\frac{5}{2}$ cups of food per day. The function $y + \frac{5}{2}x = 60$ represents the amount of food left in the bag y after x days. Graph the amount of dog food left in the bag as a function of time.

Part A

Find the x- and y-intercepts and interpret their meaning in the context of the situation.

To find the x-intercept, let $y = 0$.

$y + \frac{5}{2}x = 60$ Original equation

$0 + \frac{5}{2}x = 60$ Replace y with 0.

$\frac{5}{2}x = 60$ Simplify.

$x = 24$ Multiply each side by $\frac{2}{5}$.

The x-intercept is 24. This means that the graph intersects the x-axis at (24, 0). So, after 24 days, there is no dog food left in the bag.

To find the y-intercept, let $x = 0$.

$y + \frac{5}{2}x = 60$ Original equation

$y + \frac{5}{2}(0) = 60$ Replace x with 0.

$y = 60$ Simplify.

The y-intercept is 60. This means that the graph intersects the y-axis at (0, 60). So, after 0 days, there are 60 cups of food in the bag.

Go Online You can watch a video to see how to use a graphing calculator with this example.

Think About It! Find another point on the graph. What does it mean in the context of the problem?

Sample answer: (10, 35); After 10 days, there are 35 cups of dog food left in the bag.

(continued on the next page)

Lesson 4-1 • Graphing Linear Functions 213

Interactive Presentation

Example 6

WEB SKETCHPAD

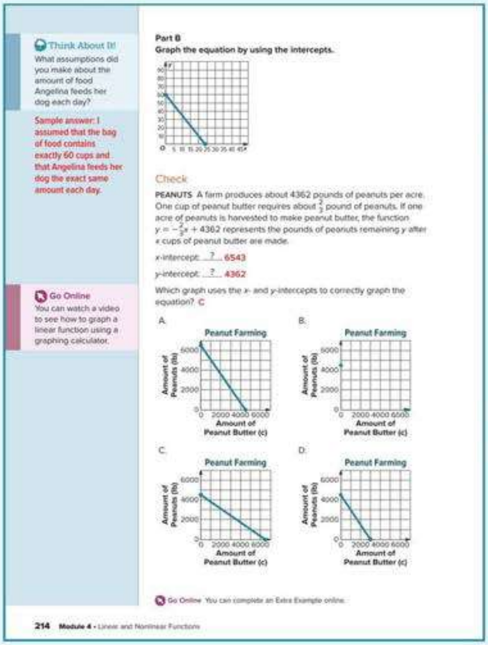
Students use a sketch to plot the intercepts and graph the line.

TYPE

Students explain the meaning of another point in context and identify the assumptions made.

WATCH

Students can watch a video to review how to graph a linear function using a graphing calculator.



Think About It! What assumptions did you make about the amount of food Angelina feeds her dog each day?

Sample answer: I assumed that the bag of food contains exactly 60 cups and that Angelina feeds her dog the exact same amount each day.

Go Online You can watch a video to see how to graph a linear function using a graphing calculator.

Part B

Graph the equation by using the intercepts.

Check

PEANUTS A farm produces about 4362 pounds of peanuts per acre. One cup of peanut butter requires about $\frac{2}{3}$ pound of peanuts. If one acre of peanuts is harvested to make peanut butter, the function $y = -\frac{2}{3}x + 4362$ represents the pounds of peanuts remaining y after x cups of peanut butter are made.

x-intercept: __?__ 6543

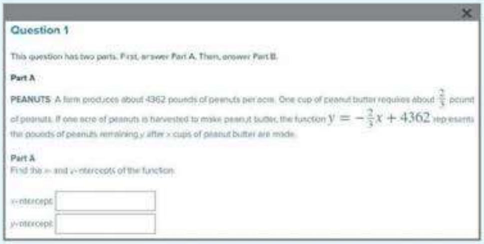
y-intercept: __?__ 4362

Which graph uses the x- and y-intercepts to correctly graph the equation? C

Go Online You can complete an Extra Example online.

214 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



Question 1

This question has two parts. First, answer Part A. Then, answer Part B.

Part A

PEANUTS A farm produces about 4362 pounds of peanuts per acre. One cup of peanut butter requires about $\frac{2}{3}$ pound of peanuts. If one acre of peanuts is harvested to make peanut butter, the function $y = -\frac{2}{3}x + 4362$ represents the pounds of peanuts remaining y after x cups of peanut butter are made.

Part A

Find the x- and y-intercepts of the function.

x-intercept:

y-intercept:

CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–16
2	exercises that use a variety of skills from this lesson	17–25
2	exercises that extend concepts learned in this lesson to new contexts	26–29
3	exercises that emphasize higher-order and critical-thinking skills	30–37

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–25 odd, 30–37
- Extension: Graphing Equations in Three Dimensions
- **ALEKS®** Ordered Pairs; Graphing Lines

OL

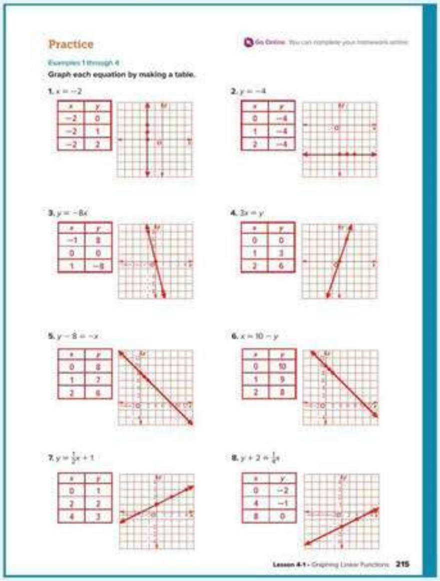
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–37 odd
- Remediation, Review Resources: Proportional Relationships and Slope
- Personal Tutors
- Extra Examples 1–6
- **ALEKS®** Proportional Relationships; Slope

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–15 odd
- Remediation, Review Resources: Proportional Relationships and Slope
- *Quick Review Math Handbook*: Linear Functions
- Arrive**MATH** Take Another Look
- **ALEKS®** Proportional Relationships; Slope



Practice

Go Online You can complete your homework online.

Examples 1 through 4

Graph each equation by making a table.

1. $x = -2$

x	y
−2	0
−2	1
−2	2

2. $y = -4$

x	y
0	−4
1	−4
2	−4

3. $y = -8x$

x	y
−1	8
0	0
1	−8

4. $3x = y$

x	y
0	0
1	3
2	6

5. $y - 8 = -x$

x	y
0	8
1	7
2	6

6. $x = 10 - y$

x	y
0	10
1	9
2	8

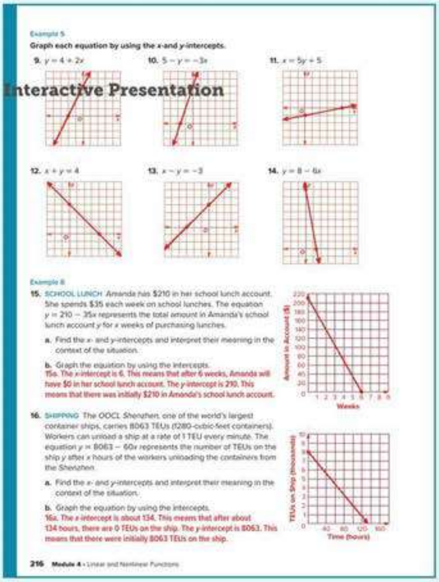
7. $y = \frac{1}{2}x + 1$

x	y
0	1
2	2
4	3

8. $y + 2 = \frac{1}{4}x$

x	y
0	−2
4	−1
8	0

Lesson 4-1 • Graphing Linear Functions 215



Example 5

Graph each equation by using the x- and y-intercepts.

9. $y = 4 + 2x$
10. $5 - y = -3x$
11. $x = 5y + 5$
12. $x + y = 4$
13. $x - y = -3$
14. $y = 8 - 6x$

Interactive Presentation

Example 6

15. SCHOOL LUNCH Amanda has $210 in her school lunch account. She spends $35 each week on school lunches. The equation $y = 210 - 35x$ represents the total amount in Amanda's school lunch account y for x weeks of purchasing lunches.
    a. Find the x- and y-intercepts and interpret their meaning in the context of the situation.
    b. Graph the equation by using the intercepts.

15a. The x-intercept is 6. This means that after 6 weeks, Amanda will have $0 in her school lunch account. The y-intercept is 210. This means that there was initially $210 in Amanda's school lunch account.

16. SHIPPING The OOCL *Shenzhen*, one of the world's largest container ships, carries 8063 TEUs (1280-cubic-feet containers). Workers can unload a ship at a rate of 1 TEU every minute. The equation $y = 8063 - 60x$ represents the number of TEUs on the ship y after x hours of the workers unloading the containers from the *Shenzhen*.
    a. Find the x- and y-intercepts and interpret their meaning in the context of the situation.
    b. Graph the equation by using the intercepts.

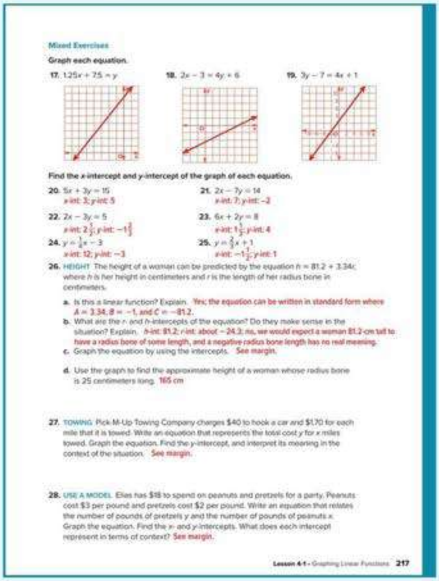
16a. The x-intercept is about 134. This means that after about 134 hours, there are 0 TEUs on the ship. The y-intercept is 8063. This means that there were initially 8063 TEUs on the ship.

216 Module 4 • Linear and Nonlinear Functions

3 REFLECT AND PRACTICE

A.REI.10, F.IF.7a, F.LE.5

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Mixed Exercises

Graph each equation.

17. $1.25x + 7.5 = y$

18. $2x - 3 = 4y + 6$

19. $3y - 7 = 4x + 1$

Find the x-intercept and y-intercept of the graph of each equation.

20. $5x + 3y = 15$
x-int: 3; y-int: 5

21. $2x - 7y = 14$
x-int: 7; y-int: −2

22. $2x - 3y = 5$
x-int: $2\frac{1}{2}$; y-int: $-1\frac{2}{3}$

23. $6x + 2y = 8$
x-int: $1\frac{1}{3}$; y-int: 4

24. $y = \frac{1}{4}x - 3$
x-int: 12; y-int: −3

25. $y = \frac{2}{3}x + 1$
x-int: $-1\frac{1}{2}$; y-int: 1

26. HEIGHT The height of a woman can be predicted by the equation $h = 81.2 + 3.34r$, where h is her height in centimeters and r is the length of her radius bone in centimeters.

a. Is this a linear function? Explain. Yes; the equation can be written in standard form where $A = 3.34$, $B = -1$, and $C = -81.2$.

b. What are the r- and h-intercepts of the equation? Do they make sense in the situation? Explain. h-int: 81.2; r-int: about −24.3; no, we would expect a woman 81.2-cm tall to have a radius bone of some length, and a negative radius bone length has no real meaning.

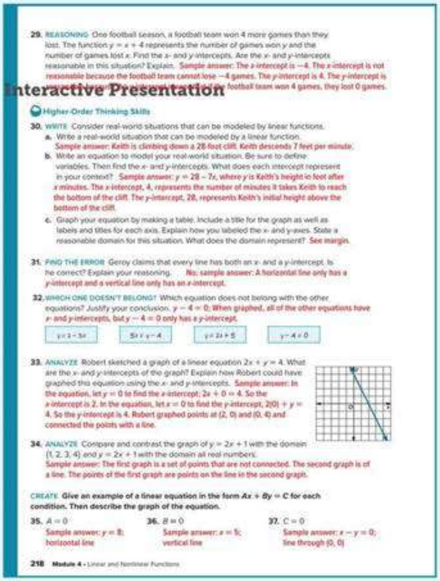
c. Graph the equation by using the intercepts. See margin.

d. Use the graph to find the approximate height of a woman whose radius bone is 25 centimeters long. 165 cm

27. TOWING Pick-M-Up Towing Company charges $40 to hook a car and $1.70 for each mile that it is towed. Write an equation that represents the total cost y for x miles towed. Graph the equation. Find the y-intercept, and interpret its meaning in the context of the situation. See margin.

28. USE A MODEL Elias has $18 to spend on peanuts and pretzels for a party. Peanuts cost $3 per pound and pretzels cost $2 per pound. Write an equation that relates the number of pounds of pretzels y and the number of pounds of peanuts x. Graph the equation. Find the x- and y-intercepts. What does each intercept represent in terms of context? See margin.

Lesson 4-1 • Graphing Linear Functions 217



29. REASONING One football season, a football team won 4 more games than they lost. The function $y = x + 4$ represents the number of games won y and the number of games lost x. Find the x- and y-intercepts. Are the x- and y-intercepts reasonable in this situation? Explain. Sample answer: The x-intercept is −4. The x-intercept is not reasonable because the football team cannot lose −4 games. The y-intercept is 4. The y-intercept is [illegible] football team won 4 games, they lost 0 games.

Interactive Presentation

Higher-Order Thinking Skills

30. WRITE Consider real-world situations that can be modeled by linear functions.

a. Write a real-world situation that can be modeled by a linear function. Sample answer: Keith is climbing down a 28-foot cliff. Keith descends 7 feet per minute.

b. Write an equation to model your real-world situation. Be sure to define variables. Then find the x- and y-intercepts. What does each intercept represent in your context? Sample answer: $y = 28 - 7x$, where y is Keith's height in feet after x minutes. The x-intercept, 4, represents the number of minutes it takes Keith to reach the bottom of the cliff. The y-intercept, 28, represents Keith's initial height above the bottom of the cliff.

c. Graph your equation by making a table. Include a title for the graph as well as labels and titles for each axis. Explain how you labeled the x- and y-axes. State a reasonable domain for this situation. What does the domain represent? See margin.

31. FIND THE ERROR Geroy claims that every line has both an x- and a y-intercept. Is he correct? Explain your reasoning. No; sample answer: A horizontal line only has a y-intercept and a vertical line only has an x-intercept.

32. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other equations? Justify your conclusion. $y - 4 = 0$; When graphed, all of the other equations have x- and y-intercepts, but $y - 4 = 0$ only has a y-intercept.

$y = 2 - 3x$ | $5x + y = 4$ | $y = 2x + 5$ | $y - 4 = 0$

33. ANALYZE Robert sketched a graph of a linear equation $2x + y = 4$. What are the x- and y-intercepts of the graph? Explain how Robert could have graphed this equation using the x- and y-intercepts. Sample answer: In the equation, let $y = 0$ to find the x-intercept; $2x + 0 = 4$. So the x-intercept is 2. In the equation, let $x = 0$ to find the y-intercept; $2(0) + y = 4$. So the y-intercept is 4. Robert graphed points at (2, 0) and (0, 4) and connected the points with a line.

34. ANALYZE Compare and contrast the graph of $y = 2x + 1$ with the domain {1, 2, 3, 4} and $y = 2x + 1$ with the domain all real numbers. Sample answer: The first graph is a set of points that are not connected. The second graph is of a line. The points of the first graph are points on the line in the second graph.

CREATE **Give an example of a linear equation in the form $Ax + By = C$ for each condition. Then describe the graph of the equation.**

35. $A = 0$
Sample answer: $y = 8$; horizontal line

36. $B = 0$
Sample answer: $x = 5$; vertical line

37. $C = 0$
Sample answer: $x - y = 0$; line through (0, 0)

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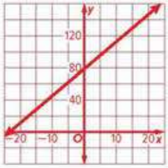
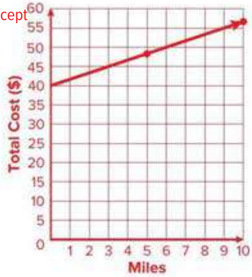
Answers

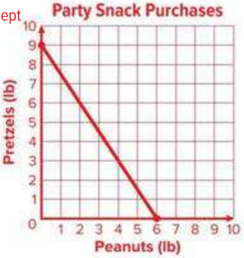
26c.



27. $y = 1.7x + 40$; The y-intercept is 40. This means that it would cost $40 to hook up the car.

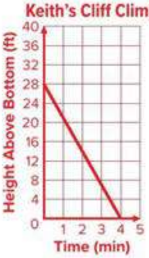


28. $3x + 2y = 18$; The x-intercept is 6. The x-intercept represents how many pounds of peanuts can be bought if no pretzels are bought. The y-intercept is 9. The y-intercept represents how many pounds of pretzels can be bought if no peanuts are bought.



30c. Sample answer: Time is the independent variable and Keith's height above the bottom of the cliff is the dependent variable, so label the x-axis as *Time (min)* and label the y-axis as *Height Above Bottom* (ft). Because height and time cannot be negative, a reasonable domain is $\{x \mid 0 \le x \le 4\}$. This represents the time it takes Keith to climb down the cliff.

Time (min) x	Height (ft) y
0	28
1	21
2	14
4	0



Lesson 4-2

Rate of Change and Slope

F.IF.6, F.LE.5

LESSON GOAL

Students find and interpret the rate of change and slopes of lines.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Rate of Change of a Linear Function

- Find the Rate of Change
- Compare Rates of Change
- Constant Rate of Change
- Rate of Change

Explore: Investigating Slope

Develop:

Slope of a Line

- Positive Slope
- Negative Slope
- Slopes of Horizontal Lines
- Slopes of Vertical Lines
- Find Coordinates Given the Slope
- Use Slope

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Order of Integer Operations	●	●		●
Extension: Treasure Hunt with Slopes		●	●	●

Language Development Handbook

Assign page 21 of the *Language Development Handbook* to help your students build mathematical language related to rates of change and slopes.

ELL You can use the tips and suggestions on page T21 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students understood the concept of slope and rate of change for linear functions.
8.EE.5, 8.EE.6

Now
Students find and interpret the rate of change and slopes of lines.
F.IF.6, F.LE.5

Next
Students will graph equations in slope-intercept form.
A.CED.2, F.IF.7a, F.LE.5

Rigor

The Three Pillars of Rigor

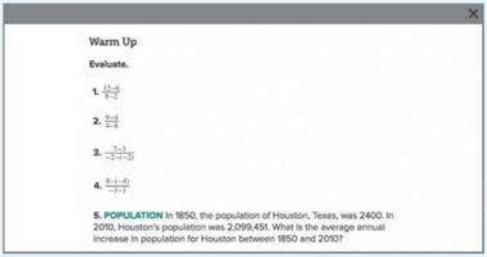
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and fluency with slope and rate of change (first studied in Grade 8). They apply their understanding of slope and rate of change by solving real-world problems.

Mathematical Background

Rate of change is a ratio that describes, on average, how one quantity changes with respect to a change in another quantity. Slope can be used to describe rate of change. The slope of a line is the ratio of the vertical change (the rise) to the horizontal change (the run). The slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points that lie on the line, can be used to find the slope of a line without graphing.

Interactive Presentation



Warm Up

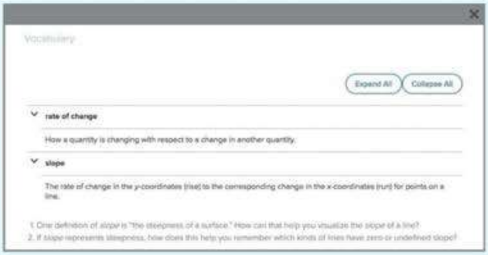
Evaluate.

5. POPULATION In 1850, the population of Houston, Texas, was 2400. In 2010, Houston's population was 2,099,451. What is the average annual increase in population for Houston between 1850 and 2010?

Warm Up

The Intimidator 305 at Kings Dominion in Doswell, Virginia, is one of the world's tallest roller coasters. A cable lift hill brings riders to a height of 305 feet before they plummet 300 feet at a slope of approximately −11. That's steep!

Launch the Lesson



rate of change

How a quantity is changing with respect to a change in another quantity.

slope

The rate of change in the y-coordinates (rise) to the corresponding change in the x-coordinates (run) for points on a line.

1. One definition of slope is "the steepness of a surface." How can that help you visualize the slope of a line?
2. If slope represents steepness, how does this help you remember which kinds of lines have zero or undefined slope?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- subtracting integers in fractions

Answers:

1. 1
2. $-\frac{5}{2}$
3. undefined
4. -3
5. 13,100 people

Launch the Lesson

Teaching the Mathematical Practices

2 Make Sense of Quantities

Mathematically proficient students need to be able to make sense of quantities, such as slope and rate of change, and their relationships.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Investigating Slope

Objective

Students use a sketch to explore how the slope of a line affects its graph.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to see how the slope of a line changes as the line is rotated. They will observe how the rise and the run are affected as the line is rotated, and how that affects the calculation of the slope. They will explore lines with positive slopes and negative slopes and will investigate the slopes of horizontal and vertical lines. Then, students will answer the Inquiry Question.

(*continued on the next page*)

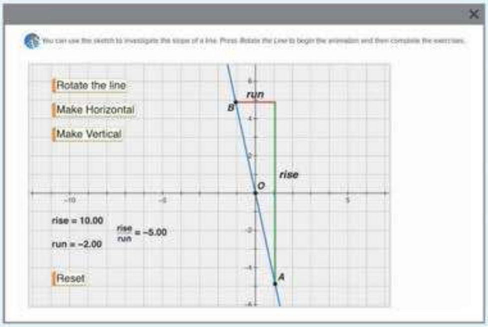
Interactive Presentation

Investigating Slope

INQUIRY How does slope help to describe a line?

You can use the sketch to investigate the slope of a line. Press *Rotate the Line* to begin the animation and then complete the exercises.

Explore



You can use the sketch to investigate the slope of a line. Press *Rotate the Line* to begin the animation and then complete the exercises.

rise = 10.00

run = −2.00

$\frac{\text{rise}}{\text{run}} = -5.00$

Explore

WEB SKETCHPAD

Students use a sketch to investigate the slope of a line.

TYPE

Students answer questions about the slope of a line.

Interactive Presentation

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Investigating Slope (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How can the words “rise” and “run” remind you whether to look for a change in *y* or a change in *x*? Sample answer: You can think of rise as something going up or down, which goes along with a change in vertical distance along the *y*-axis. You can think of “run” as something you do on the ground, which is horizontal or along the *x*-axis.
- What does a slope of −3 tell you about the line? Sample answer: The negative sign tells me that the line will be decreasing as it moves from left to right. I also know that the line will go down three units for every one unit to the right.

Inquiry

How does slope help to describe a line? Sample answer: The slope of a line can tell you whether the graph of the line will slope up or down from left to right or if it will be a horizontal or vertical line.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Rate of Change of a Linear Function

Objective

Students calculate and interpret rate of change by identifying the change in the independent and dependent variables.

Teaching the Mathematical Practices

2 Make Sense of Quantities In this Learn, help students to notice the relationship between the variables when calculating rate of change.

Example 1 Find the Rate of Change

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

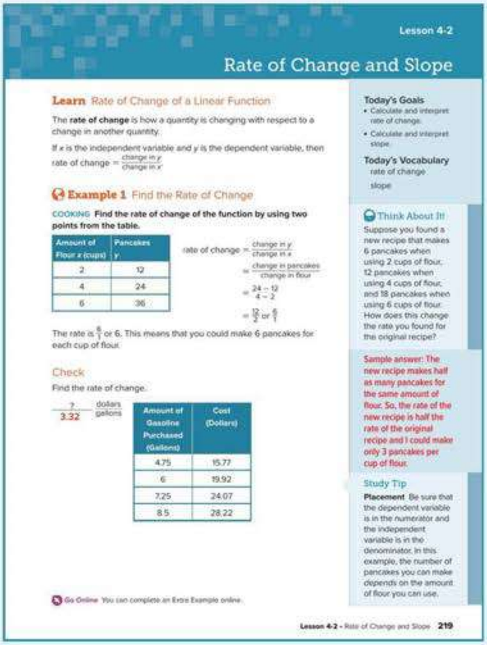
AL What are the quantities being compared in the table? the number of pancakes and the number of cups of flour

OL How do you know which is the independent variable and which is the dependent variable? Sample answer: The pancakes depend on the flour because the number of pancakes you can make depends on how much flour you use. So the number of pancakes is the dependent variable, and the amount of flour is the independent variable.

BL Would the ratio be different if you used the first and last pairs of values from the table to calculate the rate of change? Explain. No; sample answer: $\frac{36-12}{6-2}=\frac{24}{4}$ or 6.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-2

Rate of Change and Slope

Learn Rate of Change of a Linear Function

The **rate of change** is how a quantity is changing with respect to a change in another quantity.

If x is the independent variable and y is the dependent variable, then rate of change $=\frac{\text{change in } y}{\text{change in } x}$

Example 1 Find the Rate of Change

COOKING **Find the rate of change of the function by using two points from the table.**

Amount of Flour x (cups)	Pancakes y
2	12
4	24
6	36

$$\begin{aligned}\text{rate of change} &= \frac{\text{change in } y}{\text{change in } x}\\ &= \frac{\text{change in pancakes}}{\text{change in flour}}\\ &= \frac{24-12}{4-2}\\ &= \frac{12}{2} \text{ or } \frac{6}{1}\end{aligned}$$

The rate is $\frac{6}{1}$ or 6. This means that you could make 6 pancakes for each cup of flour.

Check

Find the rate of change.

$\frac{?}{3.32}$ $\frac{\text{dollars}}{\text{gallons}}$

Amount of Gasoline Purchased (Gallons)	Cost (Dollars)
4.75	15.77
6	19.92
7.25	24.07
8.5	28.22

Go Online You can complete an Extra Example online.

Today's Goals
- Calculate and interpret rate of change.
- Calculate and interpret slope.

Today's Vocabulary
rate of change
slope

Think About It!

Suppose you found a new recipe that makes 6 pancakes when using 2 cups of flour, 12 pancakes when using 4 cups of flour, and 18 pancakes when using 6 cups of flour. How does this change the rate you found for the original recipe?

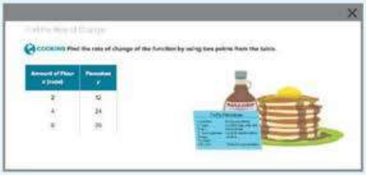
Sample answer: The new recipe makes half as many pancakes for the same amount of flour. So, the rate of the new recipe is half the rate of the original recipe and I could make only 3 pancakes per cup of flour.

Study Tip

Placement Be sure that the dependent variable is in the numerator and the independent variable is in the denominator. In this example, the number of pancakes you can make depends on the amount of flour you can use.

Lesson 4-2 • Rate of Change and Slope 219

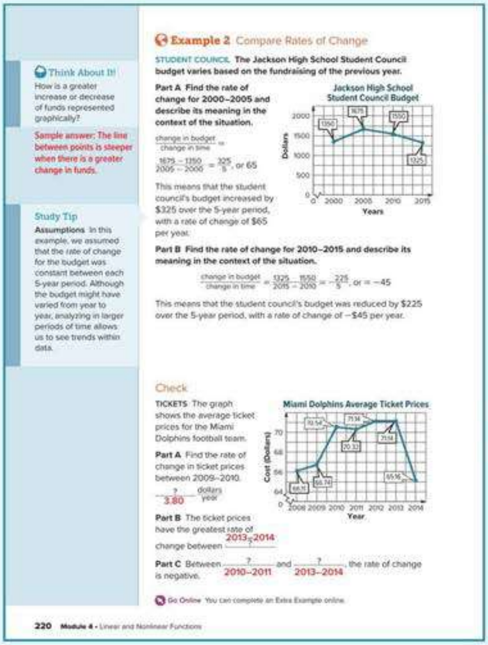
Interactive Presentation



Example 1

TYPE

Students explain how the rate of change can be used to find the number of pancakes for a given number of cups of flour.



Think About It!

How is a greater increase or decrease of funds represented graphically?

Sample answer: The line between points is steeper when there is a greater change in funds.

Study Tip

Assumptions In this example, we assumed that the rate of change for the budget was constant between each 5-year period. Although the budget might have varied from year to year, analyzing in larger periods of time allows us to see trends within data.

Example 2 Compare Rates of Change

STUDENT COUNCIL **The Jackson High School Student Council budget varies based on the fundraising of the previous year.**

Part A Find the rate of change for 2000–2005 and describe its meaning in the context of the situation.

$$\frac{\text{change in budget}}{\text{change in time}} = \frac{1675 - 1350}{2005 - 2000} = \frac{325}{5}, \text{ or } 65$$

This means that the student council's budget increased by $325 over the 5-year period, with a rate of change of $65 per year.

Part B Find the rate of change for 2010–2015 and describe its meaning in the context of the situation.

$$\frac{\text{change in budget}}{\text{change in time}} = \frac{1325 - 1550}{2015 - 2010} = -\frac{225}{5}, \text{ or } = -45$$

This means that the student council's budget was reduced by $225 over the 5-year period, with a rate of change of −$45 per year.

Check

TICKETS The graph shows the average ticket prices for the Miami Dolphins football team.

Part A Find the rate of change in ticket prices between 2009–2010.

$\frac{?}{3.80}$ $\frac{\text{dollars}}{\text{year}}$

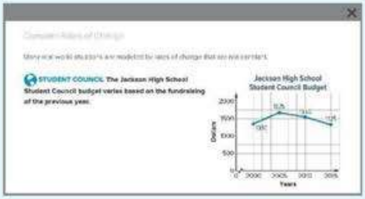
Part B The ticket prices have the greatest rate of change between $\underset{?}{2013–2014}$

Part C Between $\underset{2010–2011}{?}$ and $\underset{2013–2014}{?}$, the rate of change is negative.

Go Online You can complete an Extra Example online.

220 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



STUDENT COUNCIL The Jackson High School Student Council budget varies based on the fundraising of the previous year.

Example 2

DRAG & DROP

Students complete the rate of change formula by dragging the values to the correct bins.

TYPE

Students answer a question about how a greater increase or decrease of funds is represented graphically.

Example 2 Compare Rates of Change

Teaching the Mathematical Practices

4 Make Assumptions In the Study Tip, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL What are you trying to determine? the rate of change for two time periods: 2000–2005 and 2010–2015

OL What do *x* and *y* in the formula for the rate of change represent? *y* represents dollars, and *x* represents years

BL Can you find the rate of change by simply subtracting the numbers that are called out on the graph? Why or why not? No; sample answer: Subtracting those numbers will not produce a rate. Each listed point is at an interval of 5 years, so you need to divide by 5 to determine the rate of change per year.

Common Error

When interpreting a solution, some students may ignore the sign of a rate that is negative. Explain that in any real-world problem, the sign of a quantity has meaning. Help them to see that in this example, the negative rate of change means that the budget was reduced over that time period.

Example 3 Constant Rate of Change

Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in this example.

Questions for Mathematical Discourse

AL What do you need to know in order to determine whether the function is linear? whether there is a constant rate of change

OL How does finding the differences between successive values in the table help you determine whether a function is linear? Sample answer: If the differences are the same, then I know that the rate of change is constant, and therefore the function is linear.

BL What is another way you can use the table to determine if the rate of change is constant? Sample answer: Because consecutive *x*-values decrease by 3, I can check to see if consecutive *y*-values increase or decrease by the same number. Because consecutive *y*-values increase by 2, I know there is a constant rate of change.

Example 4 Rate of Change

Teaching the Mathematical Practices

1 Explain Correspondences Use the Study Tip to encourage students to explain the relationship between the graph and rate of change of a linear function.

Questions for Mathematical Discourse

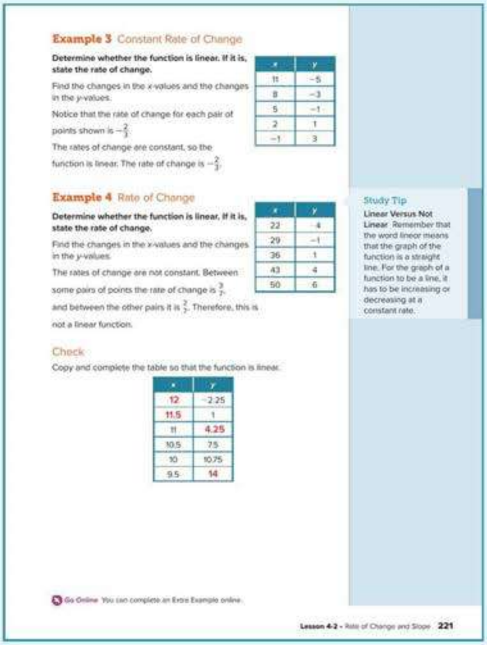
AL How will you determine whether the function is linear? Sample answer: I will find the changes in the *x*-values and the changes in the *y*-values, and see if those changes are constant.

OL Is it necessary to calculate the rate of change between every pair of points to determine linearity? Explain. No; sample answer: Once you have found two pairs that have different rates of change, you have shown that the function is not linear.

BL If you graphed the points from the table, would they lie on a straight line? How do you know? No; sample answer: Because the rates of change are not constant, the function is not linear, and therefore the graph of the points will not lie on a line.

Common Error

Some students may observe the pattern in the differences between the *y*–values (3, 2, 3, 2) and think that this regularity indicates that the function is linear. Correct this reasoning, and reinforce that when the differences in the *x*-values are the same, the differences in the *y*-values must also be the same for the function to be linear.



Example 3 Constant Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
11	−5
8	−3
5	−1
2	1
−1	3

Find the changes in the *x*-values and the changes in the *y*-values.

Notice that the rate of change for each pair of points shown is $-\frac{2}{3}$.

The rates of change are constant, so the function is linear. The rate of change is $-\frac{2}{3}$.

Example 4 Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
22	−4
29	−1
36	1
43	4
50	6

Find the changes in the *x*-values and the changes in the *y*-values.

The rates of change are not constant. Between some pairs of points the rate of change is $\frac{3}{7}$, and between the other pairs it is $\frac{2}{7}$. Therefore, this is not a linear function.

Study Tip

Linear Versus Not Linear Remember that the word *linear* means that the graph of the function is a straight line. For the graph of a function to be a line, it has to be increasing or decreasing at a constant rate.

Check

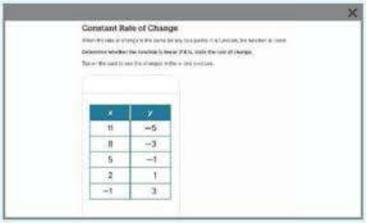
Copy and complete the table so that the function is linear.

x	y
12	−2.25
11.5	1
11	4.25
10.5	7.5
10	10.75
9.5	14

Go Online You can complete an Extra Example online.

Lesson 4-2 • Rate of Change and Slope 221

Interactive Presentation



Constant Rate of Change

x	y
11	−5
8	−3
5	−1
2	1
−1	3

Example 3

TAP

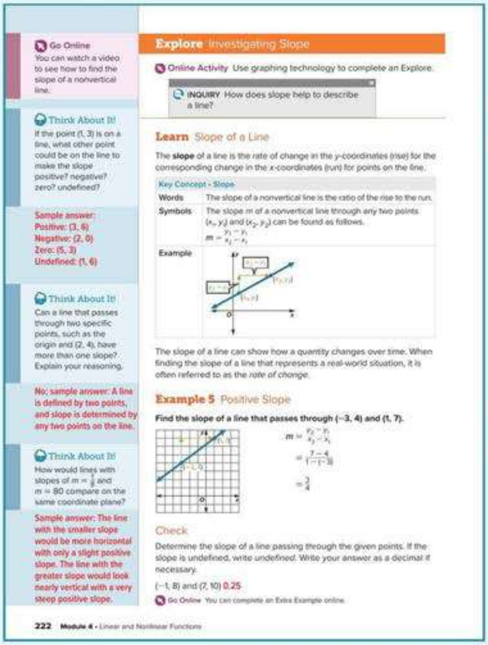
Students tap on the card to see the changes in the *x*- and *y*-values.

MULTI-SELECT

Students select the correct word to complete the sentence.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Go Online You can watch a video to see how to find the slope of a nonvertical line.

Think About It! If the point (1, 3) is on a line, what other point could be on the line to make the slope positive? negative? zero? undefined?

Sample answer:
Positive: (3, 6)
Negative: (2, 0)
Zero: (5, 3)
Undefined: (1, 6)

Think About It! Can a line that passes through two specific points, such as the origin and (2, 4), have more than one slope? Explain your reasoning.

No; sample answer: A line is defined by two points, and slope is determined by any two points on the line.

Think About It! How would lines with slopes of $m = \frac{1}{8}$ and $m = 80$ compare on the same coordinate plane?

Sample answer: The line with the smaller slope would be more horizontal with only a slight positive slope. The line with the greater slope would look nearly vertical with a very steep positive slope.

Explore Investigating Slope

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does slope help to describe a line?

Learn Slope of a Line

The **slope** of a line is the rate of change in the *y*-coordinates (rise) for the corresponding change in the *x*-coordinates (run) for points on the line.

Key Concept • Slope

Words	The slope of a nonvertical line is the ratio of the rise to the run.
Symbols	The slope m of a nonvertical line through any two points (x_1, y_1) and (x_2, y_2) can be found as follows. $m = \frac{y_2 - y_1}{x_2 - x_1}$
Example	

The slope of a line can show how a quantity changes over time. When finding the slope of a line that represents a real-world situation, it is often referred to as the *rate of change*.

Example 5 Positive Slope

Find the slope of a line that passes through (−3, 4) and (1, 7).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - 4}{1 - (-3)}$$
$$= \frac{3}{4}$$

Check

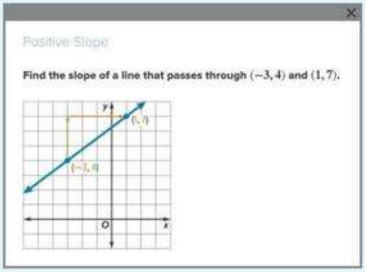
Determine the slope of a line passing through the given points. If the slope is undefined, write *undefined*. Write your answer as a decimal if necessary.

(−1, 8) and (7, 10) 0.25

Go Online You can complete an Extra Example online.

222 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Positive Slope

Find the slope of a line that passes through (−3, 4) and (1, 7).

Example 5

TYPE

Students describe and correct the error made when finding the slope of the line.

WATCH

Students can watch a video to see how find the slope of a nonvertical line.

Learn Slope of a Line

Objective

Students calculate and interpret slope by using the Slope Formula.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of slopes of lines in this Learn.

Example 5 Positive Slope

Questions for Mathematical Discourse

AL Finding the slope is the same as finding what other measure? the rate of change

OL Is the slope of this line positive, negative, or zero? How can you tell by looking at the graph? Positive; sample answer: The line slopes upward from left to right.

BL Does it matter which coordinates you use as x_1 and x_2? Explain. No; sample answer: You can use either of the *x*-coordinates as x_2, but the value for y_2 must then be the *y*-coordinate that corresponds with x_2.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students automatically assume that the left-most point has to be (x_1, y_1) and the point farther right is (x_2, y_2),
THEN explain that the designation of (x_1, y_1) and (x_2, y_2) is arbitrary. Write pairs of points on index cards. Give one card to each student. Have them find the slope both ways. Then ask which way made the subtraction easier.

DIFFERENTIATE

Language Development Activity ELL

Intermediate Instruct a small group of students to write a paragraph describing what is happening in the illustration of slope in the Key Concept box. Their paragraphs should describe all parts of the diagram in their own words. Ask for volunteers to read their paragraphs. Have students ask for clarification as needed. Then, have students revise their paragraphs based on the feedback and questions from the group.

Example 6 Negative Slope

Teaching the Mathematical Practices

8 Use Slope Help students to pay attention to the calculation of the slope of the line.

Questions for Mathematical Discourse

AL If $x_1 = -1$, what is the value of y_1? 3

OL Is the slope of this line positive, negative, or zero? How can you tell by looking at the graph? Negative; sample answer: The line slopes downward from left to right.

BL What would the value of the slope be if you used (4, 1) for (x_1, y_1) and (−1, 3) for (x_2, y_2)? It would still be $-\frac{2}{5}$.

Example 7 Slopes of Horizontal Lines

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the graph, points, and slope in this example.

Questions for Mathematical Discourse

AL How would you describe what is meant by the slope of a line? Sample answer: It is the steepness of the line.

OL Is the slope positive, negative, or zero? zero

BL Why is the slope zero? Sample answer: The slope is zero because there is no change in *y*-values, so the numerator will be zero and zero divided by any number is zero.

Example 8 Slopes of Vertical Lines

Teaching the Mathematical Practices

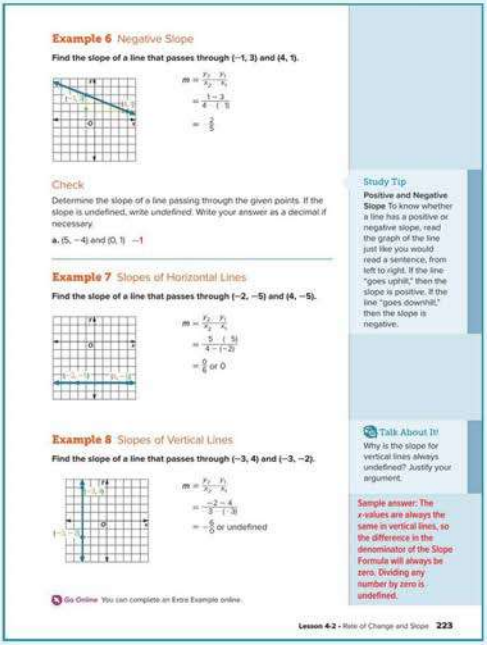
8 Use Slope Help students to pay attention to the calculation of slope for a vertical line.

Questions for Mathematical Discourse

AL Which values are the same? *x*-values: −3

OL Why is the slope undefined instead of zero? It is not possible to divide by 0. So, the slope of a vertical line is undefined.

BL Does the graph of a line with an undefined slope represent a function? Why or why not? No; sample answer: In a function, every *x*-value is paired with exactly one *y*-value. In a relation that is represented by a vertical line, there is one *x*-value paired with infinitely many *y*-values.



Example 6 Negative Slope

Find the slope of a line that passes through (−1, 3) and (4, 1).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1 - 3}{4 - (-1)}$

$= -\frac{2}{5}$

Check

Determine the slope of a line passing through the given points. If the slope is undefined, write undefined. Write your answer as a decimal if necessary.

a. (5, −4) and (0, 1) −1

Example 7 Slopes of Horizontal Lines

Find the slope of a line that passes through (−2, −5) and (4, −5).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-5 - (-5)}{4 - (-2)}$

$= \frac{0}{6}$ or 0

Example 8 Slopes of Vertical Lines

Find the slope of a line that passes through (−3, 4) and (−3, −2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2 - 4}{-3 - (-3)}$

$= -\frac{6}{0}$ or undefined

Go Online You can complete an Extra Example online.

Study Tip

Positive and Negative Slope To know whether a line has a positive or negative slope, read the graph of the line just like you would read a sentence, from left to right. If the line "goes uphill," then the slope is positive. If the line "goes downhill," then the slope is negative.

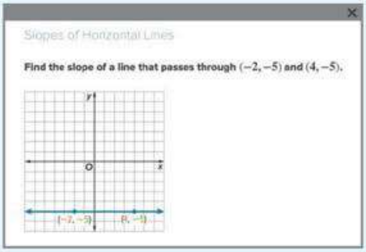
Talk About It!

Why is the slope for vertical lines always undefined? Justify your argument.

Sample answer: The *x*-values are always the same in vertical lines, so the difference in the denominator of the Slope Formula will always be zero. Dividing any number by zero is undefined.

Lesson 4-2 • Rate of Change and Slope 223

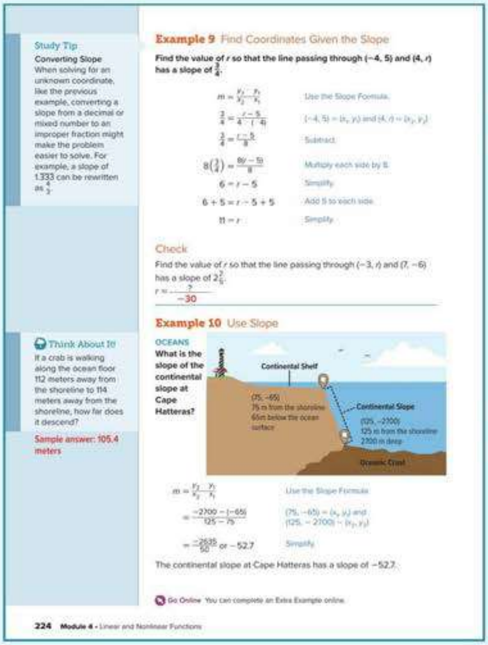
Interactive Presentation



Slopes of Horizontal Lines

Find the slope of a line that passes through (−2, −5) and (4, −5).

Example 7



Study Tip

Converting Slope When solving for an unknown coordinate, like the previous example, converting a slope from a decimal or mixed number to an improper fraction might make the problem easier to solve. For example, a slope of 1.333 can be rewritten as $\frac{4}{3}$.

Example 9 Find Coordinates Given the Slope

Find the value of r so that the line passing through $(-4, 5)$ and $(4, r)$ has a slope of $\frac{3}{4}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Use the Slope Formula.

$\frac{3}{4} = \frac{r - 5}{4 - (-4)}$ $(-4, 5) = (x_1, y_1)$ and $(4, r) = (x_2, y_2)$

$\frac{3}{4} = \frac{r - 5}{8}$ Subtract.

$8\left(\frac{3}{4}\right) = \frac{8(r - 5)}{8}$ Multiply each side by 8.

$6 = r - 5$ Simplify.

$6 + 5 = r - 5 + 5$ Add 5 to each side.

$11 = r$ Simplify.

Check

Find the value of r so that the line passing through $(-3, r)$ and $(7, -6)$ has a slope of $2\frac{2}{5}$.

$r =$? -30

Think About It!

If a crab is walking along the ocean floor 112 meters away from the shoreline to 114 meters away from the shoreline, how far does it descend?

Sample answer: 105.4 meters

Example 10 Use Slope

OCEANS **What is the slope of the continental slope at Cape Hatteras?**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Use the Slope Formula.

$= \frac{-2700 - (-65)}{125 - 75}$ $(75, -65) = (x_1, y_1)$ and $(125, -2700) = (x_2, y_2)$

$= \frac{-2635}{50}$ or -52.7 Simplify.

The continental slope at Cape Hatteras has a slope of −52.7.

Go Online You can complete an Extra Example online.

224 Module 4 • Linear and Nonlinear Functions

Interactive Presentation

Example 10

TAP

Students tap on each marker to find the points to use in the Slope Formula.

TYPE

Students use the slope to answer a question about another point along the continental slope.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 9 Find Coordinates Given the Slope

Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, “Does this make sense?” In this example, encourage students to check their answer.

Questions for Mathematical Discourse

AL For what variable in the equation do you substitute $\frac{3}{4}$? *m*

OL How could a graph help determine the missing coordinate? Sample answer: I can plot the given point and then use the slope to move to the next point. I can continue using the slope until I get to the point with the *x*-coordinate of 4.

BL Name another point on the same line. Sample answers: (0, 8), (8, 14)

Example 10 Use Slope

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL What are the two ordered pairs you can use to find the slope? (75, −65) and (125, −2700)

OL Interpret the value of the slope in the context of the problem. Sample answer: The slope means that the water gets 52.7 meters deeper for every meter you move farther from shore.

BL Do you think the continental slope is constant? Sample answer: No, there are probably places where the drop is less steep and places where it is more steep.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–49
2	exercises that use a variety of skills from this lesson	50–58
2	exercises that extend concepts learned in this lesson to new contexts	59–62
3	exercises that emphasize higher-order and critical-thinking skills	63–68

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**

THEN assign:

- Practice, Exercises 1–61 odd, 63–68
- Extension: Treasure Hunt with Slopes
- **ALEKS** Equations of Lines

IF students score 66%–89% on the Checks, **OL**

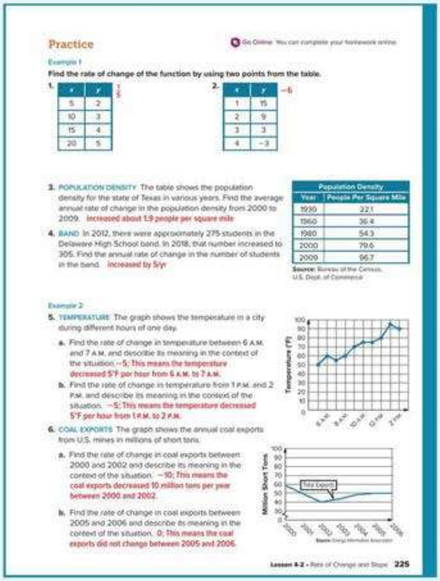
THEN assign:

- Practice, Exercises 1–67 odd
- Remediation, Review Resources: Order of Integer Operations
- Personal Tutors
- Extra Examples 1–10
- **ALEKS** Multiplication and Division with Integers

IF students score 65% or less on the Checks, **AL**

THEN assign:

- Practice, Exercises 1–49 odd
- Remediation, Review Resources: Order of Integer Operations
- *Quick Review Math Handbook*: Rate of Change and Slope
- Arrive**MATH** Take Another Look
- **ALEKS** Multiplication and Division with Integers



Practice

Go Online You can complete your homework online.

Example 1

Find the rate of change of the function by using two points from the table.

1.

x	y
5	2
10	3
15	4
20	5

$\frac{1}{5}$

2.

x	y
1	15
2	9
3	3
4	−3

−6

3. POPULATION DENSITY The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2009. increased about 1.9 people per square mile

4. BAND In 2012, there were approximately 275 students in the Delaware High School band. In 2018, that number increased to 305. Find the annual rate of change in the number of students in the band. increased by 5/yr

Population Density	
Year	People Per Square Mile
1930	22.1
1960	36.4
1980	54.3
2000	79.6
2009	96.7

Source: Bureau of the Census, U.S. Dept. of Commerce

Example 2

5. TEMPERATURE The graph shows the temperature in a city during different hours of one day.

a. Find the rate of change in temperature between 6 A.M. and 7 A.M. and describe its meaning in the context of the situation. −5; This means the temperature decreased 5°F per hour from 6 A.M. to 7 A.M.

b. Find the rate of change in temperature from 1 P.M. and 2 P.M. and describe its meaning in the context of the situation. −5; This means the temperature decreased 5°F per hour from 1 P.M. to 2 P.M.

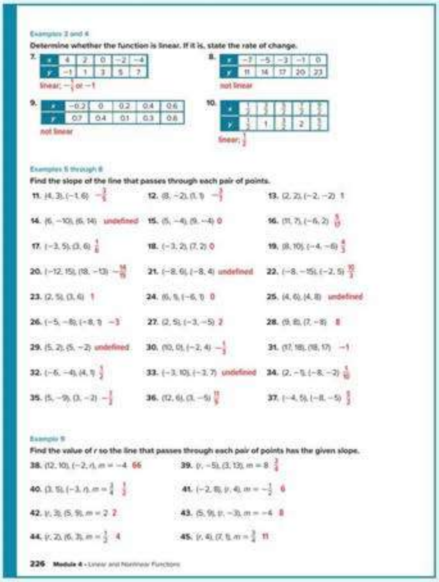
6. COAL EXPORTS The graph shows the annual coal exports from U.S. mines in millions of short tons.

a. Find the rate of change in coal exports between 2000 and 2002 and describe its meaning in the context of the situation. −10; This means the coal exports decreased 10 million tons per year between 2000 and 2002.

b. Find the rate of change in coal exports between 2005 and 2006 and describe its meaning in the context of the situation. 0; This means the coal exports did not change between 2005 and 2006.

Source: Energy Information Association

Lesson 4-2 • Rate of Change and Slope 225



Examples 3 and 4

Determine whether the function is linear. If it is, state the rate of change.

7.

x	4	2	0	−2	−4
y	−1	1	3	5	7

linear; $-\frac{2}{2}$ or −1

8.

x	−7	−5	−3	−1	0
y	11	14	17	20	23

not linear

9.

x	−0.2	0	0.2	0.4	0.6
y	0.7	0.4	0.1	0.3	0.8

not linear

10.

x	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

linear; $\frac{1}{2}$

Examples 5 through 8

Find the slope of the line that passes through each pair of points.

11. (4, 3), (−1, 6) $-\frac{3}{5}$
12. (8, −2), (1, 1) $-\frac{3}{7}$
13. (2, 2), (−2, −2) 1
14. (6, −10), (6, 14) undefined
15. (5, −4), (9, −4) 0
16. (11, 7), (−6, 2) $\frac{5}{17}$
17. (−3, 5), (3, 6) $\frac{1}{6}$
18. (−3, 2), (7, 2) 0
19. (8, 10), (−4, −6) $\frac{4}{3}$
20. (−12, 15), (18, −13) $-\frac{14}{15}$
21. (−8, 6), (−8, 4) undefined
22. (−8, −15), (−2, 5) $\frac{10}{3}$
23. (2, 5), (3, 6) 1
24. (6, 1), (−6, 1) 0
25. (4, 6), (4, 8) undefined
26. (−5, −8), (−8, 1) −3
27. (2, 5), (−3, −5) 2
28. (9, 8), (7, −8) 8
29. (5, 2), (5, −2) undefined
30. (10, 0), (−2, 4) $-\frac{1}{3}$
31. (17, 18), (18, 17) −1
32. (−6, −4), (4, 1) $\frac{1}{2}$
33. (−3, 10), (−3, 7) undefined
34. (2, −1), (−8, −2) $\frac{1}{10}$
35. (5, −9), (3, −2) $-\frac{7}{2}$
36. (12, 6), (3, −5) $\frac{11}{9}$
37. (−4, 5), (−8, −5) $\frac{5}{2}$

Example 9

Find the value of *r* so the line that passes through each pair of points has the given slope.

38. (12, 10), (−2, *r*), *m* = −4 66
39. (*r*, −5), (3, 13), *m* = 8 $\frac{3}{4}$
40. (3, 5), (−3, *r*), *m* = $\frac{3}{4}$ $\frac{1}{2}$
41. (−2, 8), (*r*, 4), *m* = $-\frac{1}{2}$ 6
42. (*r*, 3), (5, 9), *m* = 2 2
43. (5, 9), (*r*, −3), *m* = −4 8
44. (*r*, 2), (6, 3), *m* = $\frac{1}{2}$ 4
45. (*r*, 4), (7, 1), *m* = $\frac{3}{4}$ 11

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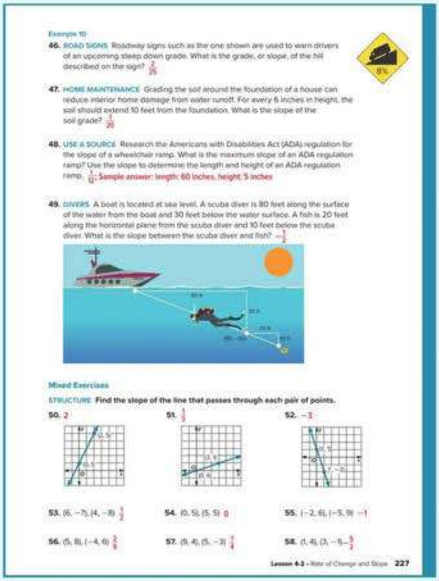
3 REFLECT AND PRACTICE

F.IF.6, F.LE.5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 10

46. ROAD SIGNS Roadway signs such as the one shown are used to warn drivers of an upcoming steep down grade. What is the grade, or slope, of the hill described on the sign? $\frac{2}{25}$

47. HOME MAINTENANCE Grading the soil around the foundation of a house can reduce interior home damage from water runoff. For every 6 inches in height, the soil should extend 10 feet from the foundation. What is the slope of the soil grade? $\frac{1}{20}$

48. USE A SOURCE Research the Americans with Disabilities Act (ADA) regulation for the slope of a wheelchair ramp. What is the maximum slope of an ADA regulation ramp? Use the slope to determine the length and height of an ADA regulation ramp. $\frac{1}{12}$; Sample answer: length: 60 inches, height: 5 inches

49. DIVERS A boat is located at sea level. A scuba diver is 80 feet along the surface of the water from the boat and 30 feet below the water surface. A fish is 20 feet along the horizontal plane from the scuba diver and 10 feet below the scuba diver. What is the slope between the scuba diver and fish? $-\frac{1}{2}$

Mixed Exercises

STRUCTURE **Find the slope of the line that passes through each pair of points.**

50. 2

51. $\frac{1}{3}$

52. -3

53. $(6, -7), (4, -8)$ $\frac{1}{2}$

54. $(0, 5), (5, 5)$ 0

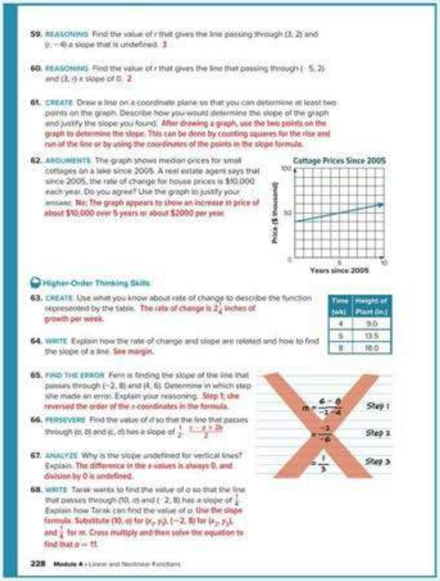
55. $(-2, 6), (-5, 9)$ -1

56. $(5, 8), (-4, 6)$ $\frac{2}{9}$

57. $(9, 4), (5, -3)$ $\frac{7}{4}$

58. $(1, 4), (3, -1)$ $-\frac{5}{2}$

Lesson 4-2 • Rate of Change and Slope 227



59. REASONING Find the value of r that gives the line passing through $(3, 2)$ and $(r, -4)$ a slope that is undefined. 3

60. REASONING Find the value of r that gives the line that passing through $(-5, 2)$ and $(3, r)$ a slope of 0. 2

61. CREATE Draw a line on a coordinate plane so that you can determine at least two points on the graph. Describe how you would determine the slope of the graph and justify the slope you found. After drawing a graph, use the two points on the graph to determine the slope. This can be done by counting squares for the rise and run of the line or by using the coordinates of the points in the slope formula.

62. ARGUMENTS The graph shows median prices for small cottages on a lake since 2005. A real estate agent says that since 2005, the rate of change for house prices is $10,000 each year. Do you agree? Use the graph to justify your answer. No; The graph appears to show an increase in price of about $10,000 over 5 years or about $2000 per year.

Higher-Order Thinking Skills

63. CREATE Use what you know about rate of change to describe the function represented by the table. The rate of change is $2\frac{1}{4}$ inches of growth per week.

Time (wk)	Height of Plant (in.)
4	9.0
6	13.5
8	18.0

64. WRITE Explain how the rate of change and slope are related and how to find the slope of a line. See margin.

65. FIND THE ERROR Fern is finding the slope of the line that passes through $(-2, 8)$ and $(4, 6)$. Determine in which step she made an error. Explain your reasoning. Step 1; she reversed the order of the x-coordinates in the formula.

$m = \frac{6-8}{-2-4}$ Step 1

$= \frac{-2}{-6}$ Step 2

$= \frac{1}{3}$ Step 3

66. PERSEVERE Find the value of d so that the line that passes through (a, b) and (c, d) has a slope of $\frac{1}{2}$. $\frac{c-a+2b}{2}$

67. ANALYZE Why is the slope undefined for vertical lines? Explain. The difference in the x-values is always 0, and division by 0 is undefined.

68. WRITE Tarak wants to find the value of a so that the line that passes through $(10, a)$ and $(-2, 8)$ has a slope of $\frac{1}{4}$. Explain how Tarak can find the value of a. Use the slope formula. Substitute $(10, a)$ for (x_1, y_1), $(-2, 8)$ for (x_2, y_2), and $\frac{1}{4}$ for m. Cross multiply and then solve the equation to find that $a = 11$.

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Answer

64. Sample answer: Slope can be used to describe a rate of change. Rate of change is a ratio that describes how much one quantity changes with respect to a change in another quantity. The slope of a line is also a ratio and it is the ratio of the change in the y-coordinates to the change in the x-coordinates.

Lesson 4-3

Slope-Intercept Form

A.CED.2, F.IF.7a, F.LE.5

LESSON GOAL

Students graph equations in slope-intercept form.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Writing Linear Equations in Slope-Intercept Form
- Write Linear Equations in Slope-Intercept Form
- Rewrite Linear Equations in Slope-Intercept Form
- Write Linear Equations

Explore: Graphing Linear Functions by Using the Slope-Intercept Form

Develop:

Graphing Linear Functions in Slope-Intercept Form
- Graph Linear Functions in Slope-Intercept Form
- Graph Linear Functions
- Graph Constant Functions
- Use Graphs of Linear Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Slope of a Line	●	●		●
Extension: Pencils of Lines		●	●	●

Language Development Handbook

Assign page 22 of the *Language Development Handbook* to help your students build mathematical language related to equations in slope-intercept form.

ELL You can use the tips and suggestions on page T22 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students found and interpreted the rate of change and slopes of lines.
8.F.4, F.IF.6, F.LE.5

Now
Students graph equations in slope-intercept form.
A.CED.2, F.IF.7a, F.LE.5

Next
Students will Identify the effects of transformations of the graphs of linear functions.
F.IF.7a, F.BF.3

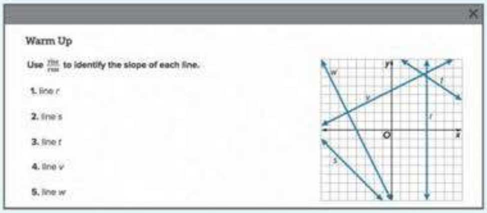
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of slope. They build fluency by rewriting equations in slope-intercept form to find the slope and *y*-intercept. They apply their understanding by solving real-world problems involving slope and *y*-intercept.

Interactive Presentation

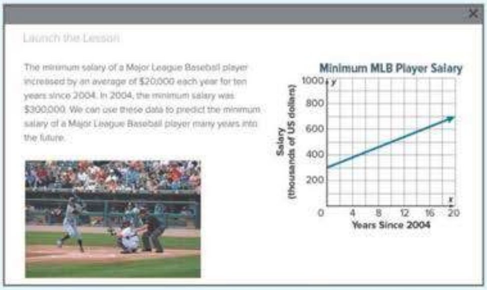


Warm Up

Use $\frac{\text{rise}}{\text{run}}$ to identify the slope of each line.

1. line r
2. line s
3. line t
4. line v
5. line w

Warm Up



Launch the Lesson

The minimum salary of a Major League Baseball player increased by an average of $20,000 each year for ten years since 2004. In 2004, the minimum salary was $300,000. We can use these data to predict the minimum salary of a Major League Baseball player many years into the future.

Launch the Lesson

Vocabulary

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying slopes

Answers:

1. undefined
2. -1
3. $-\frac{2}{3}$
4. $\frac{1}{2}$
5. -2

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal description and graphs representing the minimum salary for a Major League Baseball player.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope, and b is the y-intercept. Writing a linear equation in this form is helpful when you want to graph the function. There are two methods that can be used. The first is to select two values of x, substitute those values into the equation to calculate the corresponding values of y, plot the resulting ordered pairs, and draw the line that passes through the points. The second method is to plot the y-intercept, use it as a starting point, and then use the slope to determine another point on the line. The line can then be drawn through the two points.

Explore Graphing Linear Functions by Using the Slope-Intercept Form

Objective

Students use a sketch to explore how changing the slope and *y*-intercept changes the graph of the line.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in the Explore, students will need to use a sketch. Work with students to explore and deepen their understanding of slope-intercept form of a linear equation.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

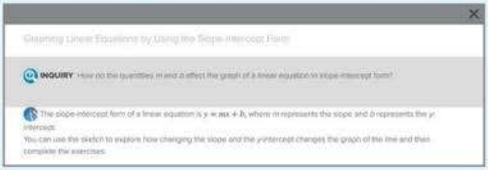
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to explore how changing the value of *m* and *b* in the equation of a line affects the graph of the function. They will use sliders and animations to change the values of *m* and/or *b* in a linear equation, and observe the change in orientation of the related line. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



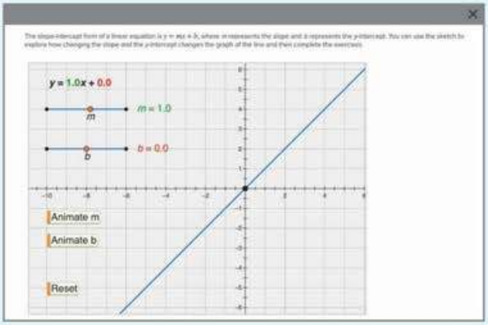
Graphing Linear Equations by Using the Slope-Intercept Form

INQUIRY How do the quantities *m* and *b* affect the graph of a linear equation in slope-intercept form?

The slope-intercept form of a linear equation is $y = mx + b$, where *m* represents the slope and *b* represents the *y*-intercept.

You can use the sketch to explore how changing the slope and the *y*-intercept changes the graph of the line and then complete the exercises.

Explore



Explore

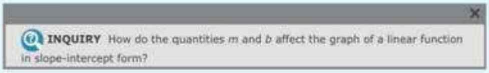
WEB SKETCHPAD

Students use a sketch to graph a line by changing the slope and *y*-intercept.

TYPE

Students answer questions about changing the parameters in the slope-intercept form of a line.

Interactive Presentation



INQUIRY How do the quantities *m* and *b* affect the graph of a linear function in slope-intercept form?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Graphing Linear Functions by Using the Slope-Intercept Form (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Describe the graph when $0 < m < 1$. Sample answer: When the slope is a fraction between 0 and 1, the run is greater than the rise. This means that the slant of the line is more gradual.
- What are the slope and *y*-intercept of $y = \frac{2}{3}x - 4$? The slope is $\frac{2}{3}$ and the *y*-intercept is -4.

Inquiry

How do the quantities *m* and *b* affect the graph of a linear function in slope-intercept form? Sample answer: Changing the slope affects the steepness of the graph. Changing the *y*-intercept determines the distance and direction that the graph is shifted from the origin.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Writing Linear Equations in Slope-Intercept Form

Objective

Students rewrite equations in slope-intercept form by applying the properties of equality.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Write Linear Equations in Slope-Intercept Form

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal description and equation in this example.

Questions for Mathematical Discourse

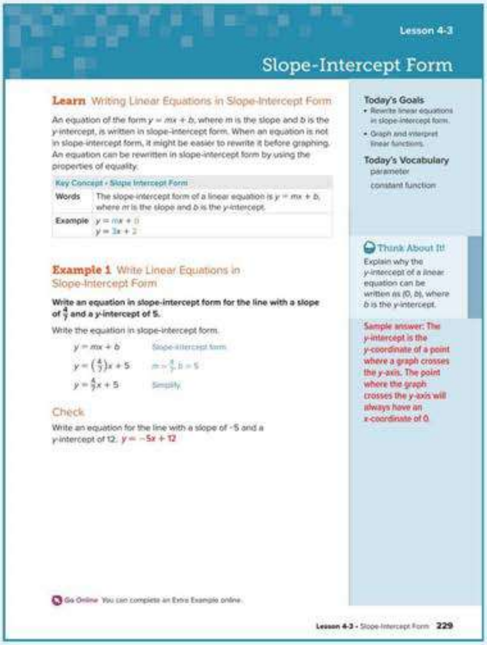
AL What is the slope of the line? $\frac{4}{7}$

OL Which variable represents the slope in $y = mx + b$? m

BL How would this equation have changed if the slope had been $-\frac{4}{7}$? It would have been $y = -\frac{4}{7}x + 5$.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-3

Slope-Intercept Form

Learn Writing Linear Equations in Slope-Intercept Form

An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is written in slope-intercept form. When an equation is not in slope-intercept form, it might be easier to rewrite it before graphing. An equation can be rewritten in slope-intercept form by using the properties of equality.

Key Concept • Slope-Intercept Form

Words	The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
Example	$y = mx + b$ $y = 3x + 2$

Example 1 Write Linear Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the line with a slope of $\frac{4}{7}$ and a y-intercept of 5.

Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = \left(\frac{4}{7}\right)x + 5$ $m = \frac{4}{7}, b = 5$

$y = \frac{4}{7}x + 5$ Simplify.

Check

Write an equation for the line with a slope of −5 and a y-intercept of 12. $y = -5x + 12$

Go Online You can complete an Extra Example online.

Today's Goals

- Rewrite linear equations in slope-intercept form.
- Graph and interpret linear functions.

Today's Vocabulary

parameter

constant function

Think About It!

Explain why the y-intercept of a linear equation can be written as $(0, b)$, where b is the y-intercept.

Sample answer: The y-intercept is the y-coordinate of a point where a graph crosses the y-axis. The point where the graph crosses the y-axis will always have an x-coordinate of 0.

Lesson 4-3 • Slope-Intercept Form 229

Interactive Presentation

Write Linear Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the line with a slope of $\frac{4}{7}$ and a y-intercept of 5.

Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

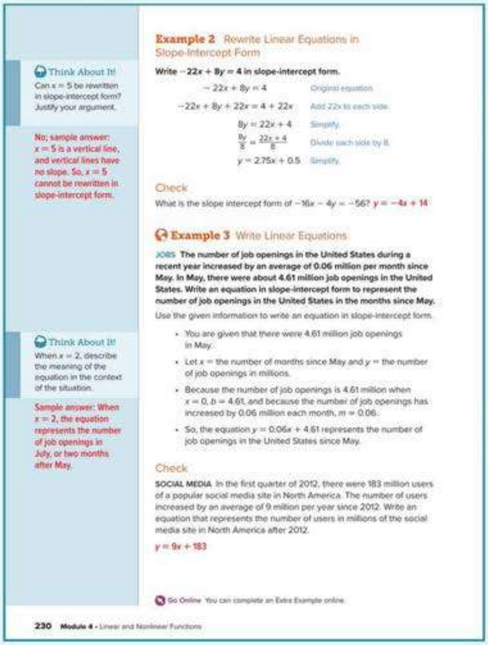
$y = \left(\frac{4}{7}\right)x + 5$ $m = \frac{4}{7}, b = 5$

$y = \frac{4}{7}x + 5$ Simplify.

Example 1

TYPE

Students explain how the equation would change if the y-intercept was negative.



Think About It!
Can $x = 5$ be rewritten in slope-intercept form? Justify your argument.

No; sample answer: $x = 5$ is a vertical line, and vertical lines have no slope. So, $x = 5$ cannot be rewritten in slope-intercept form.

Example 2 Rewrite Linear Equations in Slope-Intercept Form

Write $-22x + 8y = 4$ in slope-intercept form.

$-22x + 8y = 4$	Original equation
$-22x + 8y + 22x = 4 + 22x$	Add 22x to each side.
$8y = 22x + 4$	Simplify.
$\frac{8y}{8} = \frac{22x + 4}{8}$	Divide each side by 8.
$y = 2.75x + 0.5$	Simplify.

Check

What is the slope intercept form of $-16x - 4y = -56$? $y = -4x + 14$

Example 3 Write Linear Equations

JOBS **The number of job openings in the United States during a recent year increased by an average of 0.06 million per month since May. In May, there were about 4.61 million job openings in the United States. Write an equation in slope-intercept form to represent the number of job openings in the United States in the months since May.**

Use the given information to write an equation in slope-intercept form.

- You are given that there were 4.61 million job openings in May.
- Let x = the number of months since May and y = the number of job openings in millions.
- Because the number of job openings is 4.61 million when $x = 0$, $b = 4.61$, and because the number of job openings has increased by 0.06 million each month, $m = 0.06$.
- So, the equation $y = 0.06x + 4.61$ represents the number of job openings in the United States since May.

Think About It!
When $x = 2$, describe the meaning of the equation in the context of the situation.

Sample answer: When $x = 2$, the equation represents the number of job openings in July, or two months after May.

Check

SOCIAL MEDIA In the first quarter of 2012, there were 183 million users of a popular social media site in North America. The number of users increased by an average of 9 million per year since 2012. Write an equation that represents the number of users in millions of the social media site in North America after 2012.

$y = 9x + 183$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3

TYPE

Students explain the meaning of a certain x-value in context of the situation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Rewrite Linear Equations in Slope-Intercept Form

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. The Think About It! feature asks students to justify their conclusions.

Questions for Mathematical Discourse

AL Is this equation in slope-intercept form? Why? No; sample answer: Slope-intercept form is $y = mx + b$, and in this equation, the y-variable is not isolated.

OL How do you know if a linear equation is in slope-intercept form? Sample answer: The y-variable is isolated and it is in the form $y = mx + b$

BL How would this problem be different if the original equation had been $-22x - 8y = 4$? The last step would have involved dividing by -8 instead of 8, resulting in $y = -2.75x - 0.5$.

Example 3 Write Linear Equations

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL Which number is the y-intercept? the slope? 4.61, 0.06

OL What do the slope and y-intercept represent in the context of this situation? the increase in the number of millions of job openings per month since May; 4.61 million job openings in May

BL What would it mean if the rate of change was -0.06 in the context of the situation? Sample answer: It would mean a decrease of 0.06 million job openings per month.

Learn Graphing Linear Functions in Slope-Intercept Form

Objective

Students graph and interpret linear functions by writing them in slope-intercept form.

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between linear functions in slope-intercept form and their graphs.

Common Misconception

Some students may think that when the slope is negative, they should count down for the rise and left for the run to find additional points. Show students that this would lead to a line that is rising from left to right, not falling, as would be the orientation for a line with a negative slope. Tell them to count up and to the right for positive slopes, and down and to the right for negative slopes.

Example 4 Graph Linear Functions in Slope-Intercept Form

MP Teaching the Mathematical Practices

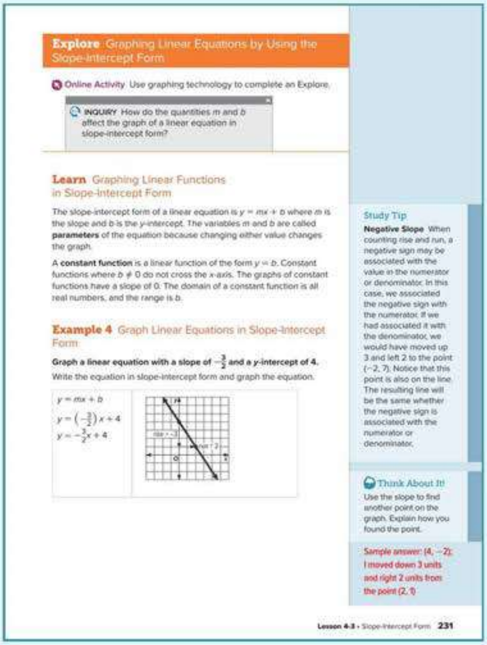
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL In slope-intercept form, which variable represents the slope? *m* the *y*-intercept? *b*

OL When graphing a line in slope-intercept form, why is *b* graphed first? Sample answer: In order to use the slope, you have to have a starting point.

BL Why do you find the next point by counting down 3 and to the right 2? Sample answer: The slope is negative, so instead of counting up and to the right, you count down and to the right.



Explore Graphing Linear Equations by Using the Slope-Intercept Form

Online Activity Use graphing technology to complete an Explore.

INQUIRY How do the quantities m and b affect the graph of a linear equation in slope-intercept form?

Learn Graphing Linear Functions in Slope-Intercept Form

The slope-intercept form of a linear equation is $y = mx + b$ where m is the slope and b is the y-intercept. The variables m and b are called **parameters** of the equation because changing either value changes the graph.

A **constant function** is a linear function of the form $y = b$. Constant functions where $b \neq 0$ do not cross the x-axis. The graphs of constant functions have a slope of 0. The domain of a constant function is all real numbers, and the range is b.

Example 4 Graph Linear Equations in Slope-Intercept Form

Graph a linear equation with a slope of $-\frac{3}{2}$ and a y-intercept of 4.

Write the equation in slope-intercept form and graph the equation.

$y = mx + b$

$y = \left(-\frac{3}{2}\right)x + 4$

$y = -\frac{3}{2}x + 4$

Study Tip

Negative Slope When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. In this case, we associated the negative sign with the numerator. If we had associated it with the denominator, we would have moved up 3 and left 2 to the point (−2, 7). Notice that this point is also on the line. The resulting line will be the same whether the negative sign is associated with the numerator or denominator.

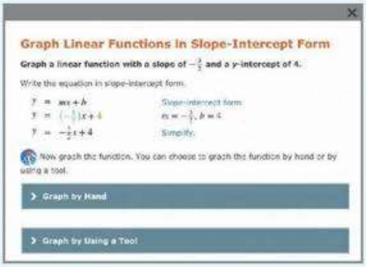
Think About It!

Use the slope to find another point on the graph. Explain how you found the point.

Sample answer: (4, −2); I moved down 3 units and right 2 units from the point (2, 1)

Lesson 4-3 • Slope-Intercept Form 231

Interactive Presentation



Graph Linear Functions in Slope-Intercept Form

Graph a linear function with a slope of $-\frac{3}{2}$ and a y-intercept of 4.

Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = \left(-\frac{3}{2}\right)x + 4$ $m = -\frac{3}{2}$, $b = 4$

$y = -\frac{3}{2}x + 4$ Simplify.

Now graph the function. You can choose to graph the function by hand or by using a tool.

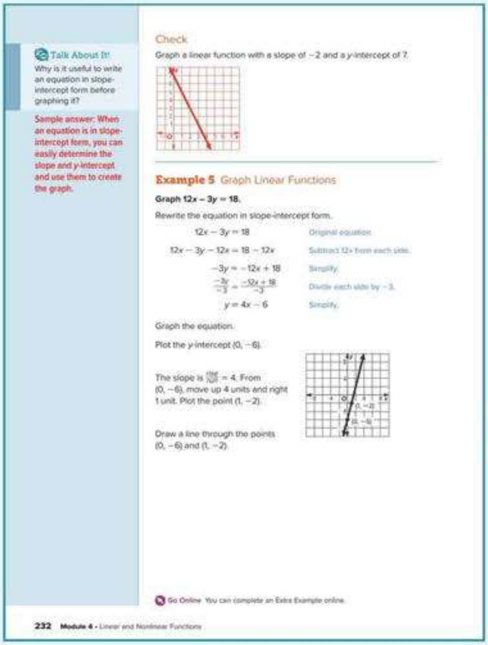
Example 4

WEB SKETCHPAD

Students use a sketch to graph a linear function in slope-intercept form.

TYPE

Students explain how to find another point on the graph by using the slope.



Talk About It!

Why is it useful to write an equation in slope-intercept form before graphing it?

Sample answer: When an equation is in slope-intercept form, you can easily determine the slope and *y*-intercept and use them to create the graph.

Check

Graph a linear function with a slope of −2 and a *y*-intercept of 7.

Example 5 Graph Linear Functions

Graph $12x - 3y = 18$.

Rewrite the equation in slope-intercept form.

$12x - 3y = 18$ Original equation

$12x - 3y - 12x = 18 - 12x$ Subtract 12x from each side.

$-3y = -12x + 18$ Simplify.

$\frac{-3y}{-3} = \frac{-12x + 18}{-3}$ Divide each side by −3.

$y = 4x - 6$ Simplify.

Graph the equation.

Plot the *y*-intercept (0, −6).

The slope is $\frac{\text{rise}}{\text{run}} = 4$. From (0, −6), move up 4 units and right 1 unit. Plot the point (1, −2).

Draw a line through the points (0, −6) and (1, −2).

Go Online You can complete an Extra Example online.

232 Module 4 • Linear and Nonlinear Functions

Interactive Presentation

Graph Linear Functions

Graph $12x - 3y = 18$.

Rewrite the equation in slope-intercept form.

$12x - 3y = 18$ Original equation

$12x - 3y - 12x = 18 - 12x$ Subtract 12x from each side.

$-3y = -12x + 18$ Simplify.

$\frac{-3y}{-3} = \frac{-12x + 18}{-3}$ Divide each side by −3.

$y = 4x - 6$ Simplify.

Example 5

TAP

Students move through the steps to graph a linear function.

TYPE

Students explain why it is useful to write an equation in slope-intercept form before graphing.

Example 5 Graph Linear Functions

Teaching the Mathematical Practices

1 Seek Information Mathematically proficient students must be able to transform algebraic expressions to reach solutions. Point out that gaining fluency in this skill is as important as learning their math facts was in the elementary grades.

Questions for Mathematical Discourse

AL What variable must you solve for in order to write the equation in slope-intercept form? *y*

OL What are the slope and the *y*-intercept of the line? The slope is 4. The *y*-intercept is −6.

BL How can the intercepts of the line be used to check your answer? Sample answer: Using the given form of the line, I know the *x*-intercept will be (1.5, 0) and the *y*-intercept will be (0, −6). My graph crosses at those points, so the graph is correct.

Common Error

For an equation such as $y = 4x - 6$, some students may state that $b = 6$. Review the general form of the slope-intercept form of a linear equation ($y = mx + b$), and highlight the plus sign. Help students to see that $y = 4x - 6$ is equivalent to $y = 4x + (-6)$, so $b = -6$. Therefore, the *y*-intercept is −6.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

IF students have difficulty distinguishing between the variables and the parameters in the equation,

THEN write several different equations on the board, each in slope-intercept form. Point out that in each case, the equation contains numbers where *m* and *b*, which are fixed values, would be the parameters while the variables *x* and *y*, which vary in value, represent the coordinates of the solutions of the equation. Examining several equations side by side helps to strengthen understanding of the concept.

DIFFERENTIATE

Enrichment Activity BL

Write $3x + 2y = 8$ and $-3x + 2y = 8$ on the board. Ask students to tell how the equations are alike and how they are different. Then, ask students to tell how the graphs of the two functions are alike and how they are different without graphing them. Finally, have them graph the functions and check their answers.

Example 6 Graph Constant Functions

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this example, students will need to use a sketch. Work with students to explore and deepen their understanding of slope-intercept form.

Questions for Mathematical Discourse

AL What is the y-intercept? x-intercept? 2; There is no x-intercept.

OL Why is the graph a horizontal line? Sample answer: Because the slope is 0, the graph will not rise, but can run left to right any amount.

BL What is the domain of this function? the range? D = all real numbers; R = 2

Common Error

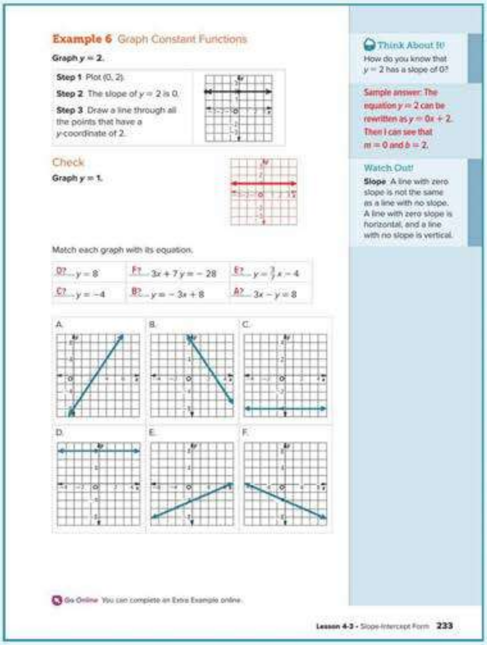
Some students may think that the slope is 2, since, when an equation is written in slope-intercept form, the slope is the number after the equal sign. Point out that if the slope were 2, the equation would be $y = 2x$. Since there is no x term, the slope is 0, and the equation is $y = 0x + 2$.

Essential Question Follow-Up

Students have explored the relationship between the parameters of a linear function and its graph.

Ask:

What can you learn about the graph of a linear function by analyzing its equation? Sample answer: If the equation is in slope-intercept form, I can tell where the graph intersects the y-axis and what the slope of the line is.



Example 6 Graph Constant Functions

Graph $y = 2$.

Step 1 Plot (0, 2).

Step 2 The slope of $y = 2$ is 0.

Step 3 Draw a line through all the points that have a y-coordinate of 2.

Check

Graph $y = 1$.

Match each graph with its equation.

D? $y = 8$	F? $3x + 7y = -28$	E? $y = \frac{3}{7}x - 4$
C? $y = -4$	B? $y = -3x + 8$	A? $3x - y = 8$

Think About It!

How do you know that $y = 2$ has a slope of 0?

Sample answer: The equation $y = 2$ can be rewritten as $y = 0x + 2$. Then I can see that $m = 0$ and $b = 2$.

Watch Out!

Slope A line with zero slope is not the same as a line with no slope. A line with zero slope is horizontal, and a line with no slope is vertical.

Go Online You can complete an Extra Example online.

Lesson 4-3 • Slope-Intercept Form 233

Interactive Presentation

Graph Constant Functions

Graph $y = 2$.

Step 1

Press *Point* to plot the y-intercept (0, 2).

Example 6

TAP

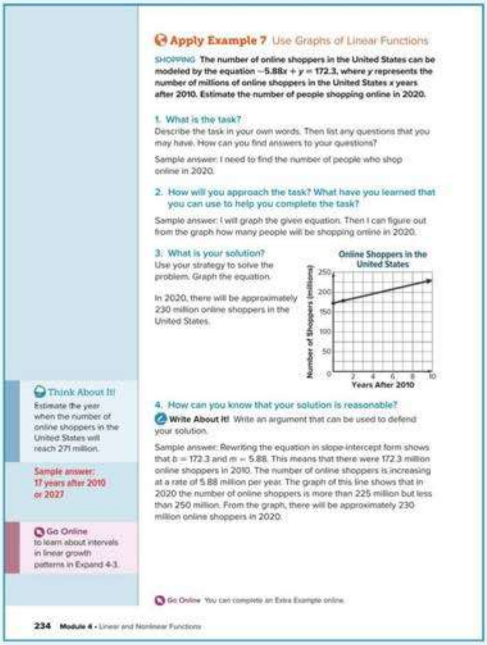
Students move through the steps of graphing a constant function.

WEB SKETCHPAD

Students use a sketch to graph a constant function.

TYPE

Students explain why the graph has a slope of 0.



Apply Example 7 Use Graphs of Linear Functions

SHOPPING **The number of online shoppers in the United States can be modeled by the equation $-5.88x + y = 172.3$, where y represents the number of millions of online shoppers in the United States x years after 2010. Estimate the number of people shopping online in 2020.**

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to find the number of people who shop online in 2020.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will graph the given equation. Then I can figure out from the graph how many people will be shopping online in 2020.

3. What is your solution?

Use your strategy to solve the problem. Graph the equation.

In 2020, there will be approximately 230 million online shoppers in the United States.

Online Shoppers in the United States
Number of Shoppers (millions)
250
200
150
100
50
0
2
4
6
8
10
Years After 2010

4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: Rewriting the equation in slope-intercept form shows that $b = 172.3$ and $m = 5.88$. This means that there were 172.3 million online shoppers in 2010. The number of online shoppers is increasing at a rate of 5.88 million per year. The graph of this line shows that in 2020 the number of online shoppers is more than 225 million but less than 250 million. From the graph, there will be approximately 230 million online shoppers in 2020.

Think About It!

Estimate the year when the number of online shoppers in the United States will reach 271 million.

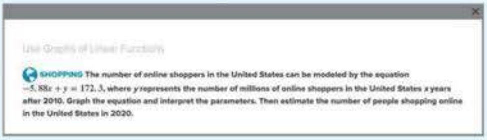
Sample answer: 17 years after 2010 or 2027

Go Online to learn about intervals in linear growth patterns in Expand 4-3.

Go Online You can complete an Extra Example online.

234 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Use Graphs of Linear Functions

SHOPPING **The number of online shoppers in the United States can be modeled by the equation $-5.88x + y = 172.3$, where y represents the number of millions of online shoppers in the United States x years after 2010. Graph the equation and interpret the parameters. Then estimate the number of people shopping online in the United States in 2020.**

Apply Example 7

TYPE

Students estimate the year when the online shoppers will reach a certain number.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Apply Example 7 Use Graphs of Linear Functions

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you determine the domain?
- How can you use the graph to estimate how many people will be shopping online in 2020?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

A.CED.2, F.IF.7a, F.LE.5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–30
2	exercises that use a variety of skills from this lesson	31–39
2	exercises that extend concepts learned in this lesson to new contexts	40–43
3	exercises that emphasize higher-order and critical-thinking skills	44–47

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: **BL**

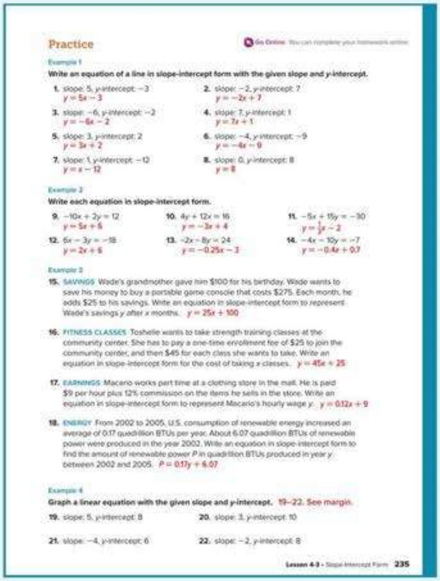
- Practice, Exercises 1–43 odd, 44–47
- Extension: Pencils of Lines
- **ALEKS** Equations of Lines

IF students score 66%–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–47 odd
- Remediation, Review Resources: Slope of a Line
- BrainPOP Video: Slope and Intercepts
- Extra Examples 1–7
- **ALEKS** Slope

IF students score 65% or less on the Checks, **THEN** assign: **AL**

- Practice, Exercises 1–29 odd
- Remediation, Review Resources: Slope of a Line
- *Quick Review Math Handbook*: Writing Equations in Slope-Intercept Form
- Arrive**MATH** Take Another Look
- **ALEKS** Slope



Practice

Go Online You can complete your homework online.

Example 1

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

1. slope: 5, y-intercept: −3 $y = 5x - 3$
2. slope: −2, y-intercept: 7 $y = -2x + 7$
3. slope: −6, y-intercept: −2 $y = -6x - 2$
4. slope: 7, y-intercept: 1 $y = 7x + 1$
5. slope: 3, y-intercept: 2 $y = 3x + 2$
6. slope: −4, y-intercept: −9 $y = -4x - 9$
7. slope: 1, y-intercept: −12 $y = x - 12$
8. slope: 0, y-intercept: 8 $y = 8$

Example 2

Write each equation in slope-intercept form.

9. $-10x + 2y = 12$ $y = 5x + 6$
10. $4y + 12x = 16$ $y = -3x + 4$
11. $-5x + 15y = -30$ $y = \frac{1}{3}x - 2$
12. $6x - 3y = -18$ $y = 2x + 6$
13. $-2x - 8y = 24$ $y = -0.25x - 3$
14. $-4x - 10y = -7$ $y = -0.4x + 0.7$

Example 3

15. SAVINGS Wade's grandmother gave him $100 for his birthday. Wade wants to save his money to buy a portable game console that costs $275. Each month, he adds $25 to his savings. Write an equation in slope-intercept form to represent Wade's savings y after x months. $y = 25x + 100$

16. FITNESS CLASSES Toshelle wants to take strength training classes at the community center. She has to pay a one-time enrollment fee of $25 to join the community center, and then $45 for each class she wants to take. Write an equation in slope-intercept form for the cost of taking x classes. $y = 45x + 25$

17. EARNINGS Macario works part time at a clothing store in the mall. He is paid $9 per hour plus 12% commission on the items he sells in the store. Write an equation in slope-intercept form to represent Macario's hourly wage y. $y = 0.12x + 9$

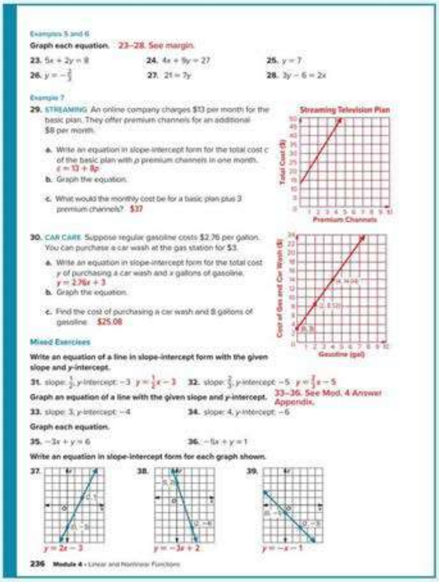
18. ENERGY From 2002 to 2005, U.S. consumption of renewable energy increased an average of 0.17 quadrillion BTUs per year. About 6.07 quadrillion BTUs of renewable power were produced in the year 2002. Write an equation in slope-intercept form to find the amount of renewable power P in quadrillion BTUs produced in year y between 2002 and 2005. $P = 0.17y + 6.07$

Example 4

Graph a linear equation with the given slope and y-intercept. 19–22. See margin.

19. slope: 5, y-intercept: 8
20. slope: 3, y-intercept: 10
21. slope: −4, y-intercept: 6
22. slope: −2, y-intercept: 8

Lesson 4-3 • Slope-Intercept Form 235



Examples 5 and 6

Graph each equation. 23–28. See margin.

23. $5x + 2y = 8$
24. $4x + 9y = 27$
25. $y = 7$
26. $y = -\frac{2}{3}$
27. $21 = 7y$
28. $3y - 6 = 2x$

Example 7

29. STREAMING An online company charges $13 per month for the basic plan. They offer premium channels for an additional $8 per month.
   a. Write an equation in slope-intercept form for the total cost c of the basic plan with p premium channels in one month. $c = 13 + 8p$
   b. Graph the equation.
   c. What would the monthly cost be for a basic plan plus 3 premium channels? $37

30. CAR CARE Suppose regular gasoline costs $2.76 per gallon. You can purchase a car wash at the gas station for $3.
   a. Write an equation in slope-intercept form for the total cost y of purchasing a car wash and x gallons of gasoline. $y = 2.76x + 3$
   b. Graph the equation.
   c. Find the cost of purchasing a car wash and 8 gallons of gasoline. $25.08

Mixed Exercises

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

31. slope: $\frac{1}{2}$, y-intercept: −3 $y = \frac{1}{2}x - 3$
32. slope: $\frac{2}{3}$, y-intercept: −5 $y = \frac{2}{3}x - 5$

Graph an equation of a line with the given slope and y-intercept. 33–36. See Mod. 4 Answer Appendix.

33. slope: 3, y-intercept: −4
34. slope: 4, y-intercept: −6

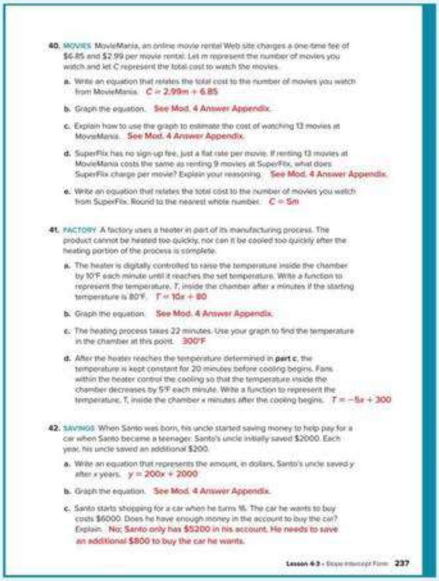
Graph each equation.

35. $-3x + y = 6$
36. $-5x + y = 1$

Write an equation in slope-intercept form for each graph shown.

37. $y = 2x - 3$
38. $y = -3x + 2$
39. $y = -x - 1$

236 Module 4 • Linear and Nonlinear Functions

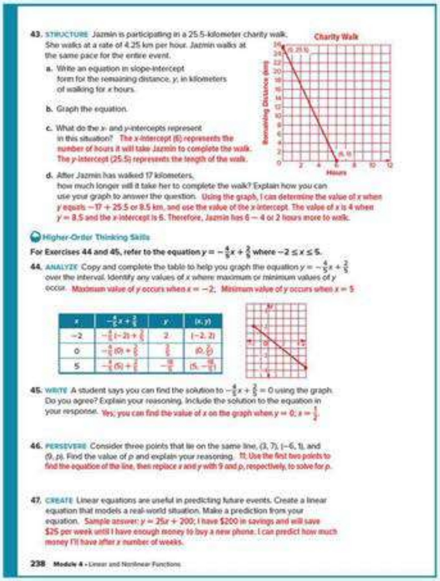


40. MOVIES MovieMania, an online movie rental Web site charges a one-time fee of \$6.85 and \$2.99 per movie rental. Let m represent the number of movies you watch and let C represent the total cost to watch the movies.
   a. Write an equation that relates the total cost to the number of movies you watch from MovieMania. $C = 2.99m + 6.85$
   b. Graph the equation. See Mod. 4 Answer Appendix.
   c. Explain how to use the graph to estimate the cost of watching 13 movies at MovieMania. See Mod. 4 Answer Appendix.
   d. SuperFlix has no sign-up fee, just a flat rate per movie. If renting 13 movies at MovieMania costs the same as renting 9 movies at SuperFlix, what does SuperFlix charge per movie? Explain your reasoning. See Mod. 4 Answer Appendix.
   e. Write an equation that relates the total cost to the number of movies you watch from SuperFlix. Round to the nearest whole number. $C = 5m$

41. FACTORY A factory uses a heater in part of its manufacturing process. The product cannot be heated too quickly, nor can it be cooled too quickly after the heating portion of the process is complete.
   a. The heater is digitally controlled to raise the temperature inside the chamber by 10°F each minute until it reaches the set temperature. Write a function to represent the temperature, T, inside the chamber after x minutes if the starting temperature is 80°F. $T = 10x + 80$
   b. Graph the equation. See Mod. 4 Answer Appendix.
   c. The heating process takes 22 minutes. Use your graph to find the temperature in the chamber at this point. 300°F
   d. After the heater reaches the temperature determined in **part c**, the temperature is kept constant for 20 minutes before cooling begins. Fans within the heater control the cooling so that the temperature inside the chamber decreases by 5°F each minute. Write a function to represent the temperature, T, inside the chamber x minutes after the cooling begins. $T = -5x + 300$

42. SAVINGS When Santo was born, his uncle started saving money to help pay for a car when Santo became a teenager. Santo's uncle initially saved \$2000. Each year, his uncle saved an additional \$200.
   a. Write an equation that represents the amount, in dollars, Santo's uncle saved y after x years. $y = 200x + 2000$
   b. Graph the equation. See Mod. 4 Answer Appendix.
   c. Santo starts shopping for a car when he turns 16. The car he wants to buy costs \$6000. Does he have enough money in the account to buy the car? Explain. No; Santo only has \$5200 in his account. He needs to save an additional \$800 to buy the car he wants.

Lesson 4-3 • Slope-Intercept Form 237



43. STRUCTURE Jazmin is participating in a 25.5-kilometer charity walk. She walks at a rate of 4.25 km per hour. Jazmin walks at the same pace for the entire event.
   a. Write an equation in slope-intercept form for the remaining distance, y, in kilometers of walking for x hours.
   b. Graph the equation.
   c. What do the x- and y-intercepts represent in this situation? The x-intercept (6) represents the number of hours it will take Jazmin to complete the walk. The y-intercept (25.5) represents the length of the walk.
   d. After Jazmin has walked 17 kilometers, how much longer will it take her to complete the walk? Explain how you can use your graph to answer the question. Using the graph, I can determine the value of x when y equals $-17 + 25.5$ or 8.5 km, and use the value of the x-intercept. The value of x is 4 when $y = 8.5$ and the x-intercept is 6. Therefore, Jazmin has $6 - 4$ or 2 hours more to walk.

Higher-Order Thinking Skills

For Exercises 44 and 45, refer to the equation $y = -\frac{4}{5}x + \frac{2}{5}$ where $-2 \le x \le 5$.

44. ANALYZE Copy and complete the table to help you graph the equation $y = -\frac{4}{5}x + \frac{2}{5}$ over the interval. Identify any values of x where maximum or minimum values of y occur. Maximum value of y occurs when $x = -2$; Minimum value of y occurs when $x = 5$

x	$-\frac{4}{5}x + \frac{2}{5}$	y	(x, y)
−2	$-\frac{4}{5}(-2) + \frac{2}{5}$	2	$(-2, 2)$
0	$-\frac{4}{5}(0) + \frac{2}{5}$	$\frac{2}{5}$	$(0, \frac{2}{5})$
5	$-\frac{4}{5}(5) + \frac{2}{5}$	$-\frac{18}{5}$	$(5, -\frac{18}{5})$

45. WRITE A student says you can find the solution to $-\frac{4}{5}x + \frac{2}{5} = 0$ using the graph. Do you agree? Explain your reasoning. Include the solution to the equation in your response. Yes; you can find the value of x on the graph when $y = 0$; $x = \frac{1}{2}$.

46. PERSEVERE Consider three points that lie on the same line, (3, 7), (−6, 1), and (9, p). Find the value of p and explain your reasoning. 11; Use the first two points to find the equation of the line, then replace x and y with 9 and p, respectively, to solve for p.

47. CREATE Linear equations are useful in predicting future events. Create a linear equation that models a real-world situation. Make a prediction from your equation. Sample answer: $y = 25x + 200$; I have \$200 in savings and will save \$25 per week until I have enough money to buy a new phone. I can predict how much money I'll have after x number of weeks.

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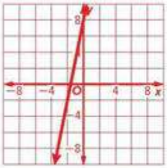
Answers

19.



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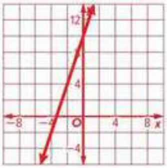

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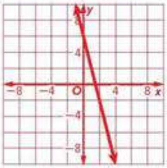

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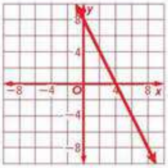

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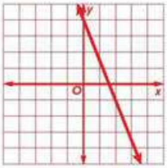

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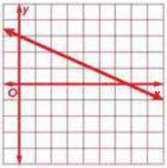

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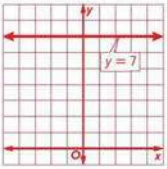

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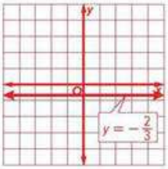

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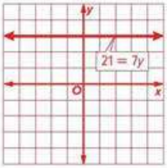

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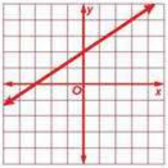


Lesson 4-4

Transformations of Linear Functions

F.IF.7a, F.BF.3

LESSON GOAL

Students identify the effects of transformations of the graphs of linear functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Transforming Linear Functions

Develop:

Translations of Linear Functions
- Vertical Translations of Linear Functions
- Horizontal Translations of Linear Functions
- Multiple Translations of Linear Functions
- Translations of Linear Functions

Dilations of Linear Functions
- Vertical Dilations of Linear Functions
- Horizontal Dilations of Linear Functions

Reflections of Linear Functions
- Reflections of Linear Functions Across the *x*-Axis
- Reflections of Linear Functions Across the *y*-Axis

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Reflections	●	●		●
Extension: Transformations of Other Families of Functions		●	●	●

Language Development Handbook

Assign page 23 of the *Language Development Handbook* to help your students build mathematical language related to transformations of the graphs of linear functions.

ELL You can use the tips and suggestions on page T23 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.
5 Use appropriate tools strategically.
7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students described the effect of transformations on two-dimensional figures using coordinates.
8.G.3

Now
Students Identify the effects of transformations of the graphs of linear functions.
F.IF.7a, F.BF.3

Next
Students will identify the effect of transformations of the graphs of nonlinear functions.
F.BF.3 (Course 1, Course 2, Course 3)

Rigor

The Three Pillars of Rigor

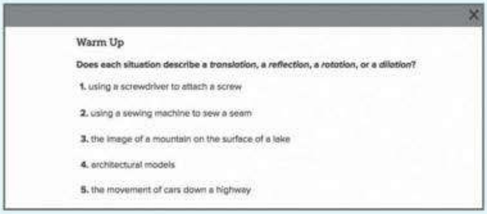
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of transformations of functions by examining the family of linear functions. They build fluency by describing transformations and identifying transformed functions. They apply their understanding by solving real-world problems.

1 LAUNCH

F.IF.7a, F.BF.3

Interactive Presentation

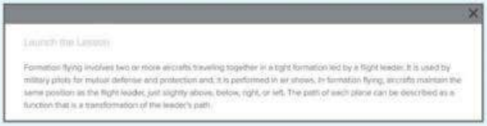


Warm Up

Does each situation describe a *translation*, a *reflection*, a *rotation*, or a *dilation*?

1. using a screwdriver to attach a screw
2. using a sewing machine to sew a seam
3. the image of a mountain on the surface of a lake
4. architectural models
5. the movement of cars down a highway

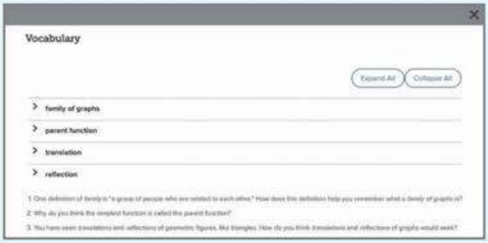
Warm Up



Launch the Lesson

Formation flying involves two or more aircrafts traveling together in a tight formation led by a flight leader. It is used by military pilots for mutual defense and protection and; it is performed in air shows. In formation flying, aircrafts maintain the same position as the flight leader, just slightly above, below, right, or left. The path of each plane can be described as a function that is a transformation of the leader's path.

Launch the Lesson



Vocabulary

- family of graphs
- parent function
- translation
- reflection

1. One definition of family is "a group of people who are related to each other." How does this definition help you remember what a family of graphs is?
2. Why do you think the simplest function is called the parent function?
3. You have seen translations and reflections of geometric figures, like triangles. How do you think translations and reflections of graphs would work?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- translating and reflecting geometric figures

Answers:

1. rotation	4. dilation
2. translation	5. translation
3. reflection	

Launch the Lesson

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of a linear function to identify the effect on the graph when replacing $f(x)$ with $f(x) + k$, $k \cdot f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

The parent function of the family of linear functions is $f(x) = x$. Transformations of the parent graph occur when a constant is added to or subtracted from the function or the argument, or when the function or the argument is multiplied by a number. These transformations alter the graph, translating it in a particular direction, dilating it, or reflecting it. Recognizing the effect produced by each type of transformation allows for the new graph to be easily obtained from the graph of the parent function.

Explore Transforming Linear Functions

Objective

Students use a sketch to explore how changing the parameters changes the graphs of linear functions.

Teaching the Mathematical Practices

3 Construct Arguments In this Explore, students will use stated assumptions, definitions, and previously established results to construct arguments.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

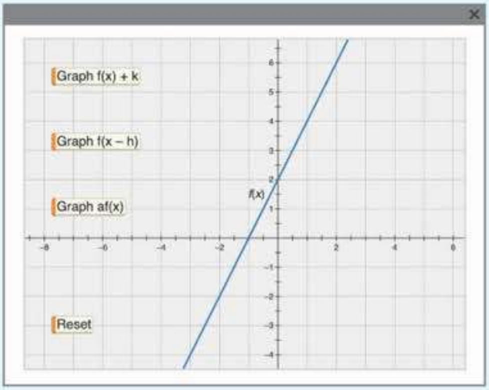
Summary of the Activity

Students complete guiding exercises throughout the Explore activity. Students use a sketch to explore how the graph of a function is affected when a number is added to the function, when a number is subtracted from the argument of the function, or when the function is multiplied by a number. They enter various values for the number and view the resulting graph. Then, students answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore the effects of addition and multiplication on a function.

TYPE

Students answer questions about transformations of linear functions.

Interactive Presentation

INQUIRY How does performing an operation on a linear function change its graph?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Transforming Linear Functions (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Does adding or subtracting a value to a function change the slope or *y*-intercept? Sample answer: The line moves up/down or left/right when you add or subtract values to the function. This means that the *y*-intercept is changing, but not the slope.
- Why does multiplying a function by a value make it more or less steep? Sample answer: If we multiply every value in a function, then we are changing the value of *y* for every *x*-value. If we multiply by a value greater than one, then the difference between the *y*-values will be greater, resulting in a greater slope and a steeper line.

Inquiry

How does performing an operation on a linear function change its graph? Sample answer: Adding a value to the function moves the graph up or down. Subtracting a value from *x* moves the graph left or right. Multiplying the function by a value makes the graph more steep or less steep.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Translations of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $f(x) + k$ and $f(x - h)$ for positive and negative values.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of translations in this Learn.

What Students Are Learning

The parent function of the family of linear functions is $f(x) = x$. Its graph is the line that passes through the origin and has a slope of 1. The graph of every other linear function is a transformation of this function. The first type of transformation students will learn about is translations. Under a translation, the graph of a line is slid to a new location.

Common Misconception

Students may believe that a translation will change the orientation of the figure. Help them to see that this is not the case. When a figure is slid in its entirety up, down, left, or right, its orientation remains the same. In the case of a line, its slope is not affected, so the new image has the same slope as the original graph.

Vertical Translations

Teaching the Mathematical Practices

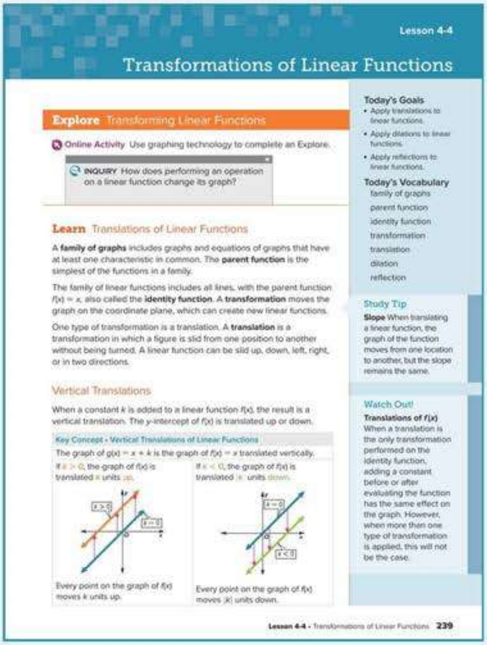
7 Use Structure Help students to explore the structure of vertical translations in this Learn.

About the Key Concept

When k is added to the function $f(x) = x$, the graph of the function is translated vertically. This is because adding k to the function increases the y-value that is associated with each x-value by k units. When k is negative, each y-value decreases, which translates the graph down $|k|$ units.

Common Misconception

Some students may think that adding k to a function increases (or decreases) the x-value in each ordered pair. Remind students that the notation $f(x)$ represents the y-value that is paired with x. Thus, $f(x) + k$ represents an increase (or decrease) in y-values, resulting in a vertical translation.



Lesson 4-4

Transformations of Linear Functions

Explore Transforming Linear Functions

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does performing an operation on a linear function change its graph?

Learn Translations of Linear Functions

A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent function** is the simplest of the functions in a family.

The family of linear functions includes all lines, with the parent function $f(x) = x$, also called the **identity function**. A **transformation** moves the graph on the coordinate plane, which can create new linear functions.

One type of transformation is a translation. A **translation** is a transformation in which a figure is slid from one position to another without being turned. A linear function can be slid up, down, left, right, or in two directions.

Vertical Translations

When a constant k is added to a linear function $f(x)$, the result is a vertical translation. The y-intercept of $f(x)$ is translated up or down.

Key Concept • Vertical Translations of Linear Functions

The graph of $g(x) = x + k$ is the graph of $f(x) = x$ translated vertically.

If $k > 0$, the graph of $f(x)$ is translated k units up.

If $k < 0$, the graph of $f(x)$ is translated $|k|$ units down.

Every point on the graph of $f(x)$ moves k units up.

Every point on the graph of $f(x)$ moves $|k|$ units down.

Today's Goals
- Apply translations to linear functions.
- Apply dilations to linear functions.
- Apply reflections to linear functions.

Today's Vocabulary

family of graphs
parent function
identity function
transformation
translation
dilation
reflection

Study Tip

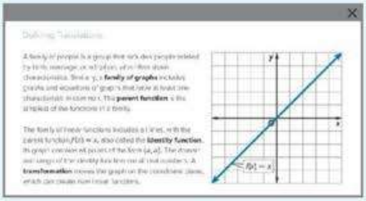
Slope When translating a linear function, the graph of the function moves from one location to another, but the slope remains the same.

Watch Out!

Translations of $f(x)$ When a translation is the only transformation performed on the identity function, adding a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Lesson 4-4 • Transformations of Linear Functions 239

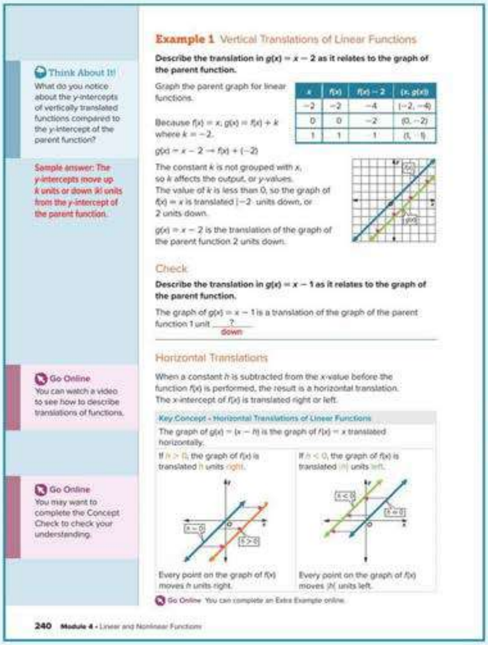
Interactive Presentation



Learn

TAP

Students tap each flash card to learn more about vertical translations.



Think About It!
What do you notice about the *y*-intercepts of vertically translated functions compared to the *y*-intercept of the parent function?

Sample answer: The *y*-intercepts move up *k* units or down |*k*| units from the *y*-intercept of the parent function.

Example 1 Vertical Translations of Linear Functions

Describe the translation in $g(x) = x - 2$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$f(x) - 2$	$(x, g(x))$
−2	−2	−4	(−2, −4)
0	0	−2	(0, −2)
1	1	−1	(1, −1)

Because $f(x) = x$, $g(x) = f(x) + k$ where $k = -2$.

$g(x) = x - 2 \rightarrow f(x) + (-2)$

The constant *k* is not grouped with *x*, so *k* affects the output, or *y*-values. The value of *k* is less than 0, so the graph of $f(x) = x$ is translated $|-2|$ units down, or 2 units down.

$g(x) = x - 2$ is the translation of the graph of the parent function 2 units down.

Check

Describe the translation in $g(x) = x - 1$ as it relates to the graph of the parent function.

The graph of $g(x) = x - 1$ is a translation of the graph of the parent function 1 unit ___?___ down

Horizontal Translations

When a constant *h* is subtracted from the *x*-value before the function *f*(*x*) is performed, the result is a horizontal translation. The *x*-intercept of *f*(*x*) is translated right or left.

Key Concept • Horizontal Translations of Linear Functions

The graph of $g(x) = (x - h)$ is the graph of $f(x) = x$ translated horizontally.

If $h > 0$, the graph of *f*(*x*) is translated *h* units right.

Every point on the graph of *f*(*x*) moves *h* units right.

If $h < 0$, the graph of *f*(*x*) is translated |*h*| units left.

Every point on the graph of *f*(*x*) moves |*h*| units left.

Go Online You can watch a video to see how to describe translations of functions.

Go Online You may want to complete the Concept Check to check your understanding.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Vertical Translations of Linear Functions

Describe the translation in $g(x) = x - 2$ as it relates to the graph of the parent function.

Example 1

TAP

Students move through the steps to graph a vertical translation.

TYPE

Students describe how the *y*-intercept of the translated function compares to the parent function.

Example 1 Vertical Translations of Linear Functions

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know the type of transformation? 2 is being subtracted from the parent function so this is a vertical translation.

OL How is the *y*-value of each ordered pair in the parent function affected? Each *y*-value decreases by 2.

BL How would you write this function as a vertical translation of the parent graph up 2 units? $g(x) = f(x) + 2$ or $g(x) = x + 2$

Horizontal Translations

Teaching the Mathematical Practices

3 Analyze Cases Guide students to examine the cases of different translations. Encourage students to familiarize themselves with all of the cases.

Common Misconception

Some students may think that the graph of $f(x + h)$, where *h* is a positive number, is a translation of the parent graph *h* units to the right. Point out that $f(x + h) = f(x - (-h))$, so the number being subtracted is a negative number. Thus, the shift is to the left, not to the right.

Example 2 Horizontal Translations of Linear Functions

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the translated function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know the type of transformation? The $+5$ is grouped with the x in the parentheses so this is a horizontal translation.

OL What are the coordinates when $g(x) = 0$? $(-5, 0)$

BL Write the function that shows a horizontal translation of the parent function 3 units right. $f(x - 3)$ 7 units left $f(x + 7)$

Example 3 Multiple Translations of Linear Functions

MP Teaching the Mathematical Practices

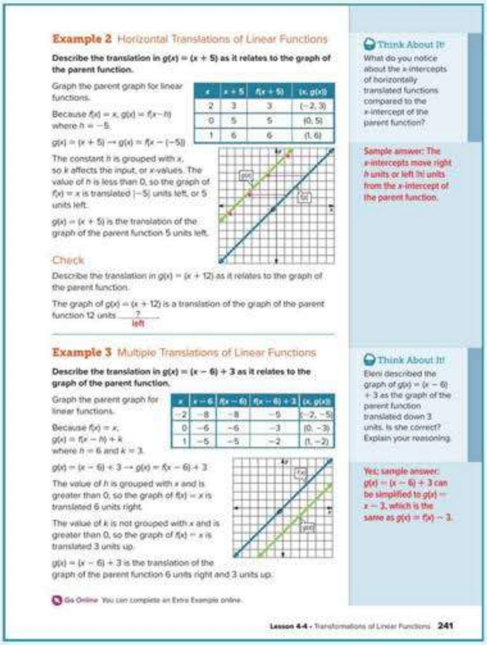
7 Use Structure Help students to use the structure of the transformed function to identify the translations in the function.

Questions for Mathematical Discourse

AL Looking at only the equation, how many translations are there? 2

OL Looking at only the equation, how do you know that the horizontal translation is to the right? because the number being subtracted from x is positive 6

BL Write a function that represents a translation 6 units left and 3 units down.
$f(x) = (x + 6) - 3$



Example 2 Horizontal Translations of Linear Functions

Describe the translation in $g(x) = (x + 5)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$x + 5$	$f(x + 5)$	$(x, g(x))$
2	3	3	(−2, 3)
0	5	5	(0, 5)
1	6	6	(1, 6)

Because $f(x) = x$, $g(x) = f(x - h)$ where $h = -5$.

$g(x) = (x + 5) \rightarrow g(x) = f(x - (-5))$

The constant h is grouped with x, so h affects the input, or x-values. The value of h is less than 0, so the graph of $f(x) = x$ is translated $|-5|$ units left, or 5 units left.

$g(x) = (x + 5)$ is the translation of the graph of the parent function 5 units left.

Check

Describe the translation in $g(x) = (x + 12)$ as it relates to the graph of the parent function.

The graph of $g(x) = (x + 12)$ is a translation of the graph of the parent function 12 units __?__ left

Example 3 Multiple Translations of Linear Functions

Describe the translation in $g(x) = (x - 6) + 3$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$x - 6$	$f(x - 6)$	$f(x - 6) + 3$	$(x, g(x))$
−2	−8	−8	−5	(−2, −5)
0	−6	−6	−3	(0, −3)
1	−5	−5	−2	(1, −2)

Because $f(x) = x$, $g(x) = f(x - h) + k$ where $h = 6$ and $k = 3$.

$g(x) = (x - 6) + 3 \rightarrow g(x) = f(x - 6) + 3$

The value of h is grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated 6 units right.

The value of k is not grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated 3 units up.

$g(x) = (x - 6) + 3$ is the translation of the graph of the parent function 6 units right and 3 units up.

Go Online You can complete an Extra Example online.

Think About It! What do you notice about the x-intercepts of horizontally translated functions compared to the x-intercept of the parent function?

Sample answer: The x-intercepts move right h units or left $|h|$ units from the x-intercept of the parent function.

Think About It! Eleni described the graph of $g(x) = (x - 6) + 3$ as the graph of the parent function translated down 3 units. Is she correct? Explain your reasoning.

Yes; sample answer: $g(x) = (x - 6) + 3$ can be simplified to $g(x) = x - 3$, which is the same as $g(x) = f(x) - 3$.

Lesson 4-4 • Transformations of Linear Functions 241

Interactive Presentation

Describe the translation in $g(x) = (x + 5)$ as it relates to the graph of the parent function.

Move through the steps to see how $g(x)$ is related to the parent function.

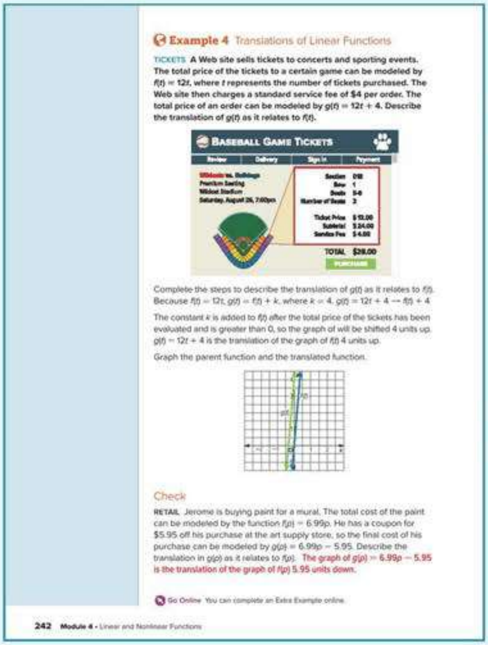
Example 2

TAP

Students move through the steps to graph a horizontal translation.

TYPE

Students describe how the x-intercept of the translated function compares to the parent.



Example 4 Translations of Linear Functions

TICKETS **A Web site sells tickets to concerts and sporting events. The total price of the tickets to a certain game can be modeled by $f(t) = 12t$, where t represents the number of tickets purchased. The Web site then charges a standard service fee of \$4 per order. The total price of an order can be modeled by $g(t) = 12t + 4$. Describe the translation of $g(t)$ as it relates to $f(t)$.**

Complete the steps to describe the translation of $g(t)$ as it relates to $f(t)$.
Because $f(t) = 12t$, $g(t) = f(t) + k$, where $k = 4$. $g(t) = 12t + 4 \rightarrow f(t) + 4$

The constant k is added to $f(t)$ after the total price of the tickets has been evaluated and is greater than 0, so the graph of will be shifted 4 units up. $g(t) = 12t + 4$ is the translation of the graph of $f(t)$ 4 units up.

Graph the parent function and the translated function.

Check

RETAIL Jerome is buying paint for a mural. The total cost of the paint can be modeled by the function $f(p) = 6.99p$. He has a coupon for \$5.95 off his purchase at the art supply store, so the final cost of his purchase can be modeled by $g(p) = 6.99p - 5.95$. Describe the translation in $g(p)$ as it relates to $f(p)$. The graph of $g(p) = 6.99p - 5.95$ is the translation of the graph of $f(p)$ 5.95 units down.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4

TYPE

Students complete the statements to identify the value for the translation.

WEB SKETCHPAD

Students use a sketch to graph the translated function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Translations of Linear Functions

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about translations of linear functions to solving a real-world problem.

Questions for Mathematical Discourse

AL What does the 12 in the function represent? the cost per ticket
What does the t in the function represent? the number of tickets

OL What does the parent function represent in the context of the situation? cost of tickets without the online service fee

BL What would the function be if, in addition to the service fee, there was also a \$5 charge for tax? $g(t) = 12t + 4 + 5$ or $g(t) = 12t + 9$

Common Error

Some students may try to work the 12 into the translation. Remind these students that translations occur when numbers are added or subtracted, not multiplied.

Essential Question Follow-Up

Students have observed how a function that models a real-world situation can be a transformation of another function.

Ask:

Why is it important to understand how the structure of a function models a situation? Sample answer: The structure helps you understand how the different quantities in the situation affect the function.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students are having difficulty determining the direction of a translation,

THEN have them create four examples of functions that represent each type of translation, and write each one on an index card. Have them sketch the transformation on a coordinate plane on the back of the card, and write the description. Then have them use the flash cards (in both directions) to practice what they have learned.

Learn Dilations of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $af(x)$ and by replacing $f(x)$ with $f(ax)$.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of vertical and horizontal dilations in this Learn.

About the Key Concept

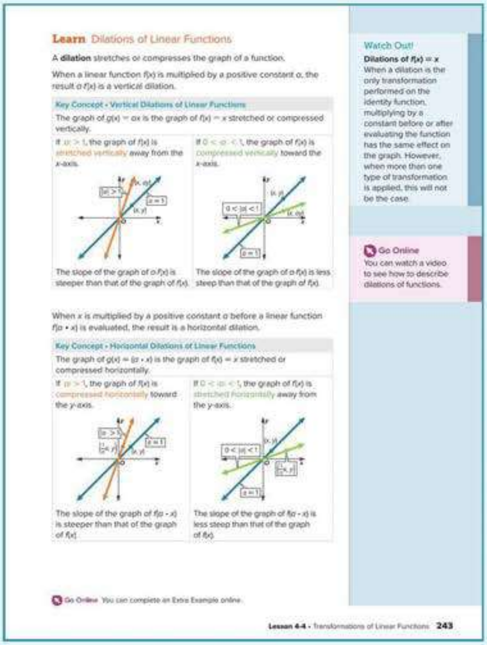
When the function $f(x) = x$ is multiplied by a number a, the graph of the function is dilated vertically. This is because multiplying the function by a number affects the y-value that is associated with each x-value. When $|a| > 1$, the graph is stretched vertically, making it steeper. When $|a| < 1$, the graph is compressed vertically, making it less steep.

Common Misconception

Some students may think that when a is positive, the dilation stretches the graph, and when it is negative, the dilation compresses the graph. Use a table of values for several functions to show students the error in this reasoning. Sample functions: $f(x) = x$, $g(x) = 2f(x)$, $g(x) = -2f(x)$, $g(x) = 0.5f(x)$, $g(x) = -0.5f(x)$

About the Key Concept

When the argument of the function $f(x) = x$ is multiplied by a number a, the graph of the function is dilated horizontally. This is because multiplying the argument by a number affects the x-value that is associated with each y-value. When $|a| > 1$, the graph is compressed horizontally, making it steeper. When $|a| < 1$, the graph is stretched horizontally, making it less steep.



Learn Dilations of Linear Functions

A **dilation** stretches or compresses the graph of a function.

When a linear function $f(x)$ is multiplied by a positive constant a, the result $a \cdot f(x)$ is a vertical dilation.

Key Concept • Vertical Dilations of Linear Functions

The graph of $g(x) = ax$ is the graph of $f(x) = x$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x)$ is stretched vertically away from the x-axis.

The slope of the graph of $a \cdot f(x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is compressed vertically toward the x-axis.

The slope of the graph of $a \cdot f(x)$ is less steep than that of the graph of $f(x)$.

When x is multiplied by a positive constant a before a linear function $f(a \cdot x)$ is evaluated, the result is a horizontal dilation.

Key Concept • Horizontal Dilations of Linear Functions

The graph of $g(x) = (a \cdot x)$ is the graph of $f(x) = x$ stretched or compressed horizontally.

If $|a| > 1$, the graph of $f(x)$ is compressed horizontally toward the y-axis.

The slope of the graph of $f(a \cdot x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is stretched horizontally away from the y-axis.

The slope of the graph of $f(a \cdot x)$ is less steep than that of the graph of $f(x)$.

Go Online You can complete an Extra Example online.

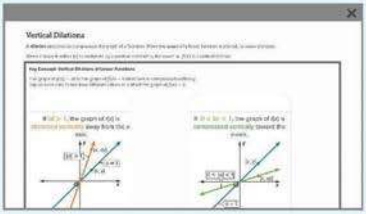
Watch Out!

Dilations of $f(x) = x$ When a dilation is the only transformation performed on the identity function, multiplying by a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Go Online You can watch a video to see how to describe dilations of functions.

Lesson 4-4 • Transformations of Linear Functions 243

Interactive Presentation



Learn

TAP

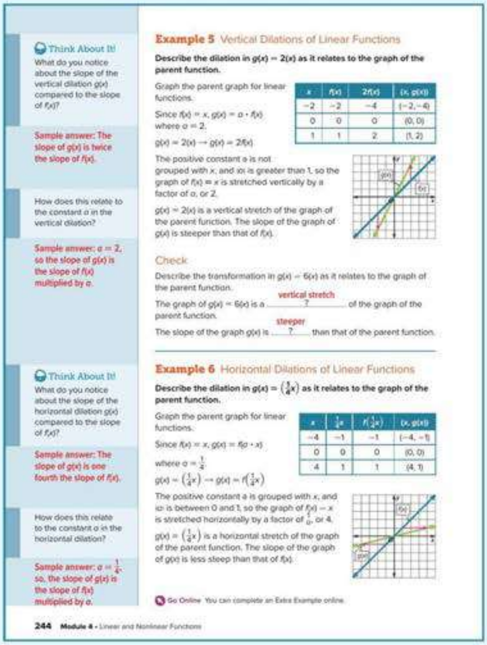
Students tap on each flash card to learn more about vertical and horizontal dilations.

DRAG & DROP

Students drag the correct term to identify vertical and horizontal dilations.

TYPE

Students explain the effect of dilations on the intercepts of the parent function.



Think About It! What do you notice about the slope of the vertical dilation $g(x)$ compared to the slope of $f(x)$?

Sample answer: The slope of $g(x)$ is twice the slope of $f(x)$.

How does this relate to the constant a in the vertical dilation?

Sample answer: $a = 2$, so the slope of $g(x)$ is the slope of $f(x)$ multiplied by a.

Example 5 Vertical Dilations of Linear Functions

Describe the dilation in $g(x) = 2(x)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$2f(x)$	$(x, g(x))$
−2	−2	−4	(−2, −4)
0	0	0	(0, 0)
1	1	2	(1, 2)

Since $f(x) = x$, $g(x) = a \cdot f(x)$ where $a = 2$.

$g(x) = 2(x) \rightarrow g(x) = 2f(x)$

The positive constant a is not grouped with x, and $|a|$ is greater than 1, so the graph of $f(x) = x$ is stretched vertically by a factor of a, or 2.

$g(x) = 2(x)$ is a vertical stretch of the graph of the parent function. The slope of the graph of $g(x)$ is steeper than that of $f(x)$.

Check

Describe the transformation in $g(x) = 6(x)$ as it relates to the graph of the parent function.

The graph of $g(x) = 6(x)$ is a vertical stretch ? of the graph of the parent function.

The slope of the graph $g(x)$ is steeper ? than that of the parent function.

Think About It! What do you notice about the slope of the horizontal dilation $g(x)$ compared to the slope of $f(x)$?

Sample answer: The slope of $g(x)$ is one fourth the slope of $f(x)$.

How does this relate to the constant a in the horizontal dilation?

Sample answer: $a = \frac{1}{4}$, so, the slope of $g(x)$ is the slope of $f(x)$ multiplied by a.

Example 6 Horizontal Dilations of Linear Functions

Describe the dilation in $g(x) = \left(\frac{1}{4}x\right)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$\frac{1}{4}x$	$f\left(\frac{1}{4}x\right)$	$(x, g(x))$
−4	−1	−1	(−4, −1)
0	0	0	(0, 0)
4	1	1	(4, 1)

Since $f(x) = x$, $g(x) = f(a \cdot x)$

where $a = \frac{1}{4}$.

$g(x) = \left(\frac{1}{4}x\right) \rightarrow g(x) = f\left(\frac{1}{4}x\right)$

The positive constant a is grouped with x, and $|a|$ is between 0 and 1, so the graph of $f(x) = x$ is stretched horizontally by a factor of $\frac{1}{a}$, or 4.

$g(x) = \left(\frac{1}{4}x\right)$ is a horizontal stretch of the graph of the parent function. The slope of the graph of $g(x)$ is less steep than that of $f(x)$.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Vertical Dilations of Linear Functions

Describe the dilation in $g(x) = 2(x)$ as it relates to the graph of the parent function.

Move through the steps to see how g(x) is related to the parent function.

Example 5

TAP

Students move through the steps to graph a vertical dilation.

TYPE

Students compare the slope and y-intercept of the vertical dilation to the parent graph.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 5 Vertical Dilations of Linear Functions

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in calculating the coordinates for $g(x)$ in this example.

Questions for Mathematical Discourse

AL Will the placement of the 2 cause a change to the x-value or to the y-value of each ordered pair of the parent function? y-value; Sample answer: Because the 2 is not grouped with the x-variable, it will change the y-value.

OL Looking at only the equation, what kind of dilation is this? a vertical stretch by a factor of 2

BL How would the transformation be different if the function had been $g(x) = \frac{1}{2}f(x)$? There would be a vertical compression instead of a vertical stretch.

Example 6 Horizontal Dilations of Linear Functions

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graphs and equations of the functions in this example.

Questions for Mathematical Discourse

AL What value is grouped with the x? $\frac{1}{4}$ Will this cause a change to the x-value or to the y-value of each ordered pair of the parent function? the x-value

OL Looking at only the equation, what kind of dilation is this? a horizontal stretch by a factor of 4

BL How is a horizontal stretch by a factor of 4 related to a vertical compression by a factor of 4? They result in the same line.

Learn Reflections of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $-af(x)$ and $f(-ax)$.

MP Teaching the Mathematical Practices

1 Explain Correspondence Encourage students to explain the relationships between the coordinates, equations, and graphs of reflected functions and the parent function.

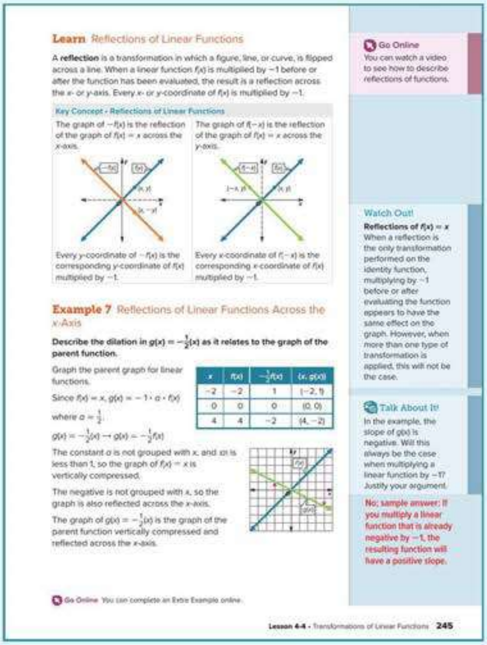
Example 7 Reflections of Linear Functions Across the *x*-Axis

MP Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument in the Talk About It! feature.

Questions for Mathematical Discourse

- **AL** Looking at only the equation, is this function a reflection? yes What other type of transformation is it? a dilation
- **OL** How do you know $-\frac{1}{2}$ is not grouped with x? Sample answer: It is not inside the parentheses with x.
- **BL** The point (2, 2) lies on the graph of the parent function. To what point does this correspond on the graph of $g(x)$? (2, −1)



Learn Reflections of Linear Functions

A **reflection** is a transformation in which a figure, line, or curve, is flipped across a line. When a linear function $f(x)$ is multiplied by −1 before or after the function has been evaluated, the result is a reflection across the x- or y-axis. Every x- or y-coordinate of $f(x)$ is multiplied by −1.

Key Concept • Reflections of Linear Functions

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x$ across the x-axis.	The graph of $f(-x)$ is the reflection of the graph of $f(x) = x$ across the y-axis.
Every y-coordinate of $-f(x)$ is the corresponding y-coordinate of $f(x)$ multiplied by −1.	Every x-coordinate of $f(-x)$ is the corresponding x-coordinate of $f(x)$ multiplied by −1.

Example 7 Reflections of Linear Functions Across the *x*-Axis

Describe the dilation in $g(x) = -\frac{1}{2}(x)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$-\frac{1}{2}f(x)$	$(x, g(x))$
−2	−2	1	(−2, 1)
0	0	0	(0, 0)
4	4	−2	(4, −2)

Since $f(x) = x$, $g(x) = -1 \cdot a \cdot f(x)$ where $a = \frac{1}{2}$.

$g(x) = -\frac{1}{2}(x) \rightarrow g(x) = -\frac{1}{2}f(x)$

The constant a is not grouped with x, and $|a|$ is less than 1, so the graph of $f(x) = x$ is vertically compressed.

The negative is not grouped with x, so the graph is also reflected across the x-axis.

The graph of $g(x) = -\frac{1}{2}(x)$ is the graph of the parent function vertically compressed and reflected across the x-axis.

Go Online You can complete an Extra Example online.

Go Online You can watch a video to see how to describe reflections of functions.

Watch Out!

Reflections of $f(x) = x$ When a reflection is the only transformation performed on the identity function, multiplying by −1 before or after evaluating the function appears to have the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Talk About It!

In the example, the slope of $g(x)$ is negative. Will this always be the case when multiplying a linear function by −1? Justify your argument.

No; sample answer: If you multiply a linear function that is already negative by −1, the resulting function will have a positive slope.

Lesson 4-4 • Transformations of Linear Functions 245

Interactive Presentation

Learn

TAP

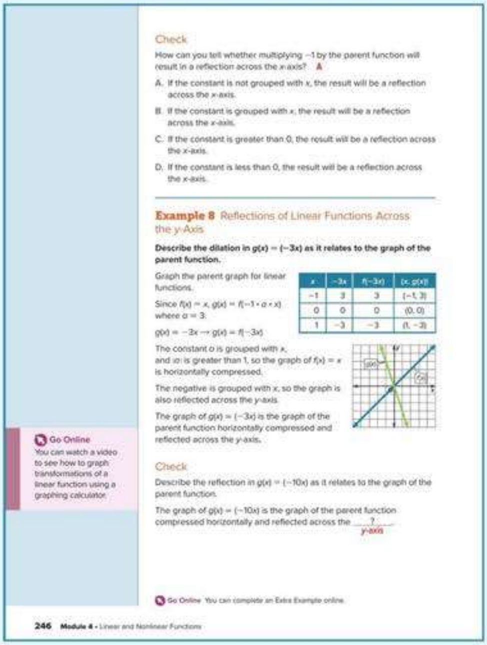
Students tap on each flash card to learn more about reflections of functions.

DRAG & DROP

Students drag the correct term to identify if a reflection is over the x- or y-axis.

TYPE

Students write a function that is a dilation and reflection of a parent function.



Check

How can you tell whether multiplying −1 by the parent function will result in a reflection across the x-axis? A

A. If the constant is not grouped with x, the result will be a reflection across the x-axis.

B. If the constant is grouped with x, the result will be a reflection across the x-axis.

C. If the constant is greater than 0, the result will be a reflection across the x-axis.

D. If the constant is less than 0, the result will be a reflection across the x-axis.

Example 8 Reflections of Linear Functions Across the y-Axis

Describe the dilation in $g(x) = (-3x)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

Since $f(x) = x$, $g(x) = f(-1 \cdot a \cdot x)$ where $a = 3$.

$g(x) = -3x \rightarrow g(x) = f(-3x)$

x	$-3x$	$f(-3x)$	$(x, g(x))$
−1	3	3	(−1, 3)
0	0	0	(0, 0)
1	−3	−3	(1, −3)

The constant a is grouped with x, and |a| is greater than 1, so the graph of $f(x) = x$ is horizontally compressed.

The negative is grouped with x, so the graph is also reflected across the y-axis.

The graph of $g(x) = (-3x)$ is the graph of the parent function horizontally compressed and reflected across the y-axis.

Go Online You can watch a video to see how to graph transformations of a linear function using a graphing calculator.

Check

Describe the reflection in $g(x) = (-10x)$ as it relates to the graph of the parent function.

The graph of $g(x) = (-10x)$ is the graph of the parent function compressed horizontally and reflected across the $\underset{y\text{-axis}}{\underline{\quad ? \quad}}$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Reflections of Linear Functions Across the y-Axis

Describe the reflection in $g(x) = (-3x)$ as it relates to the graph the parent function.

Move through the steps to see how g(x) is related to the parent function.

Example 8

TAP

Students move through the steps to graph a reflection of a linear function across the y-axis.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 8 Reflections of Linear Functions Across the y-Axis

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the reflected function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL How do you know the negative sign is grouped with the x? Sample answer: Because the negative sign is inside the parentheses with x.

OL How do you know if the parent function will be reflected over the y-axis? Sample answer: If the negative is inside the parentheses with x, the reflection will be over the y-axis.

BL How would the function have been written if the reflection was across the x-axis? $g(x) = -3f(x)$

Common Error

Students may have difficulty seeing how the graph of $g(x)$ is related to the graph of $f(x)$. For these students, you may want to show the transformation in two different steps, first dilating the graph by a factor of 3, and then reflecting the resulting graph across the y-axis.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–21
2	exercises that use a variety of skills from this lesson	22–29
2	exercises that extend concepts learned in this lesson to new contexts	30–33
3	exercises that emphasize higher-order and critical-thinking skills	34–36

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–33 odd, 34–36
- Extension: Transformations of Other Families of Functions
- **ALEKS** Equations of Lines

OL

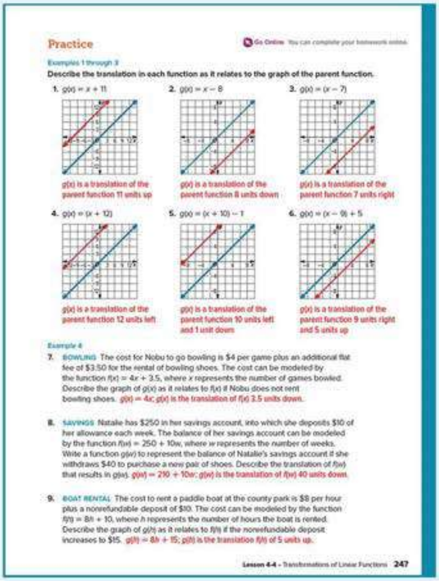
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Reflections
- Personal Tutors
- Extra Examples 1–8
- **ALEKS** Reflections

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–21 odd
- Remediation, Review Resources: Reflections
- *Quick Review Math Handbook*: Transformations of Linear Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Reflections



Practice

Go Online You can complete your homework online.

Examples 1 through 3

Describe the translation in each function as it relates to the graph of the parent function.

1. $g(x) = x + 11$
g(x) is a translation of the parent function 11 units up

2. $g(x) = x - 8$
g(x) is a translation of the parent function 8 units down

3. $g(x) = (x - 7)$
g(x) is a translation of the parent function 7 units right

4. $g(x) = (x + 12)$
g(x) is a translation of the parent function 12 units left

5. $g(x) = (x + 10) - 1$
g(x) is a translation of the parent function 10 units left and 1 unit down

6. $g(x) = (x - 9) + 5$
g(x) is a translation of the parent function 9 units right and 5 units up

Example 4

7. **BOWLING** The cost for Nobu to go bowling is \$4 per game plus an additional flat fee of \$3.50 for the rental of bowling shoes. The cost can be modeled by the function $f(x) = 4x + 3.5$, where x represents the number of games bowled. Describe the graph of $g(x)$ as it relates to $f(x)$ if Nobu does not rent bowling shoes. $g(x) = 4x$; $g(x)$ is the translation of $f(x)$ 3.5 units down.

8. **SAVINGS** Natalie has \$250 in her savings account, into which she deposits \$10 of her allowance each week. The balance of her savings account can be modeled by the function $f(w) = 250 + 10w$, where w represents the number of weeks. Write a function $g(w)$ to represent the balance of Natalie's savings account if she withdraws \$40 to purchase a new pair of shoes. Describe the translation of $f(w)$ that results in $g(w)$. $g(w) = 210 + 10w$; $g(w)$ is the translation of $f(w)$ 40 units down.

9. **BOAT RENTAL** The cost to rent a paddle boat at the county park is \$8 per hour plus a nonrefundable deposit of \$10. The cost can be modeled by the function $f(h) = 8h + 10$, where h represents the number of hours the boat is rented. Describe the graph of $g(h)$ as it relates to $f(h)$ if the nonrefundable deposit increases to \$15. $g(h) = 8h + 15$; $g(h)$ is the translation $f(h)$ of 5 units up.

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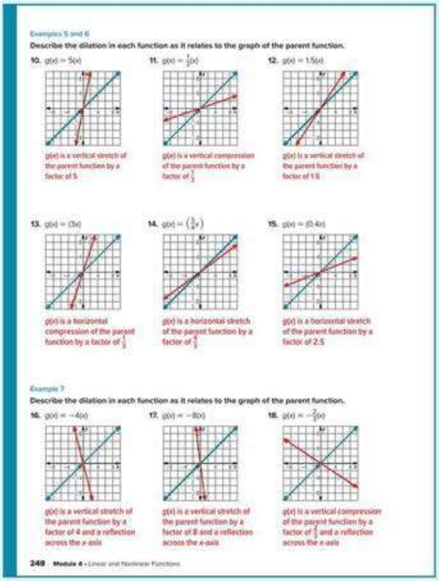

Examples 5 and 6

Describe the dilation in each function as it relates to the graph of the parent function.

10. $g(x) = 5(x)$
g(x) is a vertical stretch of the parent function by a factor of 5

11. $g(x) = \frac{1}{3}(x)$
g(x) is a vertical compression of the parent function by a factor of $\frac{1}{3}$

12. $g(x) = 1.5(x)$
g(x) is a vertical stretch of the parent function by a factor of 1.5

13. $g(x) = (3x)$
g(x) is a horizontal compression of the parent function by a factor of $\frac{1}{3}$

14. $g(x) = \left(\frac{3}{4}x\right)$
g(x) is a horizontal stretch of the parent function by a factor of $\frac{4}{3}$

15. $g(x) = (0.4x)$
g(x) is a horizontal stretch of the parent function by a factor of 2.5

Example 7

Describe the dilation in each function as it relates to the graph of the parent function.

16. $g(x) = -4(x)$
g(x) is a vertical stretch of the parent function by a factor of 4 and a reflection across the x-axis

17. $g(x) = -8(x)$
g(x) is a vertical stretch of the parent function by a factor of 8 and a reflection across the x-axis

18. $g(x) = -\frac{2}{3}(x)$
g(x) is a vertical compression of the parent function by a factor of $\frac{2}{3}$ and a reflection across the x-axis

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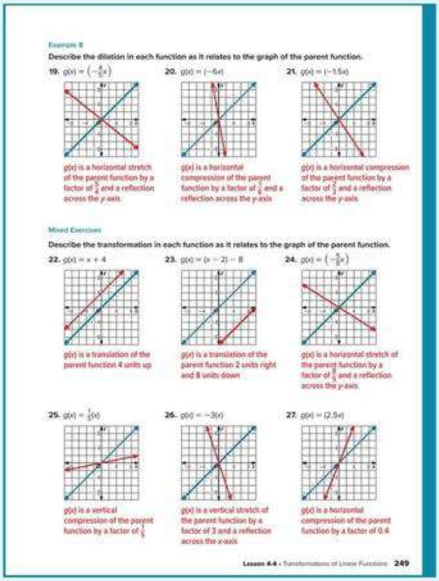
3 REFLECT AND PRACTICE

F.IF.7a, F.BF.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 8

Describe the dilation in each function as it relates to the graph of the parent function.

19. $g(x) = \left(-\frac{4}{3}x\right)$

$g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{3}{4}$ and a reflection across the y-axis.

20. $g(x) = (-6x)$

$g(x)$ is a horizontal compression of the parent function by a factor of $\frac{1}{6}$ and a reflection across the y-axis

21. $g(x) = (-1.5x)$

$g(x)$ is a horizontal compression of the parent function by a factor of $\frac{2}{3}$ and a reflection across the y-axis

Mixed Exercises

Describe the transformation in each function as it relates to the graph of the parent function.

22. $g(x) = x + 4$

$g(x)$ is a translation of the parent function 4 units up

23. $g(x) = (x - 2) - 8$

$g(x)$ is a translation of the parent function 2 units right and 8 units down

24. $g(x) = \left(-\frac{5}{8}x\right)$

$g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{8}{5}$ and a reflection across the y-axis

25. $g(x) = \frac{1}{5}(x)$

$g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{5}$

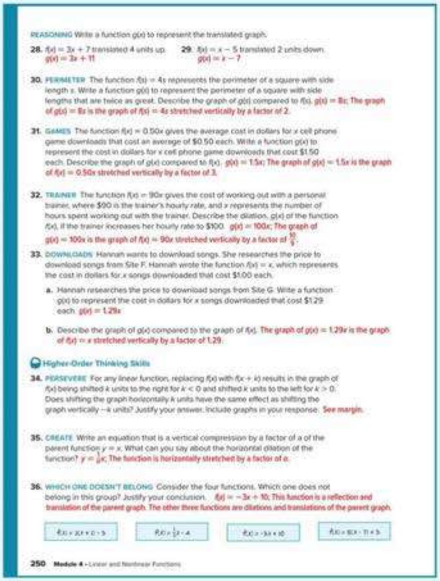
26. $g(x) = -3(x)$

$g(x)$ is a vertical stretch of the parent function by a factor of 3 and a reflection across the x-axis

27. $g(x) = (2.5x)$

$g(x)$ is a horizontal compression of the parent function by a factor of 0.4

Lesson 4-4 • Transformations of Linear Functions **249**



REASONING Write a function $g(x)$ to represent the translated graph.

28. $f(x) = 3x + 7$ translated 4 units up. $g(x) = 3x + 11$

29. $f(x) = x - 5$ translated 2 units down. $g(x) = x - 7$

30. PERIMETER The function $f(s) = 4s$ represents the perimeter of a square with side length s. Write a function $g(s)$ to represent the perimeter of a square with side lengths that are twice as great. Describe the graph of $g(s)$ compared to $f(s)$. $g(s) = 8s$; The graph of $g(s) = 8s$ is the graph of $f(s) = 4s$ stretched vertically by a factor of 2.

31. GAMES The function $f(x) = 0.50x$ gives the average cost in dollars for x cell phone game downloads that cost an average of \$0.50 each. Write a function $g(x)$ to represent the cost in dollars for x cell phone game downloads that cost \$1.50 each. Describe the graph of $g(x)$ compared to $f(x)$. $g(x) = 1.5x$; The graph of $g(x) = 1.5x$ is the graph of $f(x) = 0.50x$ stretched vertically by a factor of 3.

32. TRAINER The function $f(x) = 90x$ gives the cost of working out with a personal trainer, where \$90 is the trainer's hourly rate, and x represents the number of hours spent working out with the trainer. Describe the dilation, $g(x)$ of the function $f(x)$, if the trainer increases her hourly rate to \$100. $g(x) = 100x$; The graph of $g(x) = 100x$ is the graph of $f(x) = 90x$ stretched vertically by a factor of $\frac{10}{9}$.

33. DOWNLOADS Hannah wants to download songs. She researches the price to download songs from Site F. Hannah wrote the function $f(x) = x$, which represents the cost in dollars for x songs downloaded that cost \$1.00 each.

a. Hannah researches the price to download songs from Site G. Write a function $g(x)$ to represent the cost in dollars for x songs downloaded that cost \$1.29 each. $g(x) = 1.29x$

b. Describe the graph of $g(x)$ compared to the graph of $f(x)$. The graph of $g(x) = 1.29x$ is the graph of $f(x) = x$ stretched vertically by a factor of 1.29.

Higher-Order Thinking Skills

34. PERSEVERE For any linear function, replacing $f(x)$ with $f(x + k)$ results in the graph of $f(x)$ being shifted k units to the right for $k < 0$ and shifted k units to the left for $k > 0$. Does shifting the graph horizontally k units have the same effect as shifting the graph vertically $-k$ units? Justify your answer. Include graphs in your response. See margin.

35. CREATE Write an equation that is a vertical compression by a factor of a of the parent function $y = x$. What can you say about the horizontal dilation of the function? $y = \frac{1}{a}x$; The function is horizontally stretched by a factor of a.

36. WHICH ONE DOESN'T BELONG Consider the four functions. Which one does not belong in this group? Justify your conclusion. $f(x) = -3x + 10$; This function is a reflection and translation of the parent graph. The other three functions are dilations and translations of the parent graph.

$f(x) = 2(x + 0) - 5$	$f(x) = \frac{1}{2}x - 4$	$f(x) = -3x + 10$	$f(x) = 8(x - 7) + 5$

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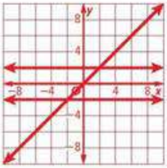
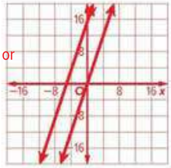
Answer

34. No; sample answer: This is true only if the slope is 1. Consider any constant function. Shifting the graph of a constant function to the right or left does not result in any vertical shift of the same graph. If the slope m of the line described by $f(x)$ is something other than −1, 0, or 1, then a horizontal shift of k units is the same as a vertical shift of $-mk$ units. For example, if $f(x) = 3x$, then $f(x + 5) = 3(x + 5)$ or $3x + 15$. $f(x + 5)$ is shifted 5 units left of $f(x)$ or 15 units up from $f(x)$. Sample graphs shown.





Lesson 4-5

Arithmetic Sequences

F.BF.1a, F.BF.2, F.LE.2

LESSON GOAL

Students write and graph equations of arithmetic sequences.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Arithmetic Sequences
- Identify Arithmetic Sequences
- Find the Next Term

Explore: Common Differences

Develop:

Arithmetic Sequences as Linear Functions
- Find the *n*th Term
- Apply Arithmetic Sequences as Linear Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Add Integers	●	●		●
Extension: Arithmetic Series		●	●	●

Language Development Handbook

Assign page 24 of the *Language Development Handbook* to help your students build mathematical language related to arithmetic sequences.

ELL You can use the tips and suggestions on page T24 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice:

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous
Students understood the initial value and constant rate of change of a linear function.
8.F.4, F.IF.6, F.LE.5

Now
Students write and graph equations of arithmetic sequences.
F.BF.1a, F.BF.2, F.LE.2

Now
Students will compare and contrast arithmetic sequences and linear functions with geometric sequences and exponential functions.
F.BF.2, F.LE.1a

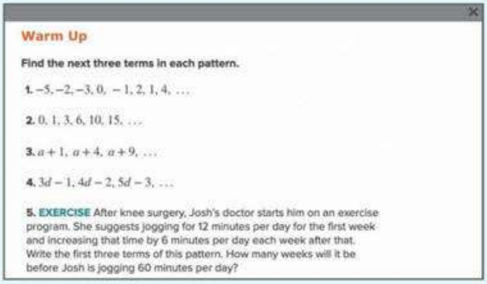
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and build fluency with sequences (first studied in Grade 4) by writing formulas for arithmetic sequences and relating them to linear functions. They apply their understanding by solving real-world problems related to arithmetic sequences.

Interactive Presentation

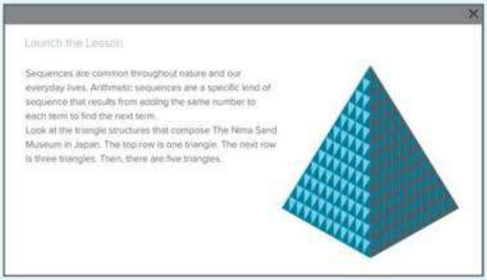


Warm Up

Find the next three terms in each pattern.

1. $-5, -2, -3, 0, -1, 2, 1, 4, \ldots$
2. $0, 1, 3, 6, 10, 15, \ldots$
3. $a + 1, a + 4, a + 9, \ldots$
4. $3d - 1, 4d - 2, 5d - 3, \ldots$
5. **EXERCISE** After knee surgery, Josh's doctor starts him on an exercise program. She suggests jogging for 12 minutes per day for the first week and increasing that time by 6 minutes per day each week after that. Write the first three terms of this pattern. How many weeks will it be before Josh is jogging 60 minutes per day?

Warm Up

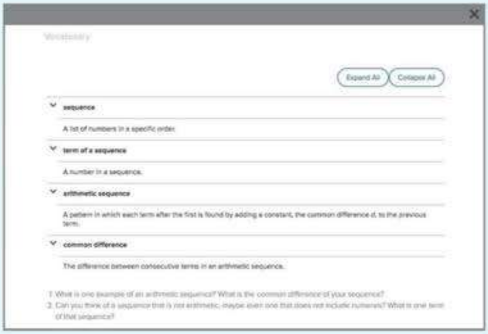


Launch the Lesson

Sequences are common throughout nature and our everyday lives. Arithmetic sequences are a specific kind of sequence that results from adding the same number to each term to find the next term.

Look at the triangle structures that compose The Nima Sand Museum in Japan. The top row is one triangle. The next row is three triangles. Then, there are five triangles.

Launch the Lesson



Vocabulary

sequence

A list of numbers in a specific order.

term of a sequence

A number in a sequence.

arithmetic sequence

A pattern in which each term after the first is found by adding a constant, the common difference d, to the previous term.

common difference

The difference between consecutive terms in an arithmetic sequence.

1. What is one example of an arithmetic sequence? What is the common difference of your sequence?
2. Can you think of a sequence that is not arithmetic, maybe even one that does not include numerals? What is one term of that sequence?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- finding the next terms in patterns

Answers:

1. 3, 6, 5
2. 21, 28, 36
3. $a + 16, a + 25, a + 36$
4. $6d - 4, 7d - 5, 8d - 6$
5. 12, 18, 24; 9 wk

Launch the Lesson

MP Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in the triangle structures that compose The Nima Sand Museum and in the Pyramid of Oranges.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

A sequence is a set of numbers in a specific order. The numbers in a sequence are called terms. If the terms of a sequence increase or decrease at a constant rate, the sequence is called an arithmetic sequence. The difference between successive terms of an arithmetic sequence is called the common difference. Any term of an arithmetic sequence can be found by adding the common difference to the preceding term. The formula for finding a specific term in an arithmetic sequence is $a_n = a_1 + (n - 1)d$, where a_n is the nth term, a_1 is the first term, and d is the common difference.

Explore Common Differences

Objective

Students use a sketch to explore the relationship between arithmetic sequences and linear functions.

Teaching the Mathematical Practices

4 Use Tools Point out that to solve the problem in this Explore, students will need to use the table and sketch.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

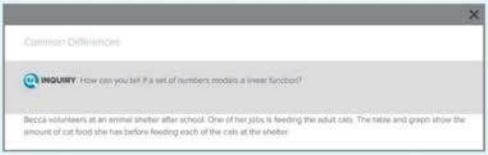
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to graph a linear function to solve a real-world problem. They will observe as the data points are plotted, and then answer questions related to the resulting graph. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

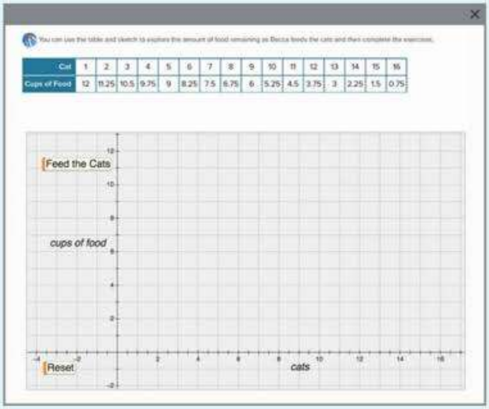


Common Differences

INQUIRY How can you tell if a set of numbers models a linear function?

Becca volunteers at an animal shelter after school. One of her jobs is feeding the adult cats. The table and graph show the amount of cat food she has before feeding each of the cats at the shelter.

Explore



You can use the table and sketch to explore the amount of food remaining as Becca feeds the cats and then complete the exercises.

Cat	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Cups of Food	12	11.25	10.5	9.75	9	8.25	7.5	6.75	6	5.25	4.5	3.75	3	2.25	1.5	0.75

Explore

WEB SKETCHPAD

Students use a sketch to explore the amount of food remaining as cats are fed.

TYPE

Students answer questions about the pattern in the data.

Interactive Presentation

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Common Differences (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is the amount of food decreasing with each cat? Sample answer: Each cat is being fed a certain amount of food, so there will be less after each cat is fed.
- How does the amount of food each cat is fed relate to the slope of the linear function that models the situation? Sample answer: The amount of food each cat is fed represents the change in the amount of food, which is the slope of the function. As long as there is a constant change, or constant slope, then you have a linear function.

Inquiry

How can you tell if a set of numbers models a linear function? Sample answer: The points are on the same line and have a constant slope.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Arithmetic Sequences

Objective

Students construct arithmetic sequences by using the common difference.

MP Teaching the Mathematical Practices

8 Look for a Pattern Help students see the pattern in this Learn.

Common Misconception

Students may think that the terms of all arithmetic sequences must increase. They may believe this because the definition of an arithmetic sequence refers to the use of addition to find successive terms. Point out that when the number being added is negative, the terms will decrease.

Example 1 Identify Arithmetic Sequences

MP Teaching the Mathematical Practices

3 Reason Inductively In this example, students will use inductive reasoning to make plausible arguments.

Questions for Mathematical Discourse

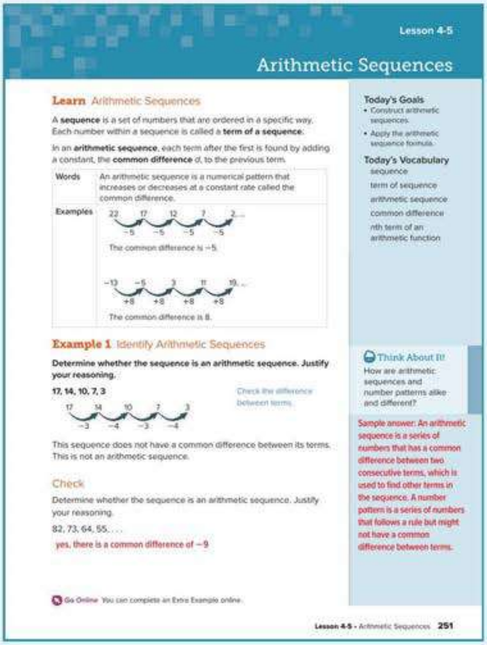
AL How are the terms of an arithmetic sequence found? The same number is added to each term to find the next term.

OL What requirement must be met for the sequence to represent an arithmetic sequence? The difference between the terms must be constant.

BL Does the sequence follow a pattern? Explain. Yes; sample answer: The difference in the numbers repeats itself, so the next difference would be −3.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-5

Arithmetic Sequences

Learn Arithmetic Sequences

A **sequence** is a set of numbers that are ordered in a specific way. Each number within a sequence is called a **term of a sequence**.

In an **arithmetic sequence**, each term after the first is found by adding a constant, the **common difference** d, to the previous term.

Words	An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
Examples	22, 17, 12, 7, 2, … −5, −5, −5, −5 The common difference is −5. −13, −5, 3, 11, 19, … +8, +8, +8, +8 The common difference is 8.

Today's Goals
- Construct arithmetic sequences.
- Apply the arithmetic sequence formula.

Today's Vocabulary
sequence
term of sequence
arithmetic sequence
common difference
nth term of an arithmetic function

Example 1 Identify Arithmetic Sequences

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

17, 14, 10, 7, 3

Check the difference between terms.

17, 14, 10, 7, 3
−3, −4, −3, −4

This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

Check

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

82, 73, 64, 55, . . .

yes, there is a common difference of −9

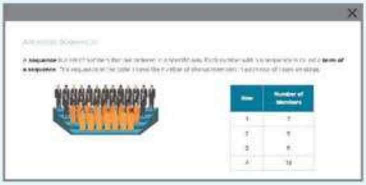
Think About It!
How are arithmetic sequences and number patterns alike and different?

Sample answer: An arithmetic sequence is a series of numbers that has a common difference between two consecutive terms, which is used to find other terms in the sequence. A number pattern is a series of numbers that follows a rule but might not have a common difference between terms.

Go Online You can complete an Extra Example online.

Lesson 4-5 • Arithmetic Sequences 251

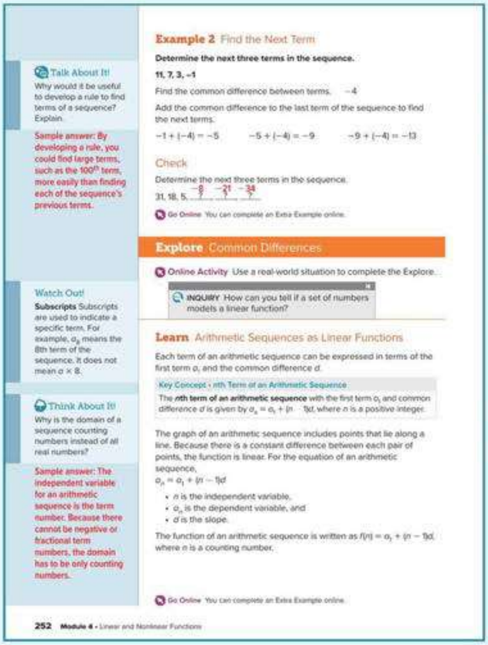
Interactive Presentation



Learn

TYPE

Students enter the number of chorus members in the 6th row.



Example 2 Find the Next Term

Determine the next three terms in the sequence.

11, 7, 3, −1

Find the common difference between terms. −4

Add the common difference to the last term of the sequence to find the next terms.

$-1 + (-4) = -5$ $-5 + (-4) = -9$ $-9 + (-4) = -13$

Check

Determine the next three terms in the sequence.

31, 18, 5, $\underset{?}{-8}$, $\underset{?}{-21}$, $\underset{?}{-34}$

Go Online You can complete an Extra Example online.

Talk About It!

Why would it be useful to develop a rule to find terms of a sequence? Explain.

Sample answer: By developing a rule, you could find large terms, such as the 100th term, more easily than finding each of the sequence's previous terms.

Explore Common Differences

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you tell if a set of numbers models a linear function?

Watch Out!

Subscripts Subscripts are used to indicate a specific term. For example, a_8 means the 8th term of the sequence. It does not mean $a \times 8$.

Learn Arithmetic Sequences as Linear Functions

Each term of an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Key Concept • nth Term of an Arithmetic Sequence

The **nth term of an arithmetic sequence** with the first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

The graph of an arithmetic sequence includes points that lie along a line. Because there is a constant difference between each pair of points, the function is linear. For the equation of an arithmetic sequence,

$a_n = a_1 + (n - 1)d$

- n is the independent variable,
- a_n is the dependent variable, and
- d is the slope.

The function of an arithmetic sequence is written as $f(n) = a_1 + (n - 1)d$, where n is a counting number.

Think About It!

Why is the domain of a sequence counting numbers instead of all real numbers?

Sample answer: The independent variable for an arithmetic sequence is the term number. Because there cannot be negative or fractional term numbers, the domain has to be only counting numbers.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Find the Next Term

Find the common difference between terms.

11 7 3 −1

common difference:

Example 2

TYPE

Students enter the consecutive members of the arithmetic sequences.

TYPE

Students explain how developing a rule or equation would be useful in finding terms of a sequence.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Find the Next Term

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress when determining the next terms in the sequence.

Questions for Mathematical Discourse

AL What is the relationship between the terms? Each term is 4 less than the previous term.

OL If this sequence continues on, will all of the subsequent terms be negative, or will they go back to being positive? Explain. Sample answer: They will stay negative because each term is less than the one before.

BL What is the tenth term in the arithmetic sequence? −25

Learn Arithmetic Sequences as Linear Functions

Objective

Students apply the arithmetic sequence formula by examining the common differences in arithmetic sequences.

Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

8 Look for a Pattern Help students to see the pattern in the formula for the nth term of an arithmetic sequence.

Important to Know

In the context of a function, the term numbers represent the input values, and the terms of the sequence represent the output values. The common difference is a constant that represents the slope. The function rule is linear, defining how each term is determined by its term number, n.

Common Misconception

Some students may think that the function rule contains more than one variable. Use several examples to show students that for any particular sequence, a_1 and d are known constants, and only n is variable.

Example 3 Find the *n*th Term

Teaching the Mathematical Practices

3 Reason Inductively In this example, students will use inductive reasoning to make plausible arguments.

Questions for Mathematical Discourse

AL What values do you need to know in order to write the equation? You need to know the first term, a_1, and the difference, d.

OL How are the values substituted to find the equation? Sample answer: $-4 + 3(n - 1) = -4 + 3n - 3 = 3n - 7$

BL Will n always be a positive number? Explain. Yes; sample answer: Since n refers to the number of the term, like the 1st term or the 15th term, it will always be a positive whole number.

DIFFERENTIATE

Reteaching Activity **AL** **ELL**

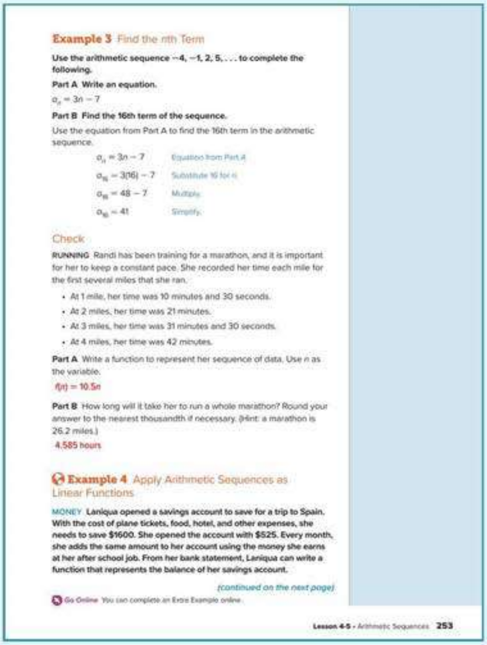
IF students have difficulty following the progression of steps that lead to the building of the equation,

THEN have them cycle through the steps again, using a simpler sequence, such as 1, 4, 7, 10, ...

DIFFERENTIATE

Enrichment Activity **BL**

Have students work with a partner. Tell them that you know of an arithmetic sequence in which the 4th term is 27 and the 8th term is 59. Ask them to find the first term and the common difference. Have pairs share how they solved the problem, and describe how they checked that their solution is correct. $a_1 = 3$, $d = 8$



Example 3 Find the *n*th Term

Use the arithmetic sequence −4, −1, 2, 5, . . . to complete the following.

Part A Write an equation.

$a_n = 3n - 7$

Part B Find the 16th term of the sequence.

Use the equation from Part A to find the 16th term in the arithmetic sequence.

$a_n = 3n - 7$ Equation from Part A

$a_{16} = 3(16) - 7$ Substitute 16 for n.

$a_{16} = 48 - 7$ Multiply.

$a_{16} = 41$ Simplify.

Check

RUNNING Randi has been training for a marathon, and it is important for her to keep a constant pace. She recorded her time each mile for the first several miles that she ran.

- At 1 mile, her time was 10 minutes and 30 seconds.
- At 2 miles, her time was 21 minutes.
- At 3 miles, her time was 31 minutes and 30 seconds.
- At 4 miles, her time was 42 minutes.

Part A Write a function to represent her sequence of data. Use n as the variable.

$f(n) = 10.5n$

Part B How long will it take her to run a whole marathon? Round your answer to the nearest thousandth if necessary. (Hint: a marathon is 26.2 miles.)

4.585 hours

Example 4 Apply Arithmetic Sequences as Linear Functions

MONEY **Laniqua opened a savings account to save for a trip to Spain. With the cost of plane tickets, food, hotel, and other expenses, she needs to save $1600. She opened the account with $525. Every month, she adds the same amount to her account using the money she earns at her after school job. From her bank statement, Laniqua can write a function that represents the balance of her savings account.**

(continued on the next page)

Go Online You can complete an Extra Example online.

Lesson 4-5 • Arithmetic Sequences 253

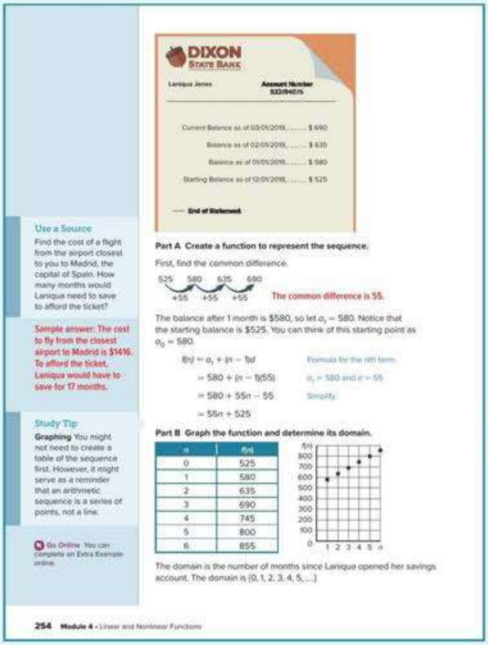
Interactive Presentation

Find the *n*th Term

Example 3

EXPAND

Students can tap to see the steps to writing and using an equation for arithmetic sequences.



DIXON STATE BANK

Laniqua Jones

Account Number 532194075

Current Balance as of 03/01/2019........ \$ 690
Balance as of 02/01/2019........ \$ 635
Balance as of 01/01/2019........ \$ 580
Starting Balance as of 12/01/2018........ \$ 525

— **End of Statement**

Use a Source
Find the cost of a flight from the airport closest to you to Madrid, the capital of Spain. How many months would Laniqua need to save to afford the ticket?

Sample answer: The cost to fly from the closest airport to Madrid is \$1416. To afford the ticket, Laniqua would have to save for 17 months.

Part A Create a function to represent the sequence.

First, find the common difference.

$$\underset{+55}{525 \curvearrowright 580} \quad \underset{+55}{580 \curvearrowright 635} \quad \underset{+55}{635 \curvearrowright 690}$$

The common difference is 55.

The balance after 1 month is \$580, so let $a_1 = 580$. Notice that the starting balance is \$525. You can think of this starting point as $a_0 = 580$.

$$\begin{aligned} f(n) &= a_1 + (n - 1)d && \text{Formula for the nth term.} \\ &= 580 + (n - 1)(55) && a_1 = 580 \text{ and } d = 55 \\ &= 580 + 55n - 55 && \text{Simplify.} \\ &= 55n + 525 \end{aligned}$$

Part B Graph the function and determine its domain.

n	$f(n)$
0	525
1	580
2	635
3	690
4	745
5	800
6	855

The domain is the number of months since Laniqua opened her savings account. The domain is {0, 1, 2, 3, 4, 5, ...}

Study Tip
Graphing You might not need to create a table of the sequence first. However, it might serve as a reminder that an arithmetic sequence is a series of points, not a line.

Go Online You can complete an Extra Example online.

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Interactive Presentation

MONEY Watch this animation to see how much money Laniqua saved for a trip to Spain

Example 4

WATCH

Students can watch an animation that describes the balance of an account over time.

TYPE

Students research costs of flights to Spain and determine the number of months they would need to save.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Apply Arithmetic Sequences as Linear Functions

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL Why does the list of balances represent an arithmetic sequence? because there is a common difference between the balances

OL What does the common difference mean in the context of the problem? Laniqua is saving \$55 each month, so her account is increasing by \$55 per month.

BL Is the function discrete or continuous? Explain. Discrete; sample answer: The domain is the counting numbers, so the graph would consist of points, not a line.

DIFFERENTIATE

Enrichment Activity BL

Arithmetic sequences can be programmed into graphing calculators with results displayed in lists. Have advanced learners locate a set of directions for programming a sequence and develop a lesson for their classmates on analyzing sequences using the calculator.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

F.BF.1a, F.BF.2, F.LE.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–34
2	exercises that extend concepts learned in this lesson to new contexts	35–37
3	exercises that emphasize higher-order and critical-thinking skills	38–46

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–37 odd, 38–46
- Extension: Arithmetic Series
- **ALEKS** Arithmetic Sequences

OL

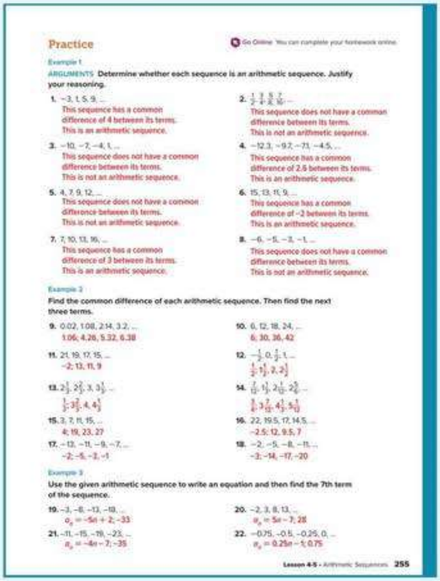
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–45 odd
- Remediation, Review Resources: Add Integers
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** Addition and Subtraction with Integers

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Add Integers
- *Quick Review Math Handbook*: Arithmetic Sequences as Linear Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Addition and Subtraction with Integers



Practice

Go Online You can complete your homework online.

Example 1

ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your reasoning.**

1. $-3, 1, 5, 9, \ldots$
This sequence has a common difference of 4 between its terms. This is an arithmetic sequence.

2. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

3. $-10, -7, -4, 1, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

4. $-12.3, -9.7, -7.1, -4.5, \ldots$
This sequence has a common difference of 2.6 between its terms. This is an arithmetic sequence.

5. $4, 7, 9, 12, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

6. $15, 13, 11, 9, \ldots$
This sequence has a common difference of -2 between its terms. This is an arithmetic sequence.

7. $7, 10, 13, 16, \ldots$
This sequence has a common difference of 3 between its terms. This is an arithmetic sequence.

8. $-6, -5, -3, -1, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

Example 2

Find the common difference of each arithmetic sequence. Then find the next three terms.

9. $0.02, 1.08, 2.14, 3.2, \ldots$
$1.06; 4.26, 5.32, 6.38$

10. $6, 12, 18, 24, \ldots$
$6; 30, 36, 42$

11. $21, 19, 17, 15, \ldots$
$-2; 13, 11, 9$

12. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots$
$\frac{1}{2}; 1\frac{1}{2}, 2, 2\frac{1}{2}$

13. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$
$\frac{1}{3}; 3\frac{2}{3}, 4, 4\frac{1}{3}$

14. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$
$\frac{3}{4}; 3\frac{7}{12}, 4\frac{1}{3}, 5\frac{1}{12}$

15. $3, 7, 11, 15, \ldots$
$4; 19, 23, 27$

16. $22, 19.5, 17, 14.5, \ldots$
$-2.5; 12, 9.5, 7$

17. $-13, -11, -9, -7, \ldots$
$2; -5, -3, -1$

18. $-2, -5, -8, -11, \ldots$
$-3; -14, -17, -20$

Example 3

Use the given arithmetic sequence to write an equation and then find the 7th term of the sequence.

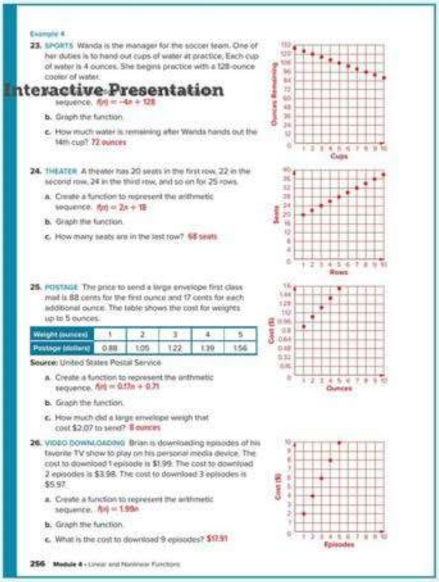
19. $-3, -8, -13, -18, \ldots$
$a_n = -5n + 2; -33$

20. $-2, 3, 8, 13, \ldots$
$a_n = 5n - 7; 28$

21. $-11, -15, -19, -23, \ldots$
$a_n = -4n - 7; -35$

22. $-0.75, -0.5, -0.25, 0, \ldots$
$a_n = 0.25n - 1; 0.75$

Lesson 4-5 • Arithmetic Sequences 255



Example 4

23. SPORTS Wanda is the manager for the soccer team. One of her duties is to hand out cups of water at practice. Each cup of water is 4 ounces. She begins practice with a 128-ounce cooler of water.

Interactive Presentation

sequence. $f(n) = -4n + 128$

b. Graph the function.

c. How much water is remaining after Wanda hands out the 14th cup? 72 ounces

24. THEATER A theater has 20 seats in the first row, 22 in the second row, 24 in the third row, and so on for 25 rows.

a. Create a function to represent the arithmetic sequence. $f(n) = 2n + 18$

b. Graph the function.

c. How many seats are in the last row? 68 seats

25. POSTAGE The price to send a large envelope first class mail is 88 cents for the first ounce and 17 cents for each additional ounce. The table shows the cost for weights up to 5 ounces.

Weight (ounces)	1	2	3	4	5
Postage (dollars)	0.88	1.05	1.22	1.39	1.56

Source: United States Postal Service

a. Create a function to represent the arithmetic sequence. $f(n) = 0.17n + 0.71$

b. Graph the function.

c. How much did a large envelope weigh that cost \$2.07 to send? 8 ounces

26. VIDEO DOWNLOADING Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is \$1.99. The cost to download 2 episodes is \$3.98. The cost to download 3 episodes is \$5.97.

a. Create a function to represent the arithmetic sequence. $f(n) = 1.99n$

b. Graph the function.

c. What is the cost to download 9 episodes? \$17.91

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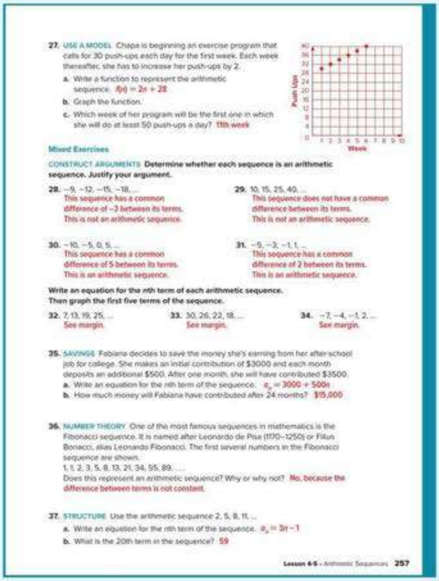


27. USE A MODEL Chapa is beginning an exercise program that calls for 30 push-ups each day for the first week. Each week thereafter, she has to increase her push-ups by 2.
    a. Write a function to represent the arithmetic sequence. $f(n) = 2n + 28$
    b. Graph the function.
    c. Which week of her program will be the first one in which she will do at least 50 push-ups a day? 11th week

Mixed Exercises

CONSTRUCT ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your argument.**

28. −9, −12, −15, −18, ...
This sequence has a common difference of −3 between its terms. This is not an arithmetic sequence.

29. 10, 15, 25, 40, ...
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

30. −10, −5, 0, 5, ...
This sequence has a common difference of 5 between its terms. This is an arithmetic sequence.

31. −5, −3, −1, 1, ...
This sequence has a common difference of 2 between its terms. This is an arithmetic sequence.

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

32. 7, 13, 19, 25, ... See margin.

33. 30, 26, 22, 18, ... See margin.

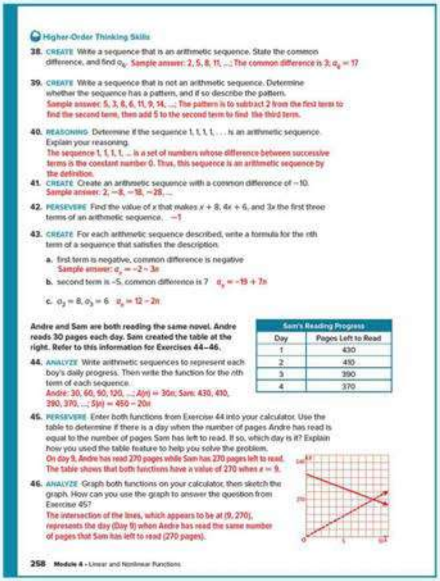
34. −7, −4, −1, 2, ... See margin.

35. SAVINGS Fabiana decides to save the money she's earning from her after-school job for college. She makes an initial contribution of \$3000 and each month deposits an additional \$500. After one month, she will have contributed \$3500.
    a. Write an equation for the *n*th term of the sequence. $a_n = 3000 + 500n$
    b. How much money will Fabiana have contributed after 24 months? \$15,000

36. NUMBER THEORY One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170–1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are shown.
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...
Does this represent an arithmetic sequence? Why or why not? No, because the difference between terms is not constant.

37. STRUCTURE Use the arithmetic sequence 2, 5, 8, 11, ...
    a. Write an equation for the *n*th term of the sequence. $a_n = 3n - 1$
    b. What is the 20th term in the sequence? 59

Lesson 4-5 • Arithmetic Sequences 257



Higher-Order Thinking Skills

38. CREATE Write a sequence that is an arithmetic sequence. State the common difference, and find a_6. Sample answer: 2, 5, 8, 11, ...; The common difference is 3; $a_6 = 17$

39. CREATE Write a sequence that is not an arithmetic sequence. Determine whether the sequence has a pattern, and if so describe the pattern.
Sample answer: 5, 3, 8, 6, 11, 9, 14, ...; The pattern is to subtract 2 from the first term to find the second term, then add 5 to the second term to find the third term.

40. REASONING Determine if the sequence 1, 1, 1, 1, . . . is an arithmetic sequence. Explain your reasoning.
The sequence 1, 1, 1, 1, ... is a set of numbers whose difference between successive terms is the constant number 0. Thus, this sequence is an arithmetic sequence by the definition.

41. CREATE Create an arithmetic sequence with a common difference of −10.
Sample answer: 2, −8, −18, −28, ...

42. PERSEVERE Find the value of *x* that makes $x + 8$, $4x + 6$, and $3x$ the first three terms of an arithmetic sequence. −1

43. CREATE For each arithmetic sequence described, write a formula for the *n*th term of a sequence that satisfies the description.
    a. first term is negative, common difference is negative
    Sample answer: $a_n = -2 - 3n$
    b. second term is −5, common difference is 7 $a_n = -19 + 7n$
    c. $a_2 = 8$, $a_3 = 6$ $a_n = 12 - 2n$

Andre and Sam are both reading the same novel. Andre reads 30 pages each day. Sam created the table at the right. Refer to this information for Exercises 44–46.

Sam's Reading Progress

Day	Pages Left to Read
1	430
2	410
3	390
4	370

44. ANALYZE Write arithmetic sequences to represent each boy's daily progress. Then write the function for the *n*th term of each sequence.
Andre: 30, 60, 90, 120, ...; $A(n) = 30n$; Sam: 430, 410, 390, 370, ...; $S(n) = 450 - 20n$

45. PERSEVERE Enter both functions from Exercise 44 into your calculator. Use the table to determine if there is a day when the number of pages Andre has read is equal to the number of pages Sam has left to read. If so, which day is it? Explain how you used the table feature to help you solve the problem.
On day 9, Andre has read 270 pages while Sam has 270 pages left to read. The table shows that both functions have a value of 270 when $x = 9$.

46. ANALYZE Graph both functions on your calculator, then sketch the graph. How can you use the graph to answer the question from Exercise 45?
The intersection of the lines, which appears to be at (9, 270), represents the day (Day 9) when Andre has read the same number of pages that Sam has left to read (270 pages).

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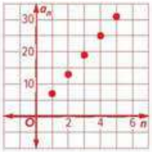
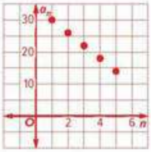
Answers

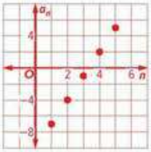
32. $a_n = 6n + 1$;



33. $a_n = -4n + 34$;



34. $a_n = 3n - 10$;



Lesson 4-6

Piecewise and Step Functions

F.IF.4, F.IF.7b

LESSON GOAL

Students graph piecewise-defined and step functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Graphing Piecewise-Defined Functions

- Graph a Piecewise-Defined Function

Explore: Age as a Function

Develop:

Graphing Step Functions

- Graph a Greatest Integer Function
- Graph a Step Function

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Construct Linear Functions	●	●		●
Extension: Taxicab Graphs		●	●	●

Language Development Handbook

Assign page 25 of the *Language Development Handbook* to help your students build mathematical language related to piecewise-defined and step functions.

ELL You can use the tips and suggestions on page T25 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Standards for Mathematical Practice:

4 Model with mathematics.

6 Attend to precision.

Coherence

Vertical Alignment

Previous

Students understood and graphed linear functions.

8.F.3, 8.F.4, F.IF.7a

Now

Students graph piecewise-defined and step functions.

F.IF.4, F.IF.7b

Next

Students will identify the effects of transformations of the graphs of absolute value functions.

F.IF.7b, F.BF.3

Rigor

The Three Pillars of Rigor

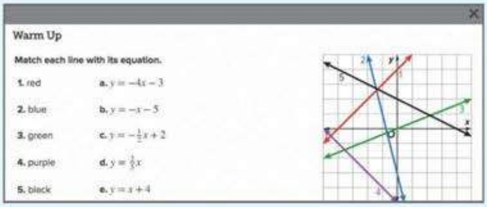
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of linear functions to piecewise-defined and step functions. They build fluency by graphing both types of functions, and they apply their understanding by solving real-world problems related to piecewise-defined and step functions.

Mathematical Background

Piecewise-defined functions are functions that are defined by two or more functions, each with its own domain. The graph consists of the graph of each piece over its domain. A step function is a function whose graph consists of segments that look like a set of steps. The graph of the greatest integer function is an example of a step function.

Interactive Presentation

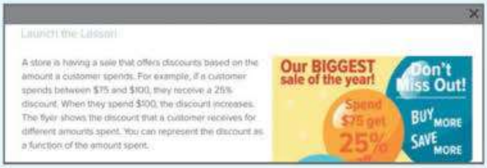


Warm Up

Match each line with its equation.

1. red a. $y = -4x - 3$
2. blue b. $y = -x - 5$
3. green c. $y = -\frac{1}{2}x + 2$
4. purple d. $y = \frac{1}{3}x$
5. black e. $y = x + 4$

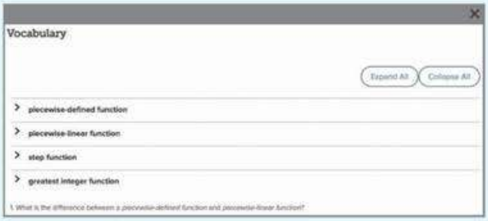
Warm Up



Launch the Lesson

A store is having a sale that offers discounts based on the amount a customer spends. For example, if a customer spends between \$75 and \$100, they receive a 25% discount. When they spend \$100, the discount increases. The flyer shows the discount that a customer receives for different amounts spent. You can represent the discount as a function of the amount spent.

Launch the Lesson



Vocabulary

- piecewise-defined function
- piecewise-linear function
- step function
- greatest integer function

1. What is the difference between a piecewise-defined function and piecewise-linear function?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- graphing linear functions

Answers:

1. e
2. a
3. d
4. b
5. c

Launch the Lesson

Teaching the Mathematical Practices

4 Apply Mathematics In this Launch, students learn how to apply what they have learned about special functions to a real-world situation about the discounts offered at a store.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Age as a Function

Objective

Students collect data to explore how real-world data can be represented by a step function.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students complete guiding exercises throughout the Explore activity. Students will explore how data in a real-world scenario involving age groups can be modeled by a step function. They will use their own age to create a table that shows the group in which they would be placed after various periods of time and answer questions regarding the data in their table. They will then explore how the graph of a step function represents this type of data. Then, students answer the Inquiry Question.

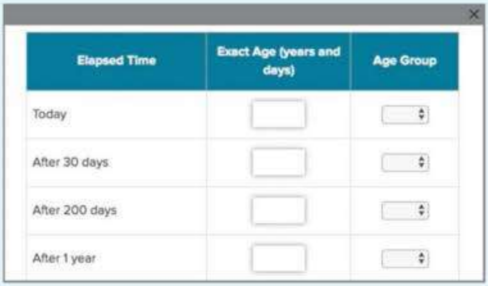
(continued on the next page)

Interactive Presentation

Age as a Function

INQUIRY When can real-world data be described using a step function?

Explore



Elapsed Time	Exact Age (years and days)	Age Group
Today		
After 30 days		
After 200 days		
After 1 year		

Explore

TYPE

Students answer the questions and complete a table based on age.

Interactive Presentation

INQUIRY When can real-world data be described using a step function?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Age as a Function (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- In which age group would you place someone who will be 13 next week? Why? 11-12; Sample answer: According to the rules, the person would be in the 11-12 group because he or she is still 12.
- What other situations could be modeled by a step function? Sample answer: Movie ticket prices that depend on age could be modeled by a step function.

Inquiry

When can real-world data be described using a step function? Sample answer: When domain values in intervals have the same range value, real-world data can be described using a step function.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphing Piecewise-Defined Functions

Objective

Students graph piecewise-defined functions and identify their domain and range by determining the intervals where each part of the function should be graphed.

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between equations and graphs of piecewise-defined and piecewise-linear functions.

Example 1 Graph a Piecewise-Defined Function

MP Teaching the Mathematical Practices

2 Different Properties Mathematically proficient students looks for different ways to solve problem. Encourage them to consider an alternate method in the Think About It! feature.

Questions for Mathematical Discourse

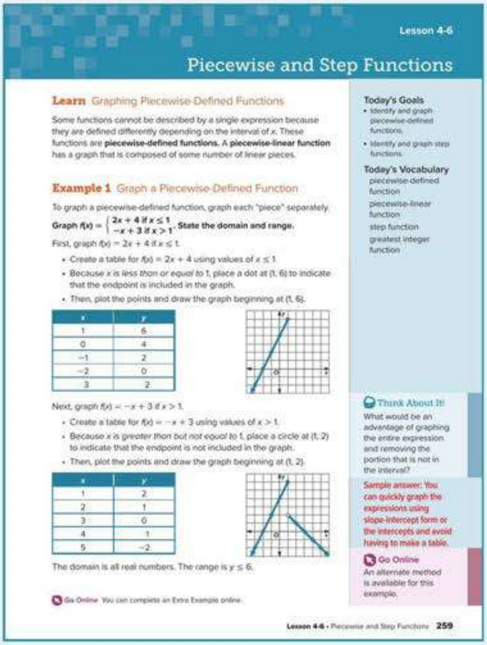
AL Why do you think this is called a piecewise-defined function? Sample answer: The function has different rules for different "pieces" of the graph.

OL Why is (1, 6) included in the graph, but (1, 2) is not? Sample answer: The first domain includes 1 because it states that $x \leq 1$ while the second domain does not include 1. So the y-value that corresponds with $x = 1$ is $2(1) + 4$, or 6.

BL Why is the range not the set of real numbers? There are no values of x that are paired with numbers greater than 6.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-6

Piecewise and Step Functions

Learn Graphing Piecewise-Defined Functions

Some functions cannot be described by a single expression because they are defined differently depending on the interval of x. These functions are **piecewise-defined functions.** A **piecewise-linear function** has a graph that is composed of some number of linear pieces.

Example 1 Graph a Piecewise-Defined Function

To graph a piecewise-defined function, graph each "piece" separately.

Graph $f(x) = \begin{cases} 2x + 4 \text{ if } x \leq 1 \\ -x + 3 \text{ if } x > 1 \end{cases}$. State the domain and range.

First, graph $f(x) = 2x + 4$ if $x \leq 1$.

- Create a table for $f(x) = 2x + 4$ using values of $x \leq 1$.
- Because x is *less than or equal to* 1, place a dot at (1, 6) to indicate that the endpoint is included in the graph.
- Then, plot the points and draw the graph beginning at (1, 6).

x	y
1	6
0	4
−1	2
−2	0
3	2

Next, graph $f(x) = -x + 3$ if $x > 1$.

- Create a table for $f(x) = -x + 3$ using values of $x > 1$.
- Because x is *greater than but not equal to* 1, place a circle at (1, 2) to indicate that the endpoint is not included in the graph.
- Then, plot the points and draw the graph beginning at (1, 2).

x	y
1	2
2	1
3	0
4	1
5	−2

The domain is all real numbers. The range is $y \leq 6$.

Go Online You can complete an Extra Example online.

Today's Goals
- Identify and graph piecewise-defined functions.
- Identify and graph step functions.

Today's Vocabulary
piecewise-defined function
piecewise-linear function
step function
greatest integer function

Think About It!
What would be an advantage of graphing the entire expression and removing the portion that is not in the interval?

Sample answer: You can quickly graph the expressions using slope-intercept form or the intercepts and avoid having to make a table.

Go Online
An alternate method is available for this example.

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Interactive Presentation

Graph $f(x) = \begin{cases} 2x + 4 & \text{if } x \leq 1 \\ -x + 3 & \text{if } x > 1 \end{cases}$ State the domain and range.

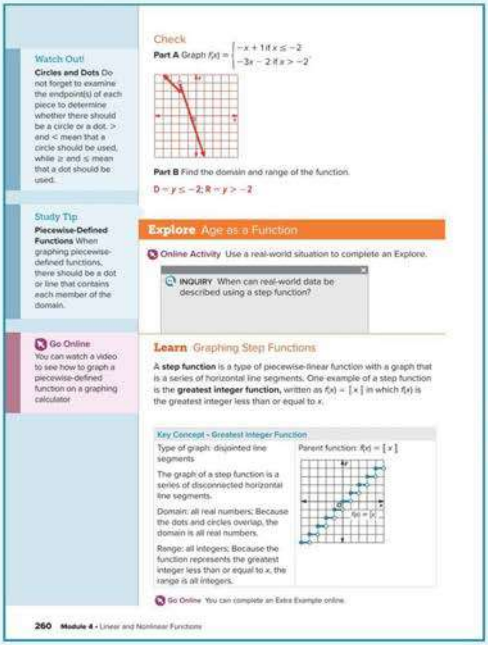
Example 1

TAP

Students select a method for graphing a piecewise-defined function.

TYPE

Students explain the advantage of graphing the entire expression and then removing the portion not in the interval.



Watch Out!

Circles and Dots Do not forget to examine the endpoint(s) of each piece to determine whether there should be a circle or a dot. > and < mean that a circle should be used, while ≥ and ≤ mean that a dot should be used.

Study Tip

Piecewise-Defined Functions When graphing piecewise-defined functions, there should be a dot or line that contains each member of the domain.

Go Online You can watch a video to see how to graph a piecewise-defined function on a graphing calculator.

Check

Part A Graph $f(x) = \begin{cases} -x + 1 \text{ if } x \leq -2 \\ -3x - 2 \text{ if } x > -2 \end{cases}$

Part B Find the domain and range of the function.

D = $y \leq -2$; R = $y > -2$

Explore Age as a Function

Online Activity Use a real-world situation to complete an Explore.

INQUIRY When can real-world data be described using a step function?

Learn Graphing Step Functions

A **step function** is a type of piecewise-linear function with a graph that is a series of horizontal line segments. One example of a step function is the **greatest integer function,** written as $f(x) = [\![x]\!]$ in which $f(x)$ is the greatest integer less than or equal to x.

Key Concept - Greatest Integer Function

Type of graph: disjointed line segments

The graph of a step function is a series of disconnected horizontal line segments.

Domain: all real numbers; Because the dots and circles overlap, the domain is all real numbers.

Range: all integers; Because the function represents the greatest integer less than or equal to x, the range is all integers.

Parent function: $f(x) = [\![x]\!]$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Learn

TAP

Students tap on each marker to learn about graphing step functions.

TYPE

Students explain the relationship between the x- and y-values of each horizontal line segment.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Learn Graphing Step Functions

Objective

Students graph step functions by making a table of values.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the x- and y-values of each horizontal line segment used in this Learn.

About the Key Concept

The graph of a greatest integer function always consists of infinitely many “steps,” each with one closed endpoint and one open endpoint. The parameters of the function determine the length of the steps. The greatest integer function is a type of piecewise-defined linear function, as the function is equal to a different constant for different intervals in the domain.

Common Misconception

Some students may think that the steps on the graph of a greatest integer function are always 1 unit long. Explain that while this is true of the graph of the parent greatest integer function, other greatest integer functions will contain parameters that may affect the length of each step.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students have difficulty understanding the nature of the graph of the greatest integer function,

THEN have them create a table of values for the function. Instruct them to include decimals and fractions in their tables. Then have them describe how they determined the y-values for the x-values that they chose.

Example 2 Graph a Greatest Integer Function

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Why do you think this is called a step function? The graph looks like steps on a staircase.

OL Why is (0, 1) included in the graph but (1, 1) is not? $\llbracket 0 + 1 \rrbracket = 1$ and $\llbracket 1 + 1 \rrbracket = 2$

BL The greatest integer function is sometimes called the floor function. Why do you think that is? Sample answer: The value truncates to the integer portion of the value, like standing on a chair on the 2nd floor still means you are on the 2nd floor.

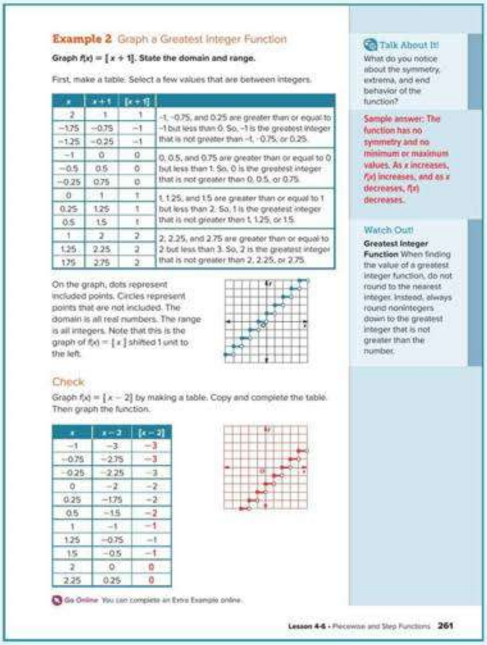
Common Error

Some students may take the greatest integer of the *x*-value before adding 1. Explain that the greatest integer symbols act as grouping symbols, requiring that the operation inside the symbols be performed first, before finding the greatest integer of the resulting value.

DIFFERENTIATE

Enrichment Activity **BL**

Have students work with a partner. Ask them to create a story problem that can be modeled using the function $f(x) = 15\llbracket x \rrbracket$. Have students construct a graph for the model, and share their problems with the class.



Example 2 Graph a Greatest Integer Function

Graph $f(x) = \llbracket x + 1 \rrbracket$. State the domain and range.

First, make a table. Select a few values that are between integers.

x	$x + 1$	$\llbracket x + 1 \rrbracket$	
2	1	1	−1, −0.75, and 0.25 are greater than or equal to −1 but less than 0. So, −1 is the greatest integer that is not greater than −1, −0.75, or 0.25.
−1.75	−0.75	−1	
−1.25	−0.25	−1	
−1	0	0	0, 0.5, and 0.75 are greater than or equal to 0 but less than 1. So, 0 is the greatest integer that is not greater than 0, 0.5, or 0.75.
−0.5	0.5	0	
−0.25	0.75	0	
0	1	1	1, 1.25, and 1.5 are greater than or equal to 1 but less than 2. So, 1 is the greatest integer that is not greater than 1, 1.25, or 1.5.
0.25	1.25	1	
0.5	1.5	1	
1	2	2	2, 2.25, and 2.75 are greater than or equal to 2 but less than 3. So, 2 is the greatest integer that is not greater than 2, 2.25, or 2.75.
1.25	2.25	2	
1.75	2.75	2	

On the graph, dots represent included points. Circles represent points that are not included. The domain is all real numbers. The range is all integers. Note that this is the graph of $f(x) = \llbracket x \rrbracket$ shifted 1 unit to the left.

Check

Graph $f(x) = \llbracket x - 2 \rrbracket$ by making a table. Copy and complete the table. Then graph the function.

x	$x - 2$	$\llbracket x - 2 \rrbracket$
−1	−3	−3
−0.75	−2.75	−3
−0.25	−2.25	−3
0	−2	−2
0.25	−1.75	−2
0.5	−1.5	−2
1	−1	−1
1.25	−0.75	−1
1.5	−0.5	−1
2	0	0
2.25	0.25	0

Go Online You can complete an Extra Example online.

Talk About It!

What do you notice about the symmetry, extrema, and end behavior of the function?

Sample answer: The function has no symmetry and no minimum or maximum values. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ decreases.

Watch Out!

Greatest Integer Function When finding the value of a greatest integer function, do not round to the nearest integer. Instead, always round nonintegers down to the greatest integer that is not greater than the number.

Lesson 4-6 • Piecewise and Step Functions 261

Interactive Presentation

Graph a Greatest Integer Function

Graph $f(x) = \llbracket x + 1 \rrbracket$. State the domain and range.

Move through the slides to learn more about the table and graph of the function.

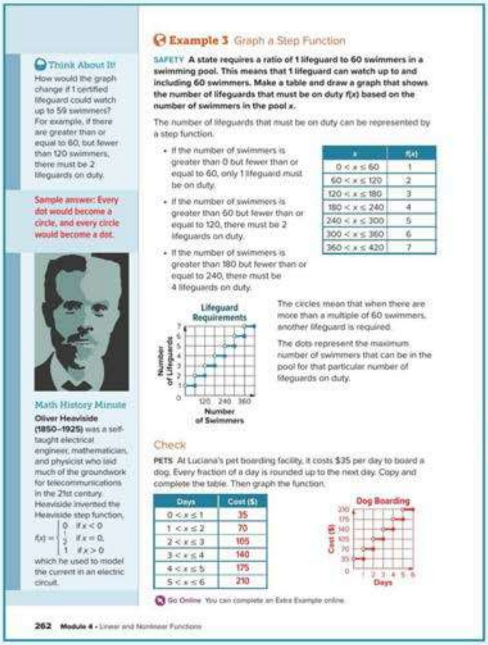
Example 2

TAP

Students move through the slides to learn more about the table and graph of a greatest integer function.

TYPE

Students describe what they noticed about the symmetry, extrema, and end behavior of the function.



Think About It!
How would the graph change if 1 certified lifeguard could watch up to 59 swimmers? For example, if there are greater than or equal to 60, but fewer than 120 swimmers, there must be 2 lifeguards on duty.

Sample answer: Every dot would become a circle, and every circle would become a dot.

Math History Minute
Oliver Heaviside (1850–1925) was a self-taught electrical engineer, mathematician, and physicist who laid much of the groundwork for telecommunications in the 21st century. Heaviside invented the Heaviside step function,

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } x = 0, \\ 1 & \text{if } x > 0 \end{cases}$$

which he used to model the current in an electric circuit.

Example 3 Graph a Step Function

SAFETY **A state requires a ratio of 1 lifeguard to 60 swimmers in a swimming pool. This means that 1 lifeguard can watch up to and including 60 swimmers. Make a table and draw a graph that shows the number of lifeguards that must be on duty $f(x)$ based on the number of swimmers in the pool x.**

The number of lifeguards that must be on duty can be represented by a step function.

- If the number of swimmers is greater than 0 but fewer than or equal to 60, only 1 lifeguard must be on duty.
- If the number of swimmers is greater than 60 but fewer than or equal to 120, there must be 2 lifeguards on duty.
- If the number of swimmers is greater than 180 but fewer than or equal to 240, there must be 4 lifeguards on duty.

x	$f(x)$
$0 < x \le 60$	1
$60 < x \le 120$	2
$120 < x \le 180$	3
$180 < x \le 240$	4
$240 < x \le 300$	5
$300 < x \le 360$	6
$360 < x \le 420$	7

The circles mean that when there are more than a multiple of 60 swimmers, another lifeguard is required.

The dots represent the maximum number of swimmers that can be in the pool for that particular number of lifeguards on duty.

Check

PETS At Luciana's pet boarding facility, it costs $35 per day to board a dog. Every fraction of a day is rounded up to the next day. Copy and complete the table. Then graph the function.

Days	Cost ($)
$0 < x \le 1$	35
$1 < x \le 2$	70
$2 < x \le 3$	105
$3 < x \le 4$	140
$4 < x \le 5$	175
$5 < x \le 6$	210

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3

DRAG & DROP

Students drag the correct values to complete the table.

TAP

Students tap on each marker to see the difference between the circles and dots.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 3 Graph a Step Function

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about step functions to solving a real-world problem.

Questions for Mathematical Discourse

AL How many lifeguards are needed for 59 swimmers? 1 For 61 swimmers? 2

OL Why is this situation represented by a step function? Sample answer: Every x-value in each interval of 60 is paired with the same y-value, forming a graph that consists of steps.

BL How would the graph of the function change if the number of lifeguards required for the number of swimmers is cut in half? Sample answer: The graph would be stretched horizontally because more swimmers could be watched by each lifeguard.

Essential Question Follow-Up

Students have analyzed and graphed step functions.

Ask:

If you know that a function is a step function, what do you know about how the elements of the domain are paired with the elements of the range? Sample answer: The domain is grouped into intervals, and every number in the interval is paired with the same number in the range.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–14
2	exercises that use a variety of skills from this lesson	15–20
2	exercises that extend concepts learned in this lesson to new contexts	21–22
3	exercises that emphasize higher-order and critical-thinking skills	23–31

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–21 odd, 23–31
- Extension: Taxicab Graphs

IF students score 66%–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–31 odd
- Remediation, Review Resources: Construct Linear Functions
- Personal Tutors
- Extra Examples 1–3
- **ALEKS** Tables and Graphs of Lines

IF students score 65% or less on the Checks, **THEN** assign: **AL**

- Practice, Exercises 1–13 odd
- Remediation, Review Resources: Construct Linear Functions
- *Quick Review Math Handbook*: Special Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Tables and Graphs of Lines

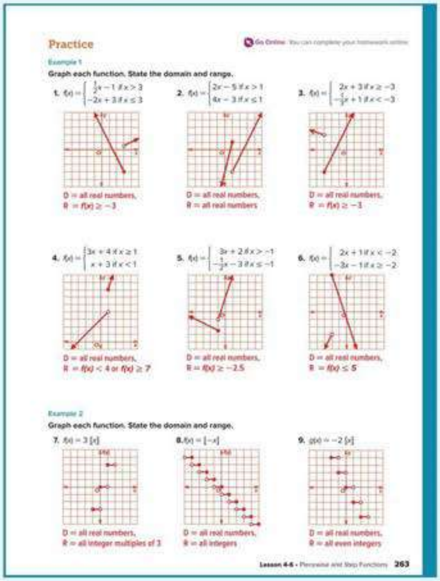
Answers

15. $$f(x) = \begin{cases} 16.20 \text{ if } 0 < x \leq 1 \\ 19.30 \text{ if } 1 < x \leq 2 \\ 22.40 \text{ if } 2 < x \leq 3 \\ 25.50 \text{ if } 3 < x \leq 4 \\ 28.60 \text{ if } 4 < x \leq 5 \end{cases}$$

16a.

x	0	4	8	12	16
$f(x)$	0	32	64	110	156

16b. $$f(x) = \begin{cases} 8x & \text{if } x \leq 8 \\ 64 + 11.5(x - 8) & \text{if } x > 8 \end{cases}$$



Practice

Go Online You can complete your homework online.

Example 1

Graph each function. State the domain and range.

1. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \leq 3 \end{cases}$

D = all real numbers, R = $f(x) \geq -3$

2. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \leq 1 \end{cases}$

D = all real numbers, R = all real numbers

3. $f(x) = \begin{cases} 2x + 3 \text{ if } x \geq -3 \\ -\frac{1}{2}x + 1 \text{ if } x < -3 \end{cases}$

D = all real numbers, R = $f(x) \geq -3$

4. $f(x) = \begin{cases} 3x + 4 \text{ if } x \geq 1 \\ x + 3 \text{ if } x < 1 \end{cases}$

D = all real numbers, R = $f(x) < 4$ or $f(x) \geq 7$

5. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \leq -1 \end{cases}$

D = all real numbers, R = $f(x) \geq -2.5$

6. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \geq -2 \end{cases}$

D = all real numbers, R = $f(x) \leq 5$

Example 2

Graph each function. State the domain and range.

7. $f(x) = 3[\![x]\!]$

D = all real numbers, R = all integer multiples of 3

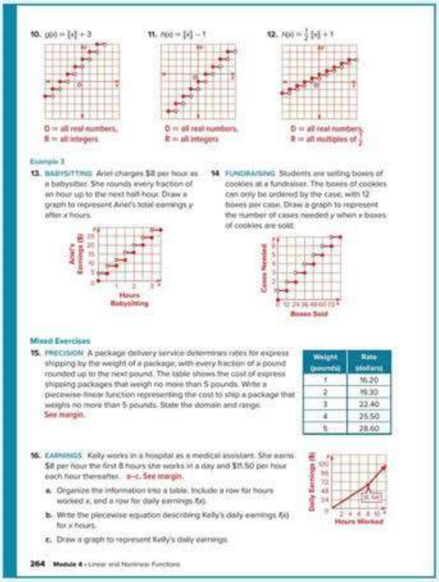
8. $f(x) = [\![-x]\!]$

D = all real numbers, R = all integers

9. $g(x) = -2[\![x]\!]$

D = all real numbers, R = all even integers

Lesson 4-6 • Piecewise and Step Functions 263



10. $g(x) = [\![x]\!] + 3$

D = all real numbers, R = all integers

11. $h(x) = [\![x]\!] - 1$

D = all real numbers, R = all integers

12. $h(x) = \frac{1}{2}[\![x]\!] + 1$

D = all real numbers, R = all multiples of $\frac{1}{2}$

Example 3

13. BABYSITTING Ariel charges $8 per hour as a babysitter. She rounds every fraction of an hour up to the next half-hour. Draw a graph to represent Ariel's total earnings y after x hours.

14. FUNDRAISING Students are selling boxes of cookies at a fundraiser. The boxes of cookies can only be ordered by the case, with 12 boxes per case. Draw a graph to represent the number of cases needed y when x boxes of cookies are sold.

Mixed Exercises

15. PRECISION A package delivery service determines rates for express shipping by the weight of a package, with every fraction of a pound rounded up to the next pound. The table shows the cost of express shipping packages that weigh no more than 5 pounds. Write a piecewise-linear function representing the cost to ship a package that weighs no more than 5 pounds. State the domain and range. See margin.

Weight (pounds)	Rate (dollars)
1	16.20
2	19.30
3	22.40
4	25.50
5	28.60

16. EARNINGS Kelly works in a hospital as a medical assistant. She earns $8 per hour the first 8 hours she works in a day and $11.50 per hour each hour thereafter. a–c. See margin.

a. Organize the information into a table. Include a row for hours worked x, and a row for daily earnings $f(x)$.

b. Write the piecewise equation describing Kelly's daily earnings $f(x)$ for x hours.

c. Draw a graph to represent Kelly's daily earnings.

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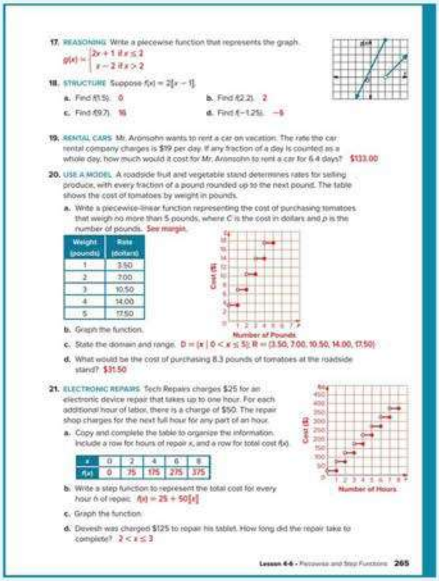
3 REFLECT AND PRACTICE

F.IF.4, F.IF.7b

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



17. REASONING Write a piecewise function that represents the graph.
$g(x) = \begin{cases} 2x + 1 \text{ if } x \leq 2 \\ x - 2 \text{ if } x > 2 \end{cases}$

18. STRUCTURE Suppose $f(x) = 2[\![x - 1]\!]$.
   - a. Find $f(1.5)$. 0
   - b. Find $f(2.2)$. 2
   - c. Find $f(9.7)$. 16
   - d. Find $f(-1.25)$. -6

19. RENTAL CARS Mr. Aronsohn wants to rent a car on vacation. The rate the car rental company charges is $19 per day. If any fraction of a day is counted as a whole day, how much would it cost for Mr. Aronsohn to rent a car for 6.4 days? $133.00

20. USE A MODEL A roadside fruit and vegetable stand determines rates for selling produce, with every fraction of a pound rounded up to the next pound. The table shows the cost of tomatoes by weight in pounds.
   - a. Write a piecewise-linear function representing the cost of purchasing tomatoes that weigh no more than 5 pounds, where C is the cost in dollars and p is the number of pounds. See margin.

Weight (pounds)	Rate (dollars)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

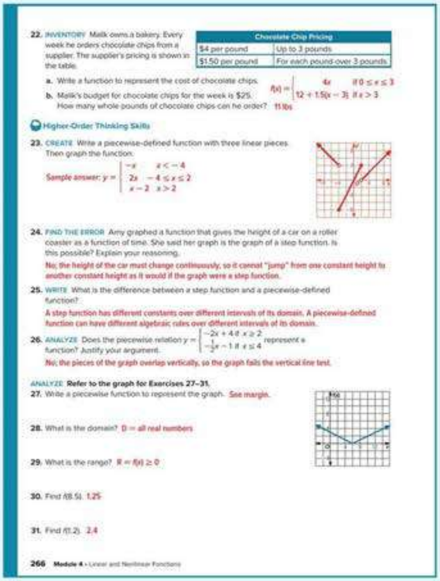
   - b. Graph the function.
   - c. State the domain and range. $D = \{x \mid 0 < x \leq 5\}$; $R = \{3.50, 7.00, 10.50, 14.00, 17.50\}$
   - d. What would be the cost of purchasing 8.3 pounds of tomatoes at the roadside stand? $31.50

21. ELECTRONIC REPAIRS Tech Repairs charges $25 for an electronic device repair that takes up to one hour. For each additional hour of labor, there is a charge of $50. The repair shop charges for the next full hour for any part of an hour.
   - a. Copy and complete the table to organize the information. Include a row for hours of repair x, and a row for total cost $f(x)$.

x	0	2	4	6	8
$f(x)$	0	75	175	275	375

   - b. Write a step function to represent the total cost for every hour h of repair. $f(x) = 25 + 50[\![x]\!]$
   - c. Graph the function.
   - d. Devesh was charged $125 to repair his tablet. How long did the repair take to complete? $2 < x \leq 3$

Lesson 4-6 • Piecewise and Step Functions 265



22. INVENTORY Malik owns a bakery. Every week he orders chocolate chips from a supplier. The supplier's pricing is shown in the table.

Chocolate Chip Pricing	
$4 per pound	Up to 3 pounds
$1.50 per pound	For each pound over 3 pounds

   - a. Write a function to represent the cost of chocolate chips. $f(x) = \begin{cases} 4x & \text{if } 0 \leq x \leq 3 \\ 12 + 1.5(x - 3); & \text{if } x > 3 \end{cases}$
   - b. Malik's budget for chocolate chips for the week is $25. How many whole pounds of chocolate chips can he order? 11 lbs

Higher-Order Thinking Skills

23. CREATE Write a piecewise-defined function with three linear pieces. Then graph the function.
Sample answer: $y = \begin{cases} -x & x < -4 \\ 2x & -4 \leq x \leq 2 \\ x - 2 & x > 2 \end{cases}$

24. FIND THE ERROR Amy graphed a function that gives the height of a car on a roller coaster as a function of time. She said her graph is the graph of a step function. Is this possible? Explain your reasoning.
No; the height of the car must change continuously, so it cannot "jump" from one constant height to another constant height as it would if the graph were a step function.

25. WRITE What is the difference between a step function and a piecewise-defined function?
A step function has different constants over different intervals of its domain. A piecewise-defined function can have different algebraic rules over different intervals of its domain.

26. ANALYZE Does the piecewise relation $y = \begin{cases} -2x + 4 \text{ if } x \geq 2 \\ -\frac{1}{2}x - 1 \text{ if } x \leq 4 \end{cases}$ represent a function? Justify your argument.
No; the pieces of the graph overlap vertically, so the graph fails the vertical line test.

ANALYZE **Refer to the graph for Exercises 27–31.**

27. Write a piecewise function to represent the graph. See margin.

28. What is the domain? D = all real numbers

29. What is the range? $R = f(x) \geq 0$

30. Find $f(8.5)$. 1.25

31. Find $f(1.2)$. 2.4

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$$20a.\ C(p) = \begin{cases} 3.50 \text{ if } 0 < x \leq 1 \\ 7.00 \text{ if } 1 < x \leq 2 \\ 10.50 \text{ if } 2 < x \leq 3 \\ 14.00 \text{ if } 3 < x \leq 4 \\ 17.50 \text{ if } 4 < x \leq 5 \end{cases}$$

$$27.\ f(x) = \begin{cases} \frac{1}{2}x - 3 \text{ if } x > 6 \\ -\frac{1}{2}x + 3 \text{ if } x \leq 6 \end{cases}$$

Lesson 4-7

Absolute Value Functions

F.IF.7b, F.BF.3

LESSON GOAL

Students identify the effects of transformations of the graphs of absolute value functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Parameters of an Absolute Value Function

Develop:

Graphing Absolute Value Functions; Translations of Absolute Value Functions

- Vertical Translations of Absolute Value Functions
- Horizontal Translations of Absolute Value Functions
- Multiple Translations of Absolute Value Functions
- Identify Absolute Value Functions from Graphs
- Identify Absolute Value Functions from Graphs (Multiple Translations)

Dilations of Absolute Value Functions

- Dilations of Form $a|x|$ When $x > 1$
- Dilations of the Form $|ax|$
- Dilations When $0 < a < 1$

Reflections of Absolute Value Functions

- Graphs of Reflections with Transformations
- Graphs of $y = -a|x|$
- Graphs of $y = |-ax|$

Transformations of Absolute Value Functions

- Graph an Absolute Value Function with Multiple Translations
- Graph an Absolute Value Function with Translations and Dilation
- Graph an Absolute Value Function with Translations and Reflection
- Apply Graphs of Absolute Value Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Integers: Opposites and Absolute Value	●	●		●
Extension: Parametric Equations		●	●	●

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

Coherence

Previous

Students solved equations involving absolute value.

A.CED.1, A.REI.3

Now

Students identify the effects of transformations of the graphs of absolute value functions.

F.IF.7b, F.BF.3

Next

Students understand, graph, and use quadratic, exponential, and other types of non-linear functions.

F.IF.4, F.IF.7, F.LE.1, F.LE.2, F.LE.3 (Course 1, Course 2, Course 3)

Rigor

The Three Pillars of Rigor

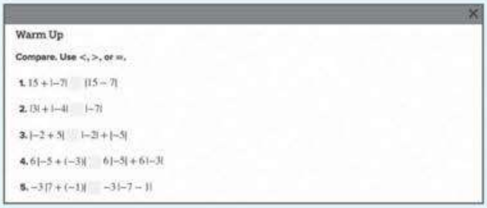
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of absolute value to absolute value functions. They build fluency by graphing absolute value functions, and they apply their understanding by solving real-world problems related to absolute value functions.

Mathematical Background

The graph of the absolute value parent function is V-shaped, with the vertex at the origin. The right side of the V is the graph of $y = x$; the left side is the graph of $y = -x$. Translations, dilations, and reflections of the graph of the absolute value parent function, $f(x) = |x|$, result in shifts, stretches or compressions, and flips (respectively), of the V-shaped graph.

Interactive Presentation

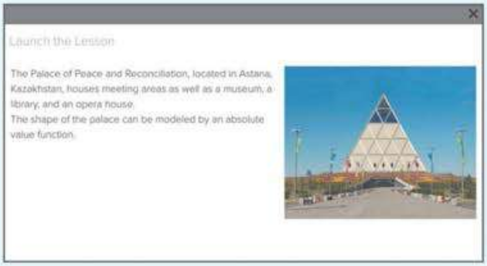


Warm Up

Compare. Use <, >, or =.

1. $|15 + |-7||$ ☐ $|15 - 7|$
2. $|3| + |-4|$ ☐ $|-7|$
3. $|-2 + 5|$ ☐ $|-2| + |-5|$
4. $6|-5 + (-3)|$ ☐ $6|-5| + 6|-3|$
5. $-3|7 + (-1)|$ ☐ $-3|-7 - 1|$

Warm Up

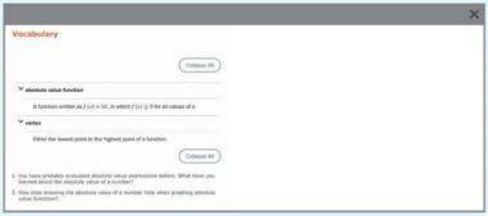


Launch the Lesson

The Palace of Peace and Reconciliation, located in Astana, Kazakhstan, houses meeting areas as well as a museum, a library, and an opera house.

The shape of the palace can be modeled by an absolute value function.

Launch the Lesson



Vocabulary

absolute value function

A function written as $f(x) = |x|$, in which $f(x) \geq 0$ for all values of x.

vertex

Either the lowest point or the highest point of a function.

1. You have probably evaluated absolute value expressions before. What have you learned about the absolute value of a number?
2. How does knowing the absolute value of a number help when graphing absolute value functions?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- evaluating absolute value expressions

Answers:

1. >
2. =
3. <
4. =
5. >

Launch the Lesson

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the shape of the Palace of Peace and Reconciliation and the graph of an absolute value function.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Language Development Handbook

Assign page 26 of the *Language Development Handbook* to help your students build mathematical language related to transformations of the graphs of absolute value functions.

ELL You can use the tips and suggestions on page T26 of the handbook to support students who are building English proficiency.

Explore Parameters of an Absolute Value Function

Objective

Students use a sketch to explore how changing the parameters changes the graphs of absolute value functions.

MP Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to complete this Explore activity, students will need to use the sketch. Work with students to explore and deepen their understanding of absolute value functions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

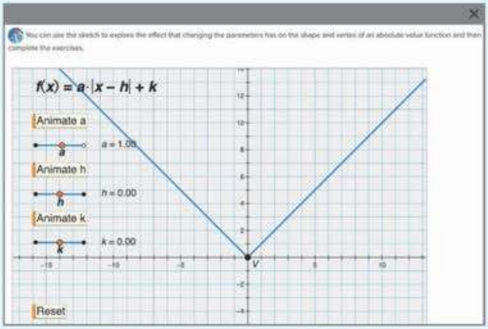
Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to explore how changing the parameters of an absolute value function affects its graph. Students explore the graphs on their own and through an animation. They will answer questions and form generalizations based on their observations. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore transformations of absolute value functions.

TYPE

Students answer questions about the transformations of absolute value functions.

Interactive Presentation

INQUIRY How does performing an operation on an absolute value function change its graph?

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Parameters of an Absolute Value Function (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How is changing the value of *a* for the absolute value graph similar to a linear function? Sample answer: The graphs get steeper as the value of *a* increases and less steep as *a* decreases.
- How can looking at point *V* help you determine the transformations in the function? Sample answer: Point *V* is moved up, down, left or right depending on how values were added or subtracted to the function.

Inquiry

How does performing an operation on an absolute value function change its graph? Sample answer: Adding a value to the function moves the graph up or down. Subtracting a value from *x* moves the graph left or right. Multiplying the function by a value makes the graph wider or narrower or flips it over the *x*-axis.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphing Absolute Value Functions

Common Misconception

Some students may think that the graph of any absolute value function will lie completely above the *x*-axis. Explain that just as with other functions, transformations of the function will relocate the graph, and the resulting graph may, in fact, contain points that lie below the *x*-axis.

Learn Translations of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $f(x) + k$ or $f(x - h)$ for positive and negative values.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Vertical Translations of Absolute Value Functions

MP Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the transformed function to identify the translation in the function.

Questions for Mathematical Discourse

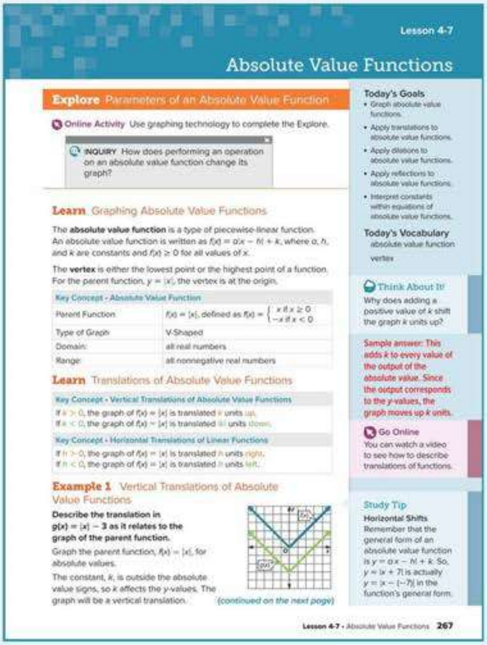
AL What type of transformation occurs in $g(x)$? a vertical translation How do you know? 3 is being subtracted from the parent function.

OL How is the *y*-value of each ordered pair in the parent function affected? Each *y*-value decreases by 3 units.

BL How would the graph of $f(x) = |x| + 3$ compare to this graph? Sample answer: It would be shifted up 3 instead of down 3.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-7

Absolute Value Functions

Explore Parameters of an Absolute Value Function

Online Activity Use graphing technology to complete the Explore.

INQUIRY How does performing an operation on an absolute value function change its graph?

Learn Graphing Absolute Value Functions

The **absolute value function** is a type of piecewise-linear function. An absolute value function is written as $f(x) = a|x - h| + k$, where a, h, and k are constants and $f(x) \geq 0$ for all values of x.

The **vertex** is either the lowest point or the highest point of a function. For the parent function, $y = |x|$, the vertex is at the origin.

Key Concept - Absolute Value Function	
Parent Function	$f(x) = \lvert x\rvert$, defined as $f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$
Type of Graph	V-Shaped
Domain:	all real numbers
Range:	all nonnegative real numbers

Learn Translations of Absolute Value Functions

Key Concept - Vertical Translations of Absolute Value Functions

If $k > 0$, the graph of $f(x) = |x|$ is translated k units up.
If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.

Key Concept - Horizontal Translations of Linear Functions

If $h > 0$, the graph of $f(x) = |x|$ is translated h units right.
If $h < 0$, the graph of $f(x) = |x|$ is translated $|h|$ units left.

Example 1 Vertical Translations of Absolute Value Functions

Describe the translation in $g(x) = |x| - 3$ as it relates to the graph of the parent function.

Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, k, is outside the absolute value signs, so k affects the y-values. The graph will be a vertical translation.

(continued on the next page)

Today's Goals
- Graph absolute value functions.
- Apply translations to absolute value functions.
- Apply dilations to absolute value functions.
- Apply reflections to absolute value functions.
- Interpret constants within equations of absolute value functions.

Today's Vocabulary
absolute value function
vertex

Think About It!
Why does adding a positive value of k shift the graph k units up?

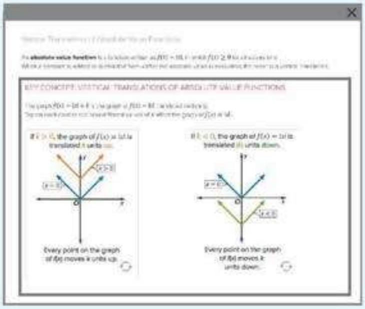
Sample answer: This adds k to every value of the output of the absolute value. Since the output corresponds to the y-values, the graph moves up k units.

Go Online
You can watch a video to see how to describe translations of functions.

Study Tip
Horizontal Shifts Remember that the general form of an absolute value function is $y = a|x - h| + k$. So, $y = |x + 7|$ is actually $y = |x - (-7)|$ in the function's general form.

Lesson 4-7 • Absolute Value Functions 267

Interactive Presentation



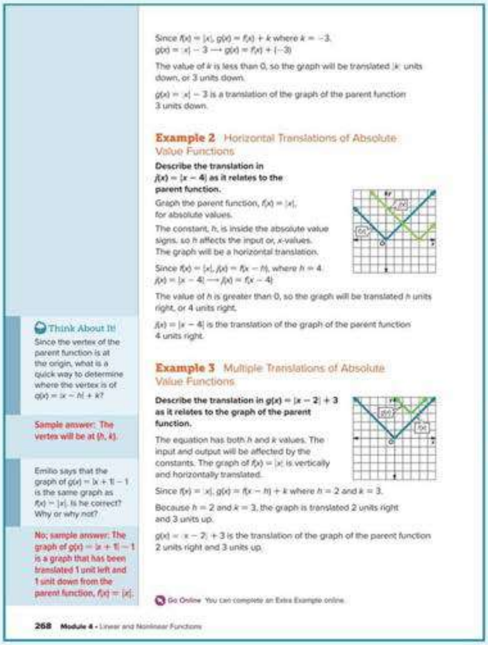
Learn

TAP

Students tap on each card to see how vertical transformations affect the graph.

TYPE

Students explain why adding a positive value shifts the graph the same number of units up.



Since $f(x) = |x|$, $g(x) = f(x) + k$ where $k = -3$.
$g(x) = |x| - 3 \longrightarrow g(x) = f(x) + (-3)$

The value of k is less than 0, so the graph will be translated $|k|$ units down, or 3 units down.

$g(x) = |x| - 3$ is a translation of the graph of the parent function 3 units down.

Example 2 Horizontal Translations of Absolute Value Functions

Describe the translation in $j(x) = |x - 4|$ as it relates to the parent function.

Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, h, is inside the absolute value signs, so h affects the input or, x-values. The graph will be a horizontal translation.

Since $f(x) = |x|$, $j(x) = f(x - h)$, where $h = 4$.
$j(x) = |x - 4| \longrightarrow j(x) = f(x - 4)$

The value of h is greater than 0, so the graph will be translated h units right, or 4 units right.

$j(x) = |x - 4|$ is the translation of the graph of the parent function 4 units right.

Example 3 Multiple Translations of Absolute Value Functions

Describe the translation in $g(x) = |x - 2| + 3$ as it relates to the graph of the parent function.

The equation has both h and k values. The input and output will be affected by the constants. The graph of $f(x) = |x|$ is vertically and horizontally translated.

Since $f(x) = |x|$, $g(x) = f(x - h) + k$ where $h = 2$ and $k = 3$.

Because $h = 2$ and $k = 3$, the graph is translated 2 units right and 3 units up.

$g(x) = |x - 2| + 3$ is the translation of the graph of the parent function 2 units right and 3 units up.

Go Online You can complete an Extra Example online.

Think About It!
Since the vertex of the parent function is at the origin, what is a quick way to determine where the vertex is of $g(x) = |x - h| + k$?

Sample answer: The vertex will be at (h, k).

Emilio says that the graph of $g(x) = |x + 1| - 1$ is the same graph as $f(x) = |x|$. Is he correct? Why or why not?

No; sample answer: The graph of $g(x) = |x + 1| - 1$ is a graph that has been translated 1 unit left and 1 unit down from the parent function, $f(x) = |x|$.

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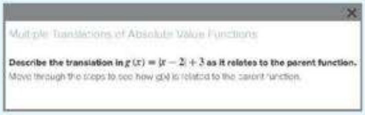
Interactive Presentation



Multiple Translations of Absolute Value Functions

Describe the translation in $g(x) = |x - 2| + 3$ as it relates to the parent function.
Move through the steps to see how g(x) is related to the parent function.

Example 3

TAP

Students move through the steps to see how the graph of the function relates to the parent function.

TYPE

Students answer questions about the graphs of translated functions.

Example 2 Horizontal Translations of Absolute Value Functions

Teaching the Mathematical Practices

7 Use Structure Help students to determine the structure of the translated absolute value function in this example.

Questions for Mathematical Discourse

AL What type of transformation occurs in $j(x)$? a horizontal translation How do you know? The -4 is inside the absolute value symbols.

OL How would the graph of $f(x) = |x + 4|$ compare to this graph? The graph of the parent function would be shifted 4 units to the left instead of to the right.

BL How would the graph of $f(x) = |x| - 4$ compare to this graph? The graph of the parent function would be shifted 4 units down instead of to the right.

Example 3 Multiple Translations of Absolute Value Functions

Questions for Mathematical Discourse

AL Looking at only the equation, which value shifts the graph vertically? $+3$

OL Looking at only the equation, how do you know that the horizontal translation is to the right? Sample answer: If you use the form $f(x - h)$ for the translation, then $|x - 2|$ means that $h = 2$. This represents a translation to the right 2 units.

BL What would the function be if it was a horizontal translation of 2 units left and 3 units down? $g(x) = |x + 2| - 3$

Common Error

As the function becomes more complex, some students may have difficulty seeing the relationship to the parent function. Encourage them to rewrite functions like the one in this example using $f(x)$. For example, for the function in this problem, students would write $f(x - 2) + 3$. In this way, they can see that 2 is being subtracted from x, and 3 is being added to the function values (i.e., the y-values).

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students are having difficulty determining the direction of a translation, **THEN** have them create four examples of absolute value functions that represent each type of translation, and write each one on an index card. Have them sketch the transformation on a coordinate plane on the back of the card, and write the description. Then have them use the flash cards (in both directions) to practice what they have learned.

Example 4 Identify Absolute Value Functions from Graphs

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and its equation used in this example.

Questions for Mathematical Discourse

AL What translation is shown on the graph? a horizontal shift of 1 to the right

OL Does this indicate that the value being added or subtracted should go inside or outside the absolute value symbols? inside

BL A classmate argues that the function should be $f(x) = |x + 1|$ because the shift is in the positive direction. Explain why this is incorrect. Sample answer: Translations are written in the form $f(x) = |x - h| + k$, so $f(x) = |x + 1|$ would be $f(x) = |x - (-1)|$, which would be a horizontal shift to the left.

Common Error

Some students may write the equation using a plus sign instead of a minus sign. Remind them that once they determine how many units and in what direction the graph is translated, they need to *subtract* that number from *x*.

Example 5 Identify Absolute Value Functions from Graphs (Multiple Translations)

Questions for Mathematical Discourse

AL How do you know that this graph represents a function with more than one transformation? Sample answer: The vertex is not on an axis.

OL How many transformations are there, and what type are they? 2; Sample answer: a horizontal translation of 2 units to the left and a vertical translation of 5 units down

BL What are the coordinates of the vertex? (−2, −5)
How does identifying the coordinates help you solve the problem? Sample answer: I can use the *x*-coordinate for *h* and the *y*-coordinate for *k* in the equation $g(x) = |x - h| + k$.

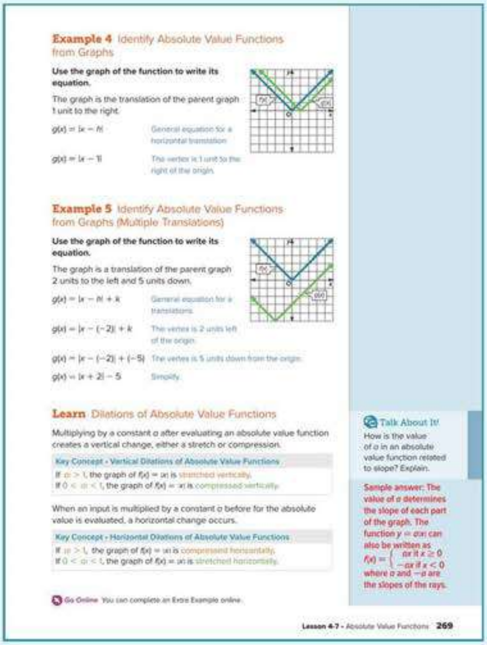
Learn Dilations of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $af(x)$ or $f(ax)$.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.



Example 4 Identify Absolute Value Functions from Graphs

Use the graph of the function to write its equation.

The graph is the translation of the parent graph 1 unit to the right.

$g(x) = |x - h|$ General equation for a horizontal translation

$g(x) = |x - 1|$ The vertex is 1 unit to the right of the origin.

Example 5 Identify Absolute Value Functions from Graphs (Multiple Translations)

Use the graph of the function to write its equation.

The graph is a translation of the parent graph 2 units to the left and 5 units down.

$g(x) = |x - h| + k$ General equation for a translations

$g(x) = |x - (-2)| + k$ The vertex is 2 units left of the origin.

$g(x) = |x - (-2)| + (-5)$ The vertex is 5 units down from the origin.

$g(x) = |x + 2| - 5$ Simplify.

Learn Dilations of Absolute Value Functions

Multiplying by a constant *a* after evaluating an absolute value function creates a vertical change, either a stretch or compression.

Key Concept • Vertical Dilations of Absolute Value Functions
If $|a| > 1$, the graph of $f(x) = a|x|$ is stretched vertically.
If $0 < |a| < 1$, the graph of $f(x) = a|x|$ is compressed vertically.

When an input is multiplied by a constant *a* before for the absolute value is evaluated, a horizontal change occurs.

Key Concept • Horizontal Dilations of Absolute Value Functions
If $|a| > 1$, the graph of $f(x) = |ax|$ is compressed horizontally.
If $0 < |a| < 1$, the graph of $f(x) = |ax|$ is stretched horizontally.

Go Online You can complete an Extra Example online.

Talk About It! How is the value of *a* in an absolute value function related to slope? Explain.

Sample answer: The value of *a* determines the slope of each part of the graph. The function $y = a|x|$ can also be written as $f(x) = \begin{cases} ax \text{ if } x \geq 0 \\ -ax \text{ if } x < 0 \end{cases}$ where *a* and $-a$ are the slopes of the rays.

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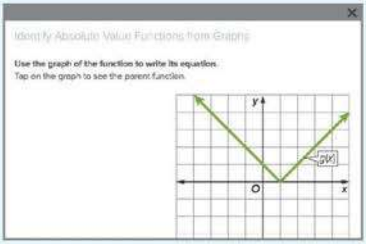
Interactive Presentation



Identify Absolute Value Functions from Graphs

Use the graph of the function to write its equation.

Tap on the graph to see the parent function.

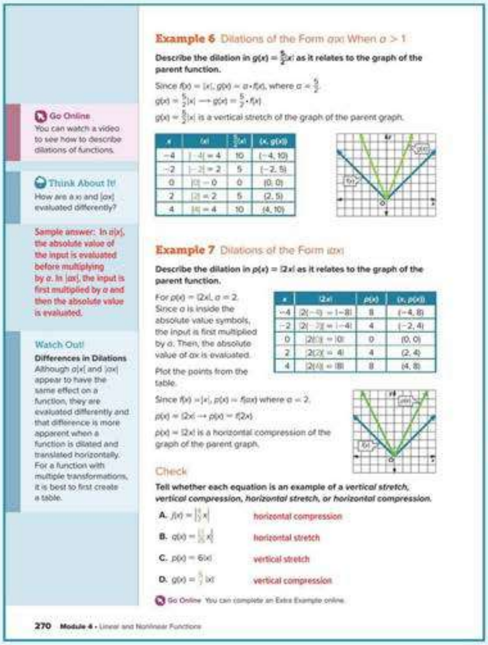
Example 4

TAP

Students tap on the graph to see the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 6 Dilations of the Form $a|x|$ When $a > 1$

Describe the dilation in $g(x) = \frac{5}{2}|x|$ as it relates to the graph of the parent function.

Since $f(x) = |x|$, $g(x) = a \cdot f(x)$, where $a = \frac{5}{2}$.

$g(x) = \frac{5}{2}|x| \longrightarrow g(x) = \frac{5}{2} \cdot f(x)$

$g(x) = \frac{5}{2}|x|$ is a vertical stretch of the graph of the parent graph.

x	$\|x\|$	$\frac{5}{2}\|x\|$	$(x, g(x))$
−4	$\|-4\| = 4$	10	(−4, 10)
−2	$\|-2\| = 2$	5	(−2, 5)
0	$\|0\| = 0$	0	(0, 0)
2	$\|2\| = 2$	5	(2, 5)
4	$\|4\| = 4$	10	(4, 10)

Go Online You can watch a video to see how to describe dilations of functions.

Think About It! How are $a|x|$ and $|ax|$ evaluated differently?

Sample answer: In $a|x|$, the absolute value of the input is evaluated before multiplying by a. In $|ax|$, the input is first multiplied by a and then the absolute value is evaluated.

Example 7 Dilations of the Form $|ax|$

Describe the dilation in $p(x) = |2x|$ as it relates to the graph of the parent function.

For $p(x) = |2x|$, $a = 2$. Since a is inside the absolute value symbols, the input is first multiplied by a. Then, the absolute value of ax is evaluated.

Plot the points from the table.

x	$\|2x\|$	$p(x)$	$(x, p(x))$
−4	$\|2(-4)\| = \|-8\|$	8	(−4, 8)
−2	$\|2(-2)\| = \|-4\|$	4	(−2, 4)
0	$\|2(0)\| = \|0\|$	0	(0, 0)
2	$\|2(2)\| = \|4\|$	4	(2, 4)
4	$\|2(4)\| = \|8\|$	8	(4, 8)

Since $f(x) = |x|$, $p(x) = f(ax)$ where $a = 2$.

$p(x) = |2x| \rightarrow p(x) = f(2x)$

$p(x) = |2x|$ is a horizontal compression of the graph of the parent graph.

Watch Out!

Differences in Dilations Although $a|x|$ and $|ax|$ appear to have the same effect on a function, they are evaluated differently and that difference is more apparent when a function is dilated and translated horizontally. For a function with multiple transformations, it is best to first create a table.

Check

Tell whether each equation is an example of a *vertical stretch, vertical compression, horizontal stretch,* or *horizontal compression.*

A. $f(x) =$ [illegible]	horizontal compression
B. $q(x) =$ [illegible]	horizontal stretch
C. $p(x) = 6\|x\|$	vertical stretch
D. $g(x) =$ [illegible] $\|x\|$	vertical compression

Go Online You can complete an Extra Example online.

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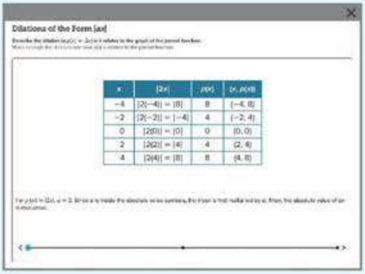
Interactive Presentation



Dilations of the Form |ax|

x	|2x|	p(x)	(x, p(x))
−4	|2(−4)| = |8|	8	(−4, 8)
−2	|2(−2)| = |−4|	4	(−2, 4)
0	|2(0)| = |0|	0	(0, 0)
2	|2(2)| = |4|	4	(2, 4)
4	|2(4)| = |8|	8	(4, 8)

Example 7

TAP

Students will move through the slides to see how to graph a dilation of an absolute value function.

Example 6 Dilations of the Form $a|x|$ When $a > 1$

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graphs and equations of the dilated function and the parent function in this example.

Questions for Mathematical Discourse

AL Is $|a|$ greater than 1 or between 0 and 1? Why? Sample answer: a is greater than 1 because $a = \frac{5}{2}$, and $\frac{5}{2} > 1$.

OL What kind of dilation does this represent? Explain. It is a vertical stretch by a factor of $\frac{5}{2}$. Sample answer: The $\frac{5}{2}$ is outside of the absolute value symbols and it is greater than 1.

BL How would the function be different if it was a horizontal compression where $a = \frac{5}{2}$? Sample answer: The function would be $g(x) = \left|\frac{5}{2}x\right|$.

Example 7 Dilations of the Form $|ax|$

Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL When the absolute value function is in the form $f(x) = |ax|$, what will be the effect of a? Sample answer: The graph will be horizontally stretched or compressed.

OL How would the transformation have changed if the function was $p(x) = \left|\frac{1}{2}x\right|$? Sample answer: It would be a horizontal stretch instead of a compression.

BL What would be an equivalent vertical dilation? Sample answer: a vertical stretch, $p(x) = 2|x|$

Example 8 Dilations When $0 < a < 1$

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and its equation used in this example.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know this is a vertical dilation and not a horizontal dilation? Sample answer: The $\frac{1}{3}$ is being multiplied on the outside of the function, not with x.

OL How would the dilation change if the function were $j(x) = 3|x|$? Sample answer: It would be a vertical stretch by a factor of 3.

BL How would this function change if it was a horizontal stretch where $a = \frac{1}{3}$? The function would be $j(x) = |\frac{1}{3}x|$.

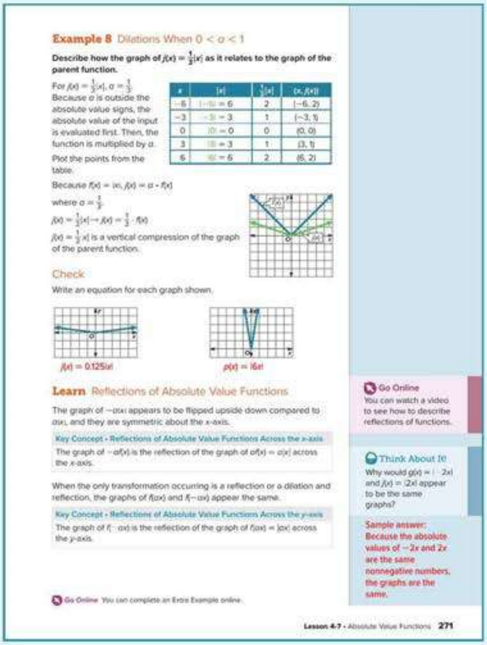
Learn Reflections of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $-af(x)$ or $f(-ax)$.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.



Example 8 Dilations When $0 < a < 1$

Describe how the graph of $j(x) = \frac{1}{3}|x|$ as it relates to the graph of the parent function.

For $j(x) = \frac{1}{3}|x|$, $a = \frac{1}{3}$.

Because a is outside the absolute value signs, the absolute value of the input is evaluated first. Then, the function is multiplied by a.

Plot the points from the table.

x	$\lvert x\rvert$	$\frac{1}{3}\lvert x\rvert$	$(x, j(x))$
−6	$\lvert -6\rvert = 6$	2	(−6, 2)
−3	$\lvert -3\rvert = 3$	1	(−3, 1)
0	$\lvert 0\rvert = 0$	0	(0, 0)
3	$\lvert 3\rvert = 3$	1	(3, 1)
6	$\lvert 6\rvert = 6$	2	(6, 2)

Because $f(x) = |x|$, $j(x) = a \cdot f(x)$ where $a = \frac{1}{3}$.

$j(x) = \frac{1}{3}|x| \rightarrow j(x) = \frac{1}{3} \cdot f(x)$

$j(x) = \frac{1}{3}|x|$ is a vertical compression of the graph of the parent function.

Check

Write an equation for each graph shown.

$j(x) = 0.125|x|$

$p(x) = |6x|$

Learn Reflections of Absolute Value Functions

The graph of $-a|x|$ appears to be flipped upside down compared to $a|x|$, and they are symmetric about the x-axis.

Key Concept • Reflections of Absolute Value Functions Across the x-axis

The graph of $-af(x)$ is the reflection of the graph of $af(x) = a|x|$ across the x-axis.

When the only transformation occuring is a reflection or a dilation and reflection, the graphs of $f(ax)$ and $f(-ax)$ appear the same.

Key Concept • Reflections of Absolute Value Functions Across the y-axis

The graph of $f(-ax)$ is the reflection of the graph of $f(ax) = |ax|$ across the y-axis.

Go Online You can complete an Extra Example online.

Go Online
You can watch a video to see how to describe reflections of functions.

Think About It!
Why would $g(x) = |-2x|$ and $j(x) = |2x|$ appear to be the same graphs?

Sample answer: Because the absolute values of $-2x$ and $2x$ are the same nonnegative numbers, the graphs are the same.

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Interactive Presentation

Dilations When $0 < a < 1$

Describe the dilation in $j(x) = \frac{1}{3}|x|$ as it relates to the graph of the parent function.

Move through the steps to see how $j(x)$ is related to the parent function.

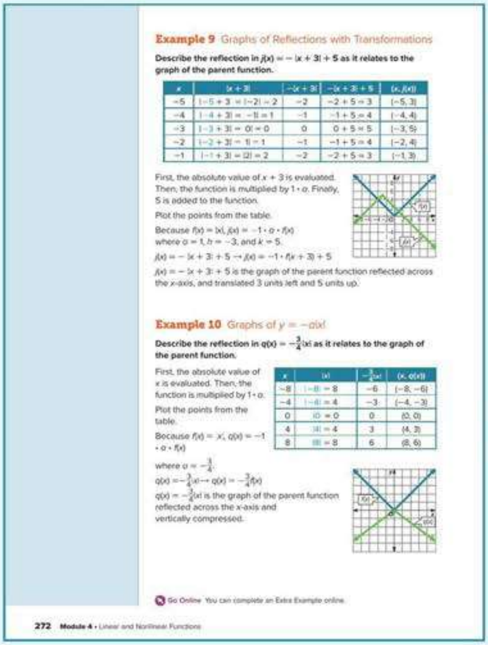
Example 8

TAP

Students move through the steps to see how the given function relates to the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 9 Graphs of Reflections with Transformations

Describe the reflection in $j(x) = -|x + 3| + 5$ as it relates to the graph of the parent function.

x	$\lvert x + 3 \rvert$	$-\lvert x + 3 \rvert$	$-\lvert x + 3 \rvert + 5$	$(x, j(x))$
−5	$\lvert -5 + 3 \rvert = \lvert -2 \rvert = 2$	−2	$-2 + 5 = 3$	(−5, 3)
−4	$\lvert -4 + 3 \rvert = \lvert -1 \rvert = 1$	−1	$-1 + 5 = 4$	(−4, 4)
−3	$\lvert -3 + 3 \rvert = \lvert 0 \rvert = 0$	0	$0 + 5 = 5$	(−3, 5)
−2	$\lvert -2 + 3 \rvert = \lvert 1 \rvert = 1$	−1	$-1 + 5 = 4$	(−2, 4)
−1	$\lvert -1 + 3 \rvert = \lvert 2 \rvert = 2$	−2	$-2 + 5 = 3$	(−1, 3)

First, the absolute value of $x + 3$ is evaluated. Then, the function is multiplied by $1 \cdot a$. Finally, 5 is added to the function.

Plot the points from the table.

Because $f(x) = |x|$, $j(x) = -1 \cdot a \cdot f(x)$ where $a = 1$, $h = -3$, and $k = 5$.

$j(x) = -|x + 3| + 5 \rightarrow j(x) = -1 \cdot f(x + 3) + 5$

$j(x) = -|x + 3| + 5$ is the graph of the parent function reflected across the x-axis, and translated 3 units left and 5 units up.

Example 10 Graphs of $y = -a|x|$

Describe the reflection in $q(x) = -\frac{3}{4}|x|$ as it relates to the graph of the parent function.

First, the absolute value of x is evaluated. Then, the function is multiplied by $1 \cdot a$.

Plot the points from the table.

x	$\lvert x \rvert$	$-\frac{3}{4}\lvert x \rvert$	$(x, q(x))$
−8	$\lvert -8 \rvert = 8$	−6	(−8, −6)
−4	$\lvert -4 \rvert = 4$	−3	(−4, −3)
0	$\lvert 0 \rvert = 0$	0	(0, 0)
4	$\lvert 4 \rvert = 4$	3	(4, 3)
8	$\lvert 8 \rvert = 8$	6	(8, 6)

Because $f(x) = |x|$, $q(x) = -1 \cdot a \cdot f(x)$ where $a = -\frac{3}{4}$.

$q(x) = -\frac{3}{4}|x| \rightarrow q(x) = -\frac{3}{4}f(x)$

$q(x) = -\frac{3}{4}|x|$ is the graph of the parent function reflected across the x-axis and vertically compressed.

Go Online You can complete an Extra Example online.

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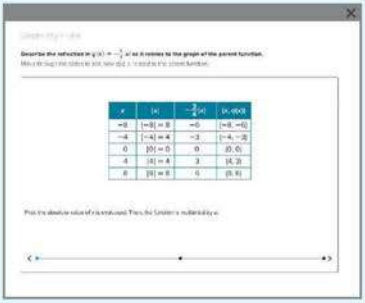
Interactive Presentation



Describe the reflection in $q(x) = -\frac{3}{4}|x|$ as it relates to the graph of the parent function.

x	$\lvert x \rvert$	$-\frac{3}{4}\lvert x \rvert$	$(x, q(x))$
−8	$\lvert -8 \rvert = 8$	−6	(−8, −6)
−4	$\lvert -4 \rvert = 4$	−3	(−4, −3)
0	$\lvert 0 \rvert = 0$	0	(0, 0)
4	$\lvert 4 \rvert = 4$	3	(4, 3)
8	$\lvert 8 \rvert = 8$	6	(8, 6)

Example 10

TAP

Students move through the slides to see how a given function is related to the parent function.

Example 9 Graphs of Reflections with Transformations

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the reflected function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL How do you know whether there is a horizontal translation? There is a 3 being added to x inside the absolute value symbols.

OL What is the effect of the negative in front of the absolute value symbols? It reflects the graph across the x-axis.

BL Why do you need to add 3 and take the absolute value before multiplying by −1? Sample answer: When evaluating to find the coordinates, you have to use the order of operations. In this case, you add 3 first because it is the operation inside the parentheses or grouping symbols.

Common Error

Remind students that the order in which they perform the operations when evaluating the function is important. Tell students that when creating the table, they must first add 3, then take the absolute value, then multiply by −1, then add 5.

Example 10 Graphs of $y = -a|x|$

Questions for Mathematical Discourse

AL How does the rule for $q(x)$ compare to the rule for the parent function? The rule for $q(x)$ is the rule for the parent function multiplied by $-\frac{3}{4}$.

OL How do you expect the vertex of $q(x)$ to compare to the vertex of the parent function? Explain. Sample answer: They will be the same because $q(x)$ has not been translated.

BL The point (12, 12) lies on the graph of the parent function. To what point does this map to on the graph of $q(x)$? (12, −9)

Common Error

Students may have difficulty seeing how the graph of $q(x)$ is related to the graph of the parent function. For these students, you may want to show the transformation in two different steps, first dilating the graph by a factor of $\frac{3}{4}$, and then reflecting the resulting graph across the x–axis.

DIFFERENTIATE

Enrichment Activity BL

Give students the function $f(x) = -|x - 4| - 2$. Have students create a step-by-step list of instructions for how to graph this function. Then have them graph the function.

F.IF.7b, F.BF.3

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 11 Graphs of $y = |-ax|$

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What is the coefficient of x? -4

OL Looking at only the equation, what type of transformations does this function represent? a horizontal compression and a reflection across the y-axis

BL How would this function be different if it was a vertical stretch where $a = 4$ and a reflection across the x-axis? The function would be $f(x) = -4|x|$.

Common Error

Some students may think that this function is equivalent to $f(x) = -|4x|$. Have them create a table of values for both functions so that they can see that the two functions produce different sets of ordered pairs.

Learn Transformations of Absolute Value Functions

Objective

Students graph absolute value functions by interpreting constants within the equation or by making a table of values.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the transformed functions and the graph of the parent function used in this Learn.

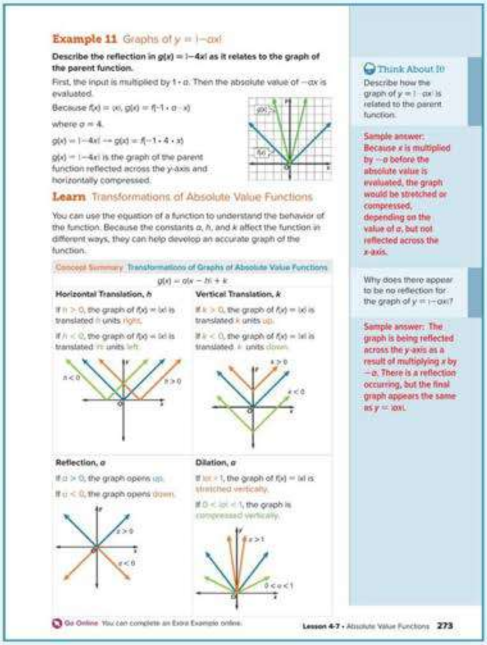
Common Misconception

Some students may think that translations should be applied before dilations and reflections. Use an example, such as $f(x) = -2|x - 3| + 4$, to show students that if they apply the vertical translation before the dilation and reflection, the resulting graph is not the same as when the transformations are applied in the correct order, with the vertical translation as the last transformation.

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner to create a poster showing examples of graphs that represent dilations of the graph of the parent function, including vertical and horizontal compressions and stretches, and have them use arrows to illustrate the stretch and compression. Have them also provide a description of each transformation.



Example 11 Graphs of $y = |-ax|$

Describe the reflection in $g(x) = |-4x|$ as it relates to the graph of the parent function.

First, the input is multiplied by $-1 \cdot a$. Then the absolute value of $-ax$ is evaluated.

Because $f(x) = |x|$, $g(x) = f(-1 \cdot a \cdot x)$

where $a = 4$.

$g(x) = |-4x| \rightarrow g(x) = f(-1 \cdot 4 \cdot x)$

$g(x) = |-4x|$ is the graph of the parent function reflected across the y-axis and horizontally compressed.

Learn Transformations of Absolute Value Functions

You can use the equation of a function to understand the behavior of the function. Because the constants a, h, and k affect the function in different ways, they can help develop an accurate graph of the function.

Concept Summary Transformations of Graphs of Absolute Value Functions

$g(x) = a|x - h| + k$

Horizontal Translation, h

If $h > 0$, the graph of $f(x) = |x|$ is translated h units right.

If $h < 0$, the graph of $f(x) = |x|$ is translated $|h|$ units left.

Vertical Translation, k

If $k > 0$, the graph of $f(x) = |x|$ is translated k units up.

If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.

Reflection, a

If $a > 0$, the graph opens up.

If $a < 0$, the graph opens down.

Dilation, a

If $|a| > 1$, the graph of $f(x) = |x|$ is stretched vertically.

If $0 < |a| < 1$, the graph is compressed vertically.

Go Online You can complete an Extra Example online.

Think About It!

Describe how the graph of $y = |-ax|$ is related to the parent function.

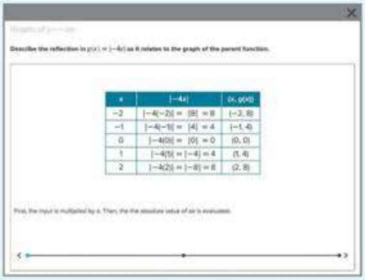
Sample answer: Because x is multiplied by $-a$ before the absolute value is evaluated, the graph would be stretched or compressed, depending on the value of a, but not reflected across the x-axis.

Why does there appear to be no reflection for the graph of $y = |-ax|$?

Sample answer: The graph is being reflected across the y-axis as a result of multiplying x by $-a$. There is a reflection occurring, but the final graph appears the same as $y = |ax|$.

Lesson 4-7 • Absolute Value Functions 273

Interactive Presentation



Describe the reflection in $g(x) = |-4x|$ as it relates to the graph of the parent function.

x	$\|-4x\|$	$(x, g(x))$
−2	$\|-4(-2)\| = \|8\| = 8$	(−2, 8)
−1	$\|-4(-1)\| = \|4\| = 4$	(−1, 4)
0	$\|-4(0)\| = \|0\| = 0$	(0, 0)
1	$\|-4(1)\| = \|-4\| = 4$	(1, 4)
2	$\|-4(2)\| = \|-8\| = 8$	(2, 8)

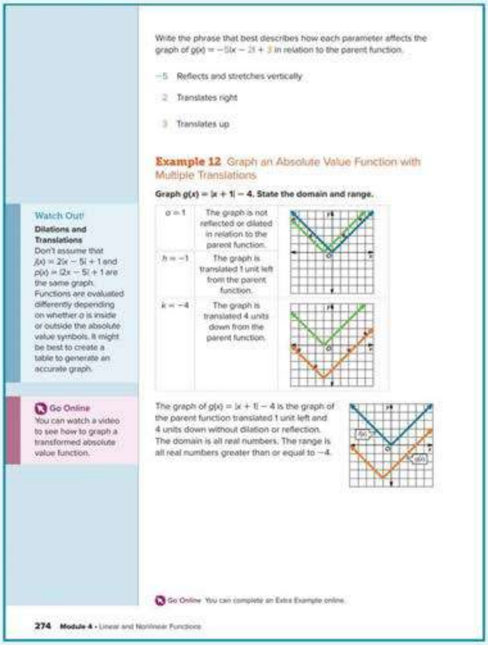
Example 11

TAP

Students move through the slides to see how a given function is related to the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Write the phrase that best describes how each parameter affects the graph of $g(x) = -5|x - 2| + 3$ in relation to the parent function.

-5 Reflects and stretches vertically

2 Translates right

3 Translates up

Example 12 Graph an Absolute Value Function with Multiple Translations

Graph $g(x) = |x + 1| - 4$. State the domain and range.

$a = 1$	The graph is not reflected or dilated in relation to the parent function.
$h = -1$	The graph is translated 1 unit left from the parent function.
$k = -4$	The graph is translated 4 units down from the parent function.

The graph of $g(x) = |x + 1| - 4$ is the graph of the parent function translated 1 unit left and 4 units down without dilation or reflection. The domain is all real numbers. The range is all real numbers greater than or equal to -4.

Watch Out!

Dilations and Translations Don't assume that $f(x) = 2|x - 5| + 1$ and $p(x) = |2x - 5| + 1$ are the same graph. Functions are evaluated differently depending on whether a is inside or outside the absolute value symbols. It might be best to create a table to generate an accurate graph.

Go Online You can watch a video to see how to graph a transformed absolute value function.

Go Online You can complete an Extra Example online.

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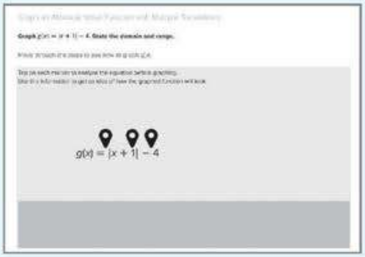
Interactive Presentation



Graph $g(x) = |x + 1| - 4$. **State the domain and range.**

$g(x) = |x + 1| - 4$

Example 12

TAP

Students tap on each marker to analyze the parameters in the function.

Example 12 Graph an Absolute Value Function with Multiple Translations

Teaching the Mathematical Practices

7 Use Structure Helps students to use the structure of the transformed function to identify the transformations in $g(x)$ and graph $g(x)$.

Questions for Mathematical Discourse

AL Looking at only the equation, what transformations occur in $g(x)$? a horizontal translation 1 unit to the left and a vertical translation 4 units down

OL How do you know that there is no reflection in this transformation? Sample answer: There are no negative coefficients in the function.

BL How would the function be different if it also represented a reflection over the x-axis? Sample answer: The function would be $f(x) = -|x + 1| - 4$.

Example 13 Graph an Absolute Value Function with Translations and Dilation

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the equations and graphs of the transformed function and the parent function.

Questions for Mathematical Discourse

AL What types of transformations occur in $j(x)$? a horizontal compression and a horizontal shift

OL What is the vertex of the graph of $j(x)$? (2, 0)

BL How could the Distributive Property help explain the horizontal shift 2 units to the right? Sample answer: If we apply the Distributive Property to factor the expression inside the absolute value function, we get $|3(x - 2)|$. This shows that we would first perform a translation of 2 units to the right, then a horizontal compression of 3.

Example 14 Graph an Absolute Value Function with Translations and Reflection

Teaching the Mathematical Practices

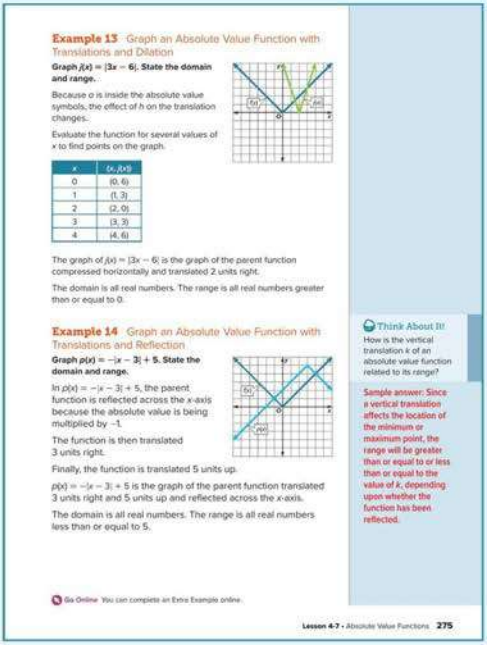
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Will the graph open up or down? How do you know? Down; sample answer: There is a negative sign in front of the absolute value symbols.

OL What types of transformations occur in $p(x)$? a horizontal translation 3 units to the right, a reflection across the x-axis, and a vertical translation 5 units up

BL How would the function be different if the graph had been translated 3 units to the right and then reflected over the y-axis instead of over the x-axis? The function would be $f(x) = |-x - 3| + 5$.



Example 13 Graph an Absolute Value Function with Translations and Dilation

Graph $j(x) = |3x - 6|$. State the domain and range.

Because a is inside the absolute value symbols, the effect of h on the translation changes.

Evaluate the function for several values of x to find points on the graph.

x	$(x, j(x))$
0	(0, 6)
1	(1, 3)
2	(2, 0)
3	(3, 3)
4	(4, 6)

The graph of $j(x) = |3x - 6|$ is the graph of the parent function compressed horizontally and translated 2 units right.

The domain is all real numbers. The range is all real numbers greater than or equal to 0.

Example 14 Graph an Absolute Value Function with Translations and Reflection

Graph $p(x) = -|x - 3| + 5$. State the domain and range.

In $p(x) = -|x - 3| + 5$, the parent function is reflected across the x-axis because the absolute value is being multiplied by −1.

The function is then translated 3 units right.

Finally, the function is translated 5 units up.

$p(x) = -|x - 3| + 5$ is the graph of the parent function translated 3 units right and 5 units up and reflected across the x-axis.

The domain is all real numbers. The range is all real numbers less than or equal to 5.

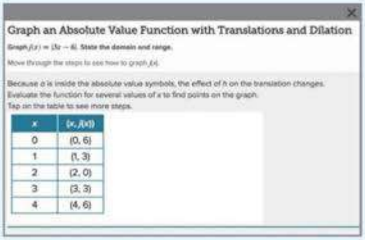
Think About It! How is the vertical translation k of an absolute value function related to its range?

Sample answer: Since a vertical translation affects the location of the minimum or maximum point, the range will be greater than or equal to or less than or equal to the value of k, depending upon whether the function has been reflected.

Go Online You can complete an Extra Example online.

Lesson 4-7 • Absolute Value Functions 275

Interactive Presentation



Graph an Absolute Value Function with Translations and Dilation

Graph $j(x) = |3x - 6|$. State the domain and range.

Move through the steps to see how to graph $j(x)$.

Because a is inside the absolute value symbols, the effect of h on the translation changes.

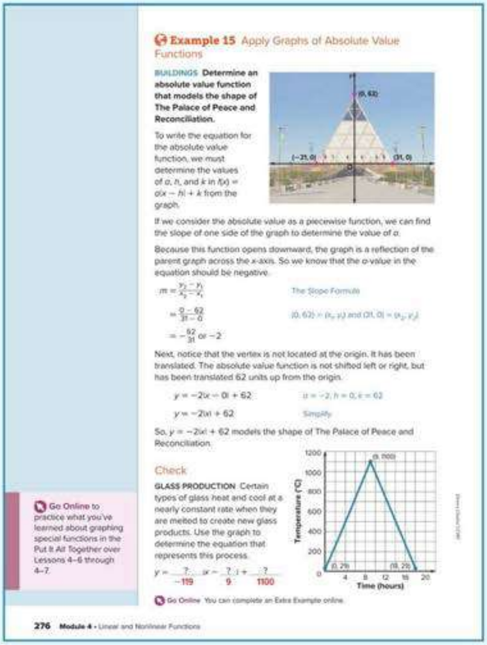
Evaluate the function for several values of x to find points on the graph.

Tap on the table to see more steps.

x	$(x, j(x))$
0	(0, 6)
1	(1, 3)
2	(2, 0)
3	(3, 3)
4	(4, 6)

Example 13

TAP Students move through the steps to graph the function.



Example 15 Apply Graphs of Absolute Value Functions

BUILDINGS **Determine an absolute value function that models the shape of The Palace of Peace and Reconciliation.**

To write the equation for the absolute value function, we must determine the values of a, h, and k in $f(x) = a|x - h| + k$ from the graph.

If we consider the absolute value as a piecewise function, we can find the slope of one side of the graph to determine the value of a.

Because this function opens downward, the graph is a reflection of the parent graph across the x-axis. So we know that the a-value in the equation should be negative.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{The Slope Formula}$$
$$= \frac{0 - 62}{31 - 0} \quad (0, 62) = (x_1, y_1) \text{ and } (31, 0) = (x_2, y_2)$$
$$= -\frac{62}{31} \text{ or } -2$$

Next, notice that the vertex is not located at the origin. It has been translated. The absolute value function is not shifted left or right, but has been translated 62 units up from the origin.

$$y = -2|x - 0| + 62 \quad a = -2, h = 0, k = 62$$
$$y = -2|x| + 62 \quad \text{Simplify.}$$

So, $y = -2|x| + 62$ models the shape of The Palace of Peace and Reconciliation.

Check

GLASS PRODUCTION Certain types of glass heat and cool at a nearly constant rate when they are melted to create new glass products. Use the graph to determine the equation that represents this process.

$y = \underline{\ ?\ }|x - \underline{\ ?\ }| + \underline{\ ?\ }$ −119 9 1100

Go Online to practice what you've learned about graphing special functions in the Put It All Together over Lessons 4-6 through 4-7.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Apply Graphs of Absolute Value Functions

BUILDINGS Determine an absolute value function that models the shape of The Palace of Peace and Reconciliation.

Example 15

TYPE

Students enter the values to find the slope of one side of the function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 15 Apply Graphs of Absolute Value Functions

Teaching the Mathematical Practices

4 Apply Mathematics Students will explore how to use an absolute value function to model the shape of a building. They will learn how to use the physical attributes of the building to calculate the parameters of the function.

Questions for Mathematical Discourse

AL How do you know that the value of a will be a negative number? Sample answer: The shape of the building is a V that opens down.

OL How do you know that the value of k will be 62? Sample answer: The vertex of the building is 62 units above the origin.

BL Why is it important to find the slope of the sides of the building? Sample answer: The slope tells you if there is a vertical or horizontal stretch or compression.

Common Error

After studying the photo, some students may try to incorporate a parameter representing a horizontal translation of 31 units. Help students to see that the diagram shows that the building is symmetric with respect to the y-axis, so there is no horizontal translation. Explain that the purpose of the marked points on the x-axis is for determining the dilation.

Exit Ticket

Recommended Use

At the end of class, have students respond to the Exit Ticket prompt using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, have students respond to the Exit Ticket prompt verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–35
2	exercises that use a variety of skills from this lesson	26–42
2	exercises that extend concepts learned in this lesson to new contexts	43–48
3	exercises that emphasize higher-order and critical-thinking skills	49–52

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–47 odd, 49–52
- Extension: Parametric Equations
- **ALEKS** Absolute Value Functions

IF students score 66%–89% on the Checks, **OL**
THEN assign:

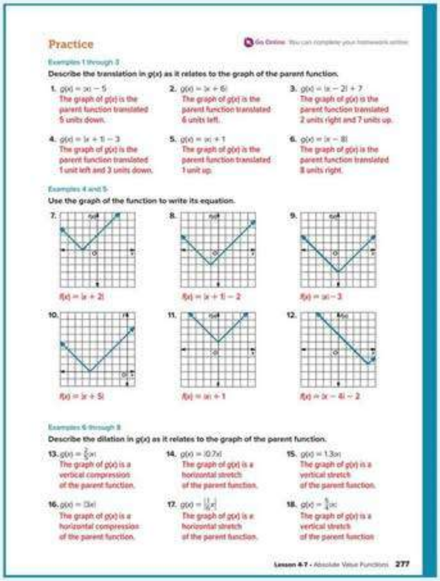
- Practice, Exercises 1–51 odd
- Remediation, Review Resources: Absolute Value and Distance
- Personal Tutors
- Extra Examples 1–15
- **ALEKS** Plotting and Comparing Signed Numbers

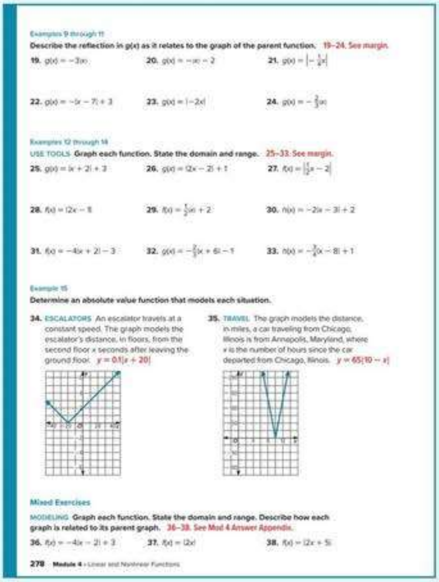
IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Absolute Value and Distance
- *Quick Review Math Handbook*: Special Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Plotting and Comparing Signed Numbers

Answers

19. The graph of $g(x)$ is a reflection of the parent function across the x-axis and a vertical stretch.
20. The graph of $g(x)$ is a reflection of the parent function across the x-axis and translated 2 units down.
21. The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal stretch.
22. The graph of $g(x)$ is a reflection of the parent function across the x-axis and translated 7 units right and 3 units up.
23. The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal compression.
24. The graph of $g(x)$ is a reflection of the parent function across the x-axis and a vertical compression.

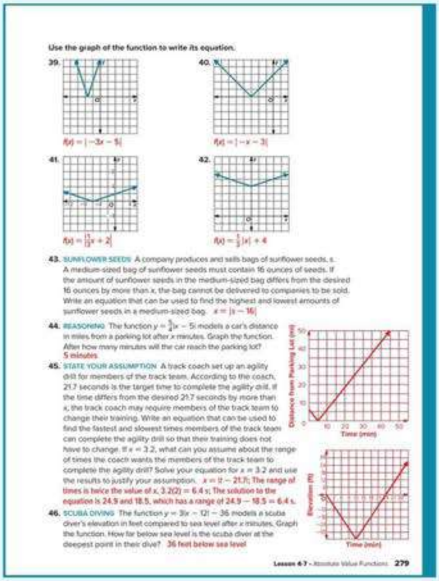




3 REFLECT AND PRACTICE

F.IF.7b, F.BF.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Use the graph of the function to write its equation.

39. $f(x) = |-3x - 5|$

40. $f(x) = |-x - 3|$

41. $f(x) = \left|\frac{1}{3}x + 2\right|$

42. $f(x) = \frac{1}{3}|x| + 4$

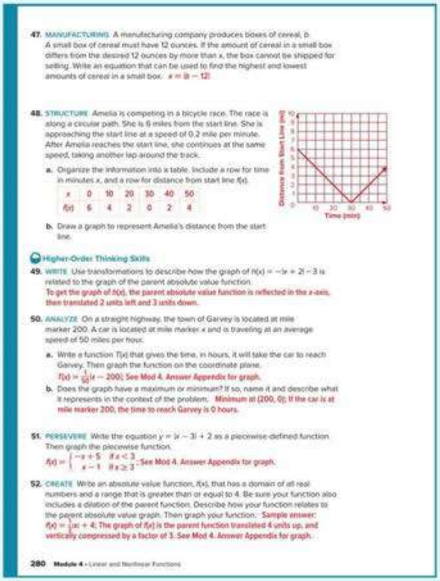
43. SUNFLOWER SEEDS A company produces and sells bags of sunflower seeds, *s*. A medium-sized bag of sunflower seeds must contain 16 ounces of seeds. If the amount of sunflower seeds in the medium-sized bag differs from the desired 16 ounces by more than *x*, the bag cannot be delivered to companies to be sold. Write an equation that can be used to find the highest and lowest amounts of sunflower seeds in a medium-sized bag. $x = |s - 16|$

44. REASONING The function $y = \frac{3}{4}|x - 5|$ models a car's distance in miles from a parking lot after *x* minutes. Graph the function. After how many minutes will the car reach the parking lot? 5 minutes

45. STATE YOUR ASSUMPTION A track coach set up an agility drill for members of the track team. According to the coach, 21.7 seconds is the target time to complete the agility drill. If the time differs from the desired 21.7 seconds by more than *x*, the track coach may require members of the track team to change their training. Write an equation that can be used to find the fastest and slowest times members of the track team can complete the agility drill so that their training does not have to change. If $x = 3.2$, what can you assume about the range of times the coach wants the members of the track team to complete the agility drill? Solve your equation for $x = 3.2$ and use the results to justify your assumption. $x = |t - 21.7|$; The range of times is twice the value of *x*, $3.2(2) = 6.4$ s; The solution to the equation is 24.9 and 18.5, which has a range of $24.9 - 18.5 = 6.4$ s.

46. SCUBA DIVING The function $y = 3|x - 12| - 36$ models a scuba diver's elevation in feet compared to sea level after *x* minutes. Graph the function. How far below sea level is the scuba diver at the deepest point in their dive? 36 feet below sea level

Lesson 4-7 • Absolute Value Functions 279



47. MANUFACTURING A manufacturing company produces boxes of cereal, *b*. A small box of cereal must have 12 ounces. If the amount of cereal in a small box differs from the desired 12 ounces by more than *x*, the box cannot be shipped for selling. Write an equation that can be used to find the highest and lowest amounts of cereal in a small box. $x = |b - 12|$

48. STRUCTURE Amelia is competing in a bicycle race. The race is along a circular path. She is 6 miles from the start line. She is approaching the start line at a speed of 0.2 mile per minute. After Amelia reaches the start line, she continues at the same speed, taking another lap around the track.

a. Organize the information into a table. Include a row for time in minutes *x*, and a row for distance from start line *f*(*x*).

x	0	10	20	30	40	50
f(*x*)	6	4	2	0	2	4

b. Draw a graph to represent Amelia's distance from the start line.

Higher-Order Thinking Skills

49. WRITE Use transformations to describe how the graph of $h(x) = -|x + 2| - 3$ is related to the graph of the parent absolute value function. To get the graph of *h*(*x*), the parent absolute value function is reflected in the *x*-axis, then translated 2 units left and 3 units down.

50. ANALYZE On a straight highway, the town of Garvey is located at mile marker 200. A car is located at mile marker *x* and is traveling at an average speed of 50 miles per hour.

a. Write a function *T*(*x*) that gives the time, in hours, it will take the car to reach Garvey. Then graph the function on the coordinate plane. $T(x) = \frac{1}{50}|x - 200|$; See Mod 4. Answer Appendix for graph.

b. Does the graph have a maximum or minimum? If so, name it and describe what it represents in the context of the problem. Minimum at (200, 0); If the car is at mile marker 200, the time to reach Garvey is 0 hours.

51. PERSEVERE Write the equation $y = |x - 3| + 2$ as a piecewise-defined function. Then graph the piecewise function.
$f(x) = \begin{cases} -x + 5 & \text{if } x < 3 \\ x - 1 & \text{if } x \geq 3 \end{cases}$; See Mod 4. Answer Appendix for graph.

52. CREATE Write an absolute value function, *f*(*x*), that has a domain of all real numbers and a range that is greater than or equal to 4. Be sure your function also includes a dilation of the parent function. Describe how your function relates to the parent absolute value graph. Then graph your function. Sample answer: $f(x) = \frac{1}{3}|x| + 4$; The graph of *f*(*x*) is the parent function translated 4 units up, and vertically compressed by a factor of 3. See Mod 4. Answer Appendix for graph.

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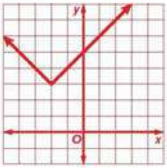
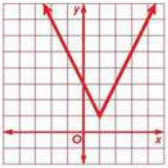
Answers

25.



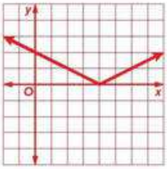
D = all real numbers,
R = $g(x) \geq 3$

26.



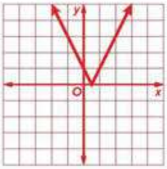
D = all real numbers,
R = $g(x) \geq 1$

27.



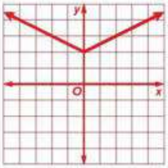
D = all real numbers,
R = $f(x) \geq 0$

28.



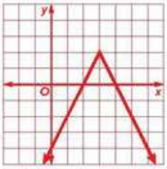
D = all real numbers,
R = $f(x) \geq 0$

29.



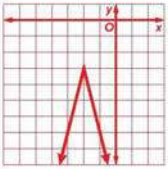
D = all real numbers,
R = $f(x) \geq 2$

30.



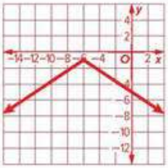
D = all real numbers,
R = $h(x) \leq 2$

31.



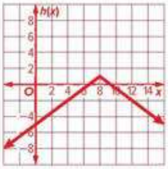
D = all real numbers,
R = $f(x) \leq -3$

32.



D = all real numbers,
R = $g(x) \leq -1$

33.



D = all real numbers,
R = $h(x) \leq 1$

Module 4 • Linear and Nonlinear Functions

Review

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it helpful to have different ways to graph linear functions?
- What can you learn about the graph of a linear function by analyzing its equation?
- Why is it important to understand how the structure of a function models a situation?
- If you know that a function is a step function, what do you know about how the elements of the domain are paired with the elements of the range?

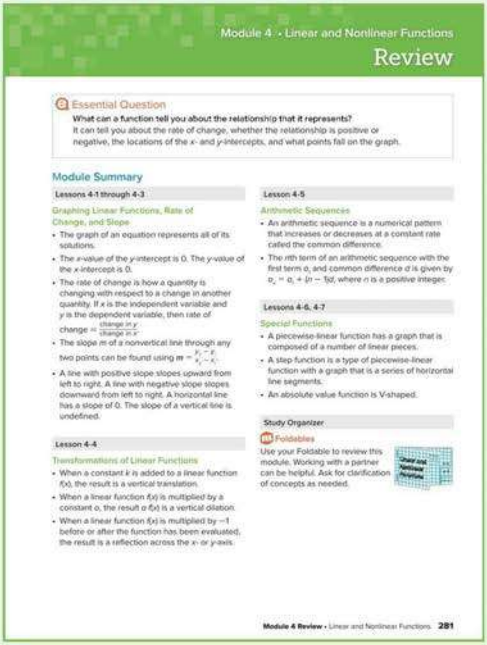
Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to linear and nonlinear functions.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Linear and Exponential Relationships, Descriptive Statistics**, and **Quadratic Functions and Modeling**.

- Interpret Expressions for Functions
- Build Linear and Exponential Function Models
- Interpret Linear Models
- Construct and Compare Linear, Quadratic, and Exponential Models and Solve Problems



Module 4 • Linear and Nonlinear Functions

Review

Essential Question

What can a function tell you about the relationship that it represents?

It can tell you about the rate of change, whether the relationship is positive or negative, the locations of the x- and y-intercepts, and what points fall on the graph.

Module Summary

Lessons 4-1 through 4-3

Graphing Linear Functions, Rate of Change, and Slope

- The graph of an equation represents all of its solutions.
- The x-value of the y-intercept is 0. The y-value of the x-intercept is 0.
- The rate of change is how a quantity is changing with respect to a change in another quantity. If x is the independent variable and y is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$.
- The slope m of a nonvertical line through any two points can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- A line with positive slope slopes upward from left to right. A line with negative slope slopes downward from left to right. A horizontal line has a slope of 0. The slope of a vertical line is undefined.

Lesson 4-4

Transformations of Linear Functions

- When a constant k is added to a linear function $f(x)$, the result is a vertical translation.
- When a linear function $f(x)$ is multiplied by a constant a, the result $a \cdot f(x)$ is a vertical dilation.
- When a linear function $f(x)$ is multiplied by -1 before or after the function has been evaluated, the result is a reflection across the x- or y-axis.

Lesson 4-5

Arithmetic Sequences

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
- The nth term of an arithmetic sequence with the first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Lessons 4-6, 4-7

Special Functions

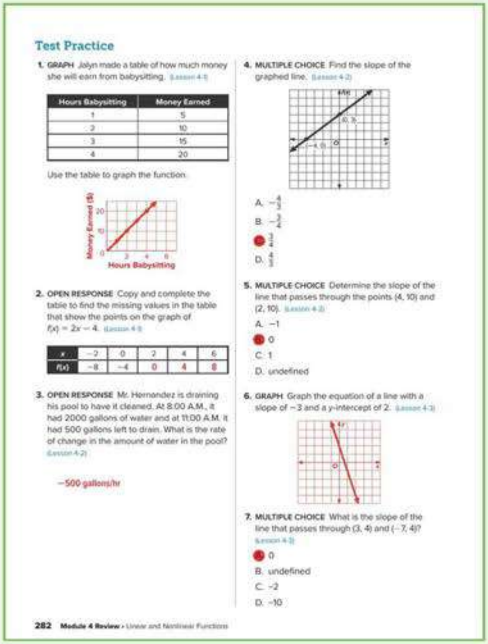
- A piecewise-linear function has a graph that is composed of a number of linear pieces.
- A step function is a type of piecewise-linear function with a graph that is a series of horizontal line segments.
- An absolute value function is V-shaped.

Study Organizer

Foldables

Use your Foldable to review this module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 4 Review • Linear and Nonlinear Functions 281



Test Practice

1. **GRAPH** Jalyn made a table of how much money she will earn from babysitting. (Lesson 4-1)

Hours Babysitting	Money Earned
1	5
2	10
3	15
4	20

Use the table to graph the function.

2. **OPEN RESPONSE** Copy and complete the table to find the missing values in the table that show the points on the graph of $f(x) = 2x - 4$. (Lesson 4-1)

x	−2	0	2	4	6
$f(x)$	−8	−4	0	4	8

3. **OPEN RESPONSE** Mr. Hernandez is draining his pool to have it cleaned. At 8:00 A.M., it had 2000 gallons of water and at 11:00 A.M. it had 500 gallons left to drain. What is the rate of change in the amount of water in the pool? (Lesson 4-2)

−500 gallons/hr

4. **MULTIPLE CHOICE** Find the slope of the graphed line. (Lesson 4-2)

A. $-\frac{4}{3}$

B. $-\frac{3}{4}$

● $\frac{3}{4}$

D. $\frac{4}{3}$

5. **MULTIPLE CHOICE** Determine the slope of the line that passes through the points (4, 10) and (2, 10). (Lesson 4-2)

A. −1

● 0

C. 1

D. undefined

6. **GRAPH** Graph the equation of a line with a slope of −3 and a y-intercept of 2. (Lesson 4-3)

7. **MULTIPLE CHOICE** What is the slope of the line that passes through (3, 4) and (−7, 4)? (Lesson 4-3)

● 0

B. undefined

C. −2

D. −10

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 4-1 through 4-7
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

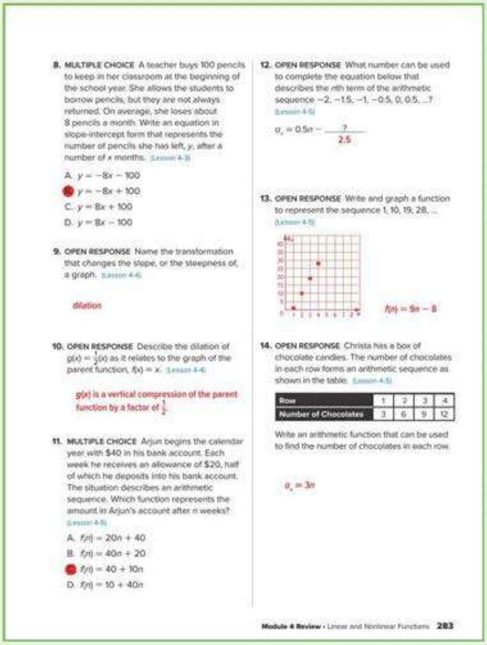
Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–21 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	4, 5, 7, 8, 11, 17
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	18
Table Item	Students complete a table by entering in the correct values.	2, 15
Graph	Students create a graph on an online coordinate plane.	1, 6, 16
Open Response	Students construct their own response.	3, 9, 10, 12, 13, 14, 19, 20, 21

To ensure that students understand the standards, check students' success on individual exercises.

Standard	Lesson(s)	Exercise(s)
A.CED.2	4-3	8
A.REI.10	4-1	2
F.IF.2	4-6	15
F.IF.6	4-2	3, 4, 5, 7
F.IF.7	4-1, 4-3, 4-4, 4-6, 4-7	1, 6, 16, 21
F.BF.1a	4-5	12
F.BF.3	4-4, 4-7	9, 10, 17, 18, 19, 20
F.LE.2	4-5	11, 13, 14



8. MULTIPLE CHOICE A teacher buys 100 pencils to keep in her classroom at the beginning of the school year. She allows the students to borrow pencils, but they are not always returned. On average, she loses about 8 pencils a month. Write an equation in slope-intercept form that represents the number of pencils she has left, y, after a number of x months. (Lesson 4-3)

A. $y = -8x - 100$

● $y = -8x + 100$

C. $y = 8x + 100$

D. $y = 8x - 100$

9. OPEN RESPONSE Name the transformation that changes the slope, or the steepness of, a graph. (Lesson 4-4)

dilation

10. OPEN RESPONSE Describe the dilation of $g(x) = \frac{1}{2}f(x)$ as it relates to the graph of the parent function, $f(x) = x$. (Lesson 4-4)

$g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{2}$.

11. MULTIPLE CHOICE Arjun begins the calendar year with $40 in his bank account. Each week he receives an allowance of $20, half of which he deposits into his bank account. The situation describes an arithmetic sequence. Which function represents the amount in Arjun's account after n weeks? (Lesson 4-5)

A. $f(n) = 20n + 40$

B. $f(n) = 40n + 20$

● $f(n) = 40 + 10n$

D. $f(n) = 10 + 40n$

12. OPEN RESPONSE What number can be used to complete the equation below that describes the nth term of the arithmetic sequence $-2, -1.5, -1, -0.5, 0, 0.5, \ldots$? (Lesson 4-5)

$a_n = 0.5n -$? 2.5

13. OPEN RESPONSE Write and graph a function to represent the sequence 1, 10, 19, 28, … (Lesson 4-5)

$f(n) = 9n - 8$

14. OPEN RESPONSE Christa has a box of chocolate candies. The number of chocolates in each row forms an arithmetic sequence as shown in the table. (Lesson 4-5)

Row	1	2	3	4
Number of Chocolates	3	6	9	12

Write an arithmetic function that can be used to find the number of chocolates in each row.

$a_n = 3n$

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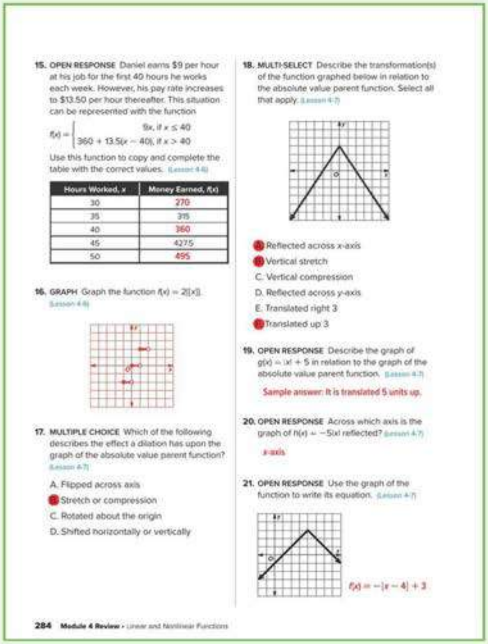

15. OPEN RESPONSE Daniel earns \$9 per hour at his job for the first 40 hours he works each week. However, his pay rate increases to \$13.50 per hour thereafter. This situation can be represented with the function

$$f(x) = \begin{cases} 9x, \text{ if } x \leq 40 \\ 360 + 13.5(x - 40), \text{ if } x > 40 \end{cases}$$

Use this function to copy and complete the table with the correct values. (Lesson 4-6)

Hours Worked, x	Money Earned, $f(x)$
30	270
35	315
40	360
45	427.5
50	495

16. GRAPH Graph the function $f(x) = 2[\![x]\!]$. (Lesson 4-6)

17. MULTIPLE CHOICE Which of the following describes the effect a dilation has upon the graph of the absolute value parent function? (Lesson 4-7)

A. Flipped across axis

[✓ B. Stretch or compression]

C. Rotated about the origin

D. Shifted horizontally or vertically

18. MULTI-SELECT Describe the transformation(s) of the function graphed below in relation to the absolute value parent function. Select all that apply. (Lesson 4-7)

[✓ A. Reflected across x-axis]

[✓ B. Vertical stretch]

C. Vertical compression

D. Reflected across y-axis

E. Translated right 3

[✓ F. Translated up 3]

19. OPEN RESPONSE Describe the graph of $g(x) = |x| + 5$ in relation to the graph of the absolute value parent function. (Lesson 4-7)

Sample answer: It is translated 5 units up.

20. OPEN RESPONSE Across which axis is the graph of $h(x) = -5|x|$ reflected? (Lesson 4-7)

x-axis

21. OPEN RESPONSE Use the graph of the function to write its equation. (Lesson 4-7)

$f(x) = -|x - 4| + 3$

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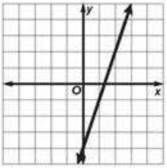
Lesson 4-3

33.



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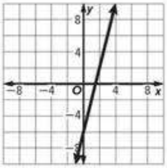

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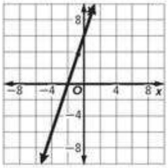

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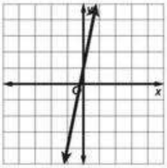
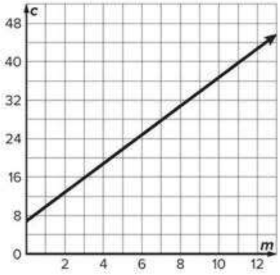


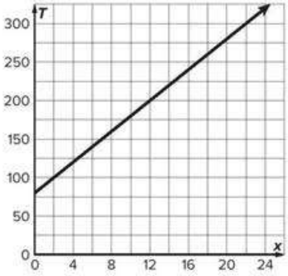
40b.



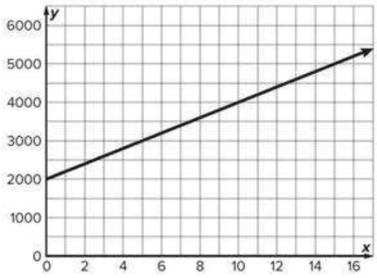
40c. Sample answer: Find 13 along the horizontal axis. Move up to the line. The corresponding value along the vertical axis is about 45. So, the cost of watching 13 movies from MovieMania is about $45.

40d. Sample answer: The cost of watching 13 movies from MovieMania is about $45, so divide $45 by 9 to get $5. So, the cost of watch a movie from SuperFlix is about $5.

41b.

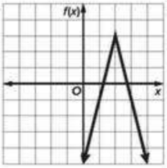


42b



Lesson 4-7

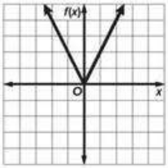
36.



D = all real numbers,
R = $f(x) \le 3$

The graph of $f(x)$ is a reflection of the parent function across the x-axis, vertically stretched by a factor of 4, and translated 2 units right and 3 units up.

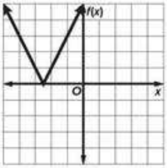
37.



D = all real numbers,
R = $f(x) \ge 0$

The graph of $f(x)$ is the parent function horizontally compressed by a factor of $\frac{1}{2}$

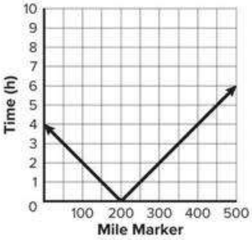
38.



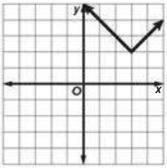
D = all real numbers,
R = $f(x) \ge 0$

The graph of $f(x)$ is the parent function horizontally compressed by a factor of $\frac{1}{2}$ and translated 2.5 units left.

50a.



51.



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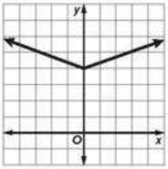


Module 5

Creating Linear Equations

Module Goals

- Students create linear equations in slope-intercept, point-slope, and standard forms.
- Students use scatter plots to make and evaluate predictions, and use best-fit lines and correlation coefficients to determine how well linear functions fit sets of data.
- Students determine whether a situation illustrates correlation or causation.
- Students find inverses of functions.

Focus

Domain: Algebra, Functions, Statistics and Probability

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

S.ID.6c Fit a linear function for a scatter plot that suggests a linear association.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Also addresses A.CED.3, S.ID.6a, S.ID.6, S.ID.8, S.ID.9, and F.BF.4a.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Coherence

Vertical Alignment

Previous

Students understood the connections between proportional relationships, lines, and linear equations.

8.EE.5

Now

Students create linear equations and analyze data to make predictions.

A.CED.2, S.ID.6c, F.BF.4a

Next

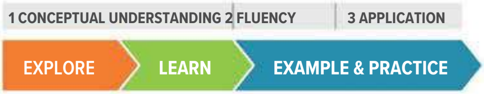
Students will use their knowledge of linear equations to build linear functions to model linear relationships.

F.BF.1(Course 1, Course 2)

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.



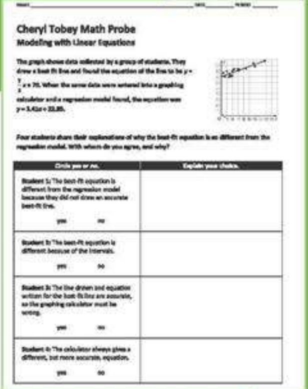
Suggested Pacing

Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
5-1 Writing Equations in Slope-Intercept Form	A.CED.2, S.ID.7	1	0.5
5-2 Writing Equations in Standard and Point-Slope Forms	A.CED.2, A.CED.3	2	1
Put It All Together: Lessons 5-1 through 5-2		1	0.5
5-3 Scatter Plots and Lines of Fit	S.ID.6a, S.ID.6c	2	1
5-4 Correlation and Causation	S.ID.9	1	0.5
5-5 Linear Regression	S.ID.6, S.ID.8	1	0.5
5-6 Inverses of Linear Functions	A.CED.2, F.BF.4a	2	1
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	13	6.5

CHERYL TOBEY
MATH
PROBES

Formative Assessment Math Probe
Modeling with Linear Equations

Module Resource



Cheryl Tobey Math Probe
Modeling with Linear Equations

Four students share their explanations of why the best-fit equation is so different from the regression model. With whom do you agree, and why?

Circle yes or no.	Explain your choice.
Student 1: The best-fit equation is different from the regression model because they did not draw an accurate best-fit line. yes no	
Student 2: The best-fit equation is different because of the intervals. yes no	
Student 3: The line drawn and equation written for the best-fit line are accurate, so the graphing calculator must be wrong. yes no	
Student 4: The calculator always gives a different, but more accurate, equation. yes no	

Correct Answers:
1. no 2. yes 3. no 4. no

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students determine why the equation for their best-fit line differs from the equation generated by their graphing calculator and explain their choices.

Targeted Concepts Understand how scale is used to determine and analyze the line of best fit.

Targeted Misconceptions

- Students may not realize the importance of scale when analyzing and interpreting lines of best fit.
- Students may not understand that the y-intercept of a line of best fit is the value of y when the x-value is equal to 0, not the left-most point on the graph.
- Students may rely on what a graph "looks" like rather than understanding a regression model as the line of best fit, generated using the data points.

Use the Probe after Lesson 5-3.

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
Student 1. yes **Student 2.** no **Student 3.** yes	• does not recognize that the y-axis is not part of the graph. • does not realize that this scale affects how the y-intercept is found and/or interpreted. • does not recognize that both equations have similar slopes but very different y-intercepts.
Student 4. yes	has generalized that the calculator is more accurate without understanding that the estimated best-fit line is inaccurate.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Scatter Plots and Lines of Best Fit
- Lesson 5-3, Learn, Example 2

Revisit the Probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

What can a function tell you about the relationship that it represents? Sample answer: Functions can tell you whether the value of dependent variable increases or decreases as the independent variable changes. They describe trends in data and can be used to make predictions.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

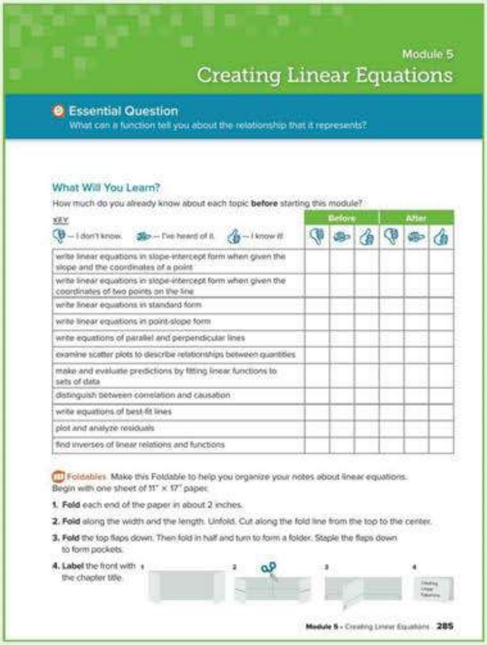
Focus As students read and study this module, they should show examples and write notes about linear equations.

Teach Have students make and label their Foldables as illustrated. Students should label the first three pockets with two lesson titles each. Students should list the vocabulary words on one index card per lesson. On the reverse of each card, students write the definitions of the vocabulary words. The cards are then placed in the appropriate pocket. The index cards can be used as flashcards for students to quiz each other.

When to Use It Encourage students to add to their Foldables as they work through the module and to use them to review for the module test.

Launch the Module

For this module, the Launch the Module video uses camping in the woods to show real-world applications of linear functions. Students learn about using linear relationships to model the heights of trees over time, the height above the ground of a person on a zip line, and other natural phenomena observable when camping.



Module 5

Creating Linear Equations

Essential Question

What can a function tell you about the relationship that it represents?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. 🤚 — I've heard of it. 👍 — I know it!

	Before			After		
	👎	🤚	👍	👎	🤚	👍
write linear equations in slope-intercept form when given the slope and the coordinates of a point						
write linear equations in slope-intercept form when given the coordinates of two points on the line						
write linear equations in standard form						
write linear equations in point-slope form						
write equations of parallel and perpendicular lines						
examine scatter plots to describe relationships between quantities						
make and evaluate predictions by fitting linear functions to sets of data						
distinguish between correlation and causation						
write equations of best-fit lines						
plot and analyze residuals						
find inverses of linear relations and functions						

Foldables Make this Foldable to help you organize your notes about linear equations. Begin with one sheet of 11" × 17" paper.

1. **Fold** each end of the paper in about 2 inches.
2. **Fold** along the width and the length. Unfold. Cut along the fold line from the top to the center.
3. **Fold** the top flaps down. Then fold in half and turn to form a folder. Staple the flaps down to form pockets.
4. **Label** the front with the chapter title.

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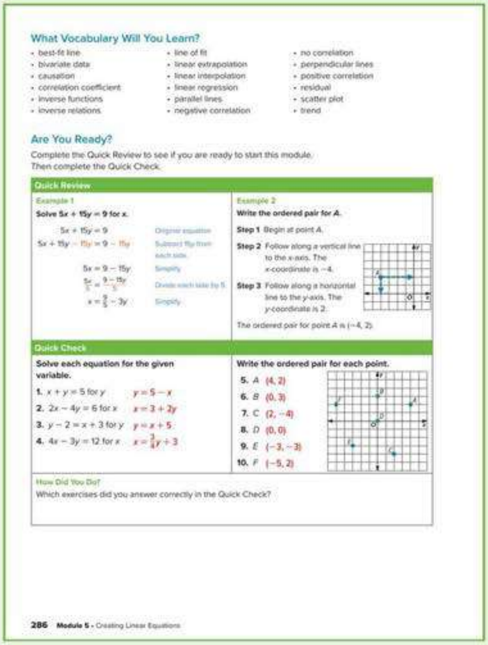
Interactive Presentation



What Vocabulary Will You Learn?

- best-fit line
- bivariate data
- causation
- correlation coefficient
- inverse functions
- inverse relations
- line of fit
- linear extrapolation
- linear interpolation
- linear regression
- parallel lines
- negative correlation
- no correlation
- perpendicular lines
- positive correlation
- residual
- scatter plot
- trend

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Solve $5x + 15y = 9$ for x.

$5x + 15y = 9$ Original equation

$5x + 15y - 15y = 9 - 15y$ Subtract 15y from each side.

$5x = 9 - 15y$ Simplify.

$\frac{5x}{5} = \frac{9 - 15y}{5}$ Divide each side by 5.

$x = \frac{9}{5} - 3y$ Simplify.

Example 2

Write the ordered pair for A.

Step 1 Begin at point A.

Step 2 Follow along a vertical line to the x-axis. The x-coordinate is -4.

Step 3 Follow along a horizontal line to the y-axis. The y-coordinate is 2.

The ordered pair for point A is $(-4, 2)$.

Quick Check

Solve each equation for the given variable.

1. $x + y = 5$ for y $y = 5 - x$
2. $2x - 4y = 6$ for x $x = 3 + 2y$
3. $y - 2 = x + 3$ for y $y = x + 5$
4. $4x - 3y = 12$ for x $x = \frac{3}{4}y + 3$

Write the ordered pair for each point.

5. A $(4, 2)$
6. B $(0, 3)$
7. C $(2, -4)$
8. D $(0, 0)$
9. E $(-3, -3)$
10. F $(-5, 2)$

How Did You Do?

Which exercises did you answer correctly in the Quick Check?

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define Slope-intercept form is an equation of the form $y = mx + b$, where m is the slope and b is the y-intercept.

Example $y = 3x + 5$ has a slope of 3 and a y-intercept of 5.

Ask Is the slope on the right side or the left side of the equation? right side

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- finding the slope and y-intercept from an equation in slope-intercept form
- finding the greatest common factor of a set of numbers
- writing linear equations given one point and the slope
- identifying patterns of association between two quantities
- using scatter plots to evaluate trends and make predictions
- making function tables

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You may want to use the **Functions and Lines** section to ensure student success in this module.

Mindset Matters

Regular Reflection

When students are asked to explain their thinking about their strategy they are engaging in thought organization, concise consolidation of knowledge, and deductive and inductive thinking.

How Can I Apply It?

Have students complete the **Exit Tickets** at the end of each lesson to reflect on their learning and communicate their thinking. Have students share by writing down their reflections or discussing with a partner or in small groups.

Lesson 5-1

A.CED.2, S.ID.7

Writing Expressions in Slope-Intercept Form

LESSON GOAL

Students create linear equations in slope-intercept form.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Slope-Intercept Form

Develop:

Creating Linear Equations in Slope-Intercept Form Given the Slope and a Point

- Write an Equation Given the Slope and a Point
- Write an Equation in Slope-Intercept Form

Creating Linear Equations in Slope-Intercept Form Given Two Points

- Write Equations Given Two Points
- Write an Equation Given Real-World Data

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Slope-Intercept Form	●	●		●
Extension: Collinearity		●	●	●

Language Development Handbook

Assign page 27 of the *Language Development Handbook* to help your students build mathematical language related to linear equations in slope-intercept form.

ELL You can use the tips and suggestions on page T27 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra, Statistics and Probability

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous

Students used similar triangles to derive the slope-intercept form of an equation.

8.EE.6

Now

Students create linear equations in slope-intercept form.

A.CED.2, S.ID.7

Next

Students will create linear equations in point-slope form and standard form.

A.CED.2, A.CED.3

Rigor

The Three Pillars of Rigor

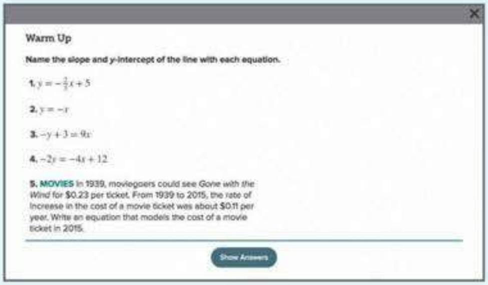
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of equations in one variable to equations in two variables and build fluency by writing these equations in slope-intercept form. They apply their understanding by interpreting slope and intercept in context.

Mathematical Background

The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. This general equation can be used to write the equation of a line when its slope and y-intercept are known.

Interactive Presentation

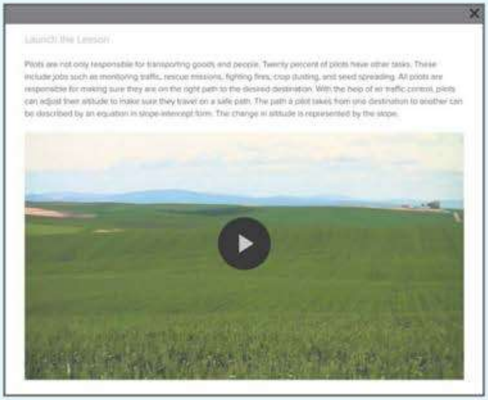


Warm Up

Name the slope and y-intercept of the line with each equation.

1. $y = -\frac{2}{3}x + 5$
2. $y = -x$
3. $-y + 3 = 9x$
4. $-2y = -4x + 12$
5. MOVIES In 1939, moviegoers could see *Gone with the Wind* for $0.23 per ticket. From 1939 to 2015, the rate of increase in the cost of a movie ticket was about $0.11 per year. Write an equation that models the cost of a movie ticket in 2015.

Warm Up



Launch the Lesson

Pilots are not only responsible for transporting goods and people. Twenty percent of pilots have other tasks. These include jobs such as monitoring traffic, rescue missions, fighting fires, crop dusting, and seed spreading. All pilots are responsible for making sure they are on the right path to the desired destination. With the help of air traffic control, pilots can adjust their altitude to make sure they travel on a safe path. The path a pilot takes from one destination to another can be described by an equation in slope-intercept form. The change in altitude is represented by the slope.

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- finding the slope and *y*-intercept from an equation in slope-intercept form

Answers:

1. $-\frac{2}{3}$, 5
2. −1, 0
3. −9, 3
4. 2, −6
5. $y = 0.11x + 0.23$

Launch the Lesson

Teaching the Mathematical Practices

4 Apply Mathematics In this Launch, students learn how to apply what they have learned about slope to a real-world situation about the flight paths of pilots.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Explore Slope-Intercept Form

Objective

Students use a sketch to explore how changing the coordinates of points on a line affects the slope of the line.

MP Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use the sketch. Work with students to explore and deepen their understanding of the slope-intercept form of a linear equation.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

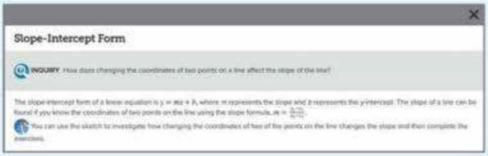
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to change the coordinates of two points on a line and observe how those changes affect the slope of the line. They will be guided through the exploration by a series of questions. They will then be asked to make generalizations about what they observed. Then, students will answer the Inquiry Question.

(continued on the next page)

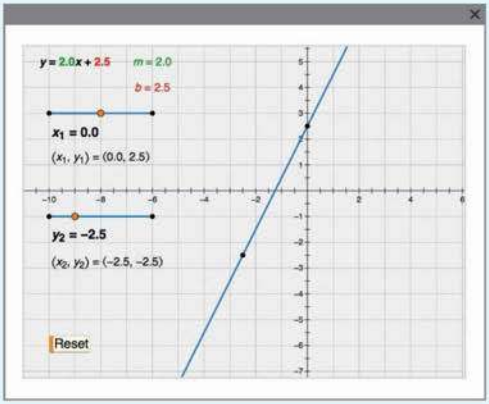
Interactive Presentation



Slope-Intercept Form

INQUIRY How does changing the coordinates of two points on a line affect the slope of the line?

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore how changing points changes slope.

TYPE

Students move through the activities and answer questions about the lines and their slope.

Interactive Presentation

INQUIRY How does changing the coordinates of two points on a line affect the slope of the line?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Slope-Intercept Form (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What is true about the coordinates of the points when the slope is undefined? The x-coordinates are both the same.
- What is true about the coordinates of the points when the slope is negative? The y-coordinate of the point with the greater x-coordinate is less than the y-coordinate of the point with the lesser x-coordinate.

Inquiry

How does changing the coordinates of two points on a line affect the slope of the line? Sample answer: As the y-coordinate approaches the same value as the other y-coordinate, the slope gets closer and closer to 0.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Creating Linear Equations in Slope-Intercept Form Given the Slope and a Point

Objective

Students create linear equations in slope-intercept form by using the slope of the line and the coordinates of a point on the line.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

About the Key Concept

To write an equation in slope-intercept form, you must know the values of *m* and *b*. If these values are given, the equation can be written simply by substituting these values into the equation. If *b* is unknown, use the point and the slope to substitute for *x*, *y*, and *m*, and solve for *b*. Then write the equation by substituting only for *m* and *b*.

Example 1 Write an Equation Given the Slope and a Point

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What two values are needed in order to write an equation in slope-intercept form? the values of *m* (the slope) and *b* (the *y*-intercept)

OL Why do you need to do Step 1? Sample answer: The value of *b* is unknown, so it has to be calculated.

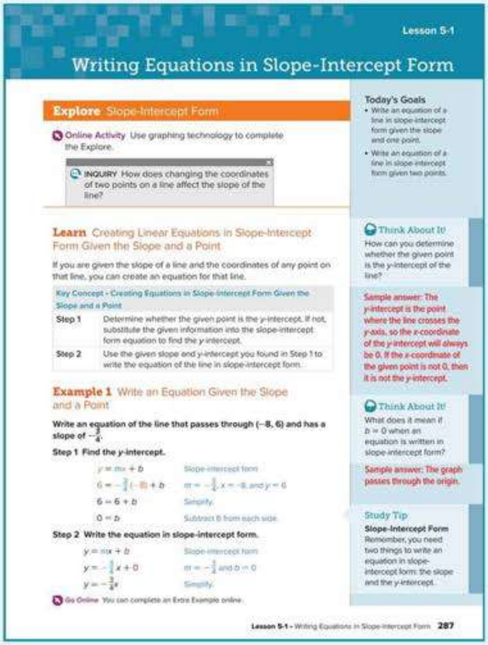
BL Why do you substitute for *x* and *y* in Step 1, but not in Step 2? In Step 1, you need the values of *x* and *y* to find *b*. In Step 2, you are writing the equation of the line, so *x* and *y* represent the coordinates of all the points on the line.

Common Error

Some students may replace *x* and *y* with the coordinates of the given point in Step 2, as well as in Step 1. Explain that the given point represents only one point on the line, and that the variables *x* and *y* are left as such in the final equation, as they represent the coordinates of all of the points on the line.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-1

Writing Equations in Slope-Intercept Form

Explore Slope-Intercept Form

Online Activity Use graphing technology to complete the Explore.

INQUIRY How does changing the coordinates of two points on a line affect the slope of the line?

Today's Goals
- Write an equation of a line in slope-intercept form given the slope and one point.
- Write an equation of a line in slope-intercept form given two points.

Learn Creating Linear Equations in Slope-Intercept Form Given the Slope and a Point

If you are given the slope of a line and the coordinates of any point on that line, you can create an equation for that line.

Key Concept • Creating Equations in Slope-Intercept Form Given the Slope and a Point	
Step 1	Determine whether the given point is the *y*-intercept. If not, substitute the given information into the slope-intercept form equation to find the *y*-intercept.
Step 2	Use the given slope and *y*-intercept you found in Step 1 to write the equation of the line in slope-intercept form.

Think About It! How can you determine whether the given point is the *y*-intercept of the line?

Sample answer: The *y*-intercept is the point where the line crosses the *y*-axis, so the *x*-coordinate of the *y*-intercept will always be 0. If the *x*-coordinate of the given point is not 0, then it is not the *y*-intercept.

Example 1 Write an Equation Given the Slope and a Point

Write an equation of the line that passes through (−8, 6) and has a slope of $-\frac{3}{4}$.

Step 1 Find the *y*-intercept.

$y = mx + b$ Slope-intercept form

$6 = -\frac{3}{4}(-8) + b$ $m = -\frac{3}{4}$, $x = -8$, and $y = 6$

$6 = 6 + b$ Simplify.

$0 = b$ Subtract 6 from each side.

Step 2 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = -\frac{3}{4}x + 0$ $m = -\frac{3}{4}$ and $b = 0$

$y = -\frac{3}{4}x$ Simplify.

Go Online You can complete an Extra Example online.

Think About It! What does it mean if $b = 0$ when an equation is written in slope-intercept form?

Sample answer: The graph passes through the origin.

Study Tip

Slope-Intercept Form Remember, you need two things to write an equation in slope-intercept form: the slope and the *y*-intercept.

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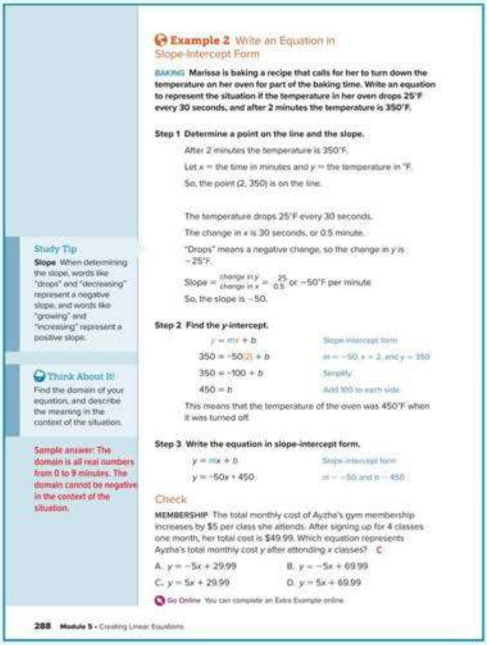
Interactive Presentation

Learn

TYPE	
a|	Students answer a question to show that they understand how to determine if a given point is the *y*-intercept of a line.



Example 2 Write an Equation in Slope-Intercept Form

BAKING **Marissa is baking a recipe that calls for her to turn down the temperature on her oven for part of the baking time. Write an equation to represent the situation if the temperature in her oven drops 25°F every 30 seconds, and after 2 minutes the temperature is 350°F.**

Step 1 Determine a point on the line and the slope.

After 2 minutes the temperature is 350°F.

Let x = the time in minutes and y = the temperature in °F.

So, the point (2, 350) is on the line.

The temperature drops 25°F every 30 seconds.

The change in x is 30 seconds, or 0.5 minute.

"Drops" means a negative change, so the change in y is −25°F.

Slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{-25}{0.5}$ or −50°F per minute

So, the slope is −50.

Study Tip

Slope When determining the slope, words like "drops" and "decreasing" represent a negative slope, and words like "growing" and "increasing" represent a positive slope.

Step 2 Find the y-intercept.

$y = mx + b$ Slope-intercept form

$350 = -50(2) + b$ $m = -50$, $x = 2$, and $y = 350$

$350 = -100 + b$ Simplify.

$450 = b$ Add 100 to each side.

This means that the temperature of the oven was 450°F when it was turned off.

Think About It!

Find the domain of your equation, and describe the meaning in the context of the situation.

Sample answer: The domain is all real numbers from 0 to 9 minutes. The domain cannot be negative in the context of the situation.

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = -50x + 450$ $m = -50$ and $b = 450$

Check

MEMBERSHIP The total monthly cost of Ayzha's gym membership increases by $5 per class she attends. After signing up for 4 classes one month, her total cost is $49.99. Which equation represents Ayzha's total monthly cost y after attending x classes? C

A. $y = -5x + 29.99$ B. $y = -5x + 69.99$

C. $y = 5x + 29.99$ D. $y = 5x + 69.99$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Question 3

○ A) $y = -5x + 29.99$
○ B) $y = -5x + 69.99$
○ C) $y = 5x + 29.99$
○ D) $y = 5x + 69.99$

Check

TAP

Students move through the steps to write an equation in slope-intercept form.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Write an Equation in Slope-Intercept Form

Teaching the Mathematical Practices

1 Analyze Givens and Constraints In this example, guide students through the steps to identify the meaning of the problem and look for entry points to its solution.

Questions for Mathematical Discourse

AL What information can be used to determine the slope? The temperature dropping 25 degrees every 30 seconds gives you information about a rate, or the slope.

OL What is the y-intercept? 450 What does it represent in this situation? the initial temperature of the oven

BL What is the temperature after 5 minutes? 200 degrees

Common Error

Some students may use −25 for the slope instead of −50. Explain that the units for both the slope and the relevant coordinate in the ordered pair must be the same. Since the point that will be used is (2, 350), and 2 represents 2 minutes, the slope should also be a rate measured in minutes.

Learn Creating Equations in Slope-Intercept Form Given Two Points

Objective

Students create linear equations in slope-intercept form by using the coordinates of two points on the line.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 3 Write Equations Given Two Points

MP Teaching the Mathematical Practices

8 Use Slope Help students to pay attention to the calculation of the slope of the line.

Questions for Mathematical Discourse

AL How can you find the slope given two points? You can use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

OL Does the order matter when substituting values into the Slope Formula? Why? No; sample answer: It does not matter which coordinate is used first as long as the *x*- and *y*-values are kept together.

BL Why is it important to find the slope first? You need to use the slope to find the *y*-intercept.

DIFFERENTIATE

Enrichment Activity **BL**

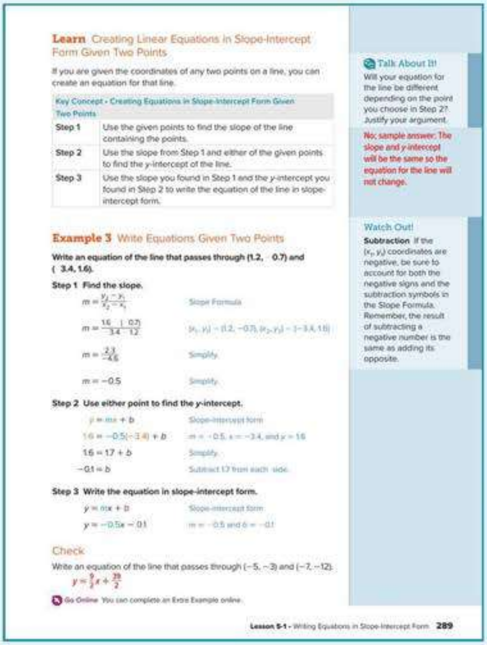
IF students are confused by learning more than one way to write a linear equation,

THEN have those students use the definition of slope to derive the slope-intercept form of a linear equation. Have them use the points (0, *b*) and (*x*, *y*) for the derivation.

DIFFERENTIATE

Enrichment Activity **AL** **BL** **ELL**

Write (3, 4) and (5, 4) on the board. Ask students to find *b*, the *y*-intercept, for the line that passes through these two points. After they have completed this, write (3, 5) and (3, 4) on the board and ask students to find *b* for the line that passes through these two points. Have them share and explain their results. For the first pair of points, $b = 4$. This is a horizontal line that passes through 4 on the *y*-axis. For the second pair of points, there is no *y*-intercept because these two points lie on the vertical line $x = 3$.



Learn Creating Linear Equations in Slope-Intercept Form Given Two Points

If you are given the coordinates of any two points on a line, you can create an equation for that line.

Key Concept • Creating Equations in Slope-Intercept Form Given Two Points	
Step 1	Use the given points to find the slope of the line containing the points.
Step 2	Use the slope from Step 1 and either of the given points to find the *y*-intercept of the line.
Step 3	Use the slope you found in Step 1 and the *y*-intercept you found in Step 2 to write the equation of the line in slope-intercept form.

Example 3 Write Equations Given Two Points

Write an equation of the line that passes through (1.2, −0.7) and (−3.4, 1.6).

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{1.6 - (-0.7)}{-3.4 - 1.2}$ $(x_1, y_1) = (1.2, -0.7), (x_2, y_2) = (-3.4, 1.6)$

$m = \frac{2.3}{-4.6}$ Simplify.

$m = -0.5$ Simplify.

Step 2 Use either point to find the *y*-intercept.

$y = mx + b$ Slope-intercept form

$1.6 = -0.5(-3.4) + b$ $m = -0.5$, $x = -3.4$, and $y = 1.6$

$1.6 = 1.7 + b$ Simplify.

$-0.1 = b$ Subtract 1.7 from each side.

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = -0.5x - 0.1$ $m = -0.5$ and $b = -0.1$

Check

Write an equation of the line that passes through (−5, −3) and (−7, −12).

$y = \frac{9}{2}x + \frac{39}{2}$

Go Online You can complete an Extra Example online.

Talk About It!

Will your equation for the line be different depending on the point you choose in Step 2? Justify your argument.

No; sample answer: The slope and *y*-intercept will be the same so the equation for the line will not change.

Watch Out!

Subtraction If the (x_1, y_1) coordinates are negative, be sure to account for both the negative signs and the subtraction symbols in the Slope Formula. Remember, the result of subtracting a negative number is the same as adding its opposite.

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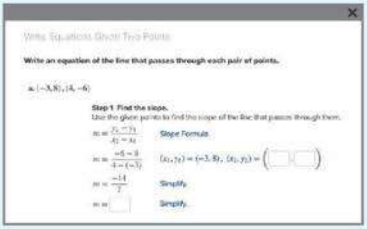
Interactive Presentation



Write Equations Given Two Points

Write an equation of the line that passes through each pair of points.

a. (−3, 8), (4, −6)

Step 1 Find the slope.

Use the given points to find the slope of the line that passes through them.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{-6 - 8}{4 - (-3)}$ $(x_1, y_1) = (-3, 8), (x_2, y_2) = (☐, ☐)$

$m = \frac{-14}{7}$ Simplify.

$m = ☐$ Simplify.

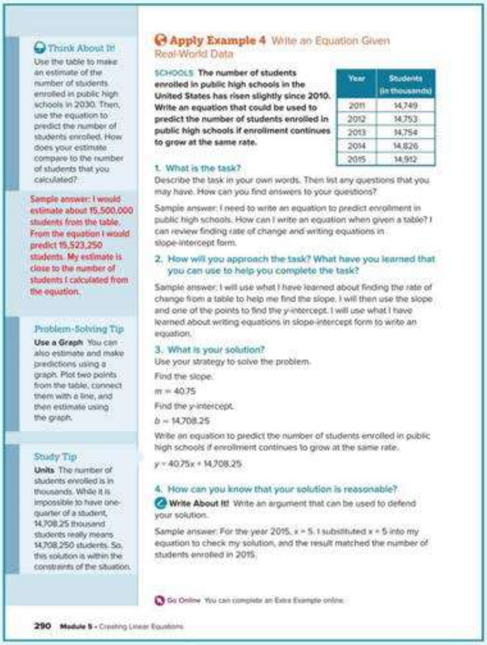
Example 3

TAP

Students move through the steps to write an equation given two points.

TYPE

Students compare two different methods of writing an equation in slope-intercept form.



Think About It! Use the table to make an estimate of the number of students enrolled in public high schools in 2030. Then, use the equation to predict the number of students enrolled. How does your estimate compare to the number of students that you calculated?

Sample answer: I would estimate about 15,500,000 students from the table. From the equation I would predict 15,523,250 students. My estimate is close to the number of students I calculated from the equation.

Problem-Solving Tip

Use a Graph You can also estimate and make predictions using a graph. Plot two points from the table, connect them with a line, and then estimate using the graph.

Study Tip

Units The number of students enrolled is in thousands. While it is impossible to have one-quarter of a student, 14,708.25 thousand students really means 14,708,250 students. So, this solution is within the constraints of the situation.

Apply Example 4 Write an Equation Given Real-World Data

SCHOOLS **The number of students enrolled in public high schools in the United States has risen slightly since 2010. Write an equation that could be used to predict the number of students enrolled in public high schools if enrollment continues to grow at the same rate.**

Year	Students (in thousands)
2011	14,749
2012	14,753
2013	14,754
2014	14,826
2015	14,912

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to write an equation to predict enrollment in public high schools. How can I write an equation when given a table? I can review finding rate of change and writing equations in slope-intercept form.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will use what I have learned about finding the rate of change from a table to help me find the slope. I will then use the slope and one of the points to find the y-intercept. I will use what I have learned about writing equations in slope-intercept form to write an equation.

3. What is your solution?

Use your strategy to solve the problem.

Find the slope.

$m = 40.75$

Find the y-intercept.

$b = 14{,}708.25$

Write an equation to predict the number of students enrolled in public high schools if enrollment continues to grow at the same rate.

$y = 40.75x + 14{,}708.25$

4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: For the year 2015, $x = 5$. I substituted $x = 5$ into my equation to check my solution, and the result matched the number of students enrolled in 2015.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Write an Equation Given Real-World Data

Year	Students (in thousands)
2011	14,749
2012	14,753
2013	14,754
2014	14,826
2015	14,912

Apply Example 4

TAP

Students move through the steps to writing an equation given real world data.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Apply Example 4 Write an Equation Given Real-World Data

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What ordered pairs can be used to find the slope and y-intercept?
- How can the equation be used to predict future enrollment?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Essential Question Follow-Up

Students have used data gathered from real-world situations to write equations that model the situation.

Ask:

Why is it useful to have an equation that models the situation, and not just the table of values? Sample answer: It is useful because you can use the equation to make predictions about data values that are not given in the table.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–49
3	exercises that emphasize higher-order and critical-thinking skills	50–53

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–49 odd, 50–53
- Extension: Collinearity
- **ALEKS** Equations of Lines

OL

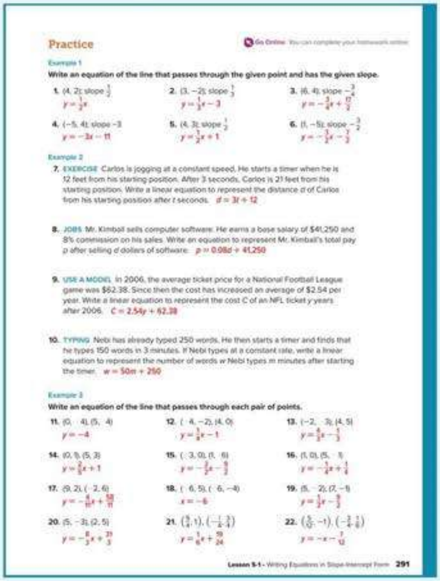
IF students score 66%–89% on the Checks,
THEN assign:

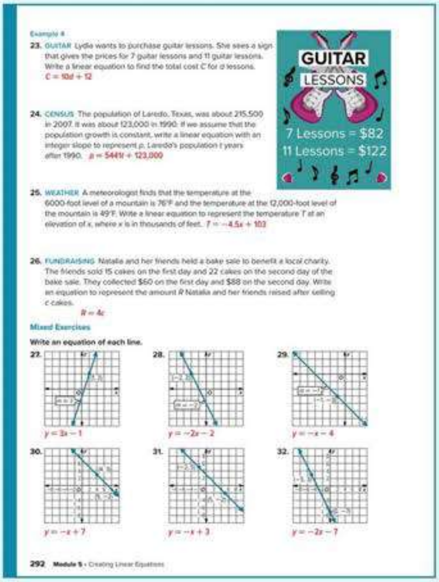
- Practice Exercises 1–53 odd
- Remediation, Review Resources: Slope-Intercept Form of a Line
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** Equations of Lines

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–25 odd
- Remediation, Review Resources: Slope-Intercept Form of a Line
- *Quick Review Math Handbook*: Writing Equations in Slope-Intercept Form
- Arrive**MATH** Take Another Look
- **ALEKS** Equations of Lines

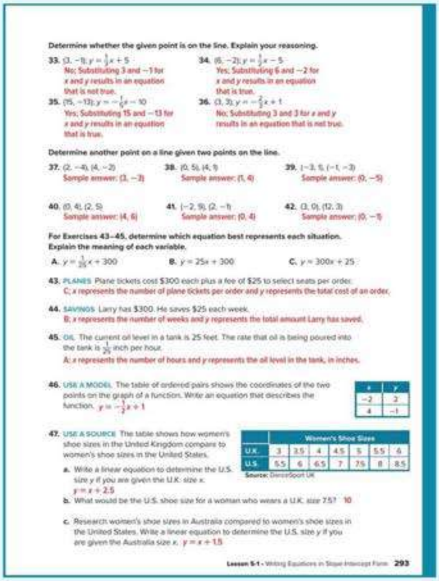




3 REFLECT AND PRACTICE

A.CED.2, S.ID.7

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Determine whether the given point is on the line. Explain your reasoning.

33. (3, −1); $y = \frac{1}{3}x + 5$
No; Substituting 3 and −1 for x and y results in an equation that is not true.

34. (6, −2); $y = \frac{1}{2}x - 5$
Yes; Substituting 6 and −2 for x and y results in an equation that is true.

35. (15, −13); $y = -\frac{1}{5}x - 10$
Yes; Substituting 15 and −13 for x and y results in an equation that is true.

36. (3, 3); $y = -\frac{2}{3}x + 1$
No; Substituting 3 and 3 for x and y results in an equation that is not true.

Determine another point on a line given two points on the line.

37. (2, −4), (4, −2)
Sample answer: (3, −3)

38. (0, 5), (4, 1)
Sample answer: (1, 4)

39. (−3, 1), (−1, −3)
Sample answer: (0, −5)

40. (0, 4), (2, 5)
Sample answer: (4, 6)

41. (−2, 9), (2, −1)
Sample answer: (0, 4)

42. (3, 0), (12, 3)
Sample answer: (0, −1)

For Exercises 43–45, determine which equation best represents each situation. Explain the meaning of each variable.

A. $y = \frac{1}{25}x + 300$ **B.** $y = 25x + 300$ **C.** $y = 300x + 25$

43. PLANES Plane tickets cost $300 each plus a fee of $25 to select seats per order.
C; x represents the number of plane tickets per order and y represents the total cost of an order.

44. SAVINGS Larry has $300. He saves $25 each week.
B; x represents the number of weeks and y represents the total amount Larry has saved.

45. OIL The current oil level in a tank is 25 feet. The rate that oil is being poured into the tank is $\frac{1}{25}$ inch per hour.
A; x represents the number of hours and y represents the oil level in the tank, in inches.

46. USE A MODEL The table of ordered pairs shows the coordinates of the two points on the graph of a function. Write an equation that describes the function. $y = -\frac{1}{2}x + 1$

x	y
−2	2
4	−1

47. USE A SOURCE The table shows how women's shoe sizes in the United Kingdom compare to women's shoe sizes in the United States.

	Women's Shoe Sizes						
U.K.	3	3.5	4	4.5	5	5.5	6
U.S.	5.5	6	6.5	7	7.5	8	8.5

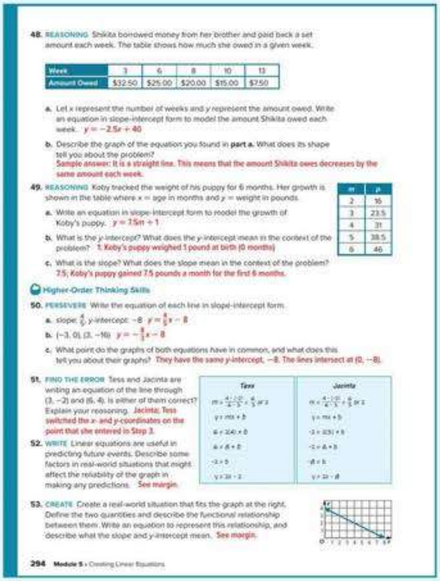
Source: DanceSport UK

a. Write a linear equation to determine the U.S. size y if you are given the U.K. size x.
$y = x + 2.5$

b. What would be the U.S. shoe size for a woman who wears a U.K. size 7.5? 10

c. Research women's shoe sizes in Australia compared to women's shoe sizes in the United States. Write a linear equation to determine the U.S. size y if you are given the Australia size x. $y = x + 1.5$

Lesson 5-1 · Writing Equations in Slope-Intercept Form 293



48. REASONING Shikita borrowed money from her brother and paid back a set amount each week. The table shows how much she owed in a given week.

Week	3	6	8	10	13
Amount Owed	$32.50	$25.00	$20.00	$15.00	$7.50

a. Let x represent the number of weeks and y represent the amount owed. Write an equation in slope-intercept form to model the amount Shikita owed each week. $y = -2.5x + 40$

b. Describe the graph of the equation you found in **part a.** What does its shape tell you about the problem?
Sample answer: It is a straight line. This means that the amount Shikita owes decreases by the same amount each week.

49. REASONING Koby tracked the weight of his puppy for 6 months. Her growth is shown in the table where x = age in months and y = weight in pounds.

m	p
2	16
3	23.5
4	31
5	38.5
6	46

a. Write an equation in slope-intercept form to model the growth of Koby's puppy. $y = 7.5m + 1$

b. What is the y-intercept? What does the y-intercept mean in the context of the problem? 1; Koby's puppy weighed 1 pound at birth (0 months)

c. What is the slope? What does the slope mean in the context of the problem?
7.5; Koby's puppy gained 7.5 pounds a month for the first 6 months.

Higher-Order Thinking Skills

50. PERSEVERE Write the equation of each line in slope-intercept form.

a. slope: $\frac{4}{5}$, y-intercept: −8 $y = \frac{4}{5}x - 8$

b. (−3, 0), (3, −16) $y = -\frac{8}{3}x - 8$

c. What point do the graphs of both equations have in common, and what does this tell you about their graphs? They have the same y-intercept, −8. The lines intersect at (0, −8).

51. FIND THE ERROR Tess and Jacinta are writing an equation of the line through (3, −2) and (6, 4). Is either of them correct? Explain your reasoning. Jacinta; Tess switched the x- and y-coordinates on the point that she entered in Step 3.

Tess	Jacinta
$m = \frac{4-(-2)}{6-3} = \frac{6}{3}$ or 2	$m = \frac{4-(-2)}{6-3} = \frac{6}{3}$ or 2
$y = mx + b$	$y = mx + b$
$6 = 2(4) + b$	$-2 = 2(3) + b$
$6 = 8 + b$	$-2 = 6 + b$
$-2 = b$	$-8 = b$
$y = 2x - 2$	$y = 2x - 8$

52. WRITE Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making any predictions. See margin.

53. CREATE Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship, and describe what the slope and y-intercept mean. See margin.

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Answers

52. Sample answer: If the problem is about something that could suddenly change, such as weather or prices, the graph could suddenly spike up. You need a constant rate of change to produce a linear graph

53. Sample answer: Let y represent the number of quarts of water in a pitcher, and let x represent the time in seconds that water is pouring from the pitcher. As time increases by 1 second, the amount of water in the pitcher decreases by $\frac{1}{2}$ qt. An equation representing this situation is $y = -\frac{1}{2}x + 4$. The slope is the rate at which the water is leaving the pitcher, $\frac{1}{2}$ quart per second. The y-intercept represents the amount of water in the pitcher when it is full, 4 qt.

Lesson 5-2

A.CED.2, A.CED.3

Writing Equations in Standard and Point-Slope Forms

LESSON GOAL

Students create linear equations in point-slope form and standard form.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Forms of Linear Equations

Develop:

Creating Linear Equations in Point-Slope Form

- Equation in Point-Slope Form Given Slope and a Point
- Equation in Point-Slope Form Given Two Points
- Change to Slope-Intercept Form
- Apply Point-Slope Form
- Change to Standard Form
- Standard Form Given Two Points

Equations of Parallel and Perpendicular Lines

- Parallel Line Through a Given Point
- Perpendicular Line Through a Given Point
- Determine Line Relationships

You may want your students to complete the Checks online

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Greatest Common Factor	●	●		●
Extension: Parallelograms on the Coordinate Plane		●	●	●

Language Development Handbook

Assign page 28 of the *Language Development Handbook* to help your students build mathematical language related to linear equations in point-slope form and standard form.

ELL You can use the tips and suggestions on page T28 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous

Students created linear equations in slope-intercept form.

A.CED.2, S.ID.7

Now

Students create linear equations in point-slope form and standard form.

A.CED.2, A.CED.3

Next

Students will use their understanding of different forms of linear equations to write equations for lines of fit that represent scatter plot data.

S.ID.6a, S.ID.6c

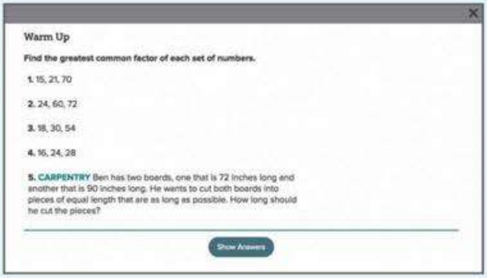
Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of linear equations in two variables to two additional forms, standard form and point-slope form. They build fluency by writing equations in these forms and apply their understanding by using linear equations to solve real-world problems.

Interactive Presentation

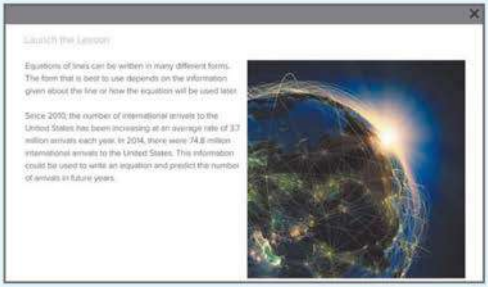


Warm Up

Find the greatest common factor of each set of numbers.

1. 15, 21, 70
2. 24, 60, 72
3. 18, 30, 54
4. 16, 24, 28
5. CARPENTRY Ben has two boards, one that is 72 inches long and another that is 90 inches long. He wants to cut both boards into pieces of equal length that are as long as possible. How long should he cut the pieces?

Warm Up

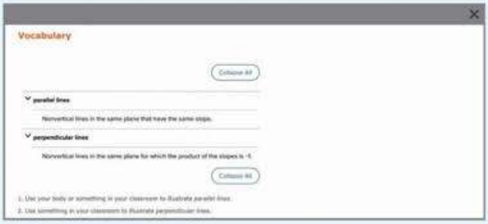


Launch the Lesson

Equations of lines can be written in many different forms. The form that is best to use depends on the information given about the line or how the equation will be used later.

Since 2010, the number of international arrivals to the United States has been increasing at an average rate of 3.7 million arrivals each year. In 2014, there were 74.8 million international arrivals to the United States. This information could be used to write an equation and predict the number of arrivals in future years.

Launch the Lesson



Vocabulary

parallel lines

Nonvertical lines in the same plane that have the same slope.

perpendicular lines

Nonvertical lines in the same plane for which the product of the slopes is −1.

1. Use your body or something in your classroom to illustrate parallel lines.
2. Use something in your classroom to illustrate perpendicular lines.

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- finding the greatest common factor of a set of numbers

Answers:

1. 1
2. 12
3. 6
4. 4
5. 18 in.

Launch the Lesson

Teaching the Mathematical Practices

2 Create Representations Students can use the information in the Launch to write an equation that models the number of international arrivals to the United States since 2010.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

Point-slope form is derived from the definition of slope using the coordinates of two points on a line. Suppose the two points on a line are given as (x, y) and (x_1, y_1). Using the definition of the slope, $m = \frac{y - y_1}{x - x_1}$. If each side of the equation is multiplied by $(x - x_1)$, the result is $y - y_1 = m(x - x_1)$, the point-slope form of a linear equation.

Explore Forms of Linear Equations

Objective

Students use a sketch to explore linear equations in point-slope form and slope-intercept form.

Teaching the Mathematical Practices

2 Represent a Situation Symbolically Guide students to define variables to solve the problem in this Explore. Help students to identify the independent and dependent variables. Then work with them to find other relationships in the problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

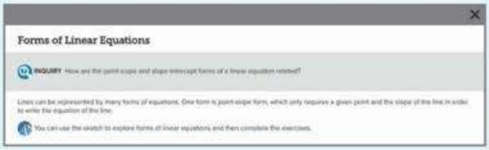
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to plot a point, and then draw lines with different slopes that pass through the point. They also will observe and compare the equations of the lines that they draw, written in both slope-intercept form and point-slope form. They will be guided through the exploration by a series of questions. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



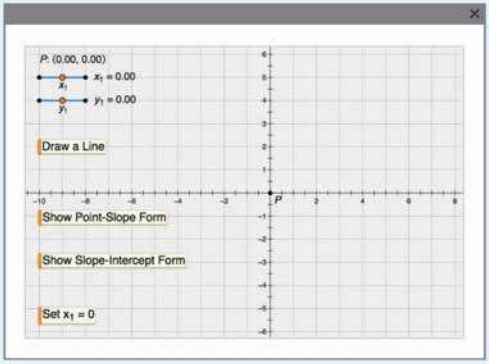
Forms of Linear Equations

INQUIRY How are the point-slope and slope-intercept forms of a linear equation related?

Lines can be represented by many forms of equations. One form is point-slope form, which only requires a given point and the slope of the line in order to write the equation of the line.

You can use the sketch to explore forms of linear equations and then complete the exercises.

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore forms of linear equations and complete the exercises.

TYPE

Students observe and compare the equations of lines in different forms.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Forms of Linear Equations (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How could you write the slope-intercept form of the equation if you know the point-slope form? Solve the equation for y.

Inquiry

How are the point-slope and slope-intercept forms of a linear equation related? Sample answer: They both contain x, y, and the slope, but the point-slope form contains the coordinates of a point on the line, while the slope-intercept form contains the y-intercept.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Creating Linear Equations in Point-Slope Form

Objective

Students create linear equations in point-slope form by applying the properties of equality.

Teaching the Mathematical Practices

2 Create Representations Guide students to write an equation of a line in point-slope form using the steps outlined in this Learn.

About the Key Concept

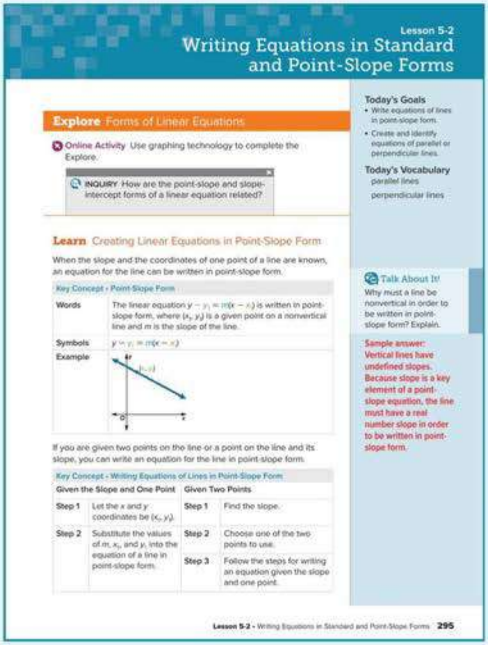
The point-slope form of a line is actually a transformation of the slope formula, $m = \frac{y - y_1}{x - x_1}$. If you multiply both sides of the formula by the expression in the denominator of the fraction, you get the point-slope form of a line, $y - y_1 = m(x - x_1)$.

Common Misconception

A common misconception some students may have is believing that one of the numbers in an equation written in point-slope form is the *y*-intercept of the line, as is the case with equations written in slope-intercept form. Provide examples of equations written in point-slope form, and have students identify what each number and each variable represent.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-2

Writing Equations in Standard and Point-Slope Forms

Explore Forms of Linear Equations

Online Activity Use graphing technology to complete the Explore.

INQUIRY How are the point-slope and slope-intercept forms of a linear equation related?

Today's Goals
- Write equations of lines in point-slope form.
- Create and identify equations of parallel or perpendicular lines.

Today's Vocabulary
parallel lines
perpendicular lines

Learn Creating Linear Equations in Point-Slope Form

When the slope and the coordinates of one point of a line are known, an equation for the line can be written in point-slope form.

Key Concept • Point-Slope Form

Words	The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.
Symbols	$y - y_1 = m(x - x_1)$
Example	

If you are given two points on the line or a point on the line and its slope, you can write an equation for the line in point-slope form.

Key Concept • Writing Equations of Lines in Point-Slope Form

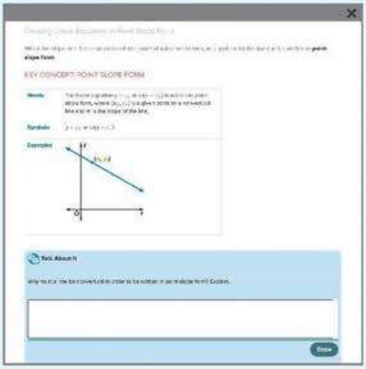
Given the Slope and One Point		Given Two Points	
Step 1	Let the x and y coordinates be (x_1, y_1).	Step 1	Find the slope.
Step 2	Substitute the values of m, x_1, and y_1 into the equation of a line in point-slope form.	Step 2	Choose one of the two points to use.
		Step 3	Follow the steps for writing an equation given the slope and one point.

Talk About It! Why must a line be nonvertical in order to be written in point-slope form? Explain.

Sample answer: Vertical lines have undefined slopes. Because slope is a key element of a point-slope equation, the line must have a real number slope in order to be written in point-slope form.

Lesson 5-2 • Writing Equations in Standard and Point-Slope Forms 295

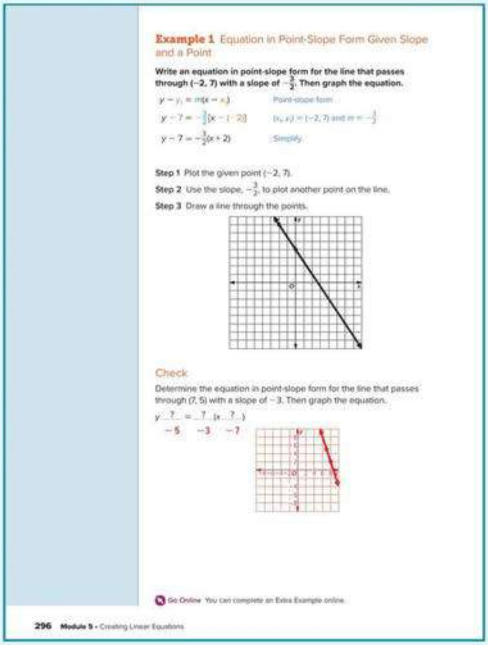
Interactive Presentation



Learn

TYPE

Students answer a question to show they understand why a line must be nonvertical to write the equation in point-slope form.



Example 1 Equation in Point-Slope Form Given Slope and a Point

Write an equation in point-slope form for the line that passes through (−2, 7) with a slope of $-\frac{3}{2}$. Then graph the equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 7 = -\frac{3}{2}[x - (-2)]$ $(x_1, y_1) = (-2, 7)$ and $m = -\frac{3}{2}$

$y - 7 = -\frac{3}{2}(x + 2)$ Simplify.

Step 1 Plot the given point (−2, 7).

Step 2 Use the slope, $-\frac{3}{2}$, to plot another point on the line.

Step 3 Draw a line through the points.

Check

Determine the equation in point-slope form for the line that passes through (7, 5) with a slope of −3. Then graph the equation.

y __?__ $=$ __?__ $(x$ __?__ $)$

−5 −3 −7

Go Online You can complete an Extra Example online.

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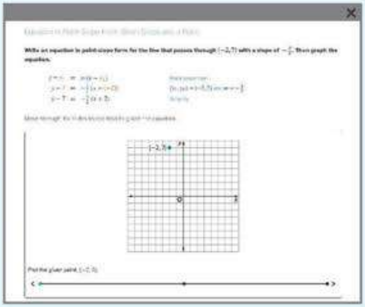
Interactive Presentation



Example 1

TAP

Students move through the steps to see how to graph the equation.

Example 1 Equation in Point-Slope Form Given Slope and a Point

Teaching the Mathematical Practices

4 Use Tools Point out that to solve the problem in this example, students will need to use the formula for point-slope form.

Questions for Mathematical Discourse

AL What is the point-slope form of a linear equation? $(y - y_1) = m(x - x_1)$ What values will you substitute into the equation? I will substitute −2 for x_1, 7 for y_1, and $-\frac{3}{2}$ for m.

OL In the final answer, why is the quantity in the parentheses $x + 2$ instead of $x - 2$? When you substitute −2 for x_1 you get $x - (-2)$, which simplifies to $x + 2$.

BL If the equation of a line in point-slope form is $y + 1 = 5(x + 7)$, what is the slope and what is an ordered pair that the line passes through? Sample answer: The slope is 5 and an ordered pair is (−7, −1).

Common Error

Students may interchange the substitutions when substituting for x_1 and y_1 because y_1 appears first in the equation while x_1 appears first in the coordinates. Remind them that the y-coordinate is subtracted from y and the x-coordinate is subtracted from x.

Example 2 Equation in Point-Slope Form Given Two Points

Teaching the Mathematical Practices

2 Different Properties Mathematically proficient students look for different ways to solve problems. Encourage them to substitute the other point into their equation to check their answer.

Questions for Mathematical Discourse

AL What two things do you need to know in order to write an equation in point-slope form? a point and the slope Do you have the information you need to write an equation in point-slope form? Explain. No; the slope is missing.

OL In Step 1, could you have used $(x_1, y_1) = (6, -3)$ and $(x_2, y_2) = (2, -7)$? yes Explain. Sample answer: Switching the ordered pairs gives $m = \frac{-7 - (-3)}{2 - 6} = \frac{-4}{-4}$ or 1 which is the same as what was calculated the first time.

BL How can you check your answer? Sample answer: I can take the coordinates of the point that I did not use to write the equation, substitute them for x and y in the final equation, and check that they satisfy the equation.

Common Error

Some students may try to use both points when substituting in Step 2. Point out that after Step 1, the procedure is identical to the procedure they learned in the previous examples, where the given information included the slope and one point.

Example 3 Change to Slope-Intercept Form

Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out in this example, students need to check their answer. Point out that they should ask themselves whether their answer makes sense and whether they have answered the problem question.

Questions for Mathematical Discourse

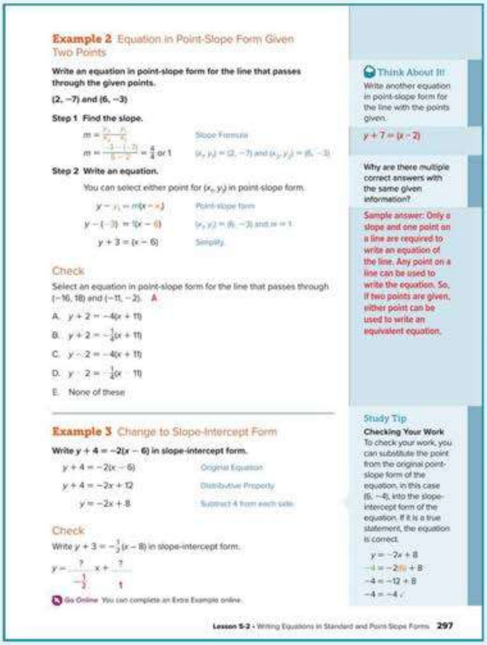
AL What operation do the parentheses in the point-slope form indicate? multiplication

OL What is the first step in rewriting the equation? Use the Distributive Property to rewrite $-2(x - 6)$ as $-2x + 12$.

BL How can you check the answer? Substitute the coordinates of the point $(6, -4)$ into the slope-intercept form of the equation to see if the equation is true.

Common Error

Some students may look at the equation that results after applying the Distributive Property, and identify the y-intercept of the equation as 12. Remind them that the equation must be solved for y in order for the number being added to the x-term to be the y-intercept.



Example 2 Equation in Point-Slope Form Given Two Points

Write an equation in point-slope form for the line that passes through the given points.

(2, −7) and (6, −3)

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{-3 - (-7)}{6 - 2} = \frac{4}{4}$ or 1 $(x_1, y_1) = (2, -7)$ and $(x_2, y_2) = (6, -3)$

Step 2 Write an equation.

You can select either point for (x_1, y_1) in point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-3) = 1(x - 6)$ $(x_1, y_1) = (6, -3)$ and $m = 1$

$y + 3 = (x - 6)$ Simplify.

Check

Select an equation in point-slope form for the line that passes through $(-16, 18)$ and $(-11, -2)$. A

A. $y + 2 = -4(x + 11)$

B. $y + 2 = -\frac{1}{4}(x + 11)$

C. $y - 2 = -4(x + 11)$

D. $y - 2 = -\frac{1}{4}(x - 11)$

E. None of these

Think About It!

Write another equation in point-slope form for the line with the points given.

$y + 7 = (x - 2)$

Why are there multiple correct answers with the same given information?

Sample answer: Only a slope and one point on a line are required to write an equation of the line. Any point on a line can be used to write the equation. So, if two points are given, either point can be used to write an equivalent equation.

Example 3 Change to Slope-Intercept Form

Write $y + 4 = -2(x - 6)$ in slope-intercept form.

$y + 4 = -2(x - 6)$ Original Equation

$y + 4 = -2x + 12$ Distributive Property

$y = -2x + 8$ Subtract 4 from each side.

Check

Write $y + 3 = -\frac{1}{2}(x - 8)$ in slope-intercept form.

$y = \underline{?}\,x + \underline{?}$

$-\frac{1}{2}$ 1

Go Online You can complete an Extra Example online.

Study Tip

Checking Your Work To check your work, you can substitute the point from the original point-slope form of the equation, in this case $(6, -4)$, into the slope-intercept form of the equation. If it is a true statement, the equation is correct.

$y = -2x + 8$

$-4 = -2(6) + 8$

$-4 = -12 + 8$

$-4 = -4$ ✓

Lesson 5-2 • Writing Equations in Standard and Point-Slope Forms 297

Interactive Presentation

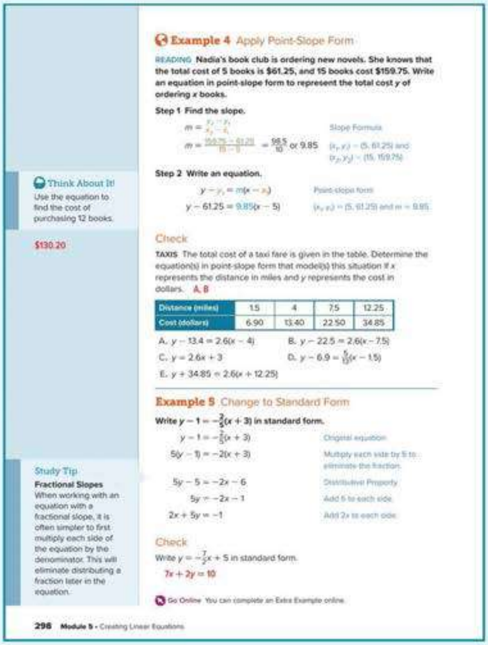
Example 3

TAP

Students select the correct property used when writing a line in slope-intercept form.

TYPE

Students enter the value subtracted from both sides of the equation.



Example 4 Apply Point-Slope Form

READING **Nadia's book club is ordering new novels. She knows that the total cost of 5 books is $61.25, and 15 books cost $159.75. Write an equation in point-slope form to represent the total cost *y* of ordering *x* books.**

Step 1 **Find the slope.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{159.75 - 61.25}{15 - 5} = \frac{98.5}{10}$ or 9.85 $(x_1, y_1) = (5, 61.25)$ and $(x_2, y_2) = (15, 159.75)$

Step 2 **Write an equation.**

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 61.25 = 9.85(x - 5)$ $(x_1, y_1) = (5, 61.25)$ and $m = 9.85$

Think About It!

Use the equation to find the cost of purchasing 12 books.

$130.20

Check

TAXIS The total cost of a taxi fare is given in the table. Determine the equation(s) in point-slope form that model(s) this situation if *x* represents the distance in miles and *y* represents the cost in dollars. A, B

Distance (miles)	1.5	4	7.5	12.25
Cost (dollars)	6.90	13.40	22.50	34.85

A. $y - 13.4 = 2.6(x - 4)$
B. $y - 22.5 = 2.6(x - 7.5)$
C. $y = 2.6x + 3$
D. $y - 6.9 = \frac{5}{13}(x - 1.5)$
E. $y + 34.85 = 2.6(x + 12.25)$

Example 5 Change to Standard Form

Write $y - 1 = -\frac{2}{5}(x + 3)$ in standard form.

$y - 1 = -\frac{2}{5}(x + 3)$ Original equation

$5(y - 1) = -2(x + 3)$ Multiply each side by 5 to eliminate the fraction.

$5y - 5 = -2x - 6$ Distributive Property

$5y = -2x - 1$ Add 5 to each side.

$2x + 5y = -1$ Add 2x to each side.

Study Tip

Fractional Slopes When working with an equation with a fractional slope, it is often simpler to first multiply each side of the equation by the denominator. This will eliminate distributing a fraction later in the equation.

Check

Write $y = -\frac{7}{2}x + 5$ in standard form.

$7x + 2y = 10$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4

TYPE

Students complete the calculations to find the cost of purchasing 12 books.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Apply Point-Slope Form

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about writing equations in point-slope form to solving a real-world problem.

Questions for Mathematical Discourse

AL What ordered pairs could you use to help write the equation? (5, 61.25) and (15, 159.75)

OL How can you calculate the slope from the information in the problem? I can use the formula for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, and the points (5, 61.25) and (15, 159.75).

BL Could you write a different equation to represent this same situation? Explain. Yes; sample answer: I could use the other point to write an equation in point-slope form: $y - 159.75 = 9.85(x - 15)$. I could also rewrite the equation in slope-intercept form: $y = 9.85x + 12$.

Essential Question Follow-Up

Students have created linear equations in different forms to represent real-world situations.

Ask:

When an equation that models a real-world situation is written in slope-intercept form or point-slope form, you can easily identify the slope. What does the slope tell you about the situation? Sample answer: It provides information about the rate at which the quantities in the situation are increasing or decreasing.

Example 5 Change to Standard Form

Teaching the Mathematical Practices

1 Seek Information Mathematically proficient students must be able to transform algebraic expressions to reach solutions. Point out that gaining fluency in this skill is as important as learning their math facts was in the elementary grades.

Questions for Mathematical Discourse

AL What is the standard form of a linear equation? $Ax + By = C$

OL Why can the equation not be left as $-2x - 5y = 1$? Sample answer: In standard form, the coefficient of *x* has to be greater than or equal to 0.

BL Could you do the operations in a different order and still arrive at $2x + 5y = -1$ as the answer? Explain. Yes; sample answer: I could distribute the $-\frac{2}{5}$ first to get $y - 1 = -\frac{2}{5}x - \frac{6}{5}$. Then I could multiply each side by 5 to get $5y - 5 = -2x - 6$.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty recognizing when an equation is written in standard form,

THEN present them with a list of 10 equations, some in standard form, others not in standard form, and have them work with a partner to sort the equations into those that fit the requirements of standard form and those that do not. For those equations that do not, have the students indicate why not.

Example 6 Standard Form Given Two Points

MP Teaching the Mathematical Practices

1 Special Cases Work with students to evaluate the two methods. Encourage students to familiarize themselves with both methods, and to know the best time to use each one.

Questions for Mathematical Discourse

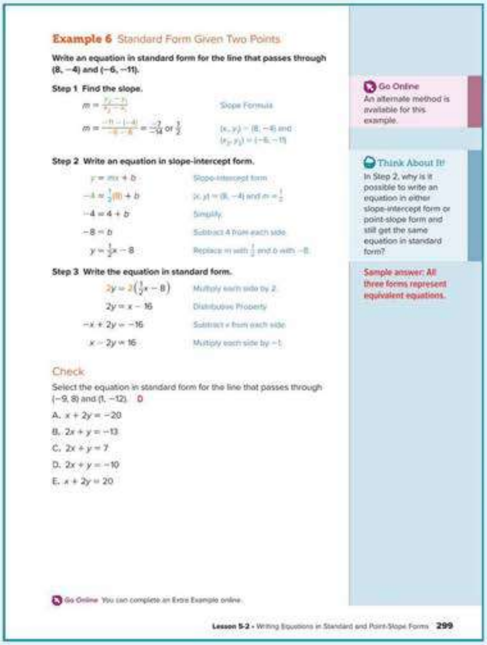
AL Does the problem provide enough information to directly write an equation in standard form? Explain. No; sample answer: If you only know two points that lie on the line, you cannot write an equation in standard form without first writing it in one of the other forms.

OL Why must you first write the equation in either slope-intercept form or point-slope form? Sample answer: The information provided cannot be used to determine the values of A, B, and C in standard form. Therefore, you must use the information to write the equation in one of the other forms first, and then rewrite the resulting equation in standard form.

BL How can you check whether the equation in slope-intercept form and the equation in point-slope form are equations for the same line? Sample answer: I can rewrite both equations in standard form and see that they produce the same equation.

Common Error

Some students may confuse the three forms of a linear equation. For these students, it may be helpful for them to see the three forms of a particular equation side-by-side so that they can compare their structures and identify what the numbers represent.



Example 6 Standard Form Given Two Points

Write an equation in standard form for the line that passes through (8, −4) and (−6, −11).

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{-11 - (-4)}{-6 - 8} = \frac{-7}{-14}$ or $\frac{1}{2}$ $(x_1, y_1) = (8, -4)$ and $(x_2, y_2) = (-6, -11)$

Step 2 Write an equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$-4 = \frac{1}{2}(8) + b$ $(x, y) = (8, -4)$ and $m = \frac{1}{2}$

$-4 = 4 + b$ Simplify.

$-8 = b$ Subtract 4 from each side.

$y = \frac{1}{2}x - 8$ Replace m with $\frac{1}{2}$ and b with -8.

Step 3 Write the equation in standard form.

$2y = 2\left(\frac{1}{2}x - 8\right)$ Multiply each side by 2.

$2y = x - 16$ Distributive Property

$-x + 2y = -16$ Subtract x from each side.

$x - 2y = 16$ Multiply each side by -1.

Check

Select the equation in standard form for the line that passes through (−9, 8) and (1, −12). D

A. $x + 2y = -20$

B. $2x + y = -13$

C. $2x + y = 7$

D. $2x + y = -10$

E. $x + 2y = 20$

Go Online An alternate method is available for this example.

Think About It! In Step 2, why is it possible to write an equation in either slope-intercept form or point-slope form and still get the same equation in standard form?

Sample answer: All three forms represent equivalent equations.

Go Online You can complete an Extra Example online.

Lesson 5-2 • Writing Equations in Standard and Point-Slope Forms 299

Interactive Presentation

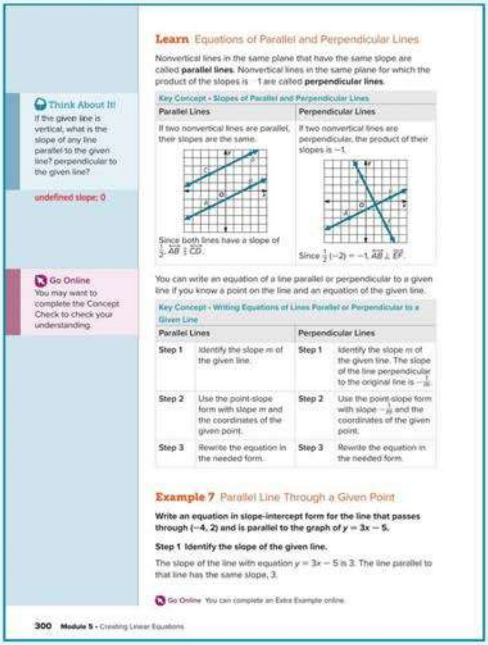
Example 6

TAP

Students select a method to write the equation in standard form.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Think About It! If the given line is vertical, what is the slope of any line parallel to the given line? perpendicular to the given line?

undefined slope; 0

Go Online You may want to complete the Concept Check to check your understanding.

Learn Equations of Parallel and Perpendicular Lines

Nonvertical lines in the same plane that have the same slope are called **parallel lines**. Nonvertical lines in the same plane for which the product of the slopes is -1 are called **perpendicular lines**.

Key Concept • Slopes of Parallel and Perpendicular Lines

Parallel Lines	Perpendicular Lines
If two nonvertical lines are parallel, their slopes are the same.	If two nonvertical lines are perpendicular, the product of their slopes is -1.
Since both lines have a slope of $\frac{1}{2}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.	Since $\frac{1}{2}(-2) = -1$, $\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$.

You can write an equation of a line parallel or perpendicular to a given line if you know a point on the line and an equation of the given line.

Key Concept • Writing Equations of Lines Parallel or Perpendicular to a Given Line

Parallel Lines		Perpendicular Lines	
Step 1	Identify the slope m of the given line.	Step 1	Identify the slope m of the given line. The slope of the line perpendicular to the original line is $-\frac{1}{m}$.
Step 2	Use the point-slope form with slope m and the coordinates of the given point.	Step 2	Use the point-slope form with slope $-\frac{1}{m}$ and the coordinates of the given point.
Step 3	Rewrite the equation in the needed form.	Step 3	Rewrite the equation in the needed form.

Example 7 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-4, 2)$ and is parallel to the graph of $y = 3x - 5$.

Step 1 Identify the slope of the given line.

The slope of the line with equation $y = 3x - 5$ is 3. The line parallel to that line has the same slope, 3.

Go Online You can complete an Extra Example online.

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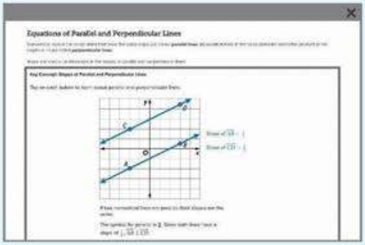
Interactive Presentation



Equations of Parallel and Perpendicular Lines

Learn

TAP

Students tap each button to learn more about parallel and perpendicular lines.

TYPE

Students determine the slope of any line parallel to a vertical line.

Learn Equations of Parallel and Perpendicular Lines

Objective

Students create and identify equations of parallel or perpendicular lines by using slope criteria.

MP Teaching the Mathematical Practices

2 Create Representations Guide students to write the equations of two parallel lines and two perpendicular lines using the information in this Learn.

About the Key Concept

If two lines are parallel, they will have the same slope. If two lines are perpendicular, their slopes will be negative reciprocals.

Common Misconception

Some students may believe that $-\frac{1}{m}$ will always result in a negative slope. Help them to see that when m is a negative number, $-\frac{1}{m}$ will be a positive number. For these students, it may be helpful for them to use the phrase "the opposite of the reciprocal" when working with the slopes of perpendicular lines.

Example 7 Parallel Line Through a Given Point

MP Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students need to check their answer. Point out that they should ask themselves whether their answer makes sense and whether they have answered the problem question.

Questions for Mathematical Discourse

AL What is the relationship between parallel lines? They have the same slope.

OL Why might you use the point-slope form of a line to write the equation, even though the problem asks for the equation in slope-intercept form? I know a point that lies on the line, and the slope of the line from the given equation. I can then rewrite the point-slope form equation in slope-intercept form.

BL When graphing these lines, which line would appear above the other? $y = 3x + 14$ How do you know? Sample answer: $y = 3x + 14$ has a positive y-intercept while $y = 3x - 5$ has a negative y-intercept, so $y = 3x + 14$ would be on top.

Common Error

Some students may try to use the *y*-intercept, as well as the slope, from the given line. Explain that only the *slopes* of parallel lines are related, and that the only information needed about the given line is its slope.

Example 8 Perpendicular Line Through a Given Point

MP Teaching the Mathematical Practices

1 Seek Information Mathematically proficient students must be able to transform algebraic expressions to reach solutions. Point out that gaining fluency in this skill is as important as learning their math facts was in the elementary grades.

Questions for Mathematical Discourse

AL Do you know the slope and *y*-intercept of the new line? no Which one should you find first? the slope

OL How do you know what to do first? Sample answer: I need to find the slope of the line. So, write the given equation in slope-intercept form by solving for *y*. The slope of the given line will be the coefficient of *x*, which is $\frac{3}{2}$.

BL How can you check the answer graphically? Sample answer: Graph both lines to verify that the lines are perpendicular and that the second line passes through the point given. Perpendicular lines meet at right angles, so if I put the corner of a piece of paper on the point of intersection and the edges of the paper align with the two lines, then the lines are perpendicular.

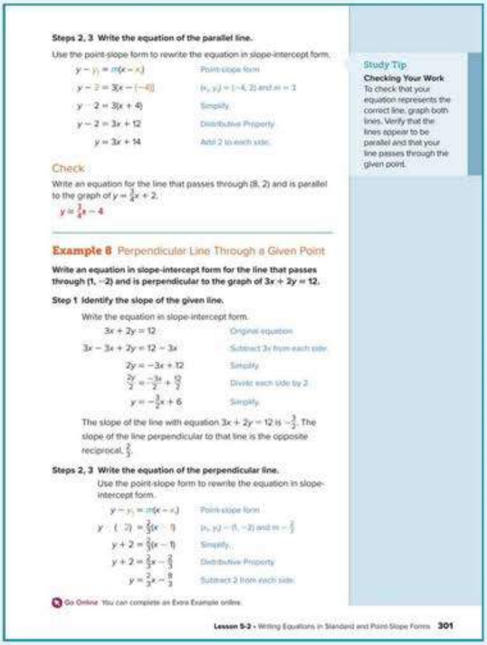
Common Error

Some students may make an error when the slope of the given line is a whole number. Remind students that a whole number can be written as a fraction with denominator 1. For example, if the slope is 4, students can write it as $\frac{4}{1}$, and then find the opposite of the reciprocal for the slope of the perpendicular line, which will be $-\frac{1}{4}$.

DIFFERENTIATE

Enrichment Activity BL

Write $4x + 3y = 8$ on the board. Ask students to rewrite the equation in slope-intercept form. Have students name the slope and draw a conclusion about the relationship between the slope and the values of *A* and *B* when an equation is written in standard form, $Ax + By = C$. Then ask students to describe why it might be useful to write equations in slope-intercept form. $y = -\frac{4}{3}x + \frac{8}{3}$; The slope is $-\frac{4}{3}$, so $m = -\frac{A}{B}$; sample answer: It is easy to identify the slope and *y*-intercept when an equation is in slope-intercept form, which can make graphing easier.



Steps 2, 3 Write the equation of the parallel line.

Use the point-slope form to rewrite the equation in slope-intercept form.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - 2 &= 3[x - (-4)] && (x_1, y_1) = (-4, 2) \text{ and } m = 3 \\ y - 2 &= 3(x + 4) && \text{Simplify.} \\ y - 2 &= 3x + 12 && \text{Distributive Property} \\ y &= 3x + 14 && \text{Add 2 to each side.} \end{aligned}$$

Check

Write an equation for the line that passes through (8, 2) and is parallel to the graph of $y = \frac{3}{4}x + 2$.

$y = \frac{3}{4}x - 4$

Example 8 Perpendicular Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through (1, −2) and is perpendicular to the graph of $3x + 2y = 12$.

Step 1 Identify the slope of the given line.

Write the equation in slope-intercept form.

$$\begin{aligned} 3x + 2y &= 12 && \text{Original equation} \\ 3x - 3x + 2y &= 12 - 3x && \text{Subtract } 3x \text{ from each side.} \\ 2y &= -3x + 12 && \text{Simplify.} \\ \frac{2y}{2} &= \frac{-3x}{2} + \frac{12}{2} && \text{Divide each side by 2.} \\ y &= -\frac{3}{2}x + 6 && \text{Simplify.} \end{aligned}$$

The slope of the line with equation $3x + 2y = 12$ is $-\frac{3}{2}$. The slope of the line perpendicular to that line is the opposite reciprocal, $\frac{2}{3}$.

Steps 2, 3 Write the equation of the perpendicular line.

Use the point-slope form to rewrite the equation in slope-intercept form.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - (-2) &= \frac{2}{3}(x - 1) && (x_1, y_1) = (1, -2) \text{ and } m = \frac{2}{3} \\ y + 2 &= \frac{2}{3}(x - 1) && \text{Simplify.} \\ y + 2 &= \frac{2}{3}x - \frac{2}{3} && \text{Distributive Property} \\ y &= \frac{2}{3}x - \frac{8}{3} && \text{Subtract 2 from each side.} \end{aligned}$$

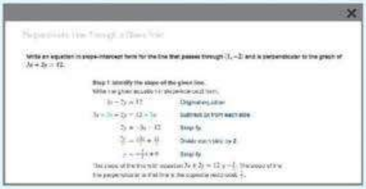
Go Online You can complete an Extra Example online.

Study Tip

Checking Your Work To check that your equation represents the correct line, graph both lines. Verify that the lines appear to be parallel and that your line passes through the given point.

Lesson 5-2 • Writing Equations in Standard and Point-Slope Forms 301

Interactive Presentation



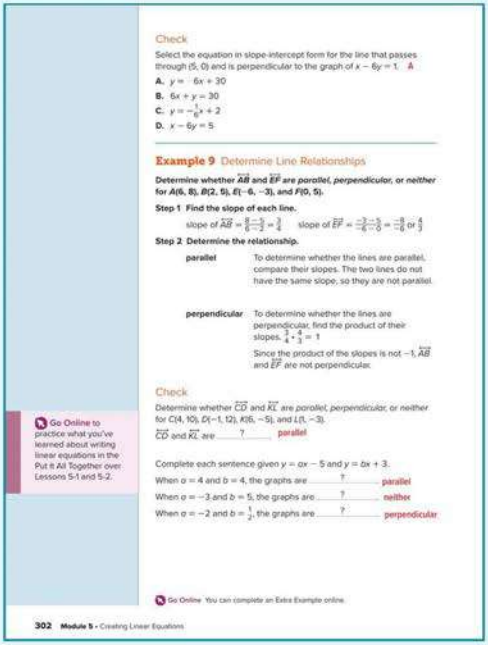
Example 8

TAP

Students move through the steps to write an equation in slope-intercept form that is perpendicular to another line.

TYPE

Students describe an error made when finding an equation of a line perpendicular and through a point.



Check

Select the equation in slope-intercept form for the line that passes through (5, 0) and is perpendicular to the graph of $x - 6y = 1$. A

A. $y = -6x + 30$

B. $6x + y = 30$

C. $y = -\frac{1}{6}x + 2$

D. $x - 6y = 5$

Example 9 Determine Line Relationships

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ are *parallel, perpendicular,* or *neither* for $A(6, 8)$, $B(2, 5)$, $E(-6, -3)$, and $F(0, 5)$.

Step 1 Find the slope of each line.

slope of $\overleftrightarrow{AB} = \frac{8-5}{6-2} = \frac{3}{4}$ slope of $\overleftrightarrow{EF} = \frac{-3-5}{-6-0} = \frac{-8}{-6}$ or $\frac{4}{3}$

Step 2 Determine the relationship.

parallel To determine whether the lines are parallel, compare their slopes. The two lines do not have the same slope, so they are not parallel.

perpendicular To determine whether the lines are perpendicular, find the product of their slopes. $\frac{3}{4} \cdot \frac{4}{3} = 1$

Since the product of the slopes is not −1, $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ are not perpendicular.

Go Online to practice what you've learned about writing linear equations in the Put It All Together over Lessons 5-1 and 5-2.

Check

Determine whether $\overleftrightarrow{CD}$ and $\overleftrightarrow{KL}$ are *parallel, perpendicular,* or *neither* for $C(4, 10)$, $D(-1, 12)$, $K(6, -5)$, and $L(1, -3)$.

$\overleftrightarrow{CD}$ and $\overleftrightarrow{KL}$ are ___?___ parallel

Complete each sentence given $y = ax - 5$ and $y = bx + 3$.

When $a = 4$ and $b = 4$, the graphs are ___?___ parallel

When $a = -3$ and $b = 5$, the graphs are ___?___ neither

When $a = -2$ and $b = \frac{1}{2}$, the graphs are ___?___ perpendicular

Go Online You can complete an Extra Example online.

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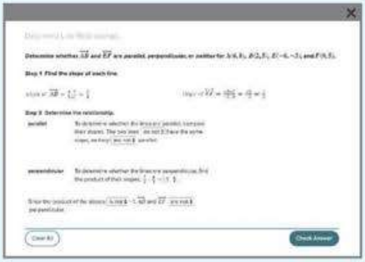
Interactive Presentation



Example 9

TAP

Students select the correct value or phrase to complete each sentence.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 9 Determine Line Relationships

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress and change course to find the ultimate solution.

Questions for Mathematical Discourse

AL What do you need to know about a pair of lines in order to determine whether they are parallel or perpendicular? their slopes

OL What is true about the slopes of parallel lines? They are the same. What is true about the slopes of perpendicular lines? They are negative reciprocals of each other, so their product is −1.

BL Why is the product of the slopes of a pair of perpendicular lines always the same number? Sample answer: Because the slopes of perpendicular lines are negative reciprocals of each other, and the product of a number and its reciprocal is always 1, the product of the slopes will always be −1.

Common Error

Some students may forget to simplify one or both slopes, and therefore may not correctly identify the relationship between them. To help students avoid making this error, encourage students to write the slopes in simplest form.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–36
2	exercises that use a variety of skills from this lesson	37–48
3	exercises that emphasize higher-order and critical-thinking skills	49–55

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

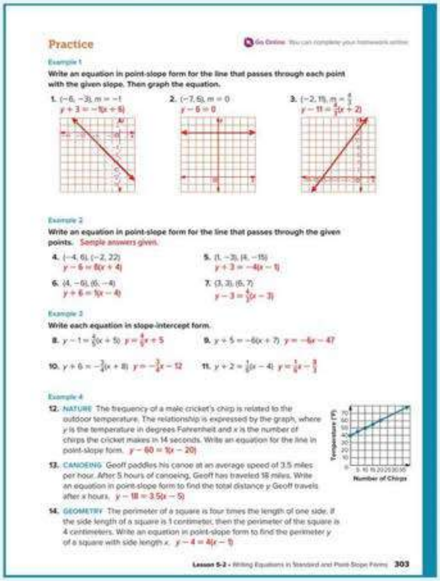
- Practice, Exercises 1–47 odd, 49–55
- Extension: Parallelograms on the Coordinate Plane
- **ALEKS** Equations of Lines

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice Exercises 1–55 odd
- Remediation, Review Resources: Greatest Common Factor
- Personal Tutors
- Extra Examples 1–9
- **ALEKS** Prime Numbers, Factors, and Multiples

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice Exercises 1–35 odd
- Remediation, Review Resources: Greatest Common Factor
- *Quick Review Math Handbook*: Writing Equations in Point-Slope Form
- Arrive**MATH** Take Another Look
- **ALEKS** Prime Numbers, Factors, and Multiples



Practice

Go Online You can complete your homework online.

Example 1

Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

1. $(-6, -3), m = -1$ $y + 3 = -1(x + 6)$
2. $(-7, 6), m = 0$ $y - 6 = 0$
3. $(-2, 11), m = \frac{4}{3}$ $y - 11 = \frac{4}{3}(x + 2)$

Example 2

Write an equation in point-slope form for the line that passes through the given points. Sample answers given.

4. $(-4, 6), (-2, 22)$ $y - 6 = 8(x + 4)$
5. $(1, -3), (4, -15)$ $y + 3 = -4(x - 1)$
6. $(4, -6), (6, -4)$ $y + 6 = 1(x - 4)$
7. $(3, 3), (6, 7)$ $y - 3 = \frac{4}{3}(x - 3)$

Example 3

Write each equation in slope-intercept form.

8. $y - 1 = \frac{4}{5}(x + 5)$ $y = \frac{4}{5}x + 5$
9. $y + 5 = -6(x + 7)$ $y = -6x - 47$
10. $y + 6 = -\frac{3}{4}(x + 8)$ $y = -\frac{3}{4}x - 12$
11. $y + 2 = \frac{1}{6}(x - 4)$ $y = \frac{1}{6}x - \frac{8}{3}$

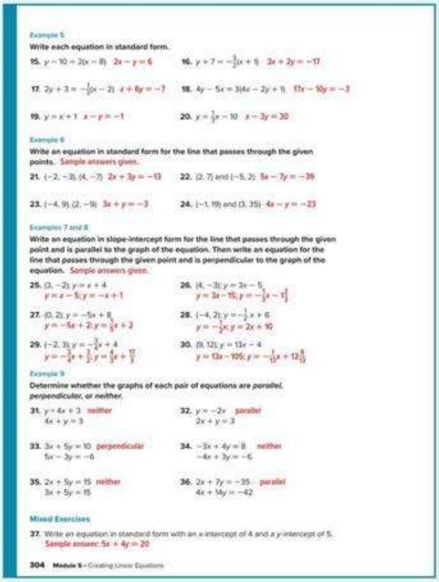
Example 4

12. NATURE The frequency of a male cricket's chirp is related to the outdoor temperature. The relationship is expressed by the graph, where y is the temperature in degrees Fahrenheit and x is the number of chirps the cricket makes in 14 seconds. Write an equation for the line in point-slope form. $y - 60 = 1(x - 20)$

13. CANOEING Geoff paddles his canoe at an average speed of 3.5 miles per hour. After 5 hours of canoeing, Geoff has traveled 18 miles. Write an equation in point-slope form to find the total distance y Geoff travels after x hours. $y - 18 = 3.5(x - 5)$

14. GEOMETRY The perimeter of a square is four times the length of one side. If the side length of a square is 1 centimeter, then the perimeter of the square is 4 centimeters. Write an equation in point-slope form to find the perimeter y of a square with side length x. $y - 4 = 4(x - 1)$

Lesson 5-2 • Writing Equations in Standard and Point-Slope Forms **303**



Example 5

Write each equation in standard form.

15. $y - 10 = 2(x - 8)$ $2x - y = 6$
16. $y + 7 = -\frac{3}{2}(x + 1)$ $3x + 2y = -17$
17. $2y + 3 = -\frac{1}{3}(x - 2)$ $x + 6y = -7$
18. $4y - 5x = 3(4x - 2y + 1)$ $17x - 10y = -3$
19. $y = x + 1$ $x - y = -1$
20. $y = \frac{1}{3}x - 10$ $x - 3y = 30$

Example 6

Write an equation in standard form for the line that passes through the given points. Sample answers given.

21. $(-2, -3), (4, -7)$ $2x + 3y = -13$
22. $(2, 7)$ and $(-5, 2)$ $5x - 7y = -39$
23. $(-4, 9), (2, -9)$ $3x + y = -3$
24. $(-1, 19)$ and $(3, 35)$ $4x - y = -23$

Examples 7 and 8

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the equation. Then write an equation for the line that passes through the given point and is perpendicular to the graph of the equation. Sample answers given.

25. $(3, -2); y = x + 4$
$y = x - 5; y = -x + 1$
26. $(4, -3); y = 3x - 5$
$y = 3x - 15; y = -\frac{1}{3}x - 1\frac{2}{3}$
27. $(0, 2); y = -5x + 8$
$y = -5x + 2; y = \frac{1}{5}x + 2$
28. $(-4, 2); y = -\frac{1}{2}x + 6$
$y = -\frac{1}{2}x; y = 2x + 10$
29. $(-2, 3); y = -\frac{3}{4}x + 4$
$y = -\frac{3}{4}x + \frac{3}{2}; y = \frac{4}{3}x + \frac{17}{3}$
30. $(9, 12); y = 13x - 4$
$y = 13x - 105; y = -\frac{1}{13}x + 12\frac{9}{13}$

Example 9

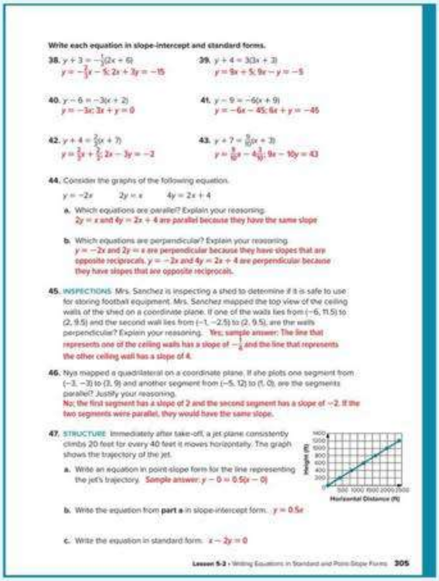
Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

31. $y = 4x + 3$ neither
$4x + y = 3$
32. $y = -2x$ parallel
$2x + y = 3$
33. $3x + 5y = 10$ perpendicular
$5x - 3y = -6$
34. $-3x + 4y = 8$ neither
$-4x + 3y = -6$
35. $2x + 5y = 15$ neither
$3x + 5y = 15$
36. $2x + 7y = -35$ parallel
$4x + 14y = -42$

Mixed Exercises

37. Write an equation in standard form with an x-intercept of 4 and a y-intercept of 5.
Sample answer: $5x + 4y = 20$

304 **Module 5** • Creating Linear Equations



Write each equation in slope-intercept and standard forms.

38. $y+3=-\frac{1}{3}(2x+6)$
$y=-\frac{2}{3}x-5$; $2x+3y=-15$

39. $y+4=3(3x+3)$
$y=9x+5$; $9x-y=-5$

40. $y-6=-3(x+2)$
$y=-3x$; $3x+y=0$

41. $y-9=-6(x+9)$
$y=-6x-45$; $6x+y=-45$

42. $y+4=\frac{2}{3}(x+7)$
$y=\frac{2}{3}x+\frac{2}{3}$; $2x-3y=-2$

43. $y+7=\frac{9}{10}(x+3)$
$y=\frac{9}{10}x-4\frac{3}{10}$; $9x-10y=43$

44. Consider the graphs of the following equation.

$y=-2x$ $\quad$ $2y=x$ $\quad$ $4y=2x+4$

a. Which equations are parallel? Explain your reasoning.
$2y=x$ and $4y=2x+4$ are parallel because they have the same slope

b. Which equations are perpendicular? Explain your reasoning.
$y=-2x$ and $2y=x$ are perpendicular because they have slopes that are opposite reciprocals. $y=-2x$ and $4y=2x+4$ are perpendicular because they have slopes that are opposite reciprocals.

45. INSPECTIONS Mrs. Sanchez is inspecting a shed to determine if it is safe to use for storing football equipment. Mrs. Sanchez mapped the top view of the ceiling walls of the shed on a coordinate plane. If one of the walls lies from (−6, 11.5) to (2, 9.5) and the second wall lies from (−1, −2.5) to (2, 9.5), are the walls perpendicular? Explain your reasoning. Yes; sample answer: The line that represents one of the ceiling walls has a slope of $-\frac{1}{4}$ and the line that represents the other ceiling wall has a slope of 4.

46. Nya mapped a quadrilateral on a coordinate plane. If she plots one segment from (−3, −3) to (3, 9) and another segment from (−5, 12) to (1, 0), are the segments parallel? Justify your reasoning.
No; the first segment has a slope of 2 and the second segment has a slope of −2. If the two segments were parallel, they would have the same slope.

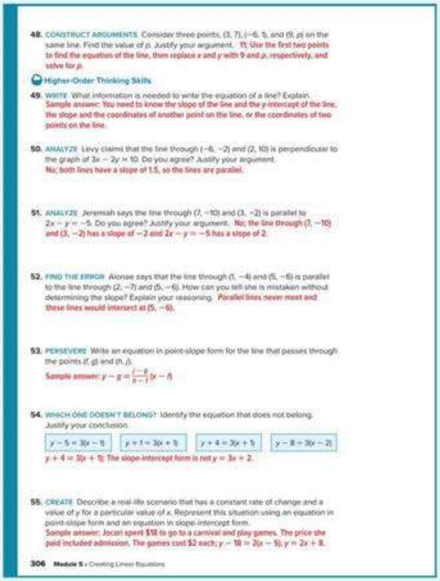
47. STRUCTURE Immediately after take-off, a jet plane consistently climbs 20 feet for every 40 feet it moves horizontally. The graph shows the trajectory of the jet.

a. Write an equation in point-slope form for the line representing the jet's trajectory. Sample answer: $y-0=0.5(x-0)$

b. Write the equation from **part a** in slope-intercept form. $y=0.5x$

c. Write the equation in standard form. $x-2y=0$

Lesson 5-3 • Writing Equations in Standard and Point-Slope Forms **305**



48. CONSTRUCT ARGUMENTS Consider three points, (3, 7), (−6, 1), and (9, *p*) on the same line. Find the value of *p*. Justify your argument. 11; Use the first two points to find the equation of the line, then replace *x* and *y* with 9 and *p*, respectively, and solve for *p*.

Higher-Order Thinking Skills

49. WRITE What information is needed to write the equation of a line? Explain.
Sample answer: You need to know the slope of the line and the *y*-intercept of the line, the slope and the coordinates of another point on the line, or the coordinates of two points on the line.

50. ANALYZE Levy claims that the line through (−6, −2) and (2, 10) is perpendicular to the graph of $3x-2y=10$. Do you agree? Justify your argument.
No; both lines have a slope of 1.5, so the lines are parallel.

51. ANALYZE Jeremiah says the line through (7, −10) and (3, −2) is parallel to $2x-y=-5$. Do you agree? Justify your argument. No; the line through (7, −10) and (3, −2) has a slope of −2 and $2x-y=-5$ has a slope of 2.

52. FIND THE ERROR Alonae says that the line through (1, −4) and (5, −6) is parallel to the line through (2, −7) and (5, −6). How can you tell she is mistaken without determining the slope? Explain your reasoning. Parallel lines never meet and these lines would intersect at (5, −6).

53. PERSEVERE Write an equation in point-slope form for the line that passes through the points (*f*, *g*) and (*h*, *j*).
Sample answer: $y-g=\frac{j-g}{h-f}(x-f)$

54. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong. Justify your conclusion.

$\boxed{y-5=3(x-1)}$ $\quad$ $\boxed{y+1=3(x+1)}$ $\quad$ $\boxed{y+4=3(x+1)}$ $\quad$ $\boxed{y-8=3(x-2)}$

$y+4=3(x+1)$; The slope-intercept form is not $y=3x+2$.

55. CREATE Describe a real-life scenario that has a constant rate of change and a value of *y* for a particular value of *x*. Represent this situation using an equation in point-slope form and an equation in slope-intercept form.
Sample answer: Jocari spent $18 to go to a carnival and play games. The price she paid included admission. The games cost $2 each; $y-18=2(x-5)$, $y=2x+8$.

306 **Module 5** • Creating Linear Equations

Lesson 5-3

Scatter Plots and Lines of Fit

S.ID.6a, S.ID.6c

LESSON GOAL

Students use scatter plots to make and evaluate predictions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Scatter Plots
- Evaluate Correlation

Explore: Making Predictions by Using a Scatter Plot

Develop:

Lines of Fit
- Write an Equation for a Line of Fit

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Equations for Lines of Best Fit	●	●		●
Extension: Latitude and Temperature		●	●	●

Language Development Handbook

Assign page 29 of the *Language Development Handbook* to help your students build mathematical language related to using scatter plots to make and evaluate predictions.

ELL You can use the tips and suggestions on page T29 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics & Probability

Standards for Mathematical Content:

S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

S.ID.6c Fit a linear function for a scatter plot that suggests a linear association.

Standards for Mathematical Practice:

5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students created linear equations in point-slope form and standard form.
A.CED.2, A.CED.3

Now
Students use scatter plots and lines of fit to make and evaluate predictions.
S.ID.6a, S.ID.6c

Next
Students will determine whether a situation illustrates correlation or causation.
S.ID.9

Rigor

The Three Pillars of Rigor

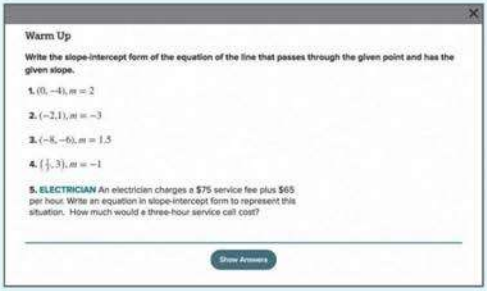
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and fluency with scatter plots (first studied in Grade 8) to prepare for using lines of fit to make predictions. They apply their understanding of linear associations by solving real-world problems.

Mathematical Background

A scatter plot consists of graphs of ordered pairs that belong to a set in which the x-coordinate represents one real-world measurement and the y-coordinate represents another. If a set of data exhibits a linear trend, a line of fit can be drawn and an equation of the line can be written to summarize the data.

Interactive Presentation

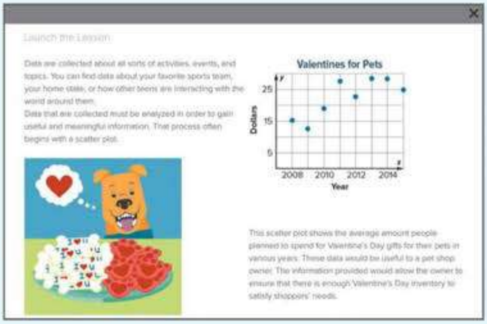


Warm Up

Write the slope-intercept form of the equation of the line that passes through the given point and has the given slope.

1. $(0, -4), m = 2$
2. $(-2, 1), m = -3$
3. $(-8, -6), m = 1.5$
4. $(\frac{1}{3}, 3), m = -1$
5. **ELECTRICIAN** An electrician charges a $75 service fee plus $65 per hour. Write an equation in slope-intercept form to represent this situation. How much would a three-hour service call cost?

Warm Up



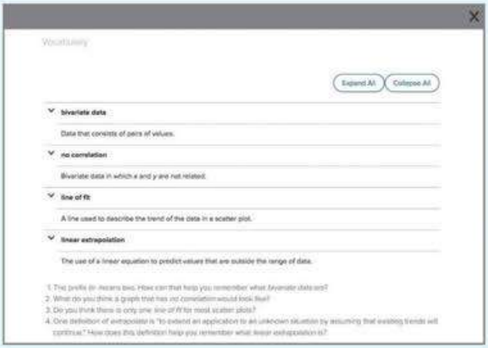
Launch the Lesson

Data are collected about all sorts of activities, events, and topics. You can find data about your favorite sports team, your home state, or how other teens are interacting with the world around them.

Data that are collected must be analyzed in order to gain useful and meaningful information. That process often begins with a scatter plot.

This scatter plot shows the average amount people planned to spend for Valentine's Day gifts for their pets in various years. These data would be useful to a pet shop owner. The information provided would allow the owner to ensure that there is enough Valentine's Day inventory to satisfy shoppers' needs.

Launch the Lesson



Vocabulary

bivariate data

Data that consists of pairs of values.

no correlation

Bivariate data in which x and y are not related.

line of fit

A line used to describe the trend of the data in a scatter plot.

linear extrapolation

The use of a linear equation to predict values that are outside the range of data.

1. The prefix *bi-* means two. How can that help you remember what *bivariate data* are?
2. What do you think a graph that has *no correlation* would look like?
3. Do you think there is only one *line of fit* for most scatter plots?
4. One definition of extrapolate is "to extend an application to an unknown situation by assuming that existing trends will continue." How does this definition help you remember what *linear extrapolation* is?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- writing linear equations given one point and the slope

Answers:

1. $y = 2x - 4$
2. $y = -3x - 5$
3. $y = 1.5x + 6$
4. $y = -x + \frac{10}{3}$
5. $y = 65x + 75$; $270

Launch the Lesson

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern of the data in the scatter plot and to explain how the data can be used to make predictions about the future.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Making Predictions by Using a Scatter Plot

Objective

Students use a sketch to explore using a line of fit to make predictions about a set of data.

Teaching the Mathematical Practices

3 Reason Inductively In this Explore, students will use inductive reasoning to make plausible arguments.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

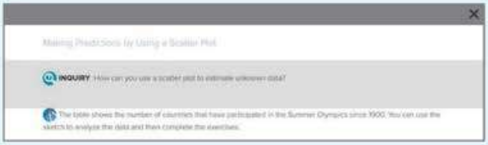
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to create a scatter plot for a given set of data. They will explore how a scatter plot can be used to approximate a line of best fit, and how that line can be used to help make predictions about additional data values. They will be guided through the exploration by a series of questions. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

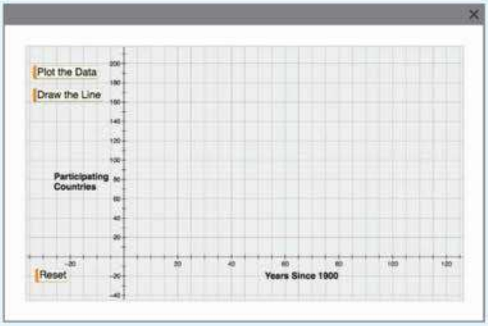


Making Predictions by Using a Scatter Plot

INQUIRY How can you use a scatter plot to estimate unknown data?

The table shows the number of countries that have participated in the Summer Olympics since 1900. You can use the sketch to analyze the data and then complete the exercises.

Explore



Explore

WEB SKETCHPAD

Students use a sketch to analyze the data and complete the exercises.

TYPE

Students answer questions about the created graph.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Making Predictions by Using a Scatter Plot (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What do you notice about how the location of the line is related to the points? Sample answer: The line is drawn in such a way so that there are about the same number of points above the line as below the line.
- Why is the line helpful? Sample answer: You can use it to estimate additional data values.

Inquiry

How can you use a scatter plot to estimate unknown data? Sample answer: If the data have a linear relationship, you can find a line to describe the data. Then, use the line to estimate unknown data.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Scatter Plots

Objective

Students categorize the correlation of a set of data in a scatter plot.

Teaching the Mathematical Practices

2 Create Representations Guide students to compare the different representations of correlations in this Learn.

Important to Know

The concept of bivariate data will be new to most students. Explain that there may or may not be a relationship between the two variables, and that a scatter plot can help identify a trend in the data. Help them to see that a trend is indicated when the general pattern in the data is somewhat linear, either increasing or decreasing.

Example 1 Evaluate Correlation

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about correlation to solving a real-world problem.

Questions for Mathematical Discourse

AL Looking at the scatter plot, do the two variables appear to be related? Explain how you know. Yes; sample answer: While the data is all over, there are several points that suggest a line with a positive slope.

OL What does the graph indicate about the data? Sample answer: In general, the taller a player, the more he weighs.

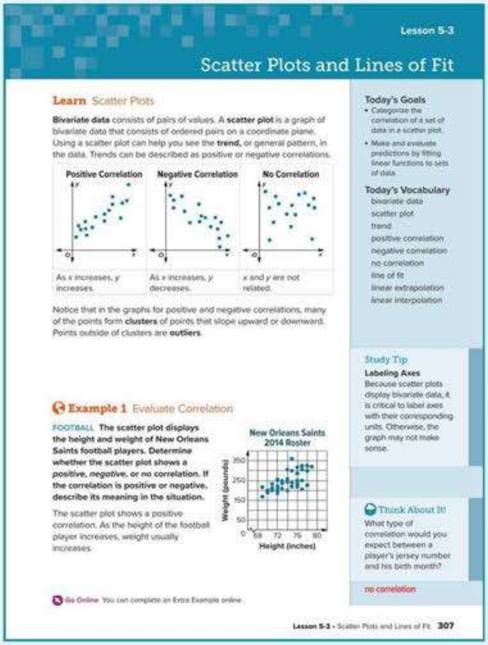
BL How is slope related to the description of a linear correlation? Sample answer: In a positive correlation, the points tend to approximate a line with a positive slope. In a negative correlation, the points tend to approximate a line with a negative slope.

Common Error

Students may have difficulty determining a trend because the data varies widely. Explain that not all of the data points need to lie on a line for there to be a correlation. The degree to which the points do approximate a line indicates the strength of the correlation.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-3

Scatter Plots and Lines of Fit

Learn Scatter Plots

Bivariate data consists of pairs of values. A **scatter plot** is a graph of bivariate data that consists of ordered pairs on a coordinate plane. Using a scatter plot can help you see the **trend,** or general pattern, in the data. Trends can be described as positive or negative correlations.

Positive Correlation	Negative Correlation	No Correlation
As x increases, y increases.	As x increases, y decreases.	x and y are not related.

Notice that in the graphs for positive and negative correlations, many of the points form **clusters** of points that slope upward or downward. Points outside of clusters are **outliers**.

Example 1 Evaluate Correlation

FOOTBALL **The scatter plot displays the height and weight of New Orleans Saints football players. Determine whether the scatter plot shows a *positive*, *negative*, or *no* correlation. If the correlation is positive or negative, describe its meaning in the situation.**

The scatter plot shows a positive correlation. As the height of the football player increases, weight usually increases.

New Orleans Saints 2014 Roster

Go Online You can complete an Extra Example online.

Today's Goals
- Categorize the correlation of a set of data in a scatter plot.
- Make and evaluate predictions by fitting linear functions to sets of data.

Today's Vocabulary
bivariate data
scatter plot
trend
positive correlation
negative correlation
no correlation
line of fit
linear extrapolation
linear interpolation

Study Tip

Labeling Axes Because scatter plots display bivariate data, it is critical to label axes with their corresponding units. Otherwise, the graph may not make sense.

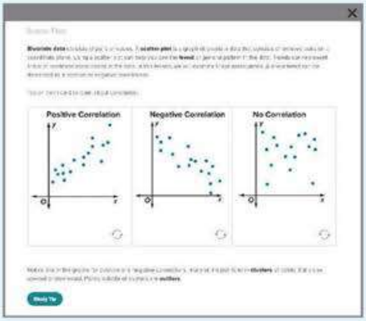
Think About It!

What type of correlation would you expect between a player's jersey number and his birth month?

no correlation

Lesson 5-3 • Scatter Plots and Lines of Fit 307

Interactive Presentation



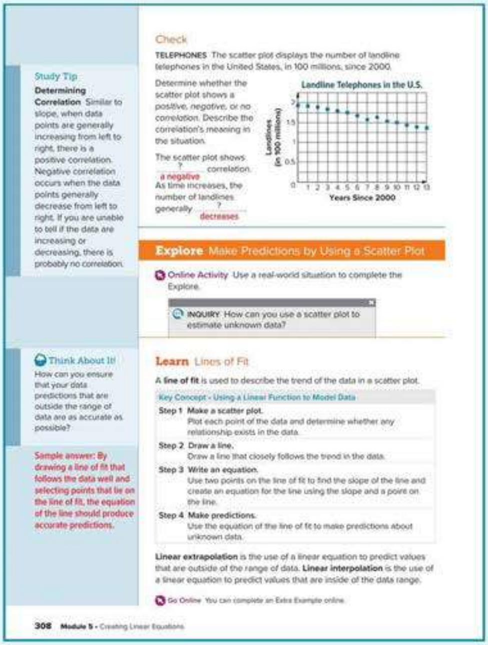
Learn

TAP

Students tap each card to learn about correlation.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Study Tip

Determining Correlation Similar to slope, when data points are generally increasing from left to right, there is a positive correlation. Negative correlation occurs when the data points generally decrease from left to right. If you are unable to tell if the data are increasing or decreasing, there is probably no correlation.

Check

TELEPHONES The scatter plot displays the number of landline telephones in the United States, in 100 millions, since 2000.

Determine whether the scatter plot shows a positive, negative, or no correlation. Describe the correlation's meaning in the situation.

The scatter plot shows ___?___ correlation. a negative

As time increases, the number of landlines generally ___?___. decreases

Explore Make Predictions by Using a Scatter Plot

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you use a scatter plot to estimate unknown data?

Think About It!

How can you ensure that your data predictions that are outside the range of data are as accurate as possible?

Sample answer: By drawing a line of fit that follows the data well and selecting points that lie on the line of fit, the equation of the line should produce accurate predictions.

Learn Lines of Fit

A **line of fit** is used to describe the trend of the data in a scatter plot.

Key Concept • Using a Linear Function to Model Data

Step 1 **Make a scatter plot.**
Plot each point of the data and determine whether any relationship exists in the data.

Step 2 **Draw a line.**
Draw a line that closely follows the trend in the data.

Step 3 **Write an equation.**
Use two points on the line of fit to find the slope of the line and create an equation for the line using the slope and a point on the line.

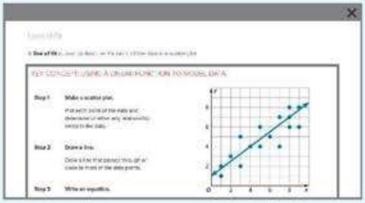
Step 4 **Make predictions.**
Use the equation of the line of fit to make predictions about unknown data.

Linear extrapolation is the use of a linear equation to predict values that are outside of the range of data. **Linear interpolation** is the use of a linear equation to predict values that are inside of the data range.

Go Online You can complete an Extra Example online.

308 **Module 5** • Creating Linear Equations

Interactive Presentation



Learn

TYPE

Students answer a question to show they understand lines of fit.

DIFFERENTIATE

Enrichment Activity BL

Write (1, 101), (2, 9.8), (3, 10), (4, 10.5), (5, 10.4), (6, 10.8), and (7, 10.3) on the board. Have students work in pairs, and challenge them to use the data to draw two scatter plots: one that indicates no correlation, and one that indicates a positive correlation. Discuss how graphs can be manipulated to lead to differing conclusions about a set of data.

Learn Lines of Fit

Objective

Students make and evaluate predictions by fitting linear functions to sets of data.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

About the Key Concept

If a scatter plot shows a linear trend, you can draw a line of fit to model that trend. Once a line is drawn, you can find an equation of that line and use that equation to make predictions about additional data points.

Common Misconception

A common misconception some students may have is that a line of fit must pass through at least one point on the scatter plot. This is not true. The line of fit must follow the trend of the data, but need not pass through any of the points.

Example 2 Write an Equation for a Line of Fit

Teaching the Mathematical Practices

4 Make Assumptions Have students explain an assumption or approximation that was made to solve the problem.

Questions for Mathematical Discourse

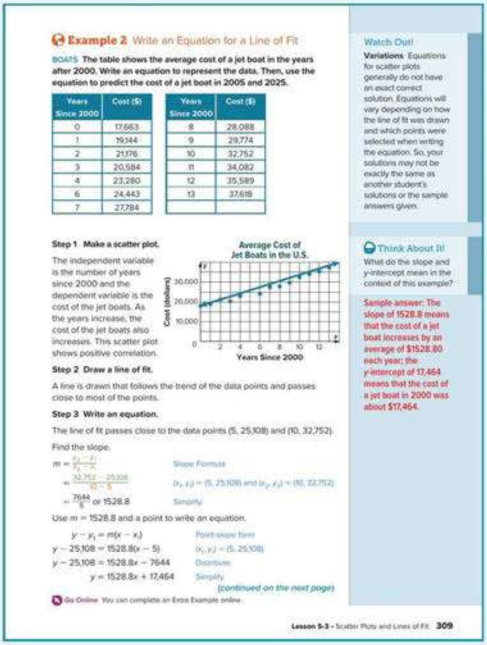
AL Once the line is drawn, how can you determine its equation? Sample answer: I can use two points that lie on the line to find its slope. Then I can use the slope and one of the points to write an equation in point-slope form.

OL Is it possible that two different people could write different equations for a line of fit and both be correct? Explain. Yes; sample answer: The two people may have drawn slightly different lines, and therefore would have used different points to create the equation.

BL How does the strength of a linear correlation affect the accuracy of the equation of a line of fit for the data? Sample answer: The stronger the relationship, the closer the points will be to the line, making it a more accurate model of the data.

Common Error

Some students may use 2005 and 2025 for the values of *x* when using the equation to make their predictions. Refer them back to the table and the graph to help them see that the *x*-value represents the number of years since 2000, not the year itself.



Example 2 Write an Equation for a Line of Fit

BOATS The table shows the average cost of a jet boat in the years after 2000. Write an equation to represent the data. Then, use the equation to predict the cost of a jet boat in 2005 and 2025.

Years Since 2000	Cost ($)
0	17,663
1	19,144
2	21,176
3	20,584
4	23,280
6	24,443
7	27,784

Years Since 2000	Cost ($)
8	28,088
9	29,774
10	32,752
11	34,082
12	35,589
13	37,618

Step 1 Make a scatter plot.

The independent variable is the number of years since 2000 and the dependent variable is the cost of the jet boats. As the years increase, the cost of the jet boats also increases. This scatter plot shows positive correlation.

Average Cost of Jet Boats in the U.S.

Step 2 Draw a line of fit.

A line is drawn that follows the trend of the data points and passes close to most of the points.

Step 3 Write an equation.

The line of fit passes close to the data points (5, 25,108) and (10, 32,752).

Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{32{,}752 - 25{,}108}{10 - 5}$ $(x_1, y_1) = (5, 25{,}108)$ and $(x_2, y_2) = (10, 32{,}752)$

$= \frac{7644}{5}$ or 1528.8 Simplify.

Use $m = 1528.8$ and a point to write an equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 25{,}108 = 1528.8(x - 5)$ $(x_1, y_1) = (5, 25{,}108)$

$y - 25{,}108 = 1528.8x - 7644$ Distribute.

$y = 1528.8x + 17{,}464$ Simplify.

(continued on the next page)

Go Online You can complete an Extra Example online.

Watch Out!

Variations Equations for scatter plots generally do not have an exact correct solution. Equations will vary depending on how the line of fit was drawn and which points were selected when writing the equation. So, your solutions may not be exactly the same as another student's solutions or the sample answers given.

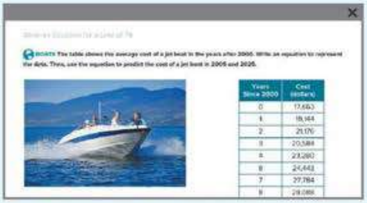
Think About It!

What do the slope and *y*-intercept mean in the context of this example?

Sample answer: The slope of 1528.8 means that the cost of a jet boat increases by an average of $1528.80 each year; the *y*-intercept of 17,464 means that the cost of a jet boat in 2000 was about $17,464.

Lesson 5-3 • Scatter Plots and Lines of Fit 309

Interactive Presentation



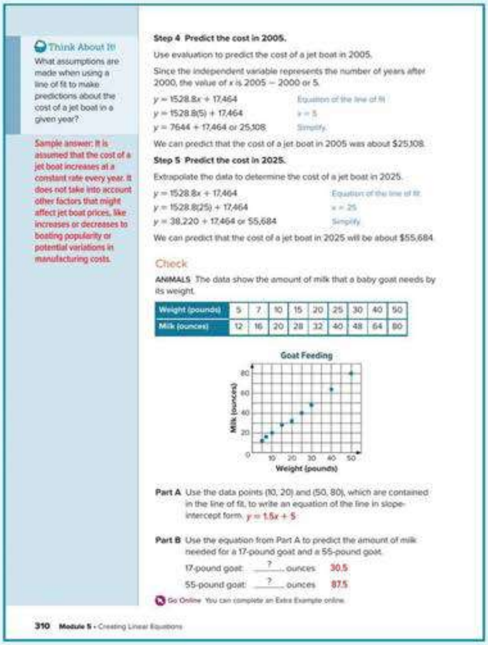
Example 2

TAP

Students move through the steps to writing an equation for a line of fit.

TYPE

Students describe what the slope and *y*-intercept mean in context and then discuss assumptions made.



Think About It!
What assumptions are made when using a line of fit to make predictions about the cost of a jet boat in a given year?

Sample answer: It is assumed that the cost of a jet boat increases at a constant rate every year. It does not take into account other factors that might affect jet boat prices, like increases or decreases to boating popularity or potential variations in manufacturing costs.

Step 4 Predict the cost in 2005.

Use evaluation to predict the cost of a jet boat in 2005.

Since the independent variable represents the number of years after 2000, the value of x is 2005 − 2000 or 5.

$y = 1528.8x + 17{,}464$ Equation of the line of fit
$y = 1528.8(5) + 17{,}464$ $x = 5$
$y = 7644 + 17{,}464$ or $25{,}108$ Simplify.

We can predict that the cost of a jet boat in 2005 was about $25,108.

Step 5 Predict the cost in 2025.

Extrapolate the data to determine the cost of a jet boat in 2025.

$y = 1528.8x + 17{,}464$ Equation of the line of fit
$y = 1528.8(25) + 17{,}464$ $x = 25$
$y = 38{,}220 + 17{,}464$ or $55{,}684$ Simplify.

We can predict that the cost of a jet boat in 2025 will be about $55,684.

Check

ANIMALS The data show the amount of milk that a baby goat needs by its weight.

Weight (pounds)	5	7	10	15	20	25	30	40	50
Milk (ounces)	12	16	20	28	32	40	48	64	80

Part A Use the data points (10, 20) and (50, 80), which are contained in the line of fit, to write an equation of the line in slope-intercept form. $y = 1.5x + 5$

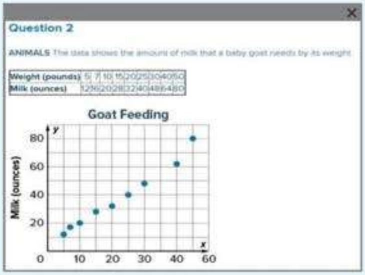
Part B Use the equation from Part A to predict the amount of milk needed for a 17-pound goat and a 55-pound goat.

17-pound goat: ___?___ ounces 30.5
55-pound goat: ___?___ ounces 87.5

Go Online You can complete an Extra Example online.

310 Module 5 • Creating Linear Equations

Interactive Presentation



Question 2

ANIMALS The data shows the amount of milk that a baby goat needs by its weight.

Weight (pounds)	5	7	10	15	20	25	30	40	50
Milk (ounces)	12	16	20	28	32	40	48	64	80

Check

CHECK

Students complete the Check online to determine whether they are ready to move on.

DIFFERENTIATE

Enrichment Activity BL

IF students need more practice making and interpreting scatter plots, **THEN** have each student make a scatter plot of their age (*x*-values) and height (*y*-values) for the first ten years of their life. Students estimate their heights as needed, but check to be sure estimates are reasonable. Ask them to draw a line of fit and write the slope-intercept form of an equation for the line. Then ask them to compare their current height with that derived from their equation.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–6
2	exercises that use a variety of skills from this lesson	7–16
3	exercises that emphasize higher-order and critical-thinking skills	17–21

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–15 odd, 17–21
- Extension: Latitude and Temperature
- **ALEKS** Scatter Plots and Lines of Best Fit

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–21 odd
- Remediation, Review Resources: Equations for Lines of Best Fit
- Personal Tutors
- Extra Examples 1, 2
- **ALEKS** Scatter Plots and Lines of Best Fit; Equations of Lines

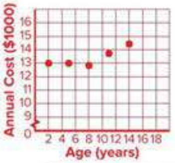
AL

IF students score 65% or less on the Checks,
THEN assign:

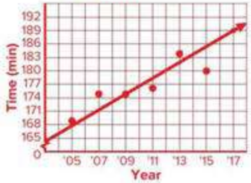
- Practice, Exercises 1–5 odd
- Remediation, Review Resources: Equations for Lines of Best Fit
- *Quick Review Math Handbook*: Scatter Plots and Lines of Fit
- Arrive**MATH** Take Another Look
- **ALEKS** Scatter Plots and Lines of Best Fit; Equations of Lines

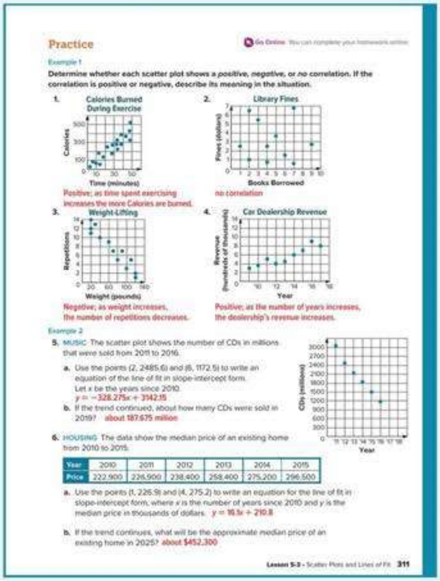
Answers

7a.



11a.





Practice

Go Online You can complete your homework online.

Example 1

Determine whether each scatter plot shows a *positive*, *negative*, or *no* correlation. If the correlation is positive or negative, describe its meaning in the situation.

1. Positive; as time spent exercising increases the more Calories are burned.

2. no correlation

3. Negative; as weight increases, the number of repetitions decreases.

4. Positive; as the number of years increases, the dealership's revenue increases.

Example 2

5. MUSIC The scatter plot shows the number of CDs in millions that were sold from 2011 to 2016.

 a. Use the points (2, 2485.6) and (6, 1172.5) to write an equation of the line of fit in slope-intercept form. Let x be the years since 2010. $y = -328.275x + 3142.15$

 b. If the trend continued, about how many CDs were sold in 2019? about 187.675 million

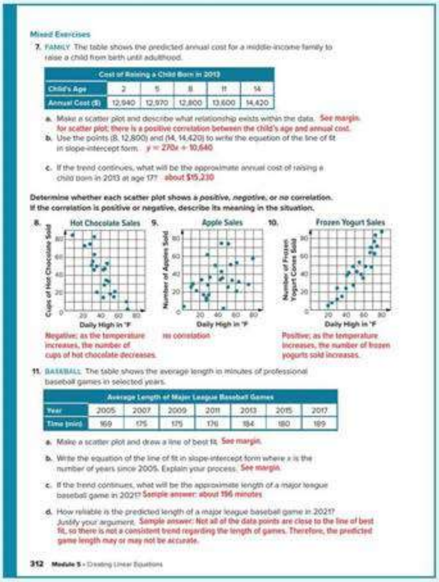
6. HOUSING The data show the median price of an existing home from 2010 to 2015.

Year	2010	2011	2012	2013	2014	2015
Price	222,900	226,900	238,400	258,400	275,200	296,500

 a. Use the points (1, 226.9) and (4, 275.2) to write an equation for the line of fit in slope-intercept form, where x is the number of years since 2010 and y is the median price in thousands of dollars. $y = 16.1x + 210.8$

 b. If the trend continues, what will be the approximate median price of an existing home in 2025? about $452,300

Lesson 5-3 • Scatter Plots and Lines of Fit 311



Mixed Exercises

7. FAMILY The table shows the predicted annual cost for a middle-income family to raise a child from birth until adulthood.

Cost of Raising a Child Born in 2013

Child's Age	2	5	8	11	14
Annual Cost ($)	12,940	12,970	12,800	13,600	14,420

 a. Make a scatter plot and describe what relationship exists within the data. See margin for scatter plot; there is a positive correlation between the child's age and annual cost.

 b. Use the points (8, 12,800) and (14, 14,420) to write the equation of the line of fit in slope-intercept form. $y = 270x + 10{,}640$

 c. If the trend continues, what will be the approximate annual cost of raising a child born in 2013 at age 17? about $15,230

Determine whether each scatter plot shows a *positive*, *negative*, or *no* correlation. If the correlation is positive or negative, describe its meaning in the situation.

8. Negative; as the temperature increases, the number of cups of hot chocolate decreases.

9. no correlation

10. Positive; as the temperature increases, the number of frozen yogurts sold increases.

11. BASEBALL The table shows the average length in minutes of professional baseball games in selected years.

Average Length of Major League Baseball Games

Year	2005	2007	2009	2011	2013	2015	2017
Time (min)	169	175	175	176	184	180	189

 a. Make a scatter plot and draw a line of best fit. See margin.

 b. Write the equation of the line of fit in slope-intercept form where x is the number of years since 2005. Explain your process. See margin.

 c. If the trend continues, what will be the approximate length of a major league baseball game in 2021? Sample answer: about 196 minutes

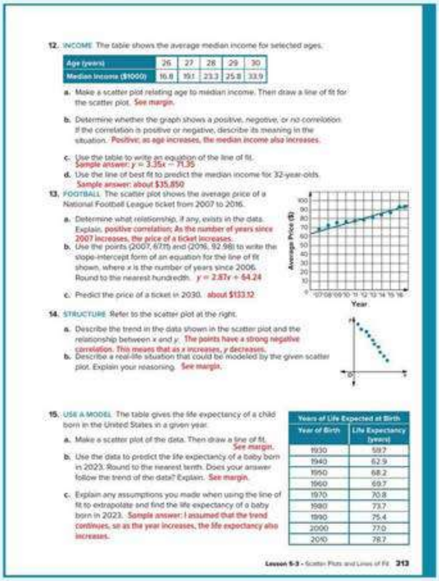
 d. How reliable is the predicted length of a major league baseball game in 2021? Justify your argument. Sample answer: Not all of the data points are close to the line of best fit, so there is not a consistent trend regarding the length of games. Therefore, the predicted game length may or may not be accurate.

312 Module 5 • Creating Linear Equations

3 REFLECT AND PRACTICE

S.ID.6a, S.ID.6c

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



12. INCOME The table shows the average median income for selected ages.

Age (years)	26	27	28	29	30
Median Income ($1000)	16.8	19.1	23.3	25.8	33.9

a. Make a scatter plot relating age to median income. Then draw a line of fit for the scatter plot. See margin.

b. Determine whether the graph shows a *positive*, *negative*, or *no correlation*. If the correlation is positive or negative, describe its meaning in the situation. Positive; as age increases, the median income also increases.

c. Use the table to write an equation of the line of fit. Sample answer: $y = 3.35x - 71.35$

d. Use the line of best fit to predict the median income for 32-year-olds. Sample answer: about $35,850

13. FOOTBALL The scatter plot shows the average price of a National Football League ticket from 2007 to 2016.

a. Determine what relationship, if any, exists in the data. Explain. positive correlation; As the number of years since 2007 increases, the price of a ticket increases.

b. Use the points (2007, 67.11) and (2016, 92.98) to write the slope-intercept form of an equation for the line of fit shown, where x is the number of years since 2006. Round to the nearest hundredth. $y = 2.87x + 64.24$

c. Predict the price of a ticket in 2030. about $133.12

14. STRUCTURE Refer to the scatter plot at the right.

a. Describe the trend in the data shown in the scatter plot and the relationship between x and y. The points have a strong negative correlation. This means that as x increases, y decreases.

b. Describe a real-life situation that could be modeled by the given scatter plot. Explain your reasoning. See margin.

15. USE A MODEL The table gives the life expectancy of a child born in the United States in a given year.

a. Make a scatter plot of the data. Then draw a line of fit. See margin.

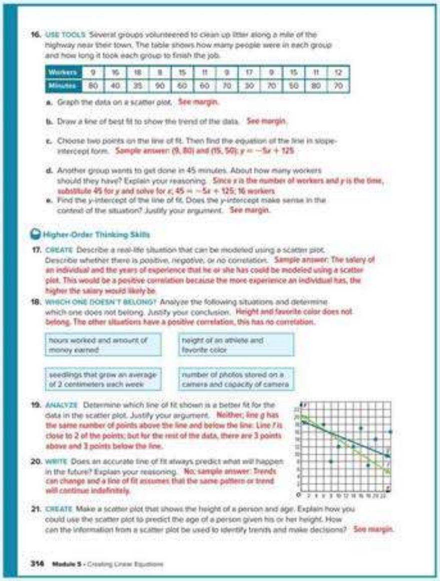
b. Use the data to predict the life expectancy of a baby born in 2023. Round to the nearest tenth. Does your answer follow the trend of the data? Explain. See margin.

c. Explain any assumptions you made when using the line of fit to extrapolate and find the life expectancy of a baby born in 2023. Sample answer: I assumed that the trend continues, so as the year increases, the life expectancy also increases.

Years of Life Expected at Birth

Year of Birth	Life Expectancy (years)
1930	59.7
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4
2000	77.0
2010	78.7

Lesson 5-3 • Scatter Plots and Lines of Fit 313



16. USE TOOLS Several groups volunteered to clean up litter along a mile of the highway near their town. The table shows how many people were in each group and how long it took each group to finish the job.

Workers	9	16	18	8	15	11	9	17	9	15	11	12
Minutes	80	40	35	90	60	60	70	30	70	50	80	70

a. Graph the data on a scatter plot. See margin.

b. Draw a line of best fit to show the trend of the data. See margin.

c. Choose two points on the line of fit. Then find the equation of the line in slope-intercept form. Sample answer: (9, 80) and (15, 50); $y = -5x + 125$

d. Another group wants to get done in 45 minutes. About how many workers should they have? Explain your reasoning. Since x is the number of workers and y is the time, substitute 45 for y and solve for x; $45 = -5x + 125$; 16 workers

e. Find the y-intercept of the line of fit. Does the y-intercept make sense in the context of the situation? Justify your argument. See margin.

Higher-Order Thinking Skills

17. CREATE Describe a real-life situation that can be modeled using a scatter plot. Describe whether there is positive, negative, or no correlation. Sample answer: The salary of an individual and the years of experience that he or she has could be modeled using a scatter plot. This would be a positive correlation because the more experience an individual has, the higher the salary would likely be.

18. WHICH ONE DOESN'T BELONG? Analyze the following situations and determine which one does not belong. Justify your conclusion. Height and favorite color does not belong. The other situations have a positive correlation, this has no correlation.

hours worked and amount of money earned	height of an athlete and favorite color
seedlings that grow an average of 2 centimeters each week	number of photos stored on a camera and capacity of camera

19. ANALYZE Determine which line of fit shown is a better fit for the data in the scatter plot. Justify your argument. Neither; line g has the same number of points above the line and below the line. Line f is close to 2 of the points; but for the rest of the data, there are 3 points above and 3 points below the line.

20. WRITE Does an accurate line of fit always predict what will happen in the future? Explain your reasoning. No; sample answer: Trends can change and a line of fit assumes that the same pattern or trend will continue indefinitely.

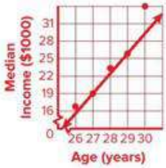
21. CREATE Make a scatter plot that shows the height of a person and age. Explain how you could use the scatter plot to predict the age of a person given his or her height. How can the information from a scatter plot be used to identify trends and make decisions? See margin.

314 Module 5 • Creating Linear Equations

Answers

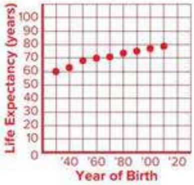
11b. Sample answer: x represents the number of years since 2005, so year 2005 is represented by $x = 0$ and year 2020 is represented by $x = 15$. Two points on the line of best fit are (4, 175) and (12, 189). Use these two points to find the slope to be 1.75 and the equation of the line of best fit to be $y = 1.75x + 168$.

12a.



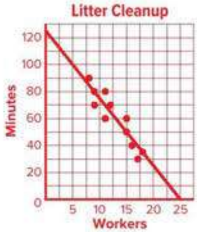
14b. Sample answer: The number of miles a car has driven and the number of gallons of gasoline in the tank; this would also be a negative correlation because as a car is driven, the amount of gasoline remaining in the tank decreases.

15a.



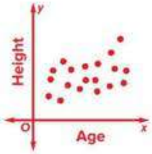
15b. Sample answer: About 81.8; The data show a positive correlation, so as the years increase, the life expectancy also increases. Therefore, the life expectancy should be higher than that of a baby born in 2010.

16a-b.



16e. Sample answer: (0, 125); The y-intercept means that 0 workers will take 125 minutes to complete the job. However, if there are no workers, the job will not get done at all. So, the equation can only be used to model the time it takes at least 1 worker to complete the job.

21. Sample answer: You can visualize a line to determine whether the data has a positive or negative correlation. The graph shows the ages and heights of people. To predict a person's age given his or her height, write a linear equation for the line of fit. Then substitute the person's height and solve for the corresponding age. You can use the pattern in the scatter plot to make decisions.



Lesson 5-4

Correlation and Causation

S.ID.9

LESSON GOAL

Students determine whether a situation illustrates correlation or causation.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Collecting Data to Determine Correlation and Causation

Develop:

Correlation and Causation

- Correlation and Causation by Graphing
- Causation and Correlation by Situation

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Scatter Plots	●	●		●
Extension: Lurking Variables		●	●	●

Language Development Handbook

Assign page 30 of the *Language Development Handbook* to help your students build mathematical language related to correlation and causation.

ELL You can use the tips and suggestions on page T30 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics & Probability

Standards for Mathematical Content:

S.ID.9 Distinguish between correlation and causation.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.
3 Construct viable arguments and critique the reasoning of others.
5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students used scatter plots and lines of fit to make and evaluate predictions.
8.SP.2, 8.SP.3, S.ID.6a, S.ID.6c

Now
Students determine whether a situation illustrates correlation or causation.
S.ID.9

Next
Students will use best-fit lines and correlation coefficients to determine how well linear functions fit sets of data.
S.ID.6, S.ID.8

Rigor

The Three Pillars of Rigor

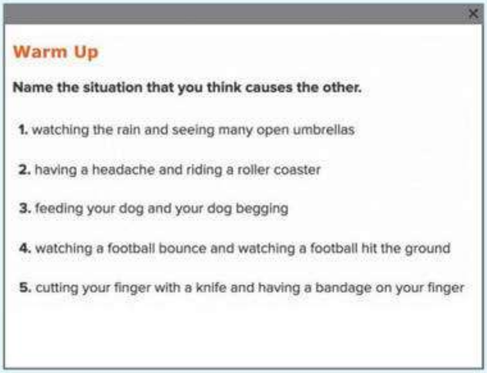
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on what they have learned about linear associations to gain understanding about the differences between correlation and causation. They apply their understanding by differentiating between situations in real-world contexts.

Mathematical Background

Some correlations may be purely coincidental, while others may have a common underlying cause. For those situations where the change in one variable is the cause for the change in the other, the relationship is one of causation. It is important to note that correlation does not imply causation. Two variables may have a positive or negative correlation, but may not have a causal relationship.

Interactive Presentation

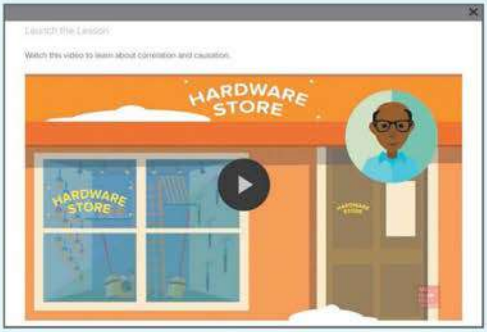


Warm Up

Name the situation that you think causes the other.

1. watching the rain and seeing many open umbrellas
2. having a headache and riding a roller coaster
3. feeding your dog and your dog begging
4. watching a football bounce and watching a football hit the ground
5. cutting your finger with a knife and having a bandage on your finger

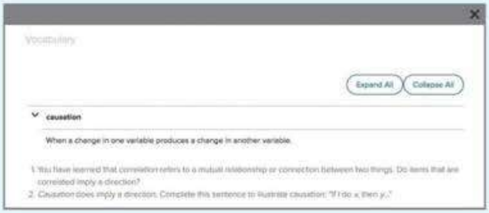
Warm Up



Launch the Lesson

Watch this video to learn about correlation and causation.

Launch the Lesson



Vocabulary

causation

When a change in one variable produces a change in another variable.

1. You have learned that correlation refers to a mutual relationship or connection between two things. Do items that are correlated imply a direction?
2. Causation does imply a direction. Complete this sentence to illustrate causation: "If I do x, then y."

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying patterns of association between two quantities

Answers:

1. positive
2. no
3. negative
4. not linear
5. negative

Launch the Lesson

MP Teaching the Mathematical Practices

3 Construct Arguments Encourage students to use the information from the video to construct an argument about whether Marcus' decision to not order any more snow shovels is reasonable.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Explore Collecting Data to Determine Correlation and Causation

Objective

Students collect data to explore the difference between correlation and causation.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

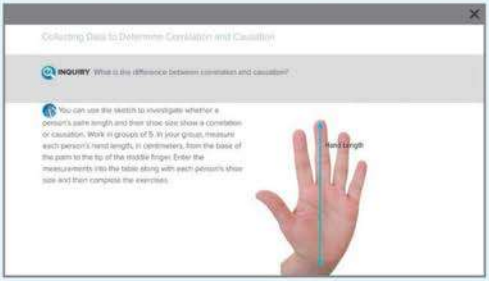
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will work in groups of 5. They will collect data on hand length and shoe size from the members of their group. They will then use the data to construct a scatter plot. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

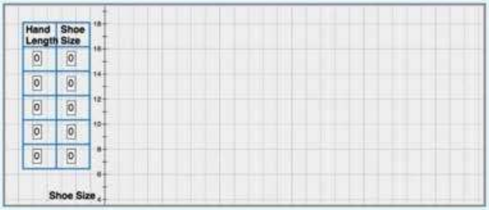


Collecting Data to Determine Correlation and Causation

INQUIRY What is the difference between correlation and causation?

You can use the sketch to investigate whether a person's palm length and their shoe size show a correlation or causation. Work in groups of 5. In your group, measure each person's hand length, in centimeters, from the base of the palm to the top of the middle finger. Enter the measurements into the table along with each person's shoe size and then complete the exercises.

Explore



Hand Length	Shoe Size
0	0
0	0
0	0
0	0
0	0

Explore

WEB SKETCHPAD Students use a sketch to graph the collected data.

TAP Students select the correct type of correlation.

TYPE Students answer questions about the sketch.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Collecting Data to Determine Correlation and Causation (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What does your graph tell you about the data? Sample answer: In general, students with bigger hands wear larger shoes.
- Why would the scatter plot for a causation situation indicate a correlation? Sample answer: Because as one variable increases, the other would increase or decrease linearly, reflecting the causation.

Inquiry

What is the difference between correlation and causation? Sample answer: Correlation means that two sets of data show a relationship while causation means that one set of data depends on the other.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Correlation and Causation

Objective

Students determine whether a data set or situation illustrates correlation or causation by analyzing the data or situation.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Important to Know

When a correlation exists between two variables, there may be a factor that is influencing both of the variables. It is important to know that this does not mean that causation exists between the two variables. For example, the number of ice cream cones sold by a vendor at the beach on a given day and the number of ocean rescues on that day may exhibit a positive correlation due to the fact that both of these variables likely increase or decrease in tandem. However neither variable causes the other. The fact that both of these variables may be affected by the temperature at the beach (which, in turn, affects the number of people at the beach) does not mean that the data exhibit causation.

Example 1 Correlation and Causation by Graphing

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about correlation and causation to solving a real-world problem.

Questions for Mathematical Discourse

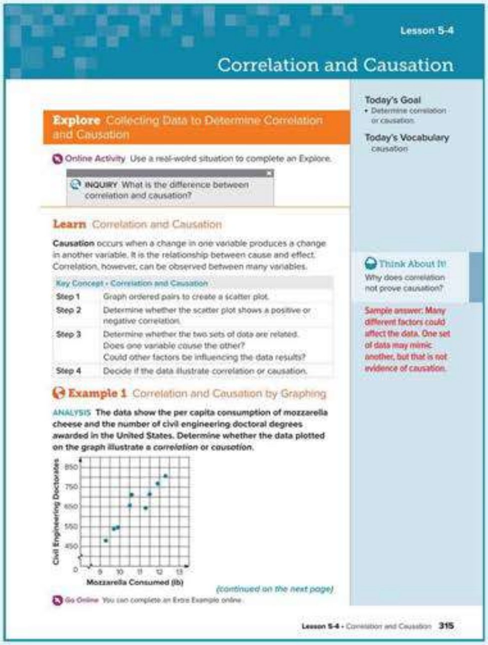
AL What type of correlation does the data exhibit? positive correlation What does this tell you about the data? Sample answer: As the amount of mozzarella consumed increases, so does the number of civil engineering doctorates awarded.

OL Does the positive correlation mean that the amount of mozzarella cheese consumed affects the number of doctorates awarded? Explain your reasoning. Sample answer: No; eating mozzarella cheese does not give you a degree. This is a correlation, not a causation.

BL What factors might have an effect on the number of doctorates awarded? Sample answers: the job market, financial aid for graduate school, engineering companies paying for their workers to continue their education

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-4

Correlation and Causation

Explore Collecting Data to Determine Correlation and Causation

Online Activity Use a real-world situation to complete an Explore.

INQUIRY What is the difference between correlation and causation?

Learn Correlation and Causation

Causation occurs when a change in one variable produces a change in another variable. It is the relationship between cause and effect. Correlation, however, can be observed between many variables.

Key Concept • Correlation and Causation	
Step 1	Graph ordered pairs to create a scatter plot.
Step 2	Determine whether the scatter plot shows a positive or negative correlation.
Step 3	Determine whether the two sets of data are related. Does one variable cause the other? Could other factors be influencing the data results?
Step 4	Decide if the data illustrate correlation or causation.

Example 1 Correlation and Causation by Graphing

ANALYSIS **The data show the per capita consumption of mozzarella cheese and the number of civil engineering doctoral degrees awarded in the United States. Determine whether the data plotted on the graph illustrate a correlation or causation.**

(continued on the next page)

Go Online You can complete an Extra Example online.

Today's Goal
- Determine correlation or causation.

Today's Vocabulary
causation

Think About It!
Why does correlation not prove causation?

Sample answer: Many different factors could affect the data. One set of data may mimic another, but that is not evidence of causation.

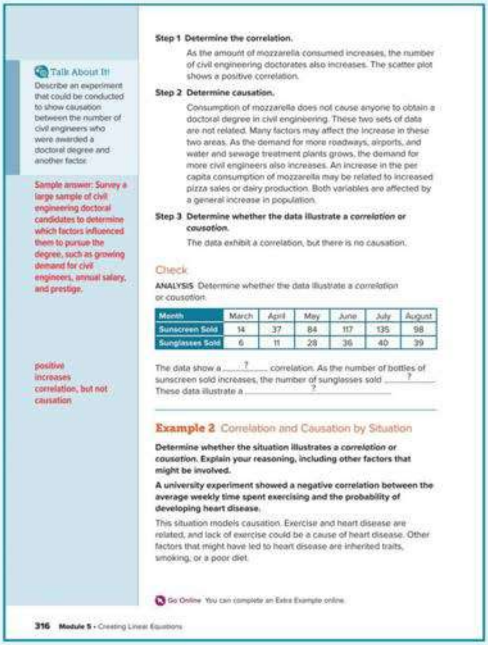
Lesson 5-4 • Correlation and Causation 315

Interactive Presentation

Learn

TYPE

Students answer a question to show they understand why correlation does not prove causation.



Talk About It!

Describe an experiment that could be conducted to show causation between the number of civil engineers who were awarded a doctoral degree and another factor.

Sample answer: Survey a large sample of civil engineering doctoral candidates to determine which factors influenced them to pursue the degree, such as growing demand for civil engineers, annual salary, and prestige.

Step 1 Determine the correlation.

As the amount of mozzarella consumed increases, the number of civil engineering doctorates also increases. The scatter plot shows a positive correlation.

Step 2 Determine causation.

Consumption of mozzarella does not cause anyone to obtain a doctoral degree in civil engineering. These two sets of data are not related. Many factors may affect the increase in these two areas. As the demand for more roadways, airports, and water and sewage treatment plants grows, the demand for more civil engineers also increases. An increase in the per capita consumption of mozzarella may be related to increased pizza sales or dairy production. Both variables are affected by a general increase in population.

Step 3 Determine whether the data illustrate a correlation or causation.

The data exhibit a correlation, but there is no causation.

Check

ANALYSIS Determine whether the data illustrate a correlation or causation.

Month	March	April	May	June	July	August
Sunscreen Sold	14	37	84	117	135	98
Sunglasses Sold	6	11	28	36	40	39

The data show a __?__ correlation. As the number of bottles of sunscreen sold increases, the number of sunglasses sold __?__. These data illustrate a __?__.

positive

increases

correlation, but not causation

Example 2 Correlation and Causation by Situation

Determine whether the situation illustrates a correlation or causation. Explain your reasoning, including other factors that might be involved.

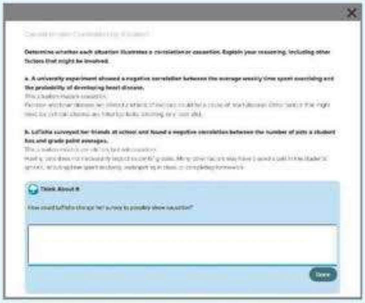
A university experiment showed a negative correlation between the average weekly time spent exercising and the probability of developing heart disease.

This situation models causation. Exercise and heart disease are related, and lack of exercise could be a cause of heart disease. Other factors that might have led to heart disease are inherited traits, smoking, or a poor diet.

Go Online You can complete an Extra Example online.

316 Module 5 • Creating Linear Equations

Interactive Presentation



Determine whether each situation illustrates a correlation or causation. Explain your reasoning, including other factors that might be involved.

a. A university experiment showed a negative correlation between the average weekly time spent exercising and the probability of developing heart disease.

b. LaTisha surveyed her friends at school and found a negative correlation between the number of jobs a student has and grade point averages.

Think About It!

How could LaTisha change her survey to possibly show causation?

Example 2

TYPE

Students answer a question to show they understand how the survey could be changed to show causation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Causation and Correlation by Situation

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

- **AL** What is the dependent factor? risk of developing heart disease
- **OL** What is the independent factor? average weekly time spent exercising
- **BL** Can the university state that this is the only cause for developing heart disease? Why? No; sample answer: There are many other factors that contribute to heart disease. This study shows only a causation between lack of exercise and heart disease.

DIFFERENTIATE

Language Development Activity **AL** **ELL**

IF students are having difficulty differentiating between causation and correlation,

THEN pair students together and have one student write situations that represent causation and situations that represent simply correlation, and have the other student determine which is which. Then have students reverse roles.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–6
2	exercises that use a variety of skills from this lesson	7–10
3	exercises that emphasize higher-order and critical-thinking skills	11–13

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–9 odd, 11–13
- Extension: Lurking Variables
- ALEKS Scatter Plots and Lines of Best Fit

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–13 odd
- Remediation, Review Resources: Scatter Plots
- Personal Tutors
- Extra Examples 1, 2
- ALEKS Scatterplots and Lines of Best Fit

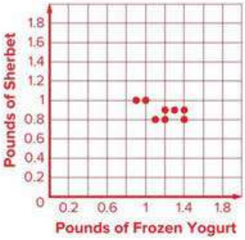
AL

IF students score 65% or less on the Checks,
THEN assign:

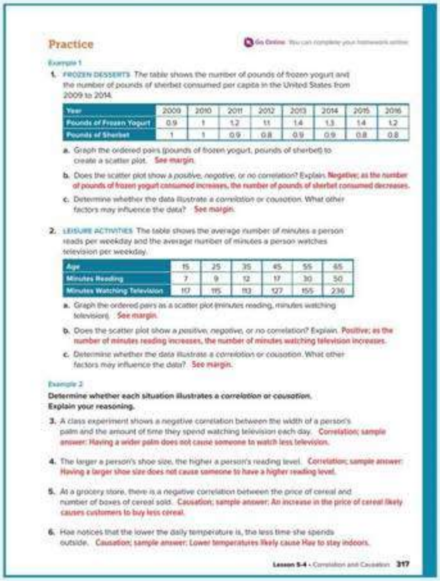
- Practice, Exercises 1–5 odd
- Remediation, Review Resources: Scatter Plots
- *Quick Review Math Handbook*: Correlation and Causation
- Arrive**MATH** Take Another Look
- ALEKS Scatterplots and Lines of Best Fit

Answers

1a.



1c. The relationship may be a causation. Since both are frozen desserts, eating more frozen yogurt may cause people to decrease the amount of sherbet they eat. Other things that might influence the data are an increase in frozen yogurt stores and a decrease in popularity or availability of sherbet.



Practice

Go Online You can complete your homework online.

Example 1

1. FROZEN DESSERTS The table shows the number of pounds of frozen yogurt and the number of pounds of sherbet consumed per capita in the United States from 2009 to 2014.

Year	2009	2010	2011	2012	2013	2014	2015	2016
Pounds of Frozen Yogurt	0.9	1	1.2	1.1	1.4	1.3	1.4	1.2
Pounds of Sherbet	1	1	0.9	0.8	0.9	0.9	0.8	0.8

a. Graph the ordered pairs (pounds of frozen yogurt, pounds of sherbet) to create a scatter plot. See margin.

b. Does the scatter plot show a positive, negative, or no correlation? Explain. Negative; as the number of pounds of frozen yogurt consumed increases, the number of pounds of sherbet consumed decreases.

c. Determine whether the data illustrate a correlation or causation. What other factors may influence the data? See margin.

2. LEISURE ACTIVITIES The table shows the average number of minutes a person reads per weekday and the average number of minutes a person watches television per weekday.

Age	15	25	35	45	55	65
Minutes Reading	7	9	12	17	30	50
Minutes Watching Television	117	115	113	127	155	236

a. Graph the ordered pairs as a scatter plot (minutes reading, minutes watching television). See margin.

b. Does the scatter plot show a positive, negative, or no correlation? Explain. Positive; as the number of minutes reading increases, the number of minutes watching television increases.

c. Determine whether the data illustrate a correlation or causation. What other factors may influence the data? See margin.

Example 2

Determine whether each situation illustrates a correlation or causation. Explain your reasoning.

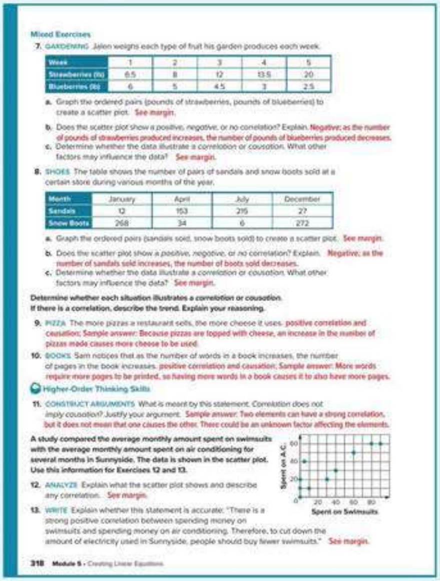
3. A class experiment shows a negative correlation between the width of a person's palm and the amount of time they spend watching television each day. Correlation; sample answer: Having a wider palm does not cause someone to watch less television.

4. The larger a person's shoe size, the higher a person's reading level. Correlation; sample answer: Having a larger shoe size does not cause someone to have a higher reading level.

5. At a grocery store, there is a negative correlation between the price of cereal and number of boxes of cereal sold. Causation; sample answer: An increase in the price of cereal likely causes customers to buy less cereal.

6. Hae notices that the lower the daily temperature is, the less time she spends outside. Causation; sample answer: Lower temperatures likely cause Hae to stay indoors.

Lesson 5-4 • Correlation and Causation 317



Mixed Exercises

7. GARDENING Jalen weighs each type of fruit his garden produces each week.

Week	1	2	3	4	5
Strawberries (lb)	6.5	8	12	13.5	20
Blueberries (lb)	6	5	4.5	3	2.5

a. Graph the ordered pairs (pounds of strawberries, pounds of blueberries) to create a scatter plot. See margin.

b. Does the scatter plot show a positive, negative, or no correlation? Explain. Negative; as the number of pounds of strawberries produced increases, the number of pounds of blueberries produced decreases.

c. Determine whether the data illustrate a correlation or causation. What other factors may influence the data? See margin.

8. SHOES The table shows the number of pairs of sandals and snow boots sold at a certain store during various months of the year.

Month	January	April	July	December
Sandals	12	153	215	27
Snow Boots	268	34	6	272

a. Graph the ordered pairs (sandals sold, snow boots sold) to create a scatter plot. See margin.

b. Does the scatter plot show a positive, negative, or no correlation? Explain. Negative; as the number of sandals sold increases, the number of boots sold decreases.

c. Determine whether the data illustrate a correlation or causation. What other factors may influence the data? See margin.

Determine whether each situation illustrates a correlation or causation. If there is a correlation, describe the trend. Explain your reasoning.

9. PIZZA The more pizzas a restaurant sells, the more cheese it uses. positive correlation and causation; Sample answer: Because pizzas are topped with cheese, an increase in the number of pizzas made causes more cheese to be used.

10. BOOKS Sam notices that as the number of words in a book increases, the number of pages in the book increases. positive correlation and causation; Sample answer: More words require more pages to be printed, so having more words in a book causes it to also have more pages.

Higher-Order Thinking Skills

11. CONSTRUCT ARGUMENTS What is meant by this statement: Correlation does not imply causation? Justify your argument. Sample answer: Two elements can have a strong correlation, but it does not mean that one causes the other. There could be an unknown factor affecting the elements.

A study compared the average monthly amount spent on swimsuits with the average monthly amount spent on air conditioning for several months in Sunnyside. The data is shown in the scatter plot. Use this information for Exercises 12 and 13.

12. ANALYZE Explain what the scatter plot shows and describe any correlation. See margin.

13. WRITE Explain whether this statement is accurate: "There is a strong positive correlation between spending money on swimsuits and spending money on air conditioning. Therefore, to cut down the amount of electricity used in Sunnyside, people should buy fewer swimsuits." See margin.

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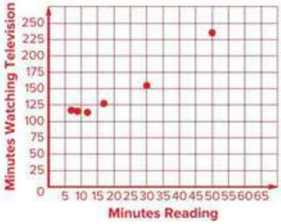
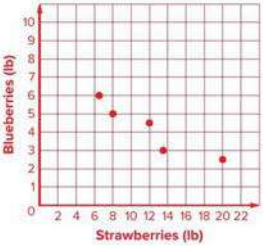
Answers

2a.



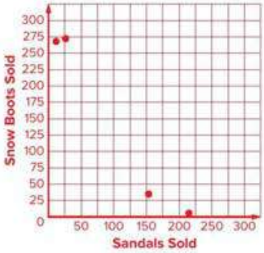
2c. The relationship is a correlation, but not a causation. An increase in minutes reading may be related to increased minutes watching television. Both activities might be affected by an increase in free time for leisure activities due to retiring and/or children are no longer being raised in their household.

7a.



7c. The relationship is a correlation, but not a causation. A better yield of strawberries does not cause the blueberries to grow poorly. Other factors, such as temperature and rain, could be affecting the plants that week.

8a.



8c. The relationship is a correlation, but not a causation. Buying sandals does not cause someone to not buy snow boots. Other factors, such as weather and sales, could be affecting the data.

12. Sample answer: The data seem to form a line with a positive slope. Most of the points follow the line fairly closely, so it shows a strong positive correlation. So, when the amount spent on swimsuits increases, so does the amount spent on AC.

13. Sample answer: Correlation does not mean causation. Even though there is a strong correlation, that does not mean buying swimsuits causes the use of air conditioners. Another factor, like the temperature, could be affecting both swimsuit sales and use of air conditioners.

Lesson 5-5

Linear Regression

S.ID.6, S.ID.8

LESSON GOAL

Students use best-fit lines and correlation coefficients to determine how well linear functions fit sets of data.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Linear Regression and Best-Fit Lines
- Find a Best-Fit Line
- Use a Best-Fit Line

Residuals
- Graph and Analyze a Residual Plot

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Draw Lines of Best Fit	●	●		●
Extension: Quadratic Regression		●	●	●

Language Development Handbook

Assign page 31 of the *Language Development Handbook* to help your students build mathematical language related to best-fit lines and correlation coefficients.

ELL You can use the tips and suggestions on page T31 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics & Probability

Standards for Mathematical Content:

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.
5 Use appropriate tools strategically.
6 Attend to precision.

Coherence

Vertical Alignment

Previous
Students determined whether a situation illustrates correlation or causation.
S.ID.9

Now
Students use best-fit lines and correlation coefficients to determine how well linear functions fit sets of data.
S.ID.6, S.ID.8

Next
Students will fit functions to data, including linear, quadratic, and exponential models.
S.ID.6a (Course 1, Course 2)

Rigor

The Three Pillars of Rigor

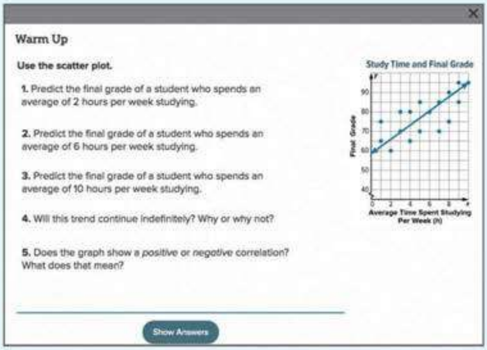
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students bring together all that they have learned about linear associations, correlations, and causation to find and interpret correlation coefficients. They build fluency in using technology to fit functions to data, and they apply their understanding by solving real-world problems.

Mathematical Background

An equation for a best-fit line can be written for any set of data, but it is only useful if the data exhibits a linear trend. A graphing calculator can be used to write an equation for the best-fit line and find the correlation coefficient. The closer the correlation coefficient is to 1 or –1, the more closely the equation models the data.

Interactive Presentation

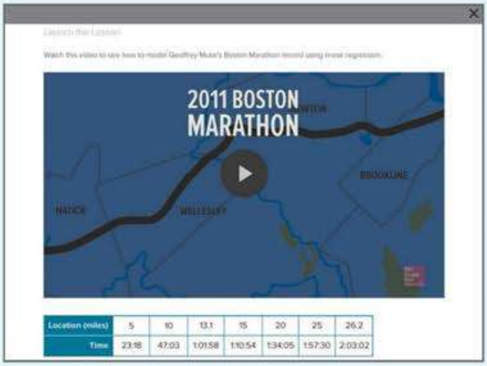

Study Time and Final Grade
y
Final Grade
90
80
70
60
50
40
0
2
4
6
8
x
Average Time Spent Studying Per Week (h)
Show Answers

Warm Up

Use the scatter plot.

1. Predict the final grade of a student who spends an average of 2 hours per week studying.
2. Predict the final grade of a student who spends an average of 6 hours per week studying.
3. Predict the final grade of a student who spends an average of 10 hours per week studying.
4. Will this trend continue indefinitely? Why or why not?
5. Does the graph show a *positive* or *negative correlation*? What does that mean?

Warm Up

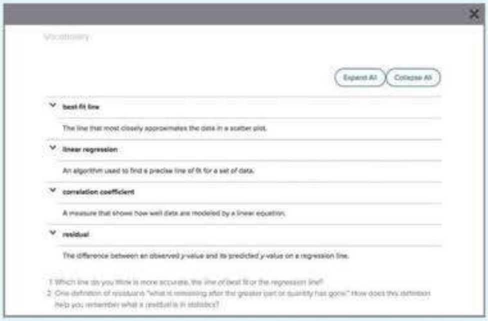


Launch the Lesson

Watch this video to see how to model Geoffrey Mutai's Boston Marathon record using linear regression.

Location (miles)	5	10	13.1	15	20	25	26.2
Time	23:18	47:03	1:01:58	1:10:54	1:34:05	1:57:30	2:03:02

Launch the Lesson



Vocabulary

best fit line	The line that most closely approximates the data in a scatter plot.
linear regression	An algorithm used to find a precise line of fit for a set of data.
correlation coefficient	A measure that shows how well data are modeled by a linear equation.
residual	The difference between an observed *y*-value and its predicted *y*-value on a regression line.

1. Which line do you think is more accurate, the line of best fit or the regression line?
2. One definition of residual is "what is remaining after the greater part or quantity has gone." How does this definition help you remember what a residual is in statistics?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- using scatter plots to evaluate trends and make predictions

Answers:

1. 66
2. 80
3. 94
4. No; once they reach a certain point, a student will not be able to increase their study time.
5. Positive; the more time a student studies, the higher the final grade.

Launch the Lesson

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to consider how the times recorded at each of the check points could be used to determine whether a runner ran at a relatively consistent rate throughout the race.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards*? and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Learn Linear Regression and Best-Fit Lines

Objective

Students compute and use best-fit lines and correlation coefficients for sets of data by using technology to perform linear regressions.

MP Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of linear regression and best-fit lines in this Learn.

What Students Are Learning

Students have already explored lines of fit. In this lesson, students learn that there is a line of best fit that can be determined using a calculator. Students use a calculator to find the line of best fit, and to identify the correlation coefficient associated with the data.

Common Misconception

A common misconception some students have is believing that the smaller the correlation coefficient, the weaker the correlation. This is not true. Explain that the closer the correlation coefficient is to 0, the weaker the correlation; the closer it is to −1 or 1, the stronger the correlation.

Example 1 Find a Best-Fit Line

MP Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the question posed in the Use a Source feature.

Questions for Mathematical Discourse

AL What patterns do you notice in the table? Sample answer: In general, as the year increases the number of hits decreases.

OL How would the equation change if the values used for List 1 were the years? Sample answer: The slope would be the same but the *y*-intercept would be different.

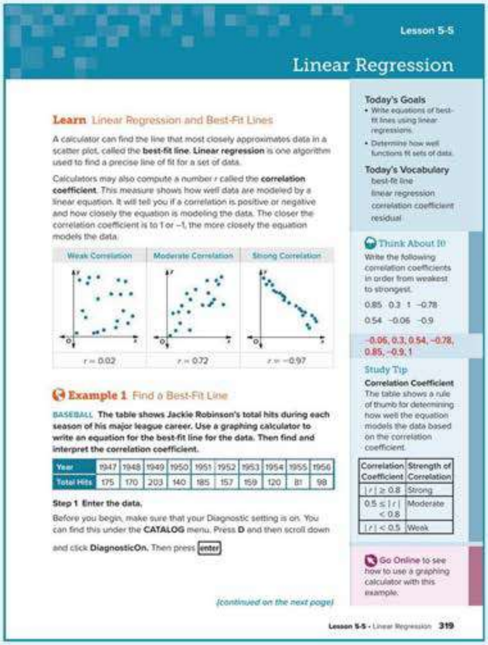
BL What is the correlation coefficient? What does it mean? −0.8022; sample answer: It means the equation models the data well. The closer this number is to 1 or to −1, the better the equation represents the data.

Common Error

Student may forget to clear L1 and L2 on their calculators before entering a new set of data. This error will produce an incorrect result. Help students to avoid making this error by making the first step in the procedure "Clear L1 and L2."

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-5

Linear Regression

Learn Linear Regression and Best-Fit Lines

A calculator can find the line that most closely approximates data in a scatter plot, called the **best-fit line**. **Linear regression** is one algorithm used to find a precise line of fit for a set of data.

Calculators may also compute a number r called the **correlation coefficient**. This measure shows how well data are modeled by a linear equation. It will tell you if a correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or −1, the more closely the equation models the data.

Example 1 Find a Best-Fit Line

BASEBALL **The table shows Jackie Robinson's total hits during each season of his major league career. Use a graphing calculator to write an equation for the best-fit line for the data. Then find and interpret the correlation coefficient.**

Year	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956
Total Hits	175	170	203	140	185	157	159	120	81	98

Step 1 Enter the data.

Before you begin, make sure that your Diagnostic setting is on. You can find this under the **CATALOG** menu. Press **D** and then scroll down and click **DiagnosticOn**. Then press enter.

(continued on the next page)

Today's Goals
- Write equations of best-fit lines using linear regressions.
- Determine how well functions fit sets of data.

Today's Vocabulary
best-fit line
linear regression
correlation coefficient
residual

Think About It!
Write the following correlation coefficients in order from weakest to strongest.

0.85 0.3 1 −0.78
0.54 −0.06 −0.9

−0.06, 0.3, 0.54, −0.78, 0.85, −0.9, 1

Study Tip
Correlation Coefficient The table shows a rule of thumb for determining how well the equation models the data based on the correlation coefficient.

Correlation Coefficient	Strength of Correlation
$\lvert r \rvert \geq 0.8$	Strong
$0.5 \leq \lvert r \rvert < 0.8$	Moderate
$\lvert r \rvert < 0.5$	Weak

Go Online to see how to use a graphing calculator with this example.

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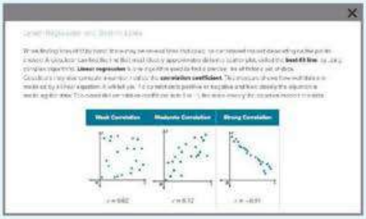
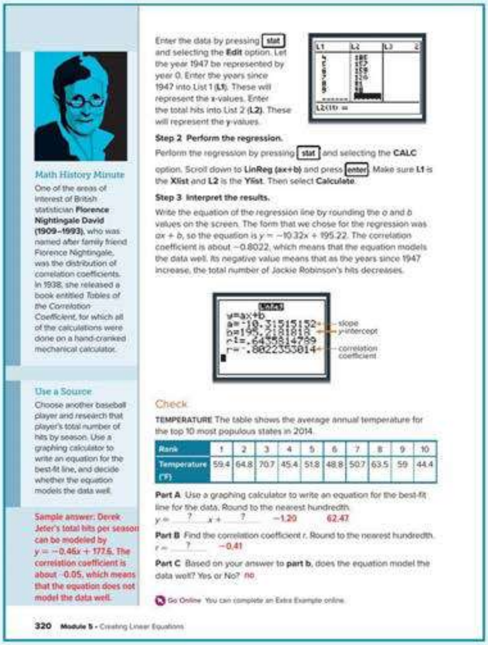
Interactive Presentation



Learn

DRAG & DROP

Students drag and drop the correlation coefficients in order from weakest to strongest.



Math History Minute

One of the areas of interest of British statistician **Florence Nightingale David (1909–1993)**, who was named after family friend Florence Nightingale, was the distribution of correlation coefficients. In 1938, she released a book entitled *Tables of the Correlation Coefficient*, for which all of the calculations were done on a hand-cranked mechanical calculator.

Use a Source

Choose another baseball player and research that player's total number of hits by season. Use a graphing calculator to write an equation for the best-fit line, and decide whether the equation models the data well.

Sample answer: Derek Jeter's total hits per season can be modeled by $y = -0.46x + 177.6$. The correlation coefficient is about -0.05, which means that the equation does not model the data well.

Enter the data by pressing stat and selecting the **Edit** option. Let the year 1947 be represented by year 0. Enter the years since 1947 into List 1 (**L1**). These will represent the *x*-values. Enter the total hits into List 2 (**L2**). These will represent the *y*-values.

Step 2 Perform the regression.

Perform the regression by pressing stat and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press enter. Make sure **L1** is the **Xlist** and **L2** is the **Ylist**. Then select **Calculate**.

Step 3 Interpret the results.

Write the equation of the regression line by rounding the *a* and *b* values on the screen. The form that we chose for the regression was $ax + b$, so the equation is $y = -10.32x + 195.22$. The correlation coefficient is about -0.8022, which means that the equation models the data well. Its negative value means that as the years since 1947 increase, the total number of Jackie Robinson's hits decreases.

```
LinReg
y=ax+b
a=-10.31515152
b=195.2181818
r²=.6435814789
r=-.8022353014
```

Check

TEMPERATURE The table shows the average annual temperature for the top 10 most populous states in 2014.

Rank	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	59.4	64.8	70.7	45.4	51.8	48.8	50.7	63.5	59	44.4

Part A Use a graphing calculator to write an equation for the best-fit line for the data. Round to the nearest hundredth.

$y =$ __?__ $x +$ __?__ −1.20 62.47

Part B Find the correlation coefficient *r*. Round to the nearest hundredth.

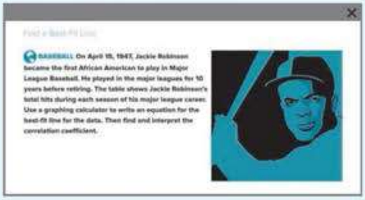
$r =$ __?__ −0.41

Part C Based on your answer to **part b**, does the equation model the data well? Yes or No? no

Go Online You can complete an Extra Example online.

320 **Module 5** • Creating Linear Equations

Interactive Presentation



Find a Best-Fit Line

BASEBALL On April 15, 1947, Jackie Robinson became the first African American to play in Major League Baseball. He played in the major leagues for 10 years before retiring. The table shows Jackie Robinson's total hits during each season of his major league career. Use a graphing calculator to write an equation for the best-fit line for the data. Then find and interpret the correlation coefficient.

Example 1

TAP

Students move through the steps to find the best-fit line using a TI-calculator.

TYPE

Students choose another baseball player and research the total number of hits per season to create a line of best fit.

Essential Question Follow-Up

Students have performed linear regression to find the equation of the best-fit line for a set of data.

Ask:

What does the best-fit line tell you about the data set it represents? Sample answer: It tells you what the relationship between the two variables is, and gives you an equation that you can use to find additional data points.

DIFFERENTIATE

Language Development Activity ELL

Beginning Help students access text by working through Examples using a graphing calculator display. Use a poster of the keyboard to point out each step as you work through the problems and explain using short phrases.

Intermediate Provide students with a study guide to make the lesson accessible to all students. Paraphrasing content helps students make connections more easily.

DIFFERENTIATE

Enrichment Activity BL

Remind students that outliers are points that are significantly distant from the other data points. Have students work with a partner to discuss how removing an outlier from a set of data impacts the line of best fit and the residual plot for the data.

Sample answer: Removing an outlier changes the line of best fit, as it changes the regression equation. It also results in a residual plot that indicates that the line is a better fit for the data.

Example 2 Use a Best-Fit Line

Teaching the Mathematical Practices

4 Make Assumptions In the Study Tip, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL What are two things that need to be done to solve this problem? Write an equation of the best-fit line, and then use the equation to find the estimate.

OL How can the regression equation be used to find the amount of sales in 2020? Substitute 11 for *x* and find the value of *y*.

BL How could the regression equation be used to find when the sales will reach 5 billion? Sample answer: Substitute 5000 for *y* and find the value of *x*. This is the years since 2009, so you would need to add 2009 to find the year.

DIFFERENTIATE

Reteaching Activity **AL**

IF students are having difficulty using a graphing calculator to perform the linear regression and graph the equation,

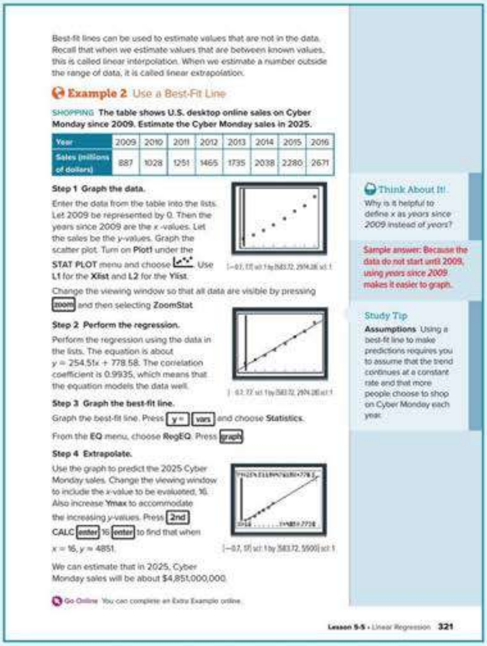
THEN use a poster of the calculator keyboard to point out each step as you work through the problems. Encourage students to take notes of the steps, using their own words to write instructions that they will find helpful.

Essential Question Follow-Up

Students have created linear functions that represent real-world data using linear regression.

Ask:

What does a linear regression equation tell you about the data that it represents? Sample answer: It gives you an approximation of how the variables are related.



Best-fit lines can be used to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called linear interpolation. When we estimate a number outside the range of data, it is called linear extrapolation.

Example 2 Use a Best-Fit Line

SHOPPING **The table shows U.S. desktop online sales on Cyber Monday since 2009. Estimate the Cyber Monday sales in 2025.**

Year	2009	2010	2011	2012	2013	2014	2015	2016
Sales (millions of dollars)	887	1028	1251	1465	1735	2038	2280	2671

Step 1 Graph the data.

Enter the data from the table into the lists. Let 2009 be represented by 0. Then the years since 2009 are the *x*-values. Let the sales be the *y*-values. Graph the scatter plot. Turn on **Plot1** under the **STAT PLOT** menu and choose ⋰. Use **L1** for the **Xlist** and **L2** for the **Ylist**.

[−0.7, 17] scl: 1 by [583.72, 2974.28] scl: 1

Change the viewing window so that all data are visible by pressing zoom and then selecting **ZoomStat**.

Step 2 Perform the regression.

Perform the regression using the data in the lists. The equation is about $y = 254.51x + 778.58$. The correlation coefficient is 0.9935, which means that the equation models the data well.

[−0.7, 17] scl: 1 by [583.72, 2974.28] scl: 1

Step 3 Graph the best-fit line.

Graph the best-fit line. Press y= vars and choose **Statistics**. From the **EQ** menu, choose **RegEQ**. Press graph.

Step 4 Extrapolate.

Use the graph to predict the 2025 Cyber Monday sales. Change the viewing window to include the *x*-value to be evaluated, 16. Also increase **Ymax** to accommodate the increasing *y*-values. Press 2nd CALC enter 16 enter to find that when $x = 16$, $y \approx 4851$.

[−0.7, 17] scl: 1 by [583.72, 5500] scl: 1

We can estimate that in 2025, Cyber Monday sales will be about $4,851,000,000.

Go Online You can complete an Extra Example online.

Think About It!

Why is it helpful to define *x* as years since 2009 instead of years?

Sample answer: Because the data do not start until 2009, using years since 2009 makes it easier to graph.

Study Tip

Assumptions Using a best-fit line to make predictions requires you to assume that the trend continues at a constant rate and that more people choose to shop on Cyber Monday each year.

Lesson 5-5 • Linear Regression 321

Interactive Presentation

Year	2007	2008	2009	2010	2011	2012	2013	2014
Sales (millions of dollars)	733	846	887	1028	1251	1465	1735	2038

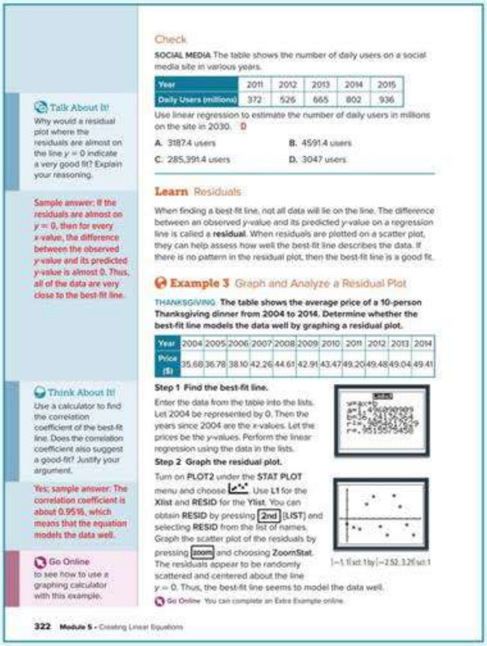
Example 2

TAP

Students move through the steps to find the best-fit line using a TI-calculator.

TYPE

Students explain why it is helpful to define *x* as years since 2007.



Talk About It!
Why would a residual plot where the residuals are almost on the line $y = 0$ indicate a very good fit? Explain your reasoning.

Sample answer: If the residuals are almost on $y = 0$, then for every x-value, the difference between the observed y-value and its predicted y-value is almost 0. Thus, all of the data are very close to the best-fit line.

Think About It!
Use a calculator to find the correlation coefficient of the best-fit line. Does the correlation coefficient also suggest a good-fit? Justify your argument.

Yes; sample answer: The correlation coefficient is about 0.9516, which means that the equation models the data well.

Go Online to see how to use a graphing calculator with this example.

Check

SOCIAL MEDIA The table shows the number of daily users on a social media site in various years.

Year	2011	2012	2013	2014	2015
Daily Users (millions)	372	526	665	802	936

Use linear regression to estimate the number of daily users in millions on the site in 2030. D

A. 3187.4 users
B. 4591.4 users
C. 285,391.4 users
D. 3047 users

Learn Residuals

When finding a best-fit line, not all data will lie on the line. The difference between an observed y-value and its predicted y-value on a regression line is called a **residual**. When residuals are plotted on a scatter plot, they can help assess how well the best-fit line describes the data. If there is no pattern in the residual plot, then the best-fit line is a good fit.

Example 3 Graph and Analyze a Residual Plot

THANKSGIVING **The table shows the average price of a 10-person Thanksgiving dinner from 2004 to 2014. Determine whether the best-fit line models the data well by graphing a residual plot.**

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Price ($)	35.68	36.78	38.10	42.26	44.61	42.91	43.47	49.20	49.48	49.04	49.41

Step 1 Find the best-fit line.

Enter the data from the table into the lists. Let 2004 be represented by 0. Then the years since 2004 are the x-values. Let the prices be the y-values. Perform the linear regression using the data in the lists.

Step 2 Graph the residual plot.

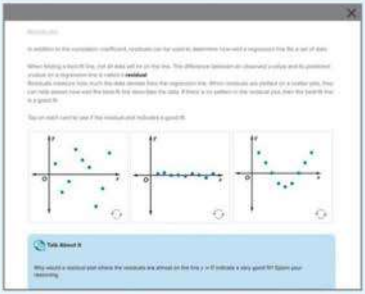
Turn on **PLOT2** under the **STAT PLOT** menu and choose ⊡. Use **L1** for the **Xlist** and **RESID** for the **Ylist**. You can obtain **RESID** by pressing 2nd [LIST] and selecting **RESID** from the list of names. Graph the scatter plot of the residuals by pressing zoom and choosing **ZoomStat**. The residuals appear to be randomly scattered and centered about the line $y = 0$. Thus, the best-fit line seems to model the data well.

[−1, 11] scl: 1 by [−2.52, 3.21] scl: 1

Go Online You can complete an Extra Example online.

322 Module 5 • Creating Linear Equations

Interactive Presentation



Learn

TAP

Students tap each card to see if the residual plot indicates a good fit.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Learn Residuals

Objective

Students determine how well functions fit sets of data by plotting and analyzing residual plots.

Teaching the Mathematical Practices

3 Construct Arguments In the Think About It! for this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

Example 3 Graph and Analyze a Residual Plot

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL Does the slope of the best-fit line mean that the price is increasing or decreasing? Why? Increasing; sample answer: If the slope is positive, then the price is increasing. If the slope is negative, then the price is decreasing.

OL What does the residual plot show? Sample answer: There are points both above and below the line of best fit.

BL Are there any outliers in the data? no What does that indicate about the data? Sample answer: It indicates that the cost of a 10-person Thanksgiving dinner increases at about the same rate from year-to-year.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

S.ID.6, S.ID.8

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–8
2	exercises that use a variety of skills from this lesson	9–13
3	exercises that emphasize higher-order and critical-thinking skills	14–17

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–13 odd, 14–17
- Extension: Quadratic Regression
- **ALEKS**® Scatter Plots and Lines of Best Fit

OL

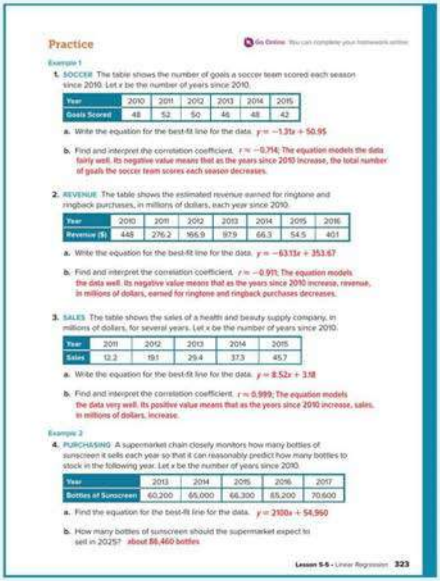
IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–17 odd
- Remediation, Review Resources: Draw Lines of Best Fit
- Personal Tutors
- Extra Examples 1–3
- **ALEKS**® Scatterplots and Lines of Best Fit

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–7 odd
- Remediation, Review Resources: Draw Lines of Best Fit
- *Quick Review Math Handbook*: Regression and Median-Fit Lines
- Arrive**MATH** Take Another Look
- **ALEKS**® Scatterplots and Lines of Best Fit



Practice

Go Online You can complete your homework online.

Example 1

1. **SOCCER** The table shows the number of goals a soccer team scored each season since 2010. Let x be the number of years since 2010.

Year	2010	2011	2012	2013	2014	2015
Goals Scored	48	52	50	46	48	42

   a. Write the equation for the best-fit line for the data. $y = -1.31x + 50.95$

   b. Find and interpret the correlation coefficient. $r \approx -0.714$; The equation models the data fairly well. Its negative value means that as the years since 2010 increase, the total number of goals the soccer team scores each season decreases.

2. **REVENUE** The table shows the estimated revenue earned for ringtone and ringback purchases, in millions of dollars, each year since 2010.

Year	2010	2011	2012	2013	2014	2015	2016
Revenue ($)	448	276.2	166.9	97.9	66.3	54.5	40.1

   a. Write the equation for the best-fit line for the data. $y = -63.13x + 353.67$

   b. Find and interpret the correlation coefficient. $r \approx -0.911$; The equation models the data well. Its negative value means that as the years since 2010 increase, revenue, in millions of dollars, earned for ringtone and ringback purchases decreases.

3. **SALES** The table shows the sales of a health and beauty supply company, in millions of dollars, for several years. Let x be the number of years since 2010.

Year	2011	2012	2013	2014	2015
Sales	12.2	19.1	29.4	37.3	45.7

   a. Write the equation for the best-fit line for the data. $y = 8.52x + 3.18$

   b. Find and interpret the correlation coefficient. $r \approx 0.999$; The equation models the data very well. Its positive value means that as the years since 2010 increase, sales, in millions of dollars, increase.

Example 2

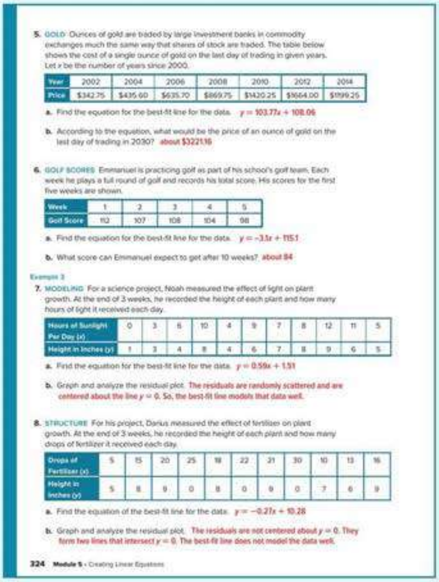
4. **PURCHASING** A supermarket chain closely monitors how many bottles of sunscreen it sells each year so that it can reasonably predict how many bottles to stock in the following year. Let x be the number of years since 2010.

Year	2013	2014	2015	2016	2017
Bottles of Sunscreen	60,200	65,000	66,300	65,200	70,600

   a. Find the equation for the best-fit line for the data. $y = 2100x + 54{,}960$

   b. How many bottles of sunscreen should the supermarket expect to sell in 2025? about 86,460 bottles

Lesson 5-5 • Linear Regression **323**



5. **GOLD** Ounces of gold are traded by large investment banks in commodity exchanges much the same way that shares of stock are traded. The table below shows the cost of a single ounce of gold on the last day of trading in given years. Let x be the number of years since 2000.

Year	2002	2004	2006	2008	2010	2012	2014
Price	$342.75	$435.60	$635.70	$869.75	$1420.25	$1664.00	$1199.25

   a. Find the equation for the best-fit line for the data. $y = 103.77x + 108.06$

   b. According to the equation, what would be the price of an ounce of gold on the last day of trading in 2030? about $3221.16

6. **GOLF SCORES** Emmanuel is practicing golf as part of his school's golf team. Each week he plays a full round of golf and records his total score. His scores for the first five weeks are shown.

Week	1	2	3	4	5
Golf Score	112	107	108	104	98

   a. Find the equation for the best-fit line for the data. $y = -3.1x + 115.1$

   b. What score can Emmanuel expect to get after 10 weeks? about 84

Example 3

7. **MODELING** For a science project, Noah measured the effect of light on plant growth. At the end of 3 weeks, he recorded the height of each plant and how many hours of light it received each day.

Hours of Sunlight Per Day (x)	0	3	6	10	4	9	7	8	12	11	5
Height in Inches (y)	1	3	4	8	4	6	7	8	9	6	5

   a. Find the equation for the best-fit line for the data. $y = 0.59x + 1.51$

   b. Graph and analyze the residual plot. The residuals are randomly scattered and are centered about the line $y = 0$. So, the best-fit line models that data well.

8. **STRUCTURE** For his project, Darius measured the effect of fertilizer on plant growth. At the end of 3 weeks, he recorded the height of each plant and how many drops of fertilizer it received each day.

Drops of Fertilizer (x)	5	15	20	25	18	22	21	30	10	13	16
Height in Inches (y)	5	8	9	0	8	0	9	0	7	6	9

   a. Find the equation of the best-fit line for the data. $y = -0.27x + 10.28$

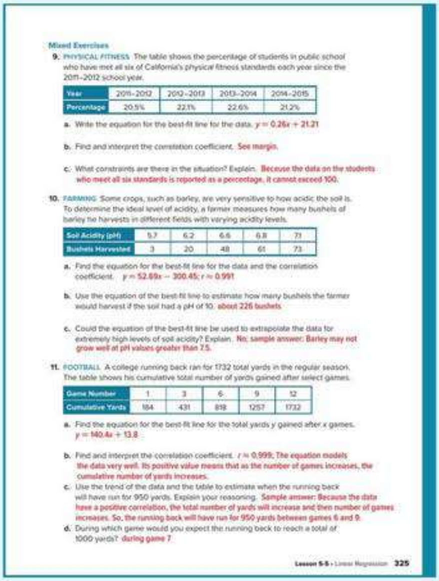
   b. Graph and analyze the residual plot. The residuals are not centered about $y = 0$. They form two lines that intersect $y = 0$. The best-fit line does not model the data well.

324 **Module 5** • Creating Linear Equations

3 REFLECT AND PRACTICE

S.ID.6, S.ID.8

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Mixed Exercises

9. PHYSICAL FITNESS The table shows the percentage of students in public school who have met all six of California's physical fitness standards each year since the 2011–2012 school year.

Year	2011–2012	2012–2013	2013–2014	2014–2015
Percentage	20.5%	22.1%	22.6%	21.2%

a. Write the equation for the best-fit line for the data. $y = 0.26x + 21.21$

b. Find and interpret the correlation coefficient. See margin.

c. What constraints are there in the situation? Explain. Because the data on the students who meet all six standards is reported as a percentage, it cannot exceed 100.

10. FARMING Some crops, such as barley, are very sensitive to how acidic the soil is. To determine the ideal level of acidity, a farmer measures how many bushels of barley he harvests in different fields with varying acidity levels.

Soil Acidity (pH)	5.7	6.2	6.6	6.8	7.1
Bushels Harvested	3	20	48	61	73

a. Find the equation for the best-fit line for the data and the correlation coefficient. $y = 52.69x - 300.45$; $r \approx 0.991$

b. Use the equation of the best-fit line to estimate how many bushels the farmer would harvest if the soil had a pH of 10. about 226 bushels

c. Could the equation of the best-fit line be used to extrapolate the data for extremely high levels of soil acidity? Explain. No; sample answer: Barley may not grow well at pH values greater than 7.5.

11. FOOTBALL A college running back ran for 1732 total yards in the regular season. The table shows his cumulative total number of yards gained after select games.

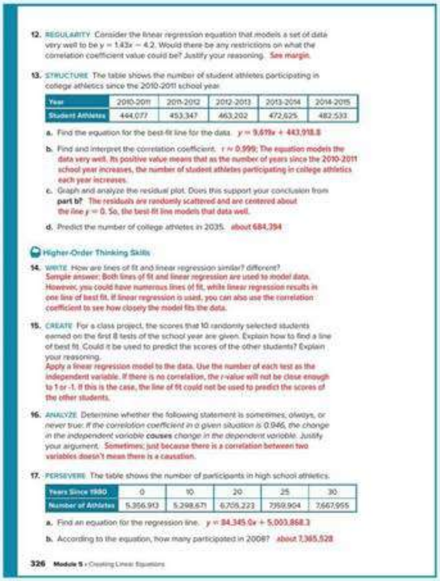
Game Number	1	3	6	9	12
Cumulative Yards	184	431	818	1257	1732

a. Find the equation for the best-fit line for the total yards y gained after x games. $y = 140.4x + 13.8$

b. Find and interpret the correlation coefficient. $r \approx 0.999$; The equation models the data very well. Its positive value means that as the number of games increases, the cumulative number of yards increases.

c. Use the trend of the data and the table to estimate when the running back will have run for 950 yards. Explain your reasoning. Sample answer: Because the data have a positive correlation, the total number of yards will increase and then number of games increases. So, the running back will have run for 950 yards between games 6 and 9.

d. During which game would you expect the running back to reach a total of 1000 yards? during game 7



12. REGULARITY Consider the linear regression equation that models a set of data very well to be $y = 1.43x - 4.2$. Would there be any restrictions on what the correlation coefficient value could be? Justify your reasoning. See margin.

13. STRUCTURE The table shows the number of student athletes participating in college athletics since the 2010-2011 school year.

Year	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015
Student Athletes	444,077	453,347	463,202	472,625	482,533

a. Find the equation for the best-fit line for the data. $y = 9,619x + 443,918.8$

b. Find and interpret the correlation coefficient. $r \approx 0.999$; The equation models the data very well. Its positive value means that as the number of years since the 2010-2011 school year increases, the number of student athletes participating in college athletics each year increases.

c. Graph and analyze the residual plot. Does this support your conclusion from **part b**? The residuals are randomly scattered and are centered about the line $y = 0$. So, the best-fit line models that data well.

d. Predict the number of college athletes in 2035. about 684,394

Higher-Order Thinking Skills

14. WRITE How are lines of fit and linear regression similar? different? Sample answer: Both lines of fit and linear regression are used to model data. However, you could have numerous lines of fit, while linear regression results in one line of best fit. If linear regression is used, you can also use the correlation coefficient to see how closely the model fits the data.

15. CREATE For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best fit. Could it be used to predict the scores of the other students? Explain your reasoning.
Apply a linear regression model to the data. Use the number of each test as the independent variable. If there is no correlation, the r-value will not be close enough to 1 or -1. If this is the case, the line of fit could not be used to predict the scores of the other students.

16. ANALYZE Determine whether the following statement is sometimes, always, or never true: *If the correlation coefficient in a given situation is 0.946, the change in the independent variable* **causes** *change in the dependent variable.* Justify your argument. Sometimes; just because there is a correlation between two variables doesn't mean there is a causation.

17. PERSEVERE The table shows the number of participants in high school athletics.

Years Since 1980	0	10	20	25	30
Number of Athletes	5,356,913	5,298,671	6,705,223	7,159,904	7,667,955

a. Find an equation for the regression line. $y = 84,345.0x + 5,003,868.3$

b. According to the equation, how many participated in 2008? about 7,365,528

Answers

9b. $r \approx 0.359$; The equation does not model the data well. Its value means that as the years since the 2011–2012 school year increase, the percentage of students in public school who met all six of California's physical fitness standards each year varies.

12. The slope of the linear regression line is positive, so the correlation coefficient should be positive. Because the equation models the data very well, there is a strong correlation, so it should be about 0.8 or greater.

Lesson 5-6

Inverses of Linear Functions

A.CED.2, F.BF.4a

LESSON GOAL

Students find inverses of functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Inverses of Relations
- Inverse Relations
- Find Inverse Relations from a Table
- Graph Inverse Relations

Explore: Comparing a Function and its Inverse

Develop:

Inverses of Linear Functions
- Find an Inverse Linear Function
- Find Inverses of Linear Functions
- Apply Inverse Linear Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Function Tables	●	●		●
Extension: One-to-One and Onto Functions		●	●	●

Language Development Handbook

Assign page 32 of the *Language Development Handbook* to help your students build mathematical language related to the inverses of functions.

ELL You can use the tips and suggestions on page T32 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students understood the concept of a function.
8.F.1

Now
Students find inverses of functions.
A.CED.2, F.BF.4a

Next
Students will understand the relationship between functions and their inverses, and whether the inverse of a function is also a function.
F.BF.4 (Course 2)

Rigor

The Three Pillars of Rigor

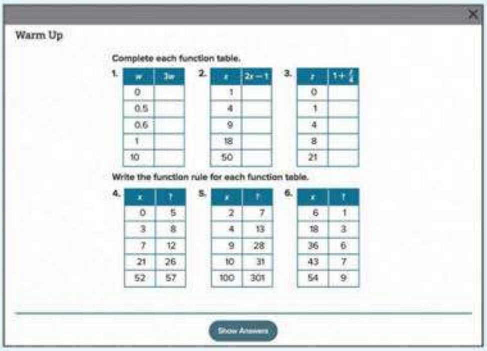
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, expand on what students have learned about linear functions to gain understanding of inverses of linear functions. They build fluency by finding inverses of relations and functions, and they apply their understanding by solving real-world problems.

Mathematical Background

The inverse of a relation is the set of ordered pairs obtained by interchanging the x-coordinate and y-coordinate of each ordered pair of the original relation. A linear function $f(x)$ has an inverse function $f^{-1}(x)$ that can be found by replacing $f(x)$ with y, interchanging x and y in the equation, solving for y, and then replacing y with $f^{-1}(x)$.

Interactive Presentation



Warm Up

Complete each function table.

1.

w	$3w$
0	
0.5	
0.6	
1	
10	

2.

x	$2x-1$
1	
4	
9	
18	
50	

3.

z	$1+\frac{z}{4}$
0	
1	
4	
8	
21	

Write the function rule for each function table.

4.

x	y
0	5
3	8
7	12
21	26
52	57

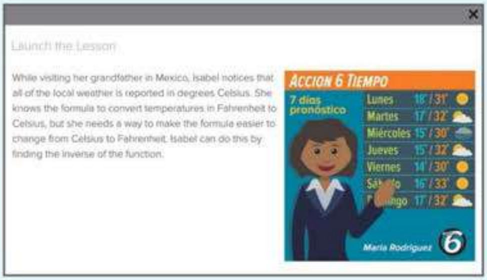
5.

x	y
2	7
4	13
9	28
10	31
100	301

6.

x	y
6	1
18	3
36	6
43	7
54	9

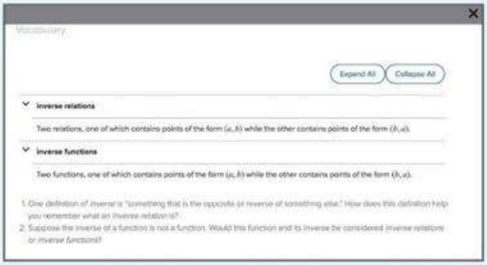
Warm Up



Launch the Lesson

While visiting her grandfather in Mexico, Isabel notices that all of the local weather is reported in degrees Celsius. She knows the formula to convert temperatures in Fahrenheit to Celsius, but she needs a way to make the formula easier to change from Celsius to Fahrenheit. Isabel can do this by finding the inverse of the function.

Launch the Lesson



Vocabulary

inverse relations

Two relations, one of which contains points of the form (a, b) while the other contains points of the form (b, a).

inverse functions

Two functions, one of which contains points of the form (a, b) while the other contains points of the form (b, a).

1. One definition of *inverse* is "something that is the opposite or reverse of something else." How does this definition help you remember what an *inverse relation* is?
2. Suppose the inverse of a function is not a function. Would this function and its inverse be considered *inverse relations* or *inverse functions*?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- making function tables

Answers

1.

w	$3w$
0	0
0.5	1.5
0.6	1.8
1	3
10	30

2.

x	$2x-1$
1	1
4	7
9	17
18	35
50	99

3.

z	$2+\frac{z}{4}$
0	1
1	1.25
4	2
8	3
21	6.25

4. $y = x + 5$ 5. $y = 3x + 1$ 6. $y = \frac{x}{6}$

Launch the Lesson

Teaching the Mathematical Practices

4 Analyze Relationships Mathematically Point out that to make the formula Isabel knows easier to use, students will need to understand the relationship between a function and its inverse.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Comparing a Function and its Inverse

Objective

Students use a sketch to explore inverse functions.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch of the graph of a function and its inverse to explore the relationship between the two graphs. A series of questions will guide students through the exploration, leading them to observe the relationship between the coordinates of points on the two graphs. Students will also explore the relationship between the graphs and the line $y = x$. Then, students will answer the Inquiry Question.

(continued on the next page)

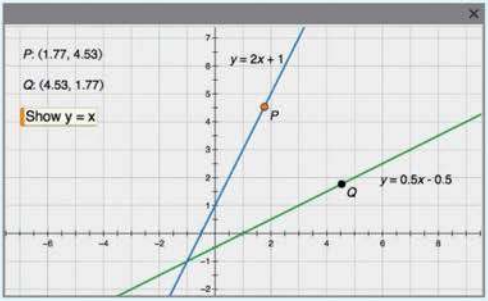
Interactive Presentation

Comparing a Function and its Inverse

INQUIRY How can you graph the inverse of a function?

You can use the sketch to explore two inverse functions and then complete the exercises.

Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore two inverse functions.

TYPE

Students answer questions regarding the relationship of inverse functions using the activity.

Interactive Presentation

INQUIRY How can you graph the inverse of a function?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Comparing a Function and Its Inverse (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- If the point (x, y) lies on the graph of a function, what point must lie on the graph of its inverse? (y, x)
- If the graph of a function is a line, will the graph of its inverse always be a line? Explain. Sample answer: Yes; because the graphs of the two functions are reflections across the line $y = x$, and the reflection of a line is another line, the graph of the inverse will also be a line.

Inquiry

How can you graph the inverse of a function? Sample answer: Reverse the coordinates of the points on the function to reflect the graph across the line $y = x$.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Inverses of Relations

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Important to Know

Every relation has an inverse. However, not every function has an inverse that is a function. The first part of this lesson deals with relations and their inverses. The second part of the lesson will address the inverses of functions.

Common Misconception

Students may think that in order for a relation to have an inverse, the relation must consist of a finite number of ordered pairs. Explain that although initially they will be working with relations consisting of a finite number of ordered pairs, they will later extend their understanding to relations containing infinitely many ordered pairs.

Example 1 Inverse Relations

Teaching the Mathematical Practices

7 Use Structure Help students to see the pattern when determining the inverse of the relation in the example. Encourage students to explain how to use this pattern to write the inverse of any relation.

Questions for Mathematical Discourse

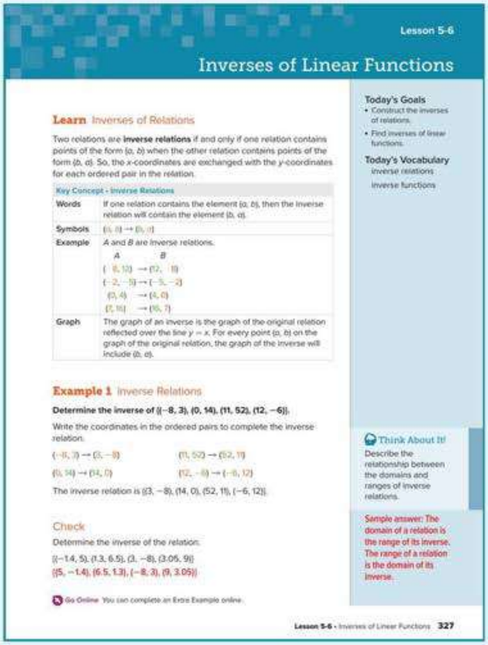
AL What is the domain of this relation? $\{-8, 0, 11, 12\}$ What is the range? $\{-6, 3, 14, 52\}$

OL What is the domain of the inverse of this relation? $\{-6, 3, 14, 52\}$ What is the range? $\{-8, 0, 11, 12\}$

BL Is it possible for a point to be in the relation and its inverse? Explain. Yes; sample answer: If a point in the relation lies on the line $y = x$, meaning that the x and y values are the same, then it will also be in the inverse relation.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 5-6

Inverses of Linear Functions

Today's Goals
- Construct the inverses of relations.
- Find inverses of linear functions.

Today's Vocabulary
inverse relations
inverse functions

Learn Inverses of Relations

Two relations are **inverse relations** if and only if one relation contains points of the form (a, b) when the other relation contains points of the form (b, a). So, the x-coordinates are exchanged with the y-coordinates for each ordered pair in the relation.

Key Concept • Inverse Relations	
Words	If one relation contains the element (a, b), then the inverse relation will contain the element (b, a).
Symbols	$(a, b) \rightarrow (b, a)$
Example	A and B are inverse relations. A → B $(-8, 12) \rightarrow (12, -8)$ $(-2, -5) \rightarrow (-5, -2)$ $(0, 4) \rightarrow (4, 0)$ $(7, 16) \rightarrow (16, 7)$
Graph	The graph of an inverse is the graph of the original relation reflected over the line $y = x$. For every point (a, b) on the graph of the original relation, the graph of the inverse will include (b, a).

Example 1 Inverse Relations

Determine the inverse of $\{(-8, 3), (0, 14), (11, 52), (12, -6)\}$.

Write the coordinates in the ordered pairs to complete the inverse relation.

$(-8, 3) \rightarrow (3, -8)$ $(11, 52) \rightarrow (52, 11)$

$(0, 14) \rightarrow (14, 0)$ $(12, -6) \rightarrow (-6, 12)$

The inverse relation is $\{(3, -8), (14, 0), (52, 11), (-6, 12)\}$.

Think About It!
Describe the relationship between the domains and ranges of inverse relations.

Sample answer: The domain of a relation is the range of its inverse. The range of a relation is the domain of its inverse.

Check

Determine the inverse of the relation.

$\{(-1.4, 5), (1.3, 6.5), (3, -8), (3.05, 9)\}$

$\{(5, -1.4), (6.5, 1.3), (-8, 3), (9, 3.05)\}$

Go Online You can complete an Extra Example online.

Lesson 5-6 • Inverses of Linear Functions 327

Interactive Presentation

Learn

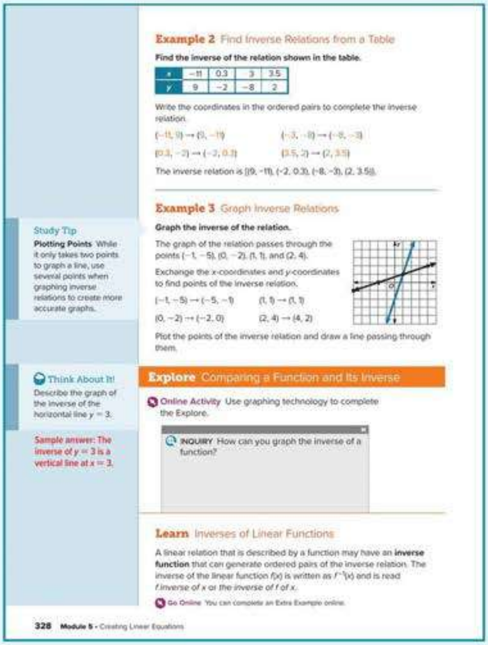
TYPE

Students answer a question to show they understand the relationship between the domains and ranges of inverse relations.

2 EXPLORE AND DEVELOP

A.CED.2, F.BF.4a

1 CONCEPTUAL UNDERSTANDING　**2 FLUENCY**　3 APPLICATION



Example 2 Find Inverse Relations from a Table

Find the inverse of the relation shown in the table.

x	−11	0.3	−3	3.5
y	9	−2	−8	2

Write the coordinates in the ordered pairs to complete the inverse relation.

(−11, 9) → (9, −11)　　(−3, −8) → (−8, −3)

(0.3, −2) → (−2, 0.3)　　(3.5, 2) → (2, 3.5)

The inverse relation is {(9, −11), (−2, 0.3), (−8, −3), (2, 3.5)}.

Example 3 Graph Inverse Relations

Graph the inverse of the relation.

The graph of the relation passes through the points (−1, −5), (0, −2), (1, 1), and (2, 4).

Exchange the x-coordinates and y-coordinates to find points of the inverse relation.

(−1, −5) → (−5, −1)　　(1, 1) → (1, 1)

(0, −2) → (−2, 0)　　(2, 4) → (4, 2)

Plot the points of the inverse relation and draw a line passing through them.

Study Tip

Plotting Points While it only takes two points to graph a line, use several points when graphing inverse relations to create more accurate graphs.

Think About It!

Describe the graph of the inverse of the horizontal line y = 3.

Sample answer: The inverse of y = 3 is a vertical line at x = 3.

Explore Comparing a Function and Its Inverse

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you graph the inverse of a function?

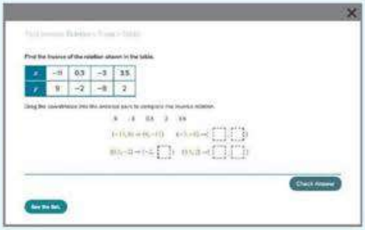
Learn Inverses of Linear Functions

A linear relation that is described by a function may have an **inverse function** that can generate ordered pairs of the inverse relation. The inverse of the linear function f(x) is written as $f^{-1}(x)$ and is read *f inverse of x* or *the inverse of f of x*.

Go Online You can complete an Extra Example online.

328 **Module 5** • Creating Linear Equations

Interactive Presentation



Example 2

DRAG & DROP

Students drag and drop values to complete the coordinates of the inverse.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Find Inverse Relations from a Table

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the table of values and the ordered pairs that comprise the inverse of the relation.

Questions for Mathematical Discourse

AL What does the table tell you about the ordered pairs that belong to the relation? Sample answer: It shows you which values make up the ordered pairs.

OL How would you make a table for the inverse relation? Switch the values in the *x*-row to be in the *y*-row, and the values in the *y*-row to be in the *x*-row.

BL What does the table tell you about the domain and range of the inverse relation? The domain of the inverse relation consists of the elements in the *y*-row, and the range of the inverse relation consists of the elements in the *x*-row.

Example 3 Graph Inverse Relations

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Why do you need to identify the coordinates of some of the points that lie on the given line? Sample answer: so that you can create some ordered pairs that belong to the inverse relation

OL Where do the graph of the given relation and the graph of the inverse relation intersect? Why? (1, 1); Sample answer: The coordinates of one of the points on the graph of the relation are the same value. That ordered pair belongs to the inverse relation, as well, so that point lies on both lines.

BL What is the graphical relationship between the inverse and original relation? The graph of the inverse is the graph of the original relation reflected across the line $y = x$.

DIFFERENTIATE

Enrichment Activity BL

Have students graph the inverse of $y = x^2$, following these instructions: First, graph $y = x^2$ on a coordinate plane by creating a table of values. Then, find points on the graph of the inverse by exchanging the *x*- and *y*-coordinates in each ordered pair from the table. Plot these new points on the same coordinate plane. Finally, connect the new points with a smooth curve, using the graph of $y = x^2$ as a guide. Remind students that the graphs of inverse relations are reflections of each other across the line $y = x$, and encourage them to use this information to help them draw their graphs.

Learn Inverses of Linear Functions

Objective

Students find inverses of linear functions by using the algorithm for finding an inverse.

MP Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of inverses of linear functions in this Learn.

Example 4 Find an Inverse Linear Function

MP Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students should check their answer by graphing $f(x)$ and $f^{-1}(x)$ on the same coordinate plane with $y = x$.

Questions for Mathematical Discourse

AL In Step 2, why do you interchange x and y? because in the inverse function, the x- and y-values from the original function are interchanged

OL How are the function and the inverse function related? Sample answer: The input of the function is the output of the inverse function and the output of the function is the input of the inverse function.

BL How do the operations performed on the input variable in the function and in its inverse compare? They are inverse operations performed in the reverse order.

Example 5 Find Inverses of Linear Functions

MP Teaching the Mathematical Practices

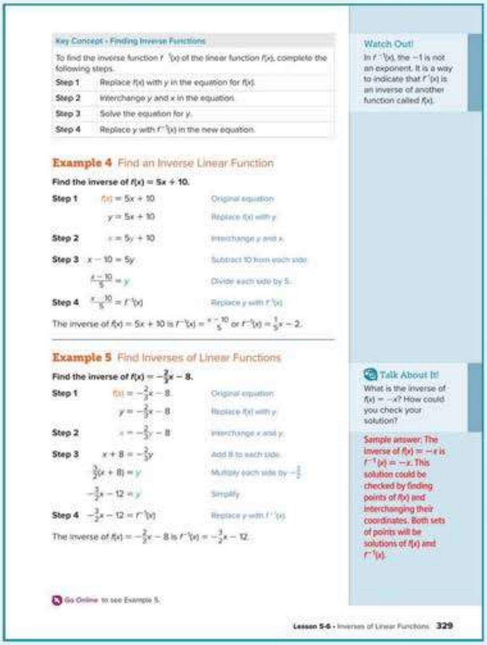
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How do you use a graph to check that two functions are inverses of each other? Sample answer: You check to see if the graphs are reflections of each other across the line $y = x$.

OL Why do you multiply each side by $-\frac{3}{2}$ instead of $\frac{3}{2}$? Sample answer: To solve for y, you must multiply by the reciprocal, which is $-\frac{3}{2}$, to undo the original operation.

BL How could you find the coordinates of the point of intersection of the graph of a function and the graph of its inverse? Sample answer: You can set the two rules equal to each other and solve for x. That answer will be both the x-coordinate and the y-coordinate of the point of intersection, as the point also lies on the graph of the line $y = x$.



Key Concept • Finding Inverse Functions

To find the inverse function $f^{-1}(x)$ of the linear function $f(x)$, complete the following steps.

Step 1	Replace $f(x)$ with y in the equation for $f(x)$.
Step 2	Interchange y and x in the equation.
Step 3	Solve the equation for y.
Step 4	Replace y with $f^{-1}(x)$ in the new equation.

Watch Out!

In $f^{-1}(x)$, the -1 is not an exponent. It is a way to indicate that $f^{-1}(x)$ is an inverse of another function called $f(x)$.

Example 4 Find an Inverse Linear Function

Find the inverse of $f(x) = 5x + 10$.

Step 1 $f(x) = 5x + 10$ Original equation

$y = 5x + 10$ Replace $f(x)$ with y.

Step 2 $x = 5y + 10$ Interchange y and x.

Step 3 $x - 10 = 5y$ Subtract 10 from each side.

$\frac{x - 10}{5} = y$ Divide each side by 5.

Step 4 $\frac{x - 10}{5} = f^{-1}(x)$ Replace y with $f^{-1}(x)$.

The inverse of $f(x) = 5x + 10$ is $f^{-1}(x) = \frac{x - 10}{5}$ or $f^{-1}(x) = \frac{1}{5}x - 2$.

Example 5 Find Inverses of Linear Functions

Find the inverse of $f(x) = -\frac{2}{3}x - 8$.

Step 1 $f(x) = -\frac{2}{3}x - 8$ Original equation

$y = -\frac{2}{3}x - 8$ Replace $f(x)$ with y.

Step 2 $x = -\frac{2}{3}y - 8$ Interchange x and y.

Step 3 $x + 8 = -\frac{2}{3}y$ Add 8 to each side.

$-\frac{3}{2}(x + 8) = y$ Multiply each side by $-\frac{3}{2}$.

$-\frac{3}{2}x - 12 = y$ Simplify.

Step 4 $-\frac{3}{2}x - 12 = f^{-1}(x)$ Replace y with $f^{-1}(x)$.

The inverse of $f(x) = -\frac{2}{3}x - 8$ is $f^{-1}(x) = -\frac{3}{2}x - 12$.

Talk About It!

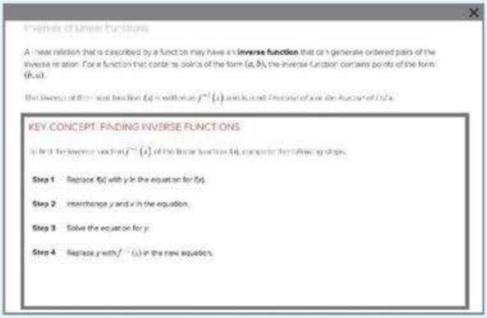
What is the inverse of $f(x) = -x$? How could you check your solution?

Sample answer: The inverse of $f(x) = -x$ is $f^{-1}(x) = -x$. This solution could be checked by finding points of $f(x)$ and interchanging their coordinates. Both sets of points will be solutions of $f(x)$ and $f^{-1}(x)$.

Go Online to see Example 5.

Lesson 5-6 • Inverses of Linear Functions 329

Interactive Presentation



Inverses of Linear Functions

A linear relation that is described by a function may have an **inverse function** that can generate ordered pairs of the inverse relation. For a function that contains points of the form (a, b), the inverse function contains points of the form (b, a).

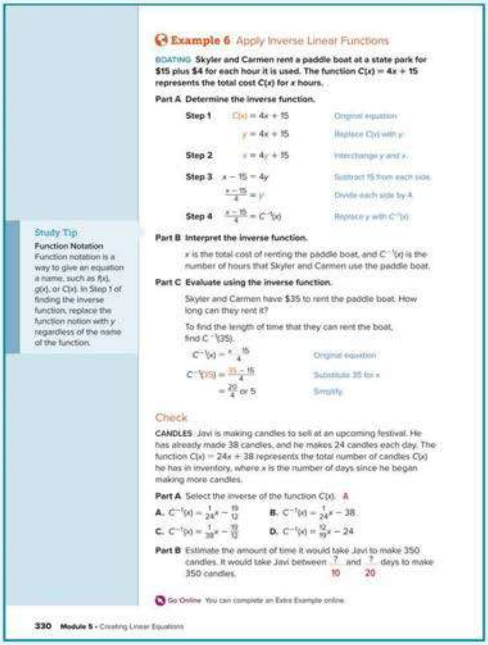
KEY CONCEPT: FINDING INVERSE FUNCTIONS

Step 1	Replace $f(x)$ with y in the equation for $f(x)$.
Step 2	Interchange y and x in the equation.
Step 3	Solve the equation for y.
Step 4	Replace y with $f^{-1}(x)$ in the new equation.

Learn

WATCH

Students can watch a video about finding inverse functions.



Example 6 Apply Inverse Linear Functions

BOATING **Skyler and Carmen rent a paddle boat at a state park for \$15 plus \$4 for each hour it is used. The function $C(x) = 4x + 15$ represents the total cost $C(x)$ for x hours.**

Part A Determine the inverse function.

Step 1 $C(x) = 4x + 15$ Original equation

$y = 4x + 15$ Replace C(x) with y.

Step 2 $x = 4y + 15$ Interchange y and x.

Step 3 $x - 15 = 4y$ Subtract 15 from each side.

$\frac{x-15}{4} = y$ Divide each side by 4.

Step 4 $\frac{x-15}{4} = C^{-1}(x)$ Replace y with $C^{-1}(x)$.

Part B Interpret the inverse function.

x is the total cost of renting the paddle boat, and $C^{-1}(x)$ is the number of hours that Skyler and Carmen use the paddle boat.

Part C Evaluate using the inverse function.

Skyler and Carmen have \$35 to rent the paddle boat. How long can they rent it?

To find the length of time that they can rent the boat, find $C^{-1}(35)$.

$C^{-1}(x) = \frac{x-15}{4}$ Original equation

$C^{-1}(35) = \frac{35-15}{4}$ Substitute 35 for x.

$= \frac{20}{4}$ or 5 Simplify.

Study Tip

Function Notation Function notation is a way to give an equation a name, such as f(x), g(x), or C(x). In Step 1 of finding the inverse function, replace the function notation with y regardless of the name of the function.

Check

CANDLES Javi is making candles to sell at an upcoming festival. He has already made 38 candles, and he makes 24 candles each day. The function $C(x) = 24x + 38$ represents the total number of candles $C(x)$ he has in inventory, where x is the number of days since he began making more candles.

Part A Select the inverse of the function C(x). A

A. $C^{-1}(x) = \frac{1}{24}x - \frac{19}{12}$ **B.** $C^{-1}(x) = \frac{1}{24}x - 38$

C. $C^{-1}(x) = \frac{1}{38}x - \frac{19}{12}$ **D.** $C^{-1}(x) = \frac{12}{19}x - 24$

Part B Estimate the amount of time it would take Javi to make 350 candles. It would take Javi between ___?___ and ___?___ days to make 350 candles. 10 20

Go Online You can complete an Extra Example online.

330 **Module 5** • Creating Linear Equations

Interactive Presentation

Apply Inverse Linear Functions

BOATING **Skyler and Carmen rent a paddle boat at a state park for \$15 plus \$4 for each hour it is used. The function $C(x) = 4x + 15$ represents the total cost $C(x)$ for x hours.**

Example 6

TAP

Students move through the step to finding and applying the inverse of a linear function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 6 Apply Inverse Linear Functions

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students will apply what they have learned about inverse functions to solving a real-world problem.

Questions for Mathematical Discourse

AL What does $C^{-1}(x)$ represent in the context of the situation? the number of hours you can rent a paddle boat with x dollars

OL What are the domain and range of $C(x)$ in the context of the situation? The domain is possible times in hours, and the range is possible costs to rent the boat.

BL What are the domain and range of $C^{-1}(x)$ in the context of the situation? The domain is possible costs to rent the boat, and the range is possible times in hours.

Common Error

Some students may make errors interpreting the function, its inverse, and the solution in the context of the situation. Students may benefit from identifying what the input and output values of the original function represent before they find the inverse. They can then reverse their labeling of these quantities (i.e., input for output, output for input) for the inverse function, clarifying the contextual interpretation of the inverse function.

Essential Question Follow-Up

Students have applied inverses in real-world situations.

Ask:

What does the inverse of a function tell you about the variables from the situation modeled by the original function? Sample answer: It tells you how to obtain what was the independent variable in the original function when you know the value of what was the dependent variable in the original function.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–41
3	exercises that emphasize higher-order and critical-thinking skills	42–51

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–41 odd, 42–51
- Extension: One-to-One and Onto Functions
- **ALEKS** Composition and Inverse Functions

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–51 odd
- Remediation, Review Resources: Function Tables
- Personal Tutors
- Extra Examples 1–6
- **ALEKS** Tables and Graphs of Lines

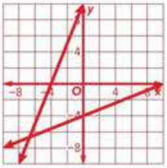
AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Function Tables
- *Quick Review Math Handbook*: Inverse Linear Functions
- Arrive**MATH** Take Another Look
- **ALEKS** Tables and Graphs of Lines

Answers

10.



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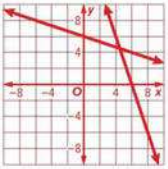
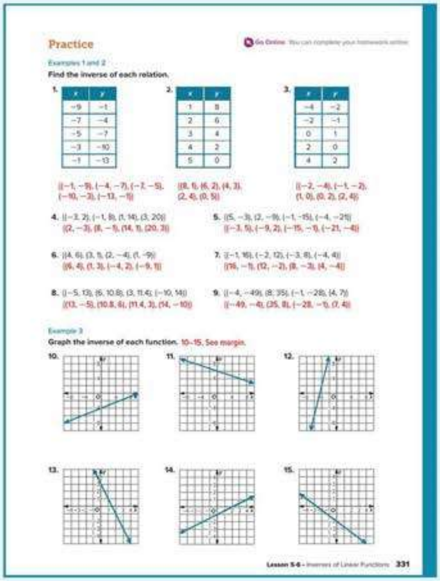




Practice

Go Online You can complete your homework online.

Examples 1 and 2

Find the inverse of each relation.

1.

x	y
−9	−1
−7	−4
−5	−7
−3	−10
−1	−13

{(−1, −9), (−4, −7), (−7, −5), (−10, −3), (−13, −1)}

2.

x	y
1	8
2	6
3	4
4	2
5	0

{(8, 1), (6, 2), (4, 3), (2, 4), (0, 5)}

3.

x	y
−4	−2
−2	−1
0	1
2	0
4	2

{(−2, −4), (−1, −2), (1, 0), (0, 2), (2, 4)}

4. {(−3, 2), (−1, 8), (1, 14), (3, 20)}
{(2, −3), (8, −1), (14, 1), (20, 3)}

5. {(5, −3), (2, −9), (−1, −15), (−4, −21)}
{(−3, 5), (−9, 2), (−15, −1), (−21, −4)}

6. {(4, 6), (3, 1), (2, −4), (1, −9)}
{(6, 4), (1, 3), (−4, 2), (−9, 1)}

7. {(−1, 16), (−2, 12), (−3, 8), (−4, 4)}
{(16, −1), (12, −2), (8, −3), (4, −4)}

8. {(−5, 13), (6, 10.8), (3, 11.4), (−10, 14)}
{(13, −5), (10.8, 6), (11.4, 3), (14, −10)}

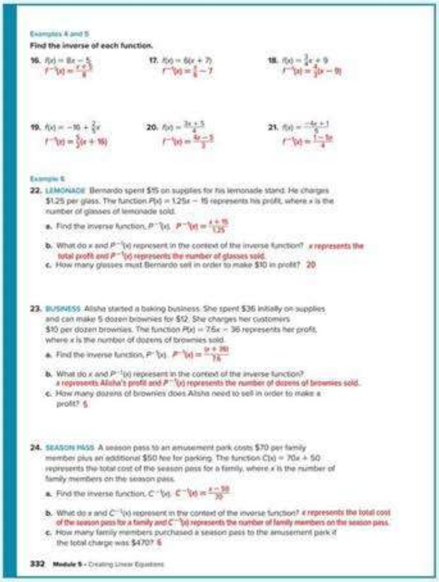
9. {(−4, −49), (8, 35), (−1, −28), (4, 7)}
{(−49, −4), (35, 8), (−28, −1), (7, 4)}

Example 3

Graph the inverse of each function. 10–15. See margin.

10. 11. 12. 13. 14. 15.

Lesson 5-6 • Inverses of Linear Functions 331



Examples 4 and 5

Find the inverse of each function.

16. $f(x) = 8x - 5$
$f^{-1}(x) = \frac{x+5}{8}$

17. $f(x) = 6(x + 7)$
$f^{-1}(x) = \frac{x}{6} - 7$

18. $f(x) = \frac{3}{4}x + 9$
$f^{-1}(x) = \frac{4}{3}(x - 9)$

19. $f(x) = -16 + \frac{3}{5}x$
$f^{-1}(x) = \frac{5}{3}(x + 16)$

20. $f(x) = \frac{3x+5}{4}$
$f^{-1}(x) = \frac{4x-5}{3}$

21. $f(x) = \frac{-4x+1}{5}$
$f^{-1}(x) = \frac{1-5x}{4}$

Example 6

22. LEMONADE Bernardo spent $15 on supplies for his lemonade stand. He charges $1.25 per glass. The function $P(x) = 1.25x - 15$ represents his profit, where x is the number of glasses of lemonade sold.
a. Find the inverse function, $P^{-1}(x)$. $P^{-1}(x) = \frac{x+15}{1.25}$
b. What do x and $P^{-1}(x)$ represent in the context of the inverse function? x represents the total profit and $P^{-1}(x)$ represents the number of glasses sold.
c. How many glasses must Bernardo sell in order to make $10 in profit? 20

23. BUSINESS Alisha started a baking business. She spent $36 initially on supplies and can make 5 dozen brownies for $12. She charges her customers $10 per dozen brownies. The function $P(x) = 7.6x - 36$ represents her profit, where x is the number of dozens of brownies sold.
a. Find the inverse function, $P^{-1}(x)$. $P^{-1}(x) = \frac{x+36}{7.6}$
b. What do x and $P^{-1}(x)$ represent in the context of the inverse function? x represents Alisha's profit and $P^{-1}(x)$ represents the number of dozens of brownies sold.
c. How many dozens of brownies does Alisha need to sell in order to make a profit? 5

24. SEASON PASS A season pass to an amusement park costs $70 per family member plus an additional $50 fee for parking. The function $C(x) = 70x + 50$ represents the total cost of the season pass for a family, where x is the number of family members on the season pass.
a. Find the inverse function, $C^{-1}(x)$. $C^{-1}(x) = \frac{x-50}{70}$
b. What do x and $C^{-1}(x)$ represent in the context of the inverse function? x represents the total cost of the season pass for a family and $C^{-1}(x)$ represents the number of family members on the season pass.
c. How many family members purchased a season pass to the amusement park if the total charge was $470? 6

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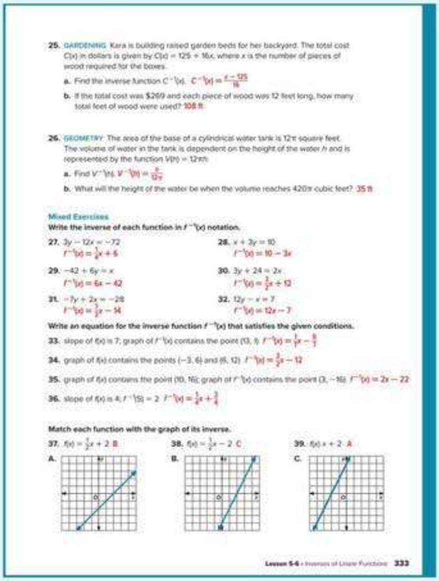


25. GARDENING Kara is building raised garden beds for her backyard. The total cost C(x) in dollars is given by C(x) = 125 + 16x, where x is the number of pieces of wood required for the boxes.

a. Find the inverse function $C^{-1}(x)$. $C^{-1}(x) = \frac{x-125}{16}$

b. If the total cost was $269 and each piece of wood was 12 feet long, how many total feet of wood were used? 108 ft

26. GEOMETRY The area of the base of a cylindrical water tank is 12π square feet. The volume of water in the tank is dependent on the height of the water h and is represented by the function V(h) = 12πh.

a. Find $V^{-1}(h)$. $V^{-1}(h) = \frac{h}{12\pi}$

b. What will the height of the water be when the volume reaches 420π cubic feet? 35 ft

Mixed Exercises

Write the inverse of each function in $f^{-1}(x)$ notation.

27. $3y - 12x = -72$
$f^{-1}(x) = \frac{1}{4}x + 6$

28. $x + 3y = 10$
$f^{-1}(x) = 10 - 3x$

29. $-42 + 6y = x$
$f^{-1}(x) = 6x - 42$

30. $3y + 24 = 2x$
$f^{-1}(x) = \frac{3}{2}x + 12$

31. $-7y + 2x = -28$
$f^{-1}(x) = \frac{7}{2}x - 14$

32. $12y - x = 7$
$f^{-1}(x) = 12x - 7$

Write an equation for the inverse function $f^{-1}(x)$ that satisfies the given conditions.

33. slope of f(x) is 7; graph of $f^{-1}(x)$ contains the point (13, 1) $f^{-1}(x) = \frac{1}{7}x - \frac{6}{7}$

34. graph of f(x) contains the points (−3, 6) and (6, 12) $f^{-1}(x) = \frac{3}{2}x - 12$

35. graph of f(x) contains the point (10, 16); graph of $f^{-1}(x)$ contains the point (3, −16) $f^{-1}(x) = 2x - 22$

36. slope of f(x) is 4; $f^{-1}(5) = 2$ $f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}$

Match each function with the graph of its inverse.

37. $f(x) = \frac{1}{2}x + 2$ B

38. $f(x) = \frac{1}{2}x - 2$ C

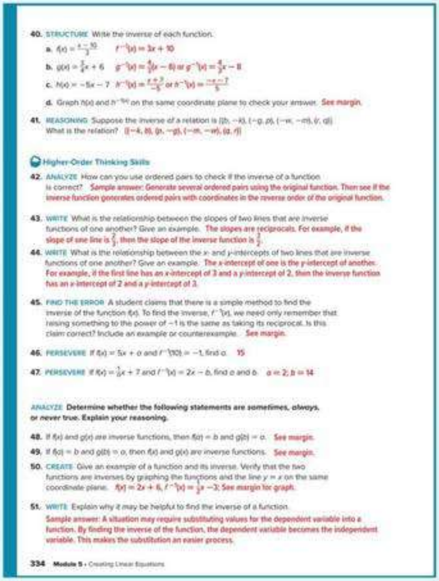
39. $f(x) = x + 2$ A

A.

B.

C.

Lesson 5-6 • Inverses of Linear Functions 333



40. STRUCTURE Write the inverse of each function.

a. $f(x) = \frac{x-10}{3}$ $f^{-1}(x) = 3x + 10$

b. $g(x) = \frac{3}{4}x + 6$ $g^{-1}(x) = \frac{4}{3}(x - 6)$ or $g^{-1}(x) = \frac{4}{3}x - 8$

c. $h(x) = -5x - 7$ $h^{-1}(x) = \frac{x+7}{-5}$ or $h^{-1}(x) = \frac{-x-7}{5}$

d. Graph h(x) and $h^{-1}(x)$ on the same coordinate plane to check your answer. See margin.

41. REASONING Suppose the inverse of a relation is {(b, −4), (−g, p), (−w, −m), (r, q)}. What is the relation? {(−4, b), (p, −g), (−m, −w), (q, r)}

Higher-Order Thinking Skills

42. ANALYZE How can you use ordered pairs to check if the inverse of a function is correct? Sample answer: Generate several ordered pairs using the original function. Then see if the inverse function generates ordered pairs with coordinates in the reverse order of the original function.

43. WRITE What is the relationship between the slopes of two lines that are inverse functions of one another? Give an example. The slopes are reciprocals. For example, if the slope of one line is $\frac{2}{3}$, then the slope of the inverse function is $\frac{3}{2}$.

44. WRITE What is the relationship between the x- and y-intercepts of two lines that are inverse functions of one another? Give an example. The x-intercept of one is the y-intercept of another. For example, if the first line has an x-intercept of 3 and a y-intercept of 2, then the inverse function has an x-intercept of 2 and a y-intercept of 3.

45. FIND THE ERROR A student claims that there is a simple method to find the inverse of the function f(x). To find the inverse, $f^{-1}(x)$, we need only remember that raising something to the power of −1 is the same as taking its reciprocal. Is this claim correct? Include an example or counterexample. See margin.

46. PERSEVERE If f(x) = 5x + a and $f^{-1}(10) = -1$, find a. 15

47. PERSEVERE If $f(x) = \frac{1}{a}x + 7$ and $f^{-1}(x) = 2x - b$, find a and b. a = 2; b = 14

ANALYZE **Determine whether the following statements are sometimes, always, or never true. Explain your reasoning.**

48. If f(x) and g(x) are inverse functions, then f(a) = b and g(b) = a. See margin.

49. If f(a) = b and g(b) = a, then f(x) and g(x) are inverse functions. See margin.

50. CREATE Give an example of a function and its inverse. Verify that the two functions are inverses by graphing the functions and the line y = x on the same coordinate plane. $f(x) = 2x + 6$, $f^{-1}(x) = \frac{1}{2}x - 3$; See margin for graph.

51. WRITE Explain why it may be helpful to find the inverse of a function. Sample answer: A situation may require substituting values for the dependent variable into a function. By finding the inverse of the function, the dependent variable becomes the independent variable. This makes the substitution an easier process.

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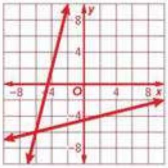
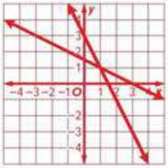
Answers

12.



13.



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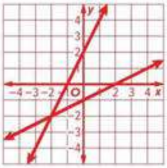
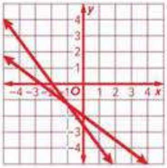
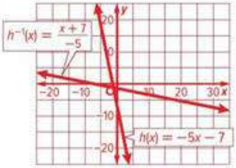


15.



40d.

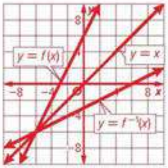


45. Sample answer: This claim is incorrect. The −1 in the inverse function notation is not an exponent. As an example, the inverse function for $y = x + 1$ is found by switching x and y and solving for y, which gives $y = x - 1$. $y = x - 1$ is not the same as $y = \frac{1}{(x+1)}$, which is not a line. This method does not work.

48. always true; Sample answer: If f(a) = b, then the graph of f(x) includes the point (a, b). If f(x) and g(x) are inverses, then the graph of g(x) includes the point (b, a). If (b, a) is included on the graph of g(x), then g(b) = a.

49. sometimes; Sample answer: f(x) and g(x) do not need to be inverse functions for f(a) = b and g(b) = a. For example, if f(x) = 2x + 10, then f(2) = 14 and if g(x) = x − 12, then g(14) = 2, but f(x) and g(x) are not inverse functions. However, if f(x) and g(x) are inverse functions, then f(a) = b and g(b) = a.

50.



Module 5 • Creating Linear Equations

Review

Rate Yourself

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it useful to have an equation that models the situation, and not just the table of values?
- When an equation that models a real-world situation is written in slope-intercept form or point-slope form, you can easily identify the slope. What does the slope tell you about the situation?
- What does the best-fit line tell you about the data set it represents?
- What does a linear regression equation tell you about the data that it represents?
- What does the inverse of a function tell you about the variables from the situation modeled by the original function?

Then have them write their answer to the Essential.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to creating linear equations.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Relationships Between Quantities and Reasoning with Equations, Expressions and Equations,** and **Descriptive Statistics**.

- Create and Solve Linear Equations
- Write Linear Equations in Different Forms
- Computing and Interpreting the Correlation Coefficient
- Distinguish Between Correlation and Causation

Module 5 • Creating Linear Equations

Review

Essential Question

What can a function tell you about the relationship that it represents?

Functions can tell you whether the value of the dependent variable increases or decreases as the independent variable changes. They describe trends in data and can be used to make predictions.

Module Summary

Lessons 5-1 and 5-2

Writing Equations

- Slope-intercept form is $y = mx + b$, where m is the slope of the line and b is the y-intercept.
- Point-slope form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.
- Standard form is $Ax + By = C$, where A, B, and C are integers, $A > 0$, A and B are both not equal to 0, and the GCF of A, B, and C is 1.
- To write a linear equation given two points on a line, first find the slope. Then use either point to write the equation in point-slope form or find the y-intercept to write the equation in slope-intercept form.

Lessons 5-3 through 5-5

Scatter Plots

- A scatter plot shows the relationship between a set of bivariate data, graphed as ordered pairs on a coordinate plane.
- A positive correlation exists when, as x increases, y increases. A negative correlation exists when, as x increases, y decreases. No correlation exists when x and y are not related.
- A line of fit is used to describe the trend of the data in a scatter plot.
- To determine causation, determine whether one variable influences the other variable.
- The correlation coefficient tells you how well the equation for the best-fit line models the data.
- A correlation coefficient close to 1 has a strong positive correlation. A correlation coefficient close to −1 has a strong negative correlation.
- Residuals measure how much the data deviate from the regression line.

Lesson 5-6

Inverses of Linear Functions

- Two relations are inverse relations if and only if one relation contains the element (a, b) when the other relation contains the element (b, a).
- In inverse relations, the x-coordinates are exchanged with the y-coordinates for each ordered pair in the relation.
- To find the inverse of $f(x)$, replace $f(x)$ with y in the equation for $f(x)$. Interchange y and x in the equation. Solve the equation for y. Replace y with $f^{-1}(x)$ in the new equation.

Study Organizer

Foldables

Use your Foldable to review the module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

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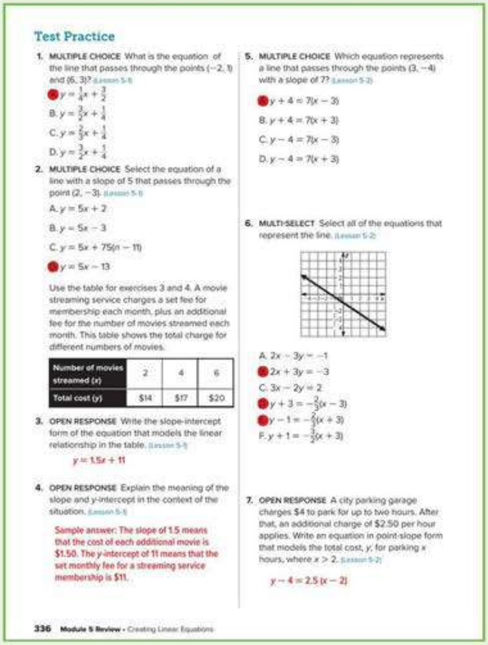

Test Practice

1. **MULTIPLE CHOICE** What is the equation of the line that passes through the points (−2, 1) and (6, 3)? (Lesson 5-1)

● $y = \frac{1}{4}x + \frac{3}{2}$

B. $y = \frac{3}{2}x + \frac{1}{4}$

C. $y = \frac{2}{3}x + \frac{1}{4}$

D. $y = \frac{3}{2}x + \frac{1}{4}$

2. **MULTIPLE CHOICE** Select the equation of a line with a slope of 5 that passes through the point (2, −3). (Lesson 5-1)

A. $y = 5x + 2$

B. $y = 5x - 3$

C. $y = 5x + 75(n - 11)$

● $y = 5x - 13$

Use the table for exercises 3 and 4. A movie streaming service charges a set fee for membership each month, plus an additional fee for the number of movies streamed each month. This table shows the total charge for different numbers of movies.

Number of movies streamed (x)	2	4	6
Total cost (y)	\$14	\$17	\$20

3. **OPEN RESPONSE** Write the slope-intercept form of the equation that models the linear relationship in the table. (Lesson 5-1)

$y = 1.5x + 11$

4. **OPEN RESPONSE** Explain the meaning of the slope and y-intercept in the context of the situation. (Lesson 5-1)

Sample answer: The slope of 1.5 means that the cost of each additional movie is \$1.50. The y-intercept of 11 means that the set monthly fee for a streaming service membership is \$11.

5. **MULTIPLE CHOICE** Which equation represents a line that passes through the points (3, −4) with a slope of 7? (Lesson 5-2)

● $y + 4 = 7(x - 3)$

B. $y + 4 = 7(x + 3)$

C. $y - 4 = 7(x - 3)$

D. $y - 4 = 7(x + 3)$

6. **MULTI-SELECT** Select all of the equations that represent the line. (Lesson 5-2)

A. $2x - 3y = -1$

● $2x + 3y = -3$

C. $3x - 2y = 2$

● $y + 3 = -\frac{2}{3}(x - 3)$

● $y - 1 = -\frac{2}{3}(x + 3)$

F. $y + 1 = -\frac{3}{2}(x + 3)$

7. **OPEN RESPONSE** A city parking garage charges \$4 to park for up to two hours. After that, an additional charge of \$2.50 per hour applies. Write an equation in point-slope form that models the total cost, y, for parking x hours, where $x > 2$. (Lesson 5-2)

$y - 4 = 2.5(x - 2)$

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 5-1 and 5-2
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

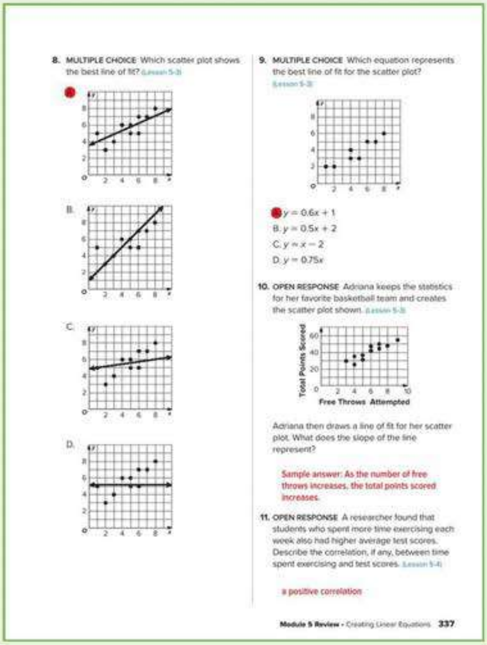
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1-17 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 2, 5, 8, 9, 13, 17
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	6, 16
Open Response	Students construct their own response.	3, 4, 7, 10, 11, 12, 14, 15

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
A.CED.2	5-1, 5-2, 5-6	1, 2, 3, 5, 6, 15, 16
A.CED.3	5-2	7
F.BF.4a	5-6	17
S.ID.6a	5-3, 5-5	10, 13
S.ID.6c	5-3	8, 9
S.ID.7	5-1	4
S.ID.8	5-5	14
S.ID.9	5-4	11, 12



8. **MULTIPLE CHOICE** Which scatter plot shows the best line of fit? (Lesson 5-3)

●

B.

C.

D.

9. **MULTIPLE CHOICE** Which equation represents the best line of fit for the scatter plot? (Lesson 5-3)

● $y = 0.6x + 1$

B. $y = 0.5x + 2$

C. $y = x - 2$

D. $y = 0.75x$

10. **OPEN RESPONSE** Adriana keeps the statistics for her favorite basketball team and creates the scatter plot shown. (Lesson 5-3)

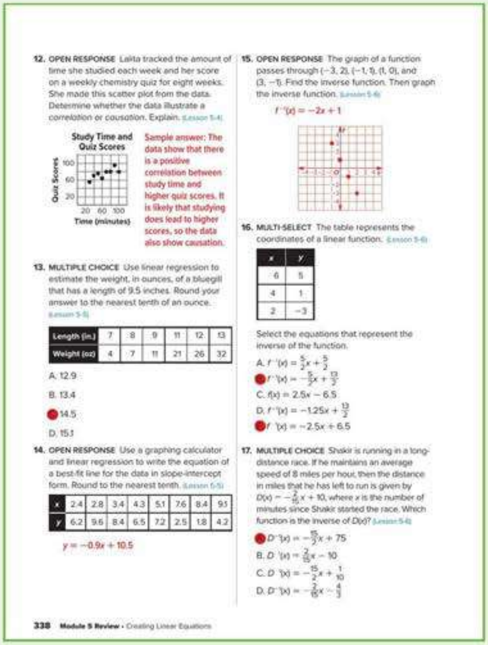
Adriana then draws a line of fit for her scatter plot. What does the slope of the line represent?

Sample answer: As the number of free throws increases, the total points scored increases.

11. **OPEN RESPONSE** A researcher found that students who spent more time exercising each week also had higher average test scores. Describe the correlation, if any, between time spent exercising and test scores. (Lesson 5-4)

a positive correlation

Module 5 Review • Creating Linear Equations 337



12. OPEN RESPONSE Lalita tracked the amount of time she studied each week and her score on a weekly chemistry quiz for eight weeks. She made this scatter plot from the data. Determine whether the data illustrate a correlation or causation. Explain. (Lesson 5-4)

Sample answer: The data show that there is a positive correlation between study time and higher quiz scores. It is likely that studying does lead to higher scores, so the data also show causation.

13. MULTIPLE CHOICE Use linear regression to estimate the weight, in ounces, of a bluegill that has a length of 9.5 inches. Round your answer to the nearest tenth of an ounce. (Lesson 5-5)

Length (in.)	7	8	9	11	12	13
Weight (oz)	4	7	11	21	26	32

A. 12.9

B. 13.4

[✓ 14.5]

D. 15.1

14. OPEN RESPONSE Use a graphing calculator and linear regression to write the equation of a best-fit line for the data in slope-intercept form. Round to the nearest tenth. (Lesson 5-5)

x	2.4	2.8	3.4	4.3	5.1	7.6	8.4	9.1
y	6.2	9.6	8.4	6.5	7.2	2.5	1.8	4.2

$y = -0.9x + 10.5$

15. OPEN RESPONSE The graph of a function passes through $(-3, 2)$, $(-1, 1)$, $(1, 0)$, and $(3, -1)$. Find the inverse function. Then graph the inverse function. (Lesson 5-6)

$f^{-1}(x) = -2x + 1$

16. MULTI-SELECT The table represents the coordinates of a linear function. (Lesson 5-6)

x	y
6	5
4	1
2	−3

Select the equations that represent the inverse of the function.

A. $f^{-1}(x) = \frac{5}{2}x + \frac{5}{2}$

[✓ $f^{-1}(x) = -\frac{5}{2}x + \frac{13}{2}$]

C. $f(x) = 2.5x - 6.5$

D. $f^{-1}(x) = -1.25x + \frac{13}{2}$

[✓ $f^{-1}(x) = -2.5x + 6.5$]

17. MULTIPLE CHOICE Shakir is running in a long-distance race. If he maintains an average speed of 8 miles per hour, then the distance in miles that he has left to run is given by $D(x) = -\frac{2}{15}x + 10$, where x is the number of minutes since Shakir started the race. Which function is the inverse of $D(x)$? (Lesson 5-6)

[✓ $D^{-1}(x) = -\frac{15}{2}x + 75$]

B. $D^{-1}(x) = \frac{2}{15}x - 10$

C. $D^{-1}(x) = -\frac{15}{2}x + \frac{1}{10}$

D. $D^{-1}(x) = -\frac{2}{15}x - \frac{4}{3}$

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Module 6

Linear Inequalities

Module Goals

- Students write and solve linear inequalities.
- Students graph linear inequalities in two variables.
- Students apply linear inequalities in problem-solving situations.

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Also addresses A.CED.3 and A.REI.12.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Coherence

Vertical Alignment

Previous

Students constructed simple one-variable inequalities to solve real-world problems.

7.EE.4

Now

Students write, solve, and graph inequalities.

A.CED.1, A.REI.3

Next

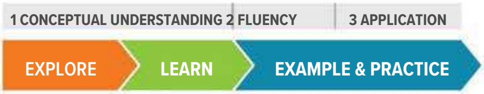
Students will solve systems of inequalities.

A.REI.12 (Course 1, Course 2)

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.



Suggested Pacing

Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
6-1 Solve One-Step Inequalities	A.CED.1, A.REI.3	2	1
6-2 Solving Multi-Step Inequalities	A.CED.1, A.REI.3	1	0.5
6-3 Solving Compound Inequalities	A.CED.1, A.CED.3	2	1
Put It All Together: Lessons 6-1 through 6-3		1	0.5
6-4 Solving Absolute Value Inequalities	A.CED.1, A.CED.3	1	0.5
6-5 Graphing Inequalities in Two Variables	A.CED.3, A.REI.12	1	0.5
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	**11**	**5.5**

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Graphs of Linear Inequalities

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students determine which graph matches the correct inequality and explain their choices.

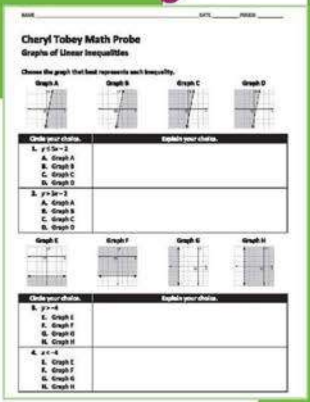
Targeted Concepts Understand the relationship between symbolic and graphic representations of linear inequalities.

Targeted Misconceptions

- Students shade the incorrect region when they lack understanding of the relationship between the graphical representation of an inequality to solutions of the algebraic representation.
- Students incorrectly use a dotted boundary line when an equality is included ($\leq$ and $\geq$) or a solid line when it is not ($<$ and $>$).
- Students may incorrectly graph equations of horizontal and vertical lines by interchanging them.

Use the Probe after Lesson 6-5.

Module Resource


Cheryl Tobey Math Probe

Graphs of Linear Inequalities

Choose the graph that best represents each inequality.

Correct Answers:
1. A 2. D 3. F 4. G

Collect and Assess Student Answers

If the student selects these responses…	**Then** the student likely…
1. C **2.** B **3.** E **4.** H	is having difficulty choosing the region that represents the correct solution of the inequality.
1. B **2.** C	is incorrectly using a solid boundary line for inequalities ($<$ and $>$) and a dotted line for inequalities that include the line ($\leq$ and $\geq$).
3. H **4.** E	is graphing a "$y =$ a constant" equation as a vertical line and an "$x =$ a constant" equation as a horizontal line.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Graphing Linear Inequalities
- Lesson 6-5, Learn, Examples 1–3

Revisit the Probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

How can writing and solving inequalities help you solve problems in the real world? Sample answer: Writing and solving inequalities can help me determine the solution sets of problems in the real world.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

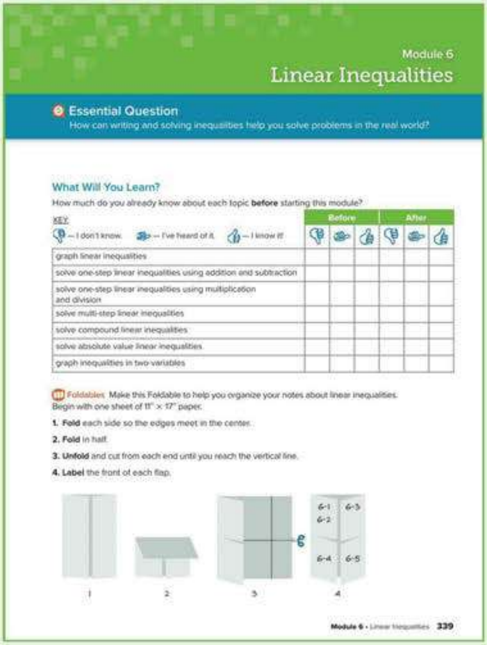
Focus Students write about the different ways inequalities can be solved as these methods are presented in the lessons of this module.

Teach Have students make and label their Foldables as illustrated. Students should fill in the appropriate sections with their notes, diagrams, and examples as they cover each lesson in this module.

When to Use It Encourage students to add to their Foldables as they work through the module and to use them to review for the chapter test.

Launch the Module

For this module, the Launch the Module video uses calculating costs and budget constraints to describe ways of using inequalities to model real-world situations.



Module 6

Linear Inequalities

Essential Question

How can writing and solving inequalities help you solve problems in the real world?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. 👉 — I've heard of it. 👍 — I know it!

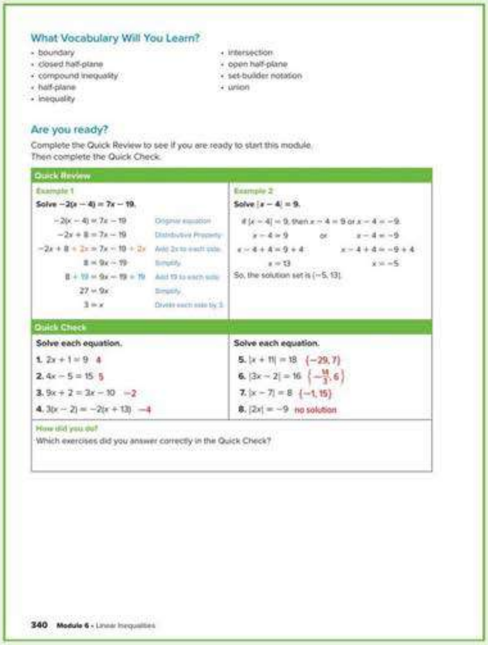
	Before			After		
	👎	👉	👍	👎	👉	👍
graph linear inequalities						
solve one-step linear inequalities using addition and subtraction						
solve one-step linear inequalities using multiplication and division						
solve multi-step linear inequalities						
solve compound linear inequalities						
solve absolute value linear inequalities						
graph inequalities in two variables						

Foldables Make this Foldable to help you organize your notes about linear inequalities. Begin with one sheet of 11" × 17" paper.

1. **Fold** each side so the edges meet in the center.
2. **Fold** in half.
3. **Unfold** and cut from each end until you reach the vertical line.
4. **Label** the front of each flap.

Module 6 • Linear Inequalities 339

Interactive Presentation



What Vocabulary Will You Learn?

- boundary
- closed half-plane
- compound inequality
- half-plane
- inequality
- intersection
- open half-plane
- set-builder notation
- union

Are you ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Solve $-2(x-4)=7x-19$.

$-2(x-4)=7x-19$ Original equation

$-2x+8=7x-19$ Distributive Property

$-2x+8+2x=7x-19+2x$ Add 2x to each side.

$8=9x-19$ Simplify.

$8+19=9x-19+19$ Add 19 to each side.

$27=9x$ Simplify.

$3=x$ Divide each side by 9.

Example 2

Solve $|x-4|=9$.

If $|x-4|=9$, then $x-4=9$ or $x-4=-9$.

$x-4=9$ or $x-4=-9$

$x-4+4=9+4$ $x-4+4=-9+4$

$x=13$ $x=-5$

So, the solution set is $\{-5, 13\}$.

Quick Check

Solve each equation.

1. $2x+1=9$ 4

2. $4x-5=15$ 5

3. $9x+2=3x-10$ −2

4. $3(x-2)=-2(x+13)$ −4

Solve each equation.

5. $|x+11|=18$ $\{-29, 7\}$

6. $|3x-2|=16$ $\left\{-\frac{14}{3}, 6\right\}$

7. $|x-7|=8$ $\{-1, 15\}$

8. $|2x|=-9$ no solution

How did you do?

Which exercises did you answer correctly in the Quick Check?

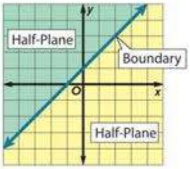
340 **Module 6** • Linear Inequalities

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define A half-plane is a region on a coordinate plane where graphs of ordered pairs are filled.

Example



Ask How many half-planes are in this graph? Two

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- solving one-step equations
- solving multi-step equations
- graphing on a number line
- solving equations involving absolute value
- recognizing solutions of inequalities

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You may want to use the **Linear Inequalities** section to ensure student success in this module.

Mindset Matters

"Not Yet" Doesn't Mean "Never"

Students with a growth mindset come to understand that just because they haven't yet found a solution, that doesn't mean they can't find one with additional effort and reasoning. It takes time to reason through the different strategies that can be used to solve a problem.

How Can I Apply It?

Assign students the **Math Probes** that are available for each module. Have them complete the probe before starting the module and again at the specified point in the module or at the end of the module so that they can see their progress.

Lesson 6-1

Solving One-Step Inequalities

A.CED.1, A.REI.3

LESSON GOAL

Students solve inequalities by using addition, subtraction, multiplication, and division.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Graphing Inequalities

Develop:

Graphing Inequalities

- Graph Inequalities
- Write Inequalities from a Graph

Explore: Properties of Inequalities

Develop:

Solving Inequalities by Using Addition and Subtraction

- Solve Inequalities by Adding
- Solve Inequalities by Subtracting
- Add or Subtract to Solve Inequalities with Variables on Each Side
- Use an Inequality to Solve a Problem

Solving Inequalities by Using Multiplication and Division

- Write and Solve an Inequality
- Solve an Inequality by Multiplying
- Solve an Inequality by Dividing
- Solve an Inequality with a Negative Coefficient

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Write and Solve One-Step Equations	●	●		●
Extension: Triangle Inequalities		●	●	●

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students constructed simple one-variable inequalities to solve real-world problems.
7.EE.4

Now
Students solve one-step inequalities.
A.CED.1, A.REI.3

Next
Students will solve multi-step inequalities.
A.CED.1, A.REI.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

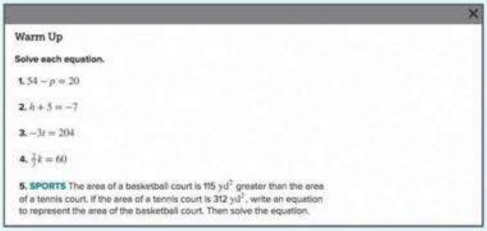
Conceptual Bridge In this lesson, students expand on their understanding of equations and use it to build fluency with solving one-step inequalities. They apply their understanding of one-step inequalities by solving real-world problems.

Language Development Handbook

Assign page 33 of the *Language Development Handbook* to help your students build mathematical language related to solving inequalities.

ELL You can use the tips and suggestions on page T33 of the handbook to support students who are building English proficiency.

Interactive Presentation

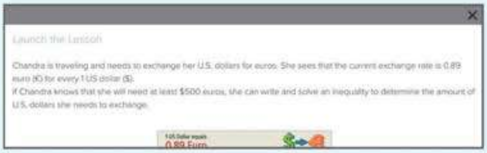


Warm Up

Solve each equation.

1. $54 - p = 20$
2. $h + 5 = -7$
3. $-3t = 204$
4. $\frac{2}{3}k = 60$
5. **SPORTS** The area of a basketball court is 115 yd^2 greater than the area of a tennis court. If the area of a tennis court is 312 yd^2, write an equation to represent the area of the basketball court. Then solve the equation.

Warm Up

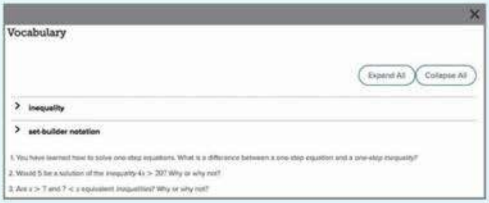


Launch the Lesson

Chandra is traveling and needs to exchange her U.S. dollars for euros. She sees that the current exchange rate is 0.89 euro (€) for every 1 US dollar ($).

If Chandra knows that she will need at least $500 euros, she can write and solve an inequality to determine the amount of U.S. dollars she needs to exchange.

Launch the lesson



Vocabulary

> inequality

> set-builder notation

1. You have learned how to solve one-step equations. What is a difference between a one-step equation and a one-step inequality?
2. Would 5 be a solution of the inequality $4x > 20$? Why or why not?
3. Are $x > 7$ and $7 < x$ equivalent inequalities? Why or why not?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- solving one-step equations

Answers:

1. 34
2. −12
3. −68
4. 90
5. $x - 115 = 312$; 427 yd^2

Launch the Lesson

Teaching the Mathematical Practices

2 Create Representations Guide students to write an inequality that can be used to model the situation. Then use the inequality to find the amount of U.S. dollars Chandra will need to exchange.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

A linear inequality is an open sentence that contains $<$, $>$, $\leq$, or $\geq$. Inequalities can be solved by using algebraic methods similar to solving equations. When solved in this way, inequalities that contain a negative coefficient for the variable require special attention when calculating the direction of the inequality in the final solution.

Explore Graphing Inequalities

Objective

Students use a sketch to explore graphing solutions of inequalities.

MP Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the exercises in this Explore, students will need to use a sketch. Work with students to explore and deepen their understanding of graphing solutions of inequalities.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

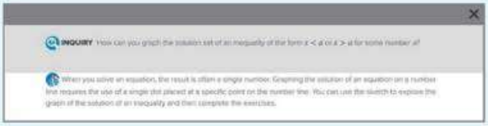
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will graph inequalities on a sketch, with different values and different directions of inequality symbols. They will then answer a series of questions related to the inequalities shown, to solidify understanding of the concepts involved. Then, students will answer the Inquiry Question.

(*continued on the next page*)

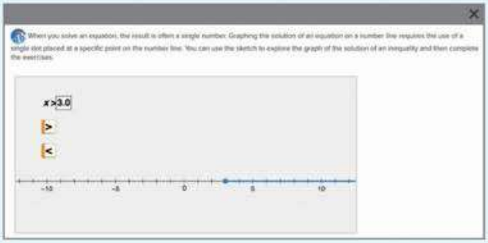
Interactive Presentation



INQUIRY How can you graph the solution set of an inequality of the form $x < a$ or $x > a$ for some number a?

When you solve an equation, the result is often a single number. Graphing the solution of an equation on a number line requires the use of a single dot placed at a specific point on the number line. You can use the sketch to explore the graph of the solution of an inequality and then complete the exercises.

Explore



When you solve an equation, the result is often a single number. Graphing the solution of an equation on a number line requires the use of a single dot placed at a specific point on the number line. You can use the sketch to explore the graph of the solution of an inequality and then complete the exercises.

Explore

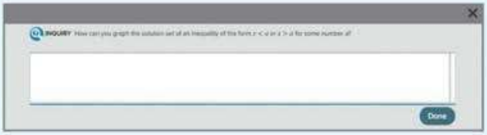
WEB SKETCHPAD

Students use a sketch to explore the solution of the graph of an inequality.

TYPE

Students answer questions to show they understand inequalities and their solutions.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Graphing Inequalities (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How is graphing the solution to an inequality different than the solution to an equation? Sample answer: An equation has only one answer, so it is just a point on the number line. Because an inequality has an infinite number of solutions, a portion of the number line needs to be shaded as the solution set.
- How do you think the graph would change for the inequality $x \geq 3$? Sample answer: 3 is now included in the solution set, so you would graph a point at 3 instead of an open circle.

Inquiry

How can you graph the solution set of an inequality of the form $x < a$ or $x > a$ for some number a? Sample answer: Draw a circle at the endpoint. Draw an arrow left if the inequality is less than; draw an arrow right if the inequality is greater than.

Explore Properties of Inequalities

Objective

Students use a sketch to explore how addition, subtraction, multiplication, and division affect inequalities.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in this Explore.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

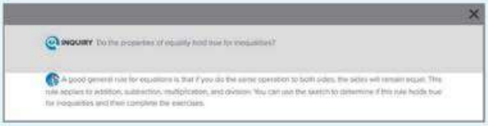
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students adjust the sliders and values for operations on the sketch, causing changes in the inequality shown to analyze the relationships when operations are applied to values on each side of an inequality. Then, students will answer the Inquiry Question.

(continued on the next page)

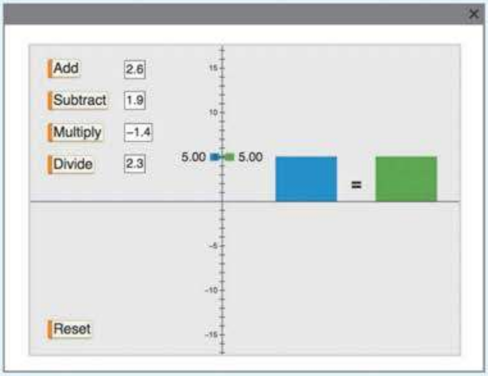
Interactive Presentation



INQUIRY Do the properties of equality hold true for inequalities?

A good general rule for equations is that if you do the same operation to both sides, the sides will remain equal. This rule applies to addition, subtraction, multiplication, and division. You can use the sketch to determine if this rule holds true for inequalities and then complete the exercises.

Explore



Explore

WEB SKETCHPAD

Students use a sketch to determine if the properties of equality hold true for inequalities.

TYPE

Students answer questions to show they understand the property of equalities.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Properties of Inequalities (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does the height of the bars stay the same for all operations? Sample answer: If the bars begin at equal heights, the values on each side of the equation are the same. Any changes applied to both sides will still make the values the same.
- Why do you think the inequality symbol changes when you multiply by a negative number? Sample answer: Because the values are not the same to begin with, multiplying will affect then differently. The greater number, multiplied by a negative number, will have a greater absolute value but will be the lesser value.

Inquiry

Do the properties of equality hold true for inequalities? Explain. No; sample answer: The Multiplication and Division Properties of Equality hold true only for positive values of c. For negative values of c, the inequality is reversed.

Learn Graphing Inequalities

Objective

Students select a graph on a number line that identifies the solution of an inequality.

MP Teaching the Mathematical Practices

4 Use Tools Point out that to solve the problem in this Learn, students will need to use graphs.

Example 1 Graph Inequalities

MP Teaching the Mathematical Practices

6 Use Quantities Use the Study Tip to guide students to clarifying their use of quantities in this example. Ensure that they specify the units of measure used in the problem and label axes appropriately.

Questions for Mathematical Discourse

AL When do you shade to the right of the endpoint on the number line? when the variable is greater than the numerical value of the endpoint

OL When do you include the endpoint as part of the graph? When the inequality is $\geq$ or $\leq$, the endpoint is included.

BL If $x < k$ has solutions for x that include all negative numbers, what has to be true about k? Explain. Sample answer: k has to be nonnegative. If k were negative, then there would be negative values greater than k that would not be included in the solution set.

Example 2 Write Inequalities from a Graph

MP Teaching the Mathematical Practices

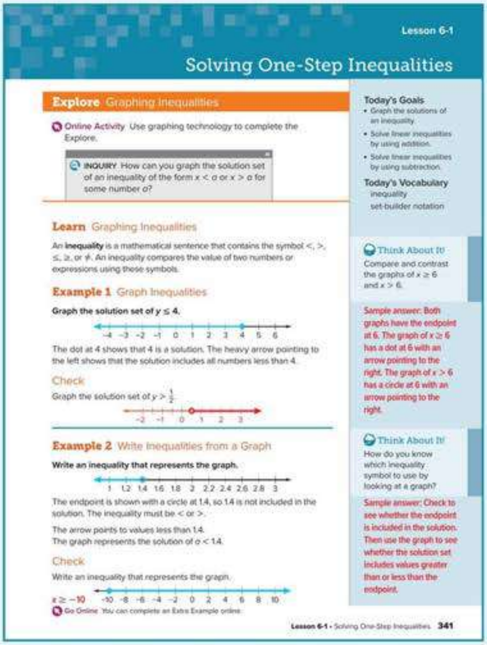
1 Explain Correspondences Encourage students to explain the relationships between the inequalities and their graphs used in this example.

Questions for Mathematical Discourse

AL Is the endpoint a solution to the inequality? Explain. No; sample answer: The graph of the inequality has an open circle at 1.4 which means that 1.4 is not included in the solution.

OL How would the inequality change if the graph had a dot at 1.4? It would be included in the solution, so $a \leq 1.4$.

BL Describe what the number line represents in this example. It represents an infinite number of solutions to the inequality, where the value of a is less than 1.4.



Lesson 6-1

Solving One-Step Inequalities

Explore Graphing Inequalities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you graph the solution set of an inequality of the form $x < a$ or $x > a$ for some number a?

Today's Goals
- Graph the solutions of an inequality.
- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

Today's Vocabulary
inequality
set-builder notation

Learn Graphing Inequalities

An **inequality** is a mathematical sentence that contains the symbol <, >, ≤, ≥, or ≠. An inequality compares the value of two numbers or expressions using these symbols.

Example 1 Graph Inequalities

Graph the solution set of $y \leq 4$.

The dot at 4 shows that 4 is a solution. The heavy arrow pointing to the left shows that the solution includes all numbers less than 4.

Check

Graph the solution set of $y > \frac{1}{2}$.

Think About It!
Compare and contrast the graphs of $x \geq 6$ and $x > 6$.

Sample answer: Both graphs have the endpoint at 6. The graph of $x \geq 6$ has a dot at 6 with an arrow pointing to the right. The graph of $x > 6$ has a circle at 6 with an arrow pointing to the right.

Example 2 Write Inequalities from a Graph

Write an inequality that represents the graph.

The endpoint is shown with a circle at 1.4, so 1.4 is not included in the solution. The inequality must be < or >.

The arrow points to values less than 1.4.
The graph represents the solution of $a < 1.4$.

Check

Write an inequality that represents the graph.

$x \geq -10$

Go Online You can complete an Extra Example online.

Think About It!
How do you know which inequality symbol to use by looking at a graph?

Sample answer: Check to see whether the endpoint is included in the solution. Then use the graph to see whether the solution set includes values greater than or less than the endpoint.

Lesson 6-1 • Solving One-Step Inequalities 341

Interactive Presentation

Learn

TAP

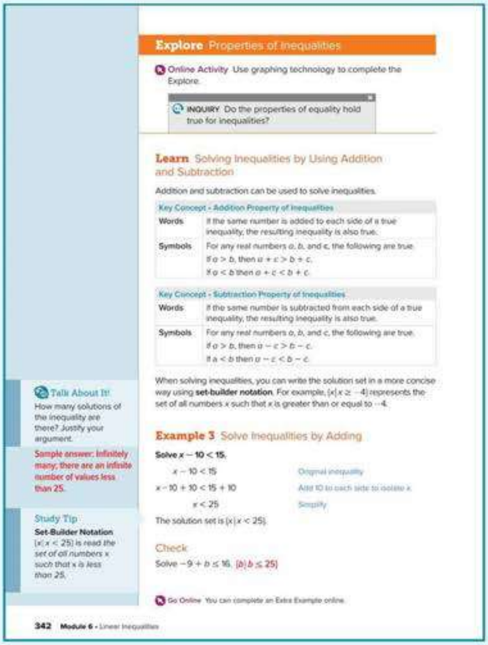
Students tap on each marker to learn how a solution set can be graphed on a number line.

TYPE

Students answer a question to show they understand how to compare and contrast the graphs of two different inequalities.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Explore Properties of Inequalities

Online Activity Use graphing technology to complete the Explore.

INQUIRY Do the properties of equality hold true for inequalities?

Learn Solving Inequalities by Using Addition and Subtraction

Addition and subtraction can be used to solve inequalities.

Key Concept • Addition Property of Inequalities	
Words	If the same number is added to each side of a true inequality, the resulting inequality is also true.
Symbols	For any real numbers a, b, and c, the following are true. If $a > b$, then $a + c > b + c$. If $a < b$ then $a + c < b + c$.

Key Concept • Subtraction Property of Inequalities	
Words	If the same number is subtracted from each side of a true inequality, the resulting inequality is also true.
Symbols	For any real numbers a, b, and c, the following are true. If $a > b$, then $a - c > b - c$. If $a < b$ then $a - c < b - c$.

When solving inequalities, you can write the solution set in a more concise way using **set-builder notation**. For example, $\{x \mid x \geq -4\}$ represents the set of all numbers x such that x is greater than or equal to -4.

Talk About It! How many solutions of the inequality are there? Justify your argument.

Sample answer: Infinitely many; there are an infinite number of values less than 25.

Example 3 Solve Inequalities by Adding

Solve $x - 10 < 15$.

$x - 10 < 15$	Original inequality
$x - 10 + 10 < 15 + 10$	Add 10 to each side to isolate x.
$x < 25$	Simplify.

The solution set is $\{x \mid x < 25\}$.

Study Tip **Set-Builder Notation** $\{x \mid x < 25\}$ is read *the set of all numbers x such that x is less than 25.*

Check

Solve $-9 + b \leq 16$. $\{b \mid b \leq 25\}$

Go Online You can complete an Extra Example online.

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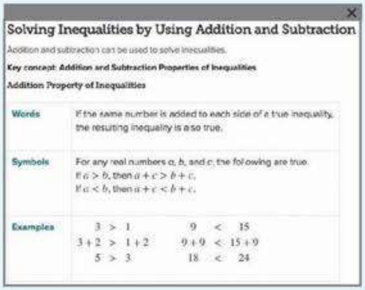
Interactive Presentation



Solving Inequalities by Using Addition and Subtraction

Addition and subtraction can be used to solve inequalities.

Key concept: Addition and Subtraction Properties of Inequalities

Addition Property of Inequalities

Words	If the same number is added to each side of a true inequality, the resulting inequality is also true.
Symbols	For any real numbers a, b, and c, the following are true. If $a > b$, then $a + c > b + c$. If $a < b$, then $a + c < b + c$.
Examples	$3 > 1$ $9 < 15$ $3 + 2 > 1 + 2$ $9 + 9 < 15 + 9$ $5 > 3$ $18 < 24$

Learn

TYPE

Students answer a question to show they understand that the Addition and Subtraction Properties of Inequalities hold true for all inequalities.

Learn Solving Inequalities by Using Addition and Subtraction

Objective

Students solve linear inequalities by applying the Addition Property of Inequality or the Subtraction Property of Inequality.

MP Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

DIFFERENTIATE

Language Development Activity AL ELL

IF students have trouble understanding some of the phrases used to indicate inequalities, such as “at most” or “no less than”,
THEN have them work in pairs or groups to write problems using the phrases. Have students write inequalities to represent the situations.

Example 3 Solve Inequalities by Adding

MP Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. This example asks students to justify their conclusions.

Questions for Mathematical Discourse

- AL What is the inverse operation of subtraction? addition
- OL Which property should be used to solve the inequality? Addition Property of Inequality
- BL How would you rewrite the solution with the x on the right side of the inequality? $25 > x$

DIFFERENTIATE

Enrichment Activity BL

Write these three linear inequalities on the board:

$$y > 3 \qquad y + 1 > 4 \qquad 5 < y + 2$$

Have students solve each linear inequality and compare the solutions. Ask students to formulate three more linear inequalities that are equivalent to $y > 3$.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Solve Inequalities by Subtracting

Teaching the Mathematical Practices

1 Check Answers Use the Think About It! feature to encourage students to check their solution.

Questions for Mathematical Discourse

AL What are two values that make the inequality true? Sample answers: 50, 65 (any number greater than 37)

OL Will the point on the graph be opened or closed? Explain. Closed; sample answer: The value 37 is included in the solution set because the inequality is "greater than or equal to."

BL How many solutions are there of the inequality? Explain. Infinitely many; sample answer: There are an infinite number of values greater than or equal to 37.

Example 5 Add or Subtract to Solve Inequalities with Variables on Each Side

Teaching the Mathematical Practices

2 Attend to Quantities Point out the order in which to write the solution in set builder notation.

Questions for Mathematical Discourse

AL Why is it important to isolate the variable? Sample answer: In order to solve for a variable, all like terms need to be together on one side of the inequality.

OL How can you check to make sure your solution is correct? Sample answer: Replace *y* with a number less than or equal to 3 in the original inequality and check that it is true.

BL Why do we not subtract 10*y* from each side? Sample answer: We would be left with 0 on the right side. We need to have variables on one side and the constants on the other side.

Example 6 Use an Inequality to Solve a Problem

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

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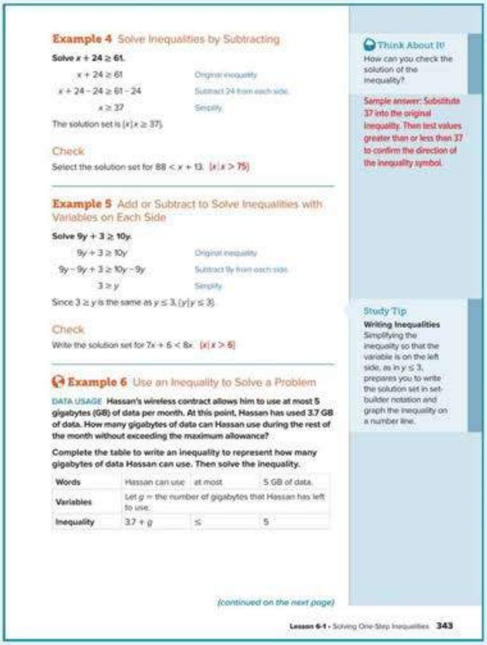

Example 4 Solve Inequalities by Subtracting

Solve $x + 24 \geq 61$.

$x + 24 \geq 61$	Original inequality
$x + 24 - 24 \geq 61 - 24$	Subtract 24 from each side.
$x \geq 37$	Simplify.

The solution set is $\{x \mid x \geq 37\}$.

Check

Select the solution set for $88 < x + 13$. $\{x \mid x > 75\}$

Think About It!

How can you check the solution of the inequality?

Sample answer: Substitute 37 into the original inequality. Then test values greater than or less than 37 to confirm the direction of the inequality symbol.

Example 5 Add or Subtract to Solve Inequalities with Variables on Each Side

Solve $9y + 3 \geq 10y$.

$9y + 3 \geq 10y$	Original inequality
$9y - 9y + 3 \geq 10y - 9y$	Subtract 9y from each side.
$3 \geq y$	Simplify.

Since $3 \geq y$ is the same as $y \leq 3$, $\{y \mid y \leq 3\}$.

Check

Write the solution set for $7x + 6 < 8x$. $\{x \mid x > 6\}$

Study Tip

Writing Inequalities Simplifying the inequality so that the variable is on the left side, as in $y \leq 3$, prepares you to write the solution set in set-builder notation and graph the inequality on a number line.

Example 6 Use an Inequality to Solve a Problem

DATA USAGE **Hassan's wireless contract allows him to use at most 5 gigabytes (GB) of data per month. At this point, Hassan has used 3.7 GB of data. How many gigabytes of data can Hassan use during the rest of the month without exceeding the maximum allowance?**

Complete the table to write an inequality to represent how many gigabytes of data Hassan can use. Then solve the inequality.

Words	Hassan can use	at most	5 GB of data.
Variables	Let g = the number of gigabytes that Hassan has left to use.		
Inequality	$3.7 + g$	$\leq$	5

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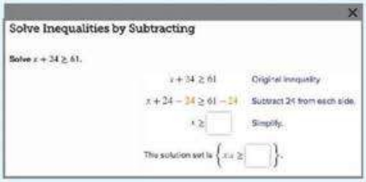
Interactive Presentation



Solve Inequalities by Subtracting

Solve $x + 24 \geq 61$.

$x + 24 \geq 61$	Original inequality
$x + 24 - 24 \geq 61 - 24$	Subtract 24 from each side.
$x \geq$ ☐	Simplify.

The solution set is $\{x \mid x \geq$ ☐ $\}$.

Example 4

TYPE

Students complete the statements to solve the inequality.

TYPE

Students answer a question to show they understand how to check the solution of the inequality.

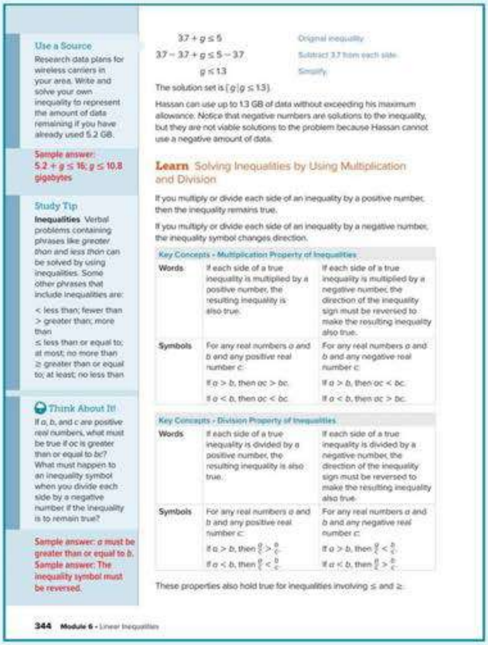
CHECK

Students complete the Check online to determine whether they are ready to move on.

2 EXPLORE AND DEVELOP

A.CED.1, A.REI.3

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Use a Source

Research data plans for wireless carriers in your area. Write and solve your own inequality to represent the amount of data remaining if you have already used 5.2 GB.

Sample answer: $5.2 + g \leq 16$; $g \leq 10.8$ gigabytes

Study Tip

Inequalities Verbal problems containing phrases like *greater than* and *less than* can be solved by using inequalities. Some other phrases that include inequalities are:

$<$ less than; fewer than
$>$ greater than; more than
$\leq$ less than or equal to; at most; no more than
$\geq$ greater than or equal to; at least; no less than

Think About It!

If a, b, and c are positive real numbers, what must be true if ac is greater than or equal to bc? What must happen to an inequality symbol when you divide each side by a negative number if the inequality is to remain true?

Sample answer: a must be greater than or equal to b. Sample answer: The inequality symbol must be reversed.

$$\begin{aligned} 3.7 + g &\leq 5 && \text{Original inequality} \\ 3.7 - 3.7 + g &\leq 5 - 3.7 && \text{Subtract 3.7 from each side.} \\ g &\leq 1.3 && \text{Simplify.} \end{aligned}$$

The solution set is $\{g \mid g \leq 1.3\}$.

Hassan can use up to 1.3 GB of data without exceeding his maximum allowance. Notice that negative numbers are solutions to the inequality, but they are not viable solutions to the problem because Hassan cannot use a negative amount of data.

Learn Solving Inequalities by Using Multiplication and Division

If you multiply or divide each side of an inequality by a positive number, then the inequality remains true.

If you multiply or divide each side of an inequality by a negative number, the inequality symbol changes direction.

Key Concepts • Multiplication Property of Inequalities

Words	If each side of a true inequality is multiplied by a positive number, the resulting inequality is also true.	If each side of a true inequality is multiplied by a negative number, the direction of the inequality sign must be reversed to make the resulting inequality also true.
Symbols	For any real numbers a and b and any positive real number c: If $a > b$, then $ac > bc$. If $a < b$, then $ac < bc$.	For any real numbers a and b and any negative real number c: If $a > b$, then $ac < bc$. If $a < b$, then $ac > bc$.

Key Concepts • Division Property of Inequalities

Words	If each side of a true inequality is divided by a positive number, the resulting inequality is also true.	If each side of a true inequality is divided by a negative number, the direction of the inequality sign must be reversed to make the resulting inequality also true.
Symbols	For any real numbers a and b and any positive real number c: If $a > b$, then $\frac{a}{c} > \frac{b}{c}$. If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	For any real numbers a and b and any negative real number c: If $a > b$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

These properties also hold true for inequalities involving $\leq$ and $\geq$.

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Interactive Presentation

Learn

TYPE

Students answer questions to show they understand what must be true about the product of three positive real numbers, and what happens to the inequality sign when each side is divided by a negative number.

Questions for Mathematical Discourse

AL What phrase tells you this is an inequality? Explain. At most; Sample answer: This means that there is a range of values that are acceptable and an endpoint.

OL What equation could be used to determine data usage that uses exactly all 5 GB of data? $3.7 + g = 5$

BL Could we rewrite the inequality as $g + 3.7 \leq 5$? Explain. Yes; sample answer: Addition is commutative.

Learn Solving Inequalities by Using Multiplication and Division

Objective

Students solve linear inequalities by using multiplication and division.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Important to Know

If an inequality is multiplied or divided by a positive number, the inequality symbol stays the same. If an inequality is multiplied or divided by a negative number, the inequality symbol changes direction.

Essential Question Follow-Up

Students have explored using and solving inequalities.

Ask:

Why is it important to understand what the symbols in a mathematical sentence represent? Sample answer: If you are unsure of what the symbols represent, you may not translate a verbal representation into the correct algebraic representation.

DIFFERENTIATE

Language Development Activity ELL

Beginning Read the lesson opener or an Example aloud one sentence at a time. At the end of each sentence, ask students to say a word or short phrase that describes an important piece of information from the sentence. Model recording the information in preparation for solving the problem. Have students use your model to record information in their notes.

Intermediate Slowly read the lesson opener or an Example aloud. After each sentence or two, pause and ask volunteers to identify an important piece of information. Have students write the important ideas in their notes.

Advanced Tell students to listen without taking notes while you read aloud. After you have finished, have students write down what they remember from your reading. Have students work in small groups to compare their notes. Then have each group discuss the problem and its solution.

Advanced High Have students practice active listening as you read aloud by taking notes. Then have students work in pairs to summarize the information and solve the problem. Have pairs share with the class.

Apply Example 7 Write and Solve an Inequality

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- H ow can you write the related equation?
- Which inequality symbol can be used to represent this situation?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Example 8 Solve an Inequality by Multiplying

Teaching the Mathematical Practices

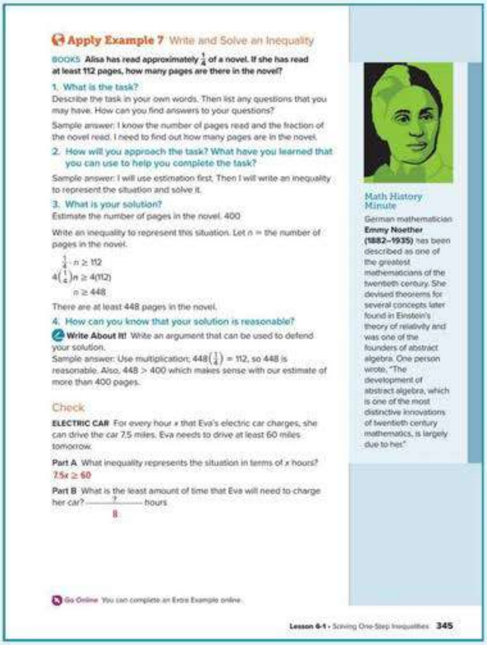
6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What operation is being performed on the variable? multiplication by a fraction

OL How can you solve the inequality? Multiply each side by the reciprocal.

BL Why do we reverse the inequality symbol? We are multiplying by a negative number.



Apply Example 7 Write and Solve an Inequality

BOOKS **Alisa has read approximately $\frac{1}{4}$ of a novel. If she has read at least 112 pages, how many pages are there in the novel?**

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I know the number of pages read and the fraction of the novel read. I need to find out how many pages are in the novel.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will use estimation first. Then I will write an inequality to represent the situation and solve it.

3. What is your solution?

Estimate the number of pages in the novel. 400

Write an inequality to represent this situation. Let n = the number of pages in the novel.

$$\begin{aligned} \tfrac{1}{4}\cdot n &\ge 112 \\ 4\left(\tfrac{1}{4}\right)n &\ge 4(112) \\ n &\ge 448 \end{aligned}$$

There are at least 448 pages in the novel.

4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: Use multiplication: $448\left(\frac{1}{4}\right) = 112$, so 448 is reasonable. Also, $448 > 400$ which makes sense with our estimate of more than 400 pages.

Check

ELECTRIC CAR For every hour x that Eva's electric car charges, she can drive the car 7.5 miles. Eva needs to drive at least 60 miles tomorrow.

Part A What inequality represents the situation in terms of x hours?
$7.5x \ge 60$

Part B What is the least amount of time that Eva will need to charge her car? ___?___ hours
8

Math History Minute

German mathematician **Emmy Noether (1882–1935)** has been described as one of the greatest mathematicians of the twentieth century. She devised theorems for several concepts later found in Einstein's theory of relativity and was one of the founders of abstract algebra. One person wrote, "The development of abstract algebra, which is one of the most distinctive innovations of twentieth century mathematics, is largely due to her."

Go Online You can complete an Extra Example online.

Lesson 6-1 • Solving One-Step Inequalities 345

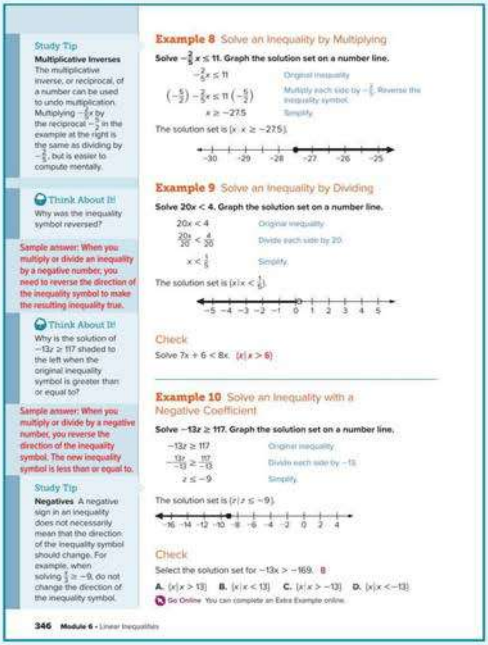
Interactive Presentation

BOOKS Alisa has read $\frac{1}{4}$ of a novel. If she has read at least 112 pages, how many pages are there in the novel?

Apply Example 7

TAP

Students move through the steps to solve a problem involving an inequality.



Study Tip

Multiplicative Inverses The multiplicative inverse, or reciprocal, of a number can be used to undo multiplication. Multiplying $-\frac{2}{5}x$ by the reciprocal $-\frac{5}{2}$ in the example at the right is the same as dividing by $-\frac{2}{5}$, but is easier to compute mentally.

Example 8 Solve an Inequality by Multiplying

Solve $-\frac{2}{5}x \le 11$. Graph the solution set on a number line.

$-\frac{2}{5}x \le 11$ Original inequality

$\left(-\frac{5}{2}\right) -\frac{2}{5}x \le 11\left(-\frac{5}{2}\right)$ Multiply each side by $-\frac{5}{2}$. Reverse the inequality symbol.

$x \ge -27.5$ Simplify.

The solution set is $\{x \mid x \ge -27.5\}$.

Think About It! Why was the inequality symbol reversed?

Sample answer: When you multiply or divide an inequality by a negative number, you need to reverse the direction of the inequality symbol to make the resulting inequality true.

Example 9 Solve an Inequality by Dividing

Solve $20x < 4$. Graph the solution set on a number line.

$20x < 4$ Original inequality

$\frac{20x}{20} < \frac{4}{20}$ Divide each side by 20.

$x < \frac{1}{5}$ Simplify.

The solution set is $\{x \mid x < \frac{1}{5}\}$.

Check

Solve $7x + 6 < 8x$. $\{x \mid x > 6\}$

Think About It! Why is the solution of $-13z \ge 117$ shaded to the left when the original inequality symbol is greater than or equal to?

Sample answer: When you multiply or divide by a negative number, you reverse the direction of the inequality symbol. The new inequality symbol is less than or equal to.

Example 10 Solve an Inequality with a Negative Coefficient

Solve $-13z \ge 117$. Graph the solution set on a number line.

$-13z \ge 117$ Original inequality

$\frac{-13z}{-13} \ge \frac{117}{-13}$ Divide each side by −13.

$z \le -9$ Simplify.

The solution set is $\{z \mid z \le -9\}$.

Study Tip

Negatives A negative sign in an inequality does not necessarily mean that the direction of the inequality symbol should change. For example, when solving $\frac{z}{3} \ge -9$, do not change the direction of the inequality symbol.

Check

Select the solution set for $-13x > -169$. B

A. $\{x \mid x > 13\}$ **B.** $\{x \mid x < 13\}$ **C.** $\{x \mid x > -13\}$ **D.** $\{x \mid x < -13\}$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Solve an Inequality by Multiplying

Complete each step to solve $-\frac{2}{5}x \le 11$. Graph the solution set on a number line.

You can use a multiplicative inverse with the Multiplication Property of Inequalities to solve this inequality.

$-\frac{2}{5}x \le$ ☐ Original inequality

Example 8

TYPE

Students complete the statement to solve the inequality.

TYPE

Students answer a question to show they understand why the inequality symbol was reversed.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 9 Solve an Inequality by Dividing

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the inequality and the graph in this example.

Questions for Mathematical Discourse

- **AL** Will the endpoint be a dot or a circle? How do you know? circle; The inequality is <.
- **OL** How can you solve the inequality? Divide each side by 20.
- **BL** By what could you multiply the original inequality to isolate the variable? $\frac{1}{20}$

Example 10 Solve an Inequality with a Negative Coefficient

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions to the Talk About It! feature.

Questions for Mathematical Discourse

- **AL** What is the inverse operation of multiplication? division
- **OL** What happens to the inequality symbol when you multiply or divide by a negative number? The inequality symbol reverses direction.
- **BL** What value can you use to check the inequality solution? Sample answer: −10 is in the solution region, so −13(−10) = 130 and 130 ≥ 117.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–54
2	exercises that use a variety of skills from this lesson	55–78
3	exercises that emphasize higher-order and critical-thinking skills	79–82

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–77 odd, 79–82
- Extension: Triangle Inequalities
- **ALEKS** Writing and Graphing Inequalities; Linear Inequalities and Applications

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–81 odd
- Remediation, Review Resources: Write and Solve One-Step Equations
- Personal Tutors
- Extra Examples 1–10
- **ALEKS** Solving One-Step Equations

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–53 odd
- Remediation, Review Resources: Write and Solve One-Step Equations
- *Quick Review Math Handbook*: Solving Inequalities by Multiplication and Division
- ArriveMATH Take Another Look
- **ALEKS** Solving One-Step Equations

Answers

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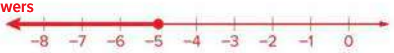

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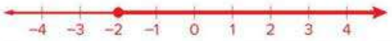

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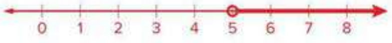

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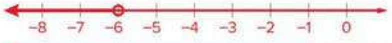

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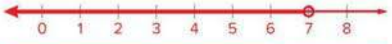

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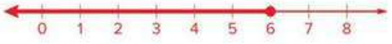

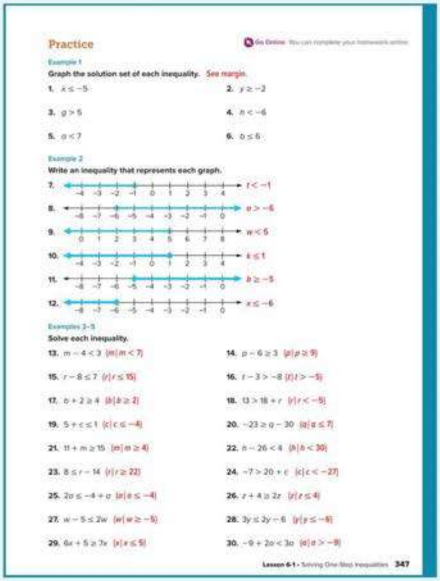

Practice

Go Online You can complete your homework online.

Example 1

Graph the solution set of each inequality. See margin.

1. $x \le -5$
2. $y \ge -2$
3. $g > 5$
4. $h < -6$
5. $a < 7$
6. $b \le 6$

Example 2

Write an inequality that represents each graph.

7. $r < -1$
8. $a > -6$
9. $w < 5$
10. $k \le 1$
11. $b \ge -5$
12. $x \le -6$

Examples 3–5

Solve each inequality.

13. $m - 4 < 3$ $\{m \mid m < 7\}$
14. $p - 6 \ge 3$ $\{p \mid p \ge 9\}$
15. $r - 8 \le 7$ $\{r \mid r \le 15\}$
16. $t - 3 > -8$ $\{t \mid t > -5\}$
17. $b + 2 \ge 4$ $\{b \mid b \ge 2\}$
18. $13 > 18 + r$ $\{r \mid r < -5\}$
19. $5 + c \le 1$ $\{c \mid c \le -4\}$
20. $-23 \ge q - 30$ $\{q \mid q \le 7\}$
21. $11 + m \ge 15$ $\{m \mid m \ge 4\}$
22. $h - 26 < 4$ $\{h \mid h < 30\}$
23. $8 \le r - 14$ $\{r \mid r \ge 22\}$
24. $-7 > 20 + c$ $\{c \mid c < -27\}$
25. $2a \le -4 + a$ $\{a \mid a \le -4\}$
26. $z + 4 \ge 2z$ $\{z \mid z \le 4\}$
27. $w - 5 \le 2w$ $\{w \mid w \ge -5\}$
28. $3y \le 2y - 6$ $\{y \mid y \le -6\}$
29. $6x + 5 \ge 7x$ $\{x \mid x \le 5\}$
30. $-9 + 2a < 3a$ $\{a \mid a > -9\}$

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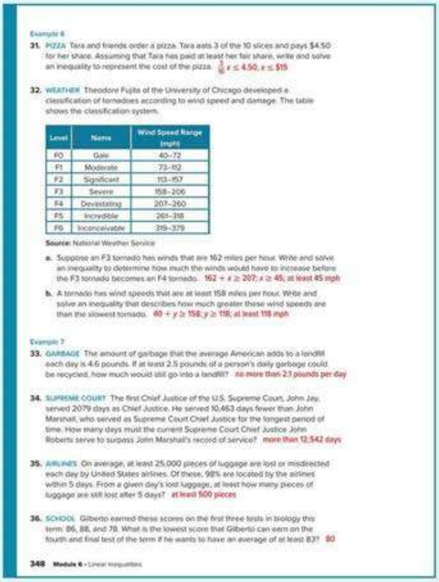

Example 6

31. PIZZA Tara and friends order a pizza. Tara eats 3 of the 10 slices and pays $4.50 for her share. Assuming that Tara has paid at least her fair share, write and solve an inequality to represent the cost of the pizza. $\frac{3}{10}x \le 4.50$, $x \le \$15$

32. WEATHER Theodore Fujita of the University of Chicago developed a classification of tornadoes according to wind speed and damage. The table shows the classification system.

Level	Name	Wind Speed Range (mph)
F0	Gale	40–72
F1	Moderate	73–112
F2	Significant	113–157
F3	Severe	158–206
F4	Devastating	207–260
F5	Incredible	261–318
F6	Inconceivable	319–379

Source: National Weather Service

a. Suppose an F3 tornado has winds that are 162 miles per hour. Write and solve an inequality to determine how much the winds would have to increase before the F3 tornado becomes an F4 tornado. $162 + x \ge 207$; $x \ge 45$; at least 45 mph

b. A tornado has wind speeds that are at least 158 miles per hour. Write and solve an inequality that describes how much greater these wind speeds are than the slowest tornado. $40 + y \ge 158$; $y \ge 118$; at least 118 mph

Example 7

33. GARBAGE The amount of garbage that the average American adds to a landfill each day is 4.6 pounds. If at least 2.5 pounds of a person's daily garbage could be recycled, how much would still go into a landfill? no more than 2.1 pounds per day

34. SUPREME COURT The first Chief Justice of the U.S. Supreme Court, John Jay, served 2079 days as Chief Justice. He served 10,463 days fewer than John Marshall, who served as Supreme Court Chief Justice for the longest period of time. How many days must the current Supreme Court Chief Justice John Roberts serve to surpass John Marshall's record of service? more than 12,542 days

35. AIRLINES On average, at least 25,000 pieces of luggage are lost or misdirected each day by United States airlines. Of these, 98% are located by the airlines within 5 days. From a given day's lost luggage, at least how many pieces of luggage are still lost after 5 days? at least 500 pieces

36. SCHOOL Gilberto earned these scores on the first three tests in biology this term: 86, 88, and 78. What is the lowest score that Gilberto can earn on the fourth and final test of the term if he wants to have an average of at least 83? 80

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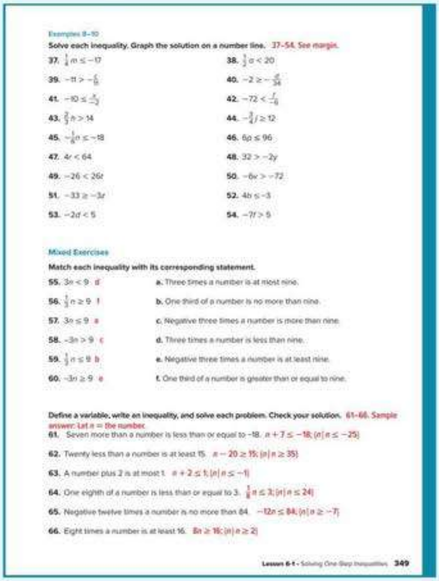


Examples 8–10

Solve each inequality. Graph the solution on a number line. 37–54. See margin.

37. $\frac{1}{4}m \leq -17$
38. $\frac{1}{2}a < 20$
39. $-11 > -\frac{c}{11}$
40. $-2 \geq -\frac{d}{34}$
41. $-10 \leq \frac{x}{-2}$
42. $-72 < \frac{f}{-6}$
43. $\frac{2}{3}h > 14$
44. $-\frac{3}{4}j \geq 12$
45. $-\frac{1}{6}n \leq -18$
46. $6p \leq 96$
47. $4r < 64$
48. $32 > -2y$
49. $-26 < 26t$
50. $-6v > -72$
51. $-33 \geq -3z$
52. $4b \leq -3$
53. $-2d < 5$
54. $-7f > 5$

Mixed Exercises

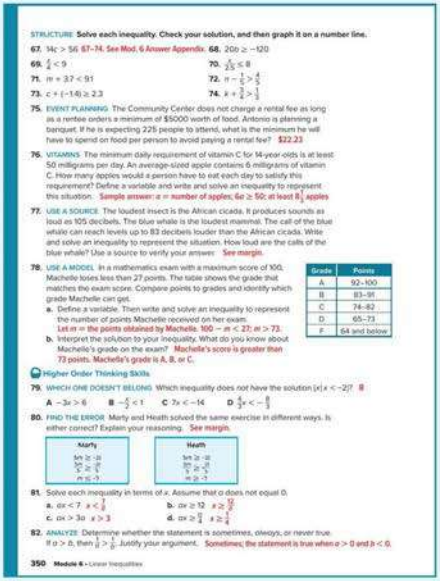
Match each inequality with its corresponding statement.

55. $3n < 9$ d	a. Three times a number is at most nine.
56. $\frac{1}{3}n \geq 9$ f	b. One third of a number is no more than nine.
57. $3n \leq 9$ a	c. Negative three times a number is more than nine.
58. $-3n > 9$ c	d. Three times a number is less than nine.
59. $\frac{1}{3}n \leq 9$ b	e. Negative three times a number is at least nine.
60. $-3n \geq 9$ e	f. One third of a number is greater than or equal to nine.

Define a variable, write an inequality, and solve each problem. Check your solution. 61–66. Sample answer: Let n = the number.

61. Seven more than a number is less than or equal to −18. $n + 7 \leq -18$; $\{n \mid n \leq -25\}$
62. Twenty less than a number is at least 15. $n - 20 \geq 15$; $\{n \mid n \geq 35\}$
63. A number plus 2 is at most 1. $n + 2 \leq 1$; $\{n \mid n \leq -1\}$
64. One eighth of a number is less than or equal to 3. $\frac{1}{8}n \leq 3$; $\{n \mid n \leq 24\}$
65. Negative twelve times a number is no more than 84. $-12n \leq 84$; $\{n \mid n \geq -7\}$
66. Eight times a number is at least 16. $8n \geq 16$; $\{n \mid n \geq 2\}$

Lesson 6-1 • Solving One-Step Inequalities 349



STRUCTURE **Solve each inequality. Check your solution, and then graph it on a number line.**

67. $14c > 56$ 67–74. See Mod. 6 Answer Appendix.
68. $20b \geq -120$
69. $\frac{c}{4} < 9$
70. $\frac{x}{25} \leq 8$
71. $m + 3.7 < 9.1$
72. $n - \frac{1}{4} > \frac{4}{5}$
73. $c + (-1.4) \geq 2.3$
74. $k + \frac{3}{4} > \frac{1}{3}$

75. EVENT PLANNING The Community Center does not charge a rental fee as long as a rentee orders a minimum of \$5000 worth of food. Antonio is planning a banquet. If he is expecting 225 people to attend, what is the minimum he will have to spend on food per person to avoid paying a rental fee? \$22.23

76. VITAMINS The minimum daily requirement of vitamin C for 14-year-olds is at least 50 milligrams per day. An average-sized apple contains 6 milligrams of vitamin C. How many apples would a person have to eat each day to satisfy this requirement? Define a variable and write and solve an inequality to represent this situation. Sample answer: a = number of apples; $6a \geq 50$; at least $8\frac{1}{3}$ apples

77. USE A SOURCE The loudest insect is the African cicada. It produces sounds as loud as 105 decibels. The blue whale is the loudest mammal. The call of the blue whale can reach levels up to 83 decibels louder than the African cicada. Write and solve an inequality to represent the situation. How loud are the calls of the blue whale? Use a source to verify your answer. See margin.

78. USE A MODEL In a mathematics exam with a maximum score of 100, Machelle loses less than 27 points. The table shows the grade that matches the exam score. Compare points to grades and identify which grade Machelle can get.
    a. Define a variable. Then write and solve an inequality to represent the number of points Machelle received on her exam. Let m = the points obtained by Machelle. $100 - m < 27$; $m > 73$.
    b. Interpret the solution to your inequality. What do you know about Machelle's grade on the exam? Machelle's score is greater than 73 points. Machelle's grade is A, B, or C.

Grade	Points
A	92–100
B	83–91
C	74–82
D	65–73
F	64 and below

Higher Order Thinking Skills

79. WHICH ONE DOESN'T BELONG Which inequality does *not* have the solution $\{x \mid x < -2\}$? B

A $-3x > 6$ B $-\frac{x}{2} < 1$ C $7x < -14$ D $\frac{4}{3}x < -\frac{8}{3}$

80. FIND THE ERROR Marty and Heath solved the same exercise in different ways. Is either correct? Explain your reasoning. See margin.

Marty	Heath
$5m \geq -35$ $\frac{5m}{5} \geq \frac{-35}{5}$ $m \leq -7$	$5m \geq -35$ $\frac{5m}{5} \geq \frac{-35}{5}$ $m \geq -7$

81. Solve each inequality in terms of x. Assume that a does not equal 0.
    a. $ax < 7$ $x < \frac{7}{a}$
    b. $ax \geq 12$ $x \geq \frac{12}{a}$
    c. $ax > 3a$ $x > 3$
    d. $ax \geq \frac{a}{4}$ $x \geq \frac{1}{4}$

82. ANALYZE Determine whether the statement is sometimes, always, or never true. If $a > b$, then $\frac{1}{a} > \frac{1}{b}$. Justify your argument. Sometimes; the statement is true when $a > 0$ and $b < 0$.

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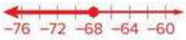
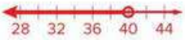
Answers

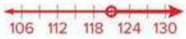
37. $\{m \mid m \leq -68\}$



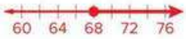
38. $\{a \mid a < 40\}$



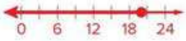
39. $\{c \mid c > 121\}$



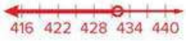
40. $\{d \mid d \geq 68\}$



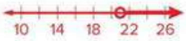
41. $\{x \mid x \leq 20\}$



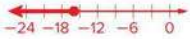
42. $\{f \mid f < 432\}$



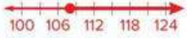
43. $\{h \mid h > 21\}$



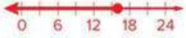
44. $\{j \mid j \leq -16\}$



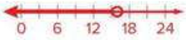
45. $\{n \mid n \geq 108\}$



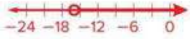
46. $\{p \mid p \leq 16\}$



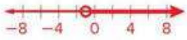
47. $\{r \mid r < 16\}$



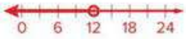
48. $\{y \mid y > -16\}$



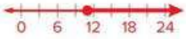
49. $\{t \mid t > -1\}$



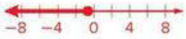
50. $\{v \mid v < 12\}$



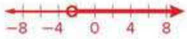
51. $\{z \mid z \geq 11\}$



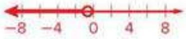
52. $\{b \mid b \leq -\frac{3}{4}\}$



53. $\{d \mid d > -2\frac{1}{2}\}$



54. $\{f \mid f < -\frac{5}{7}\}$



77. Sample answer: Let x represent the decibel level of the calls of a blue whale; $x - 83 \leq 105$; $x \leq 188$. The calls of a blue whale are less than or equal to 188 decibels.

80. Sample answer: Heath is correct. Marty solved the inequality incorrectly because the inequality symbol was reversed when dividing by a positive number.

Lesson 6-2

Solving Multi-Step Inequalities

A.CED.1, A.REI.3

LESSON GOAL

Students solve inequalities by using more than one step.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Modeling Multi-Step Inequalities

Develop:

Solving Inequalities Involving More Than One Step
- Apply Multi-Step Inequalities
- Write and Solve a Multi-Step Inequality

Solving Inequalities Involving the Distributive Property
- Solve an Inequality with the Distributive Property

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Solve Two-Step Equations: $px + q = r$	●	●		●
Extension: Graphing Linear Equations to Solve Multi-Step Inequalities		●	●	●

Language Development Handbook

Assign page 34 of the *Language Development Handbook* to help your students build mathematical language related to solving multi-step inequalities.

ELL You can use the tips and suggestions on page T34 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

Coherence

Vertical Alignment

Previous
Students solved one-step inequalities.
7.EE.4, A.CED.1, A.REI.3

Now
Students solve multi-step inequalities.
A.CED.1, A.REI.3

Next
Students will solve compound inequalities.
A.CED.1, A.CED.3

Rigor

The Three Pillars of Rigor

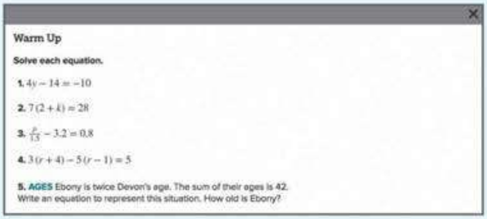
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their understanding of solving one-step inequalities and build fluency with solving multi-step inequalities. They apply their understanding of multi-step inequalities by solving real-world problems.

Mathematical Background

A linear inequality is an open sentence that contains $<$, $>$, $\leq$, or $\geq$, which can be solved by using algebraic methods similar to solving equations. Inequalities containing more elaborate expressions require additional steps of computations to isolate the variables in a solution inequality.

Interactive Presentation

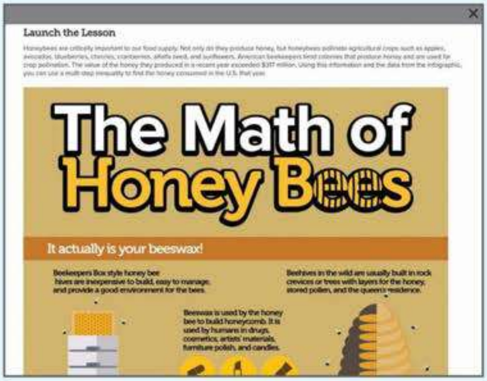


Warm Up

Solve each equation.

1. $4y - 14 = -10$
2. $7(2 + k) = 28$
3. $\frac{p}{13} - 3.2 = 0.8$
4. $3(r + 4) - 5(r - 1) = 5$
5. AGES Ebony is twice Devon's age. The sum of their ages is 42. Write an equation to represent this situation. How old is Ebony?

Warm Up



Launch the Lesson

Honeybees are critically important to our food supply. Not only do they produce honey, but honeybees pollinate agricultural crops such as apples, avocados, blueberries, cherries, cranberries, alfalfa seed, and sunflowers. American beekeepers tend colonies that produce honey and are used for crop pollination. The value of the honey they produced in a recent year exceeded $317 million. Using this information and the data from the infographic, you can use a multi-step inequality to find the honey consumed in the U.S. that year.

The Math of Honey Bees
It actually is your beeswax!
Beekeepers Box style honey bee hives are inexpensive to build, easy to manage, and provide a good environment for the bees.
Beehives in the wild are usually built in rock crevices or trees with layers for the honey, stored pollen, and the queen's residence.
Beeswax is used by the honey bee to build honeycomb. It is used by humans in drugs, cosmetics, artists' materials, furniture polish, and candles.

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- solving multi-step equations

Answers:

1. 1
2. 2
3. 6
4. 6
5. $d + 2d = 42$; 28 yr

Launch the Lesson

Teaching the Mathematical Practices

2 Make Sense of Quantities Mathematically proficient students need to be able to make sense of quantities and their relationships. Students can make sense of the information given and the information found in the infographic to write an inequality representing the honey consumed in the U.S. in a given year.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Explore Modeling Multi-Step Inequalities

Objective

Students use algebra tiles to explore solving multi-step inequalities.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use algebra tiles. Work with students to explore and deepen their understanding of multi-step inequalities.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

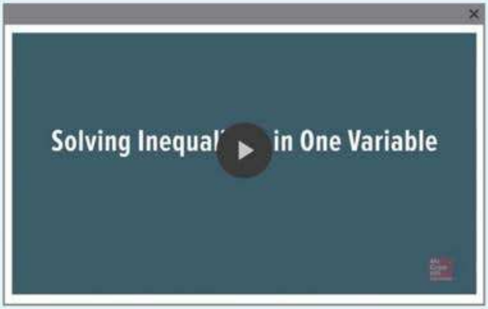
Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Student use algebra tiles with inequalities, to solidify understanding of how multi-step expressions can be used with inequalities. As the students are guided through the series of questions, they develop an understanding of how their known methods for solving multi-step equations apply to solving multi-step inequalities, with the aid of modeling by algebra tiles. Then, students will answer the Inquiry Question.

(*continued on the next page*)

Interactive Presentation

Explore



Explore

WATCH	Students can watch a video that explains how algebra tiles can be used to model and solve multi-step inequalities.
TYPE	Students answer questions to show they understand the video.
SELECT	Students select the correct property to answer the question.

Interactive Presentation

INQUIRY How can you model and solve a multi-step inequality?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Modeling Multi-Step Inequalities (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why should you begin by adding or subtracting 1-tiles? Sample answer: Just like solving an equation, you need to isolate the x-tiles using zero pairs.
- How would you model and solve the inequality $9 \leq 2x - 5$? Sample answer: Place 9 1-tiles on the left side of the mat, then place two x-tiles and five negative 1-tiles on the right side. Add five 1-tiles to each side of the mat and clear out the zero pairs. Then divide the tiles into two equal groups to get $7 \leq x$ or $x \geq 7$.

Inquiry

How can you model and solve a multi-step inequality? Sample answer: Use algebra tiles to model the inequality. Then add positive or negative 1-tiles to form zero pairs and isolate the x-tiles. Finally, separate the remaining tiles into equal groups.

Learn Solving Inequalities Involving More Than One Step

Objective

Students solve multi-step linear inequalities by applying properties of inequalities.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in solving multi-step inequalities in this Learn.

Example 1 Apply Multi-Step Inequalities

Teaching the Mathematical Practices

6 State Meanings of Symbols Guide students to define variables to solve the problem in this example. Help students to identify the independent and dependent variables. Then work with them to find the other relationships in the problem.

Questions for Mathematical Discourse

AL What are you trying to determine? What phrase indicates an inequality? How many comic book copies Suzy can afford to self-publish; maximum budget

OL What expression represents the total cost for printing *x* copies of Suzy's comic book? $220 + 3x$

BL How would the inequality change if Suzy decided to spend $25 of her budget on internet advertising? The $220 + 3x$ would need to change to $245 + 3x$.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF you have students who are interested in science,
THEN point out that there are many natural settings that can be connected to linear inequalities. Have students write observations about possible connections in their notebooks and then share their observations with the class.

Essential Question Follow-Up

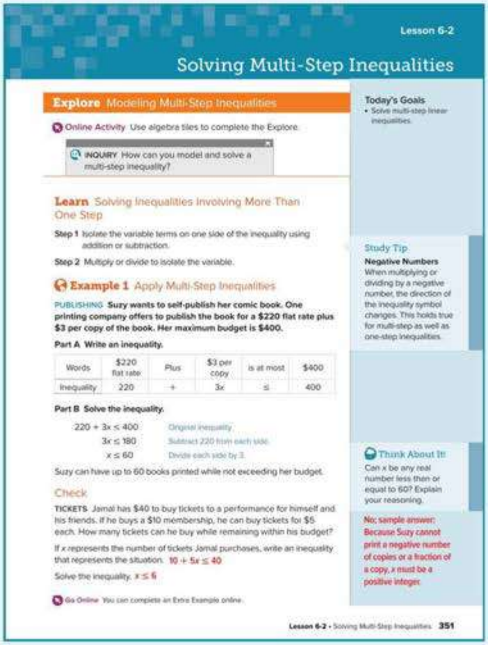
Students have explored expressions, equations, and inequalities.

Ask:

How are symbols used to write expressions, equations, and inequalities? Sample answer: If only operational symbols are used without a symbol that denotes equality or inequality, the statement is an expression. If an equal sign is used to show that two or more expressions are equal, the statement is an equation. If an inequality symbol is used to show that two or more expressions are unequal, the statement is an inequality.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 6-2

Solving Multi-Step Inequalities

Explore Modeling Multi-Step Inequalities

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you model and solve a multi-step inequality?

Learn Solving Inequalities Involving More Than One Step

Step 1 Isolate the variable terms on one side of the inequality using addition or subtraction.

Step 2 Multiply or divide to isolate the variable.

Example 1 Apply Multi-Step Inequalities

PUBLISHING **Suzy wants to self-publish her comic book. One printing company offers to publish the book for a $220 flat rate plus $3 per copy of the book. Her maximum budget is $400.**

Part A Write an inequality.

Words	$220 flat rate	Plus	$3 per copy	is at most	$400
Inequality	220	+	3x	≤	400

Part B Solve the inequality.

$220 + 3x \le 400$ Original inequality
$3x \le 180$ Subtract 220 from each side.
$x \le 60$ Divide each side by 3.

Suzy can have up to 60 books printed while not exceeding her budget.

Check

TICKETS Jamal has $40 to buy tickets to a performance for himself and his friends. If he buys a $10 membership, he can buy tickets for $5 each. How many tickets can he buy while remaining within his budget?

If *x* represents the number of tickets Jamal purchases, write an inequality that represents the situation. $10 + 5x \le 40$

Solve the inequality. $x \le 6$

Go Online You can complete an Extra Example online.

Today's Goals
- Solve multi-step linear inequalities.

Study Tip

Negative Numbers When multiplying or dividing by a negative number, the direction of the inequality symbol changes. This holds true for multi-step as well as one-step inequalities.

Think About It!

Can *x* be any real number less than or equal to 60? Explain your reasoning.

No; sample answer: Because Suzy cannot print a negative number of copies or a fraction of a copy, *x* must be a positive integer.

Lesson 6-2 • Solving Multi-Step Inequalities 351

Interactive Presentation

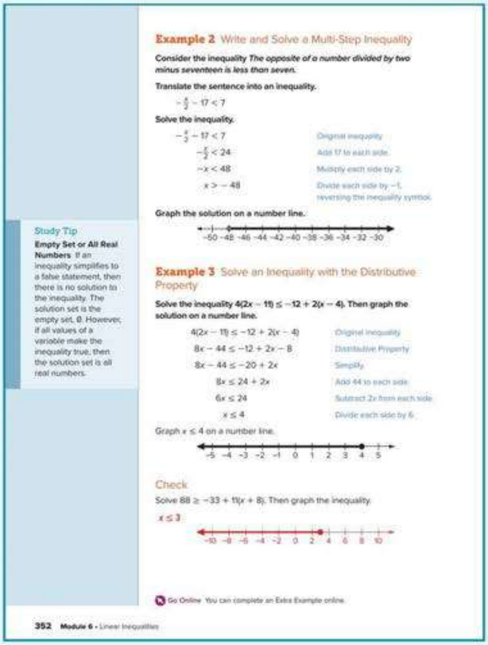
Example 1

SELECT

Students complete the table to write an inequality that represents the situation.

TYPE

Students answer a question to show they understand that *x* cannot be any real number less than or equal to 60.



Example 2 Write and Solve a Multi-Step Inequality

Consider the inequality *The opposite of a number divided by two minus seventeen is less than seven.*

Translate the sentence into an inequality.

$-\frac{x}{2} - 17 < 7$

Solve the inequality.

$-\frac{x}{2} - 17 < 7$	Original inequality
$-\frac{x}{2} < 24$	Add 17 to each side.
$-x < 48$	Multiply each side by 2.
$x > -48$	Divide each side by −1, reversing the inequality symbol.

Graph the solution on a number line.

Study Tip

Empty Set or All Real Numbers If an inequality simplifies to a false statement, then there is no solution to the inequality. The solution set is the empty set, ∅. However, if all values of a variable make the inequality true, then the solution set is all real numbers.

Example 3 Solve an Inequality with the Distributive Property

Solve the inequality $4(2x - 11) \le -12 + 2(x - 4)$. Then graph the solution on a number line.

$4(2x - 11) \le -12 + 2(x - 4)$	Original inequality
$8x - 44 \le -12 + 2x - 8$	Distributive Property
$8x - 44 \le -20 + 2x$	Simplify.
$8x \le 24 + 2x$	Add 44 to each side.
$6x \le 24$	Subtract 2x from each side.
$x \le 4$	Divide each side by 6.

Graph $x \le 4$ on a number line.

Check

Solve $88 \ge -33 + 11(x + 8)$. Then graph the inequality.

$x \le 3$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 2

SELECT

Students select the correct symbol to complete the inequality.

TAP

Students move through the slides to solve the inequality.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Write and Solve a Multi-Step Inequality

Teaching the Mathematical Practices

2 Create Representations Guide students to write an inequality that represents the verbal description of the inequality in this example. Then use the inequality to solve the problem.

Questions for Mathematical Discourse

AL What phrase in the description indicates a variable? *a number*

OL What is the coefficient of the variable? $-\frac{1}{2}$

BL How is solving this inequality similar to solving a multi-step equation? Sample answer: Because there are multiple operations applied to x, you still need to undo each using the reverse order of operations.

Example 3 Solve an Inequality with the Distributive Property

Teaching the Mathematical Practices

1 Explain Correspondences In this example students should be able to explain the relationship between the solution of the inequality and its number line.

Questions for Mathematical Discourse

AL What operation is indicated by the number outside the parentheses? multiplication

OL What is the first step in simplifying this problem? Sample answer: Distribute the 4 and the 2 to the terms in parentheses.

BL Describe another way to solve the inequality. Sample answer: Use the Division Property to divide each side by 2. This will eliminate the need to distribute on the right side, though you will have to take half of the −12, turning it into a −6, and then still distribute on the left side.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–21
2	exercises that use a variety of skills from this lesson	22–41
3	exercises that emphasize higher-order and critical-thinking skills	42–52

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–41 odd, 42–52
- Extension: Graph Linear Equations to Solve Multi-Step Inequalities
- ALEKS Writing and Graphing Inequalities; Linear Inequalities, and Applications

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–51 odd
- Remediation, Review Resources: Solve Two-Step Equations: $px + q = r$
- Personal Tutors
- Extra Examples 1–3
- ALEKS Solving Multi-Step Equations

AL

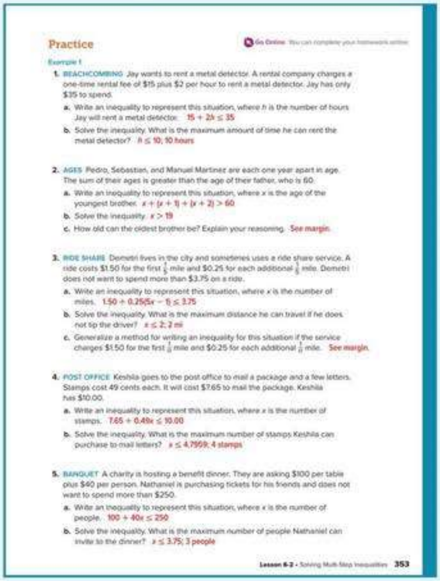
IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–21 odd
- Remediation, Review Resources: Solve Two-Step Equations: $px + q = r$
- *Quick Review Math Handbook*: Solving Multi-Step Inequalities
- Arrive**MATH** Take Another Look
- ALEKS Solving Multi-Step Equations

Answers

2c. no younger than 22; The solution to the inequality is $x > 19$, so the youngest brother is no younger than 20. The oldest brother is 2 years older than the youngest brother since each are one year apart. Therefore, the oldest brother is no younger than 20 + 2, or 22.

3c. Because the service charges per $\frac{1}{a}$ mile, multiply a by the number of miles, x, to find the number of $\frac{1}{a}$ miles. Subtract 1 from the total number of $\frac{1}{a}$ miles, ax, to find the number of additional $\frac{1}{a}$ miles. Multiply the difference by the cost per additional $\frac{1}{a}$ mile, 0.25, and add the cost for the first $\frac{1}{a}$ mile, 1.50. This sum is less than or equal to \$3.75, so $1.50 + 0.25(ax - 1) \leq 3.75$.



Practice

Go Online You can complete your homework online.

Example 1

1. BEACHCOMBING Jay wants to rent a metal detector. A rental company charges a one-time rental fee of \$15 plus \$2 per hour to rent a metal detector. Jay has only \$35 to spend.
   a. Write an inequality to represent this situation, where h is the number of hours Jay will rent a metal detector. $15 + 2h \leq 35$
   b. Solve the inequality. What is the maximum amount of time he can rent the metal detector? $h \leq 10$; 10 hours

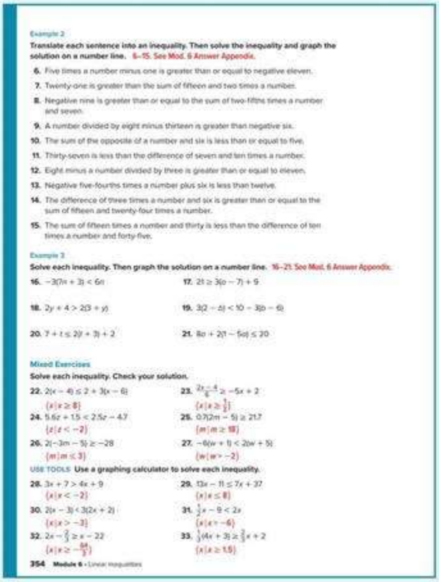
2. AGES Pedro, Sebastian, and Manuel Martinez are each one year apart in age. The sum of their ages is greater than the age of their father, who is 60.
   a. Write an inequality to represent this situation, where x is the age of the youngest brother. $x + (x + 1) + (x + 2) > 60$
   b. Solve the inequality. $x > 19$
   c. How old can the oldest brother be? Explain your reasoning. See margin.

3. RIDE SHARE Demetri lives in the city and sometimes uses a ride share service. A ride costs \$1.50 for the first $\frac{1}{5}$ mile and \$0.25 for each additional $\frac{1}{5}$ mile. Demetri does not want to spend more than \$3.75 on a ride.
   a. Write an inequality to represent this situation, where x is the number of miles. $1.50 + 0.25(5x - 1) \leq 3.75$
   b. Solve the inequality. What is the maximum distance he can travel if he does not tip the driver? $x \leq 2$; 2 mi
   c. Generalize a method for writing an inequality for this situation if the service charges \$1.50 for the first $\frac{1}{a}$ mile and \$0.25 for each additional $\frac{1}{a}$ mile. See margin.

4. POST OFFICE Keshia goes to the post office to mail a package and a few letters. Stamps cost 49 cents each. It will cost \$7.65 to mail the package. Keshia has \$10.00.
   a. Write an inequality to represent this situation, where x is the number of stamps. $7.65 + 0.49x \leq 10.00$
   b. Solve the inequality. What is the maximum number of stamps Keshia can purchase to mail letters? $x \leq 4.7959$; 4 stamps

5. BANQUET A charity is hosting a benefit dinner. They are asking \$100 per table plus \$40 per person. Nathaniel is purchasing tickets for his friends and does not want to spend more than \$250.
   a. Write an inequality to represent this situation, where x is the number of people. $100 + 40x \leq 250$
   b. Solve the inequality. What is the maximum number of people Nathaniel can invite to the dinner? $x \leq 3.75$; 3 people

Lesson 6-2 • Solving Multi-Step Inequalities 353



Example 2

Translate each sentence into an inequality. Then solve the inequality and graph the solution on a number line. 6–15. See Mod. 6 Answer Appendix.

6. Five times a number minus one is greater than or equal to negative eleven.
7. Twenty-one is greater than the sum of fifteen and two times a number.
8. Negative nine is greater than or equal to the sum of two-fifths times a number and seven.
9. A number divided by eight minus thirteen is greater than negative six.
10. The sum of the opposite of a number and six is less than or equal to five.
11. Thirty-seven is less than the difference of seven and ten times a number.
12. Eight minus a number divided by three is greater than or equal to eleven.
13. Negative five-fourths times a number plus six is less than twelve.
14. The difference of three times a number and six is greater than or equal to the sum of fifteen and twenty-four times a number.
15. The sum of fifteen times a number and thirty is less than the difference of ten times a number and forty-five.

Example 3

Solve each inequality. Then graph the solution on a number line. 16–21. See Mod. 6 Answer Appendix.

16. $-3(7n + 3) < 6n$
17. $2t \geq 3(t - 7) + 9$
18. $2y + 4 > 2(3 + y)$
19. $3(2 - 4b) < 10 - 3(b - 6)$
20. $7 + t \leq 2(t + 3) + 2$
21. $8a + 2(1 - 5a) \leq 20$

Mixed Exercises

Solve each inequality. Check your solution.

22. $2(x - 4) \leq 2 + 3(x - 6)$ $\{x \mid x \geq 8\}$
23. $\frac{2x - 4}{6} \geq -5x + 2$ $\{x \mid x \geq \frac{1}{2}\}$
24. $5.6z + 1.5 < 2.5z - 4.7$ $\{z \mid z < -2\}$
25. $0.7(2m - 5) \geq 21.7$ $\{m \mid m \geq 18\}$
26. $2(-3m - 5) \geq -28$ $\{m \mid m \leq 3\}$
27. $-6(w + 1) < 2(w + 5)$ $\{w \mid w > -2\}$

USE TOOLS **Use a graphing calculator to solve each inequality.**

28. $3x + 7 > 4x + 9$ $\{x \mid x < -2\}$
29. $13x - 11 \leq 7x + 37$ $\{x \mid x \leq 8\}$
30. $2(x - 3) < 3(2x + 2)$ $\{x \mid x > -3\}$
31. $\frac{1}{2}x - 9 < 2x$ $\{x \mid x > -6\}$
32. $2x - \frac{2}{3} \geq x - 22$ $\{x \mid x \geq -\frac{64}{3}\}$
33. $\frac{1}{3}(4x + 3) \geq \frac{2}{3}x + 2$ $\{x \mid x \geq 1.5\}$

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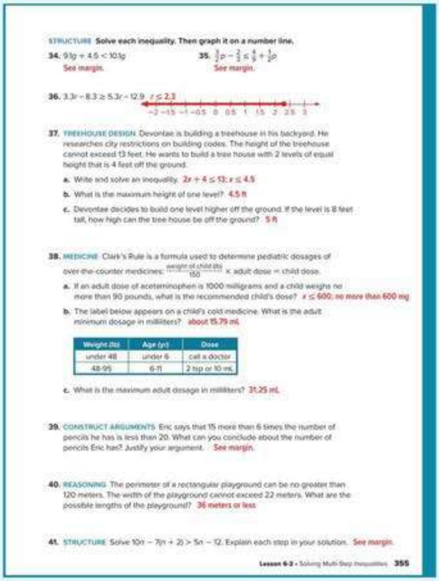
3 REFLECT AND PRACTICE



STRUCTURE **Solve each inequality. Then graph it on a number line.**

34. $9.1g + 4.5 < 10.1g$ See margin.

35. $\frac{3}{2}p - \frac{2}{3} \le \frac{4}{9} + \frac{1}{2}p$ See margin.

36. $3.3r - 8.3 \ge 5.3r - 12.9$ $r \le 2.3$

37. **TREEHOUSE DESIGN** Devontae is building a treehouse in his backyard. He researches city restrictions on building codes. The height of the treehouse cannot exceed 13 feet. He wants to build a tree house with 2 levels of equal height that is 4 feet off the ground.
   - a. Write and solve an inequality. $2x + 4 \le 13$; $x \le 4.5$
   - b. What is the maximum height of one level? 4.5 ft
   - c. Devontae decides to build one level higher off the ground. If the level is 8 feet tall, how high can the tree house be off the ground? 5 ft

38. **MEDICINE** Clark's Rule is a formula used to determine pediatric dosages of over-the-counter medicines: $\frac{\text{weight of child (lb)}}{150} \times$ adult dose = child dose.
   - a. If an adult dose of acetaminophen is 1000 milligrams and a child weighs no more than 90 pounds, what is the recommended child's dose? $x \le 600$; no more than 600 mg
   - b. The label below appears on a child's cold medicine. What is the adult minimum dosage in milliliters? about 15.79 mL

Weight (lb)	Age (yr)	Dose
under 48	under 6	call a doctor
48-95	6-11	2 tsp or 10 mL

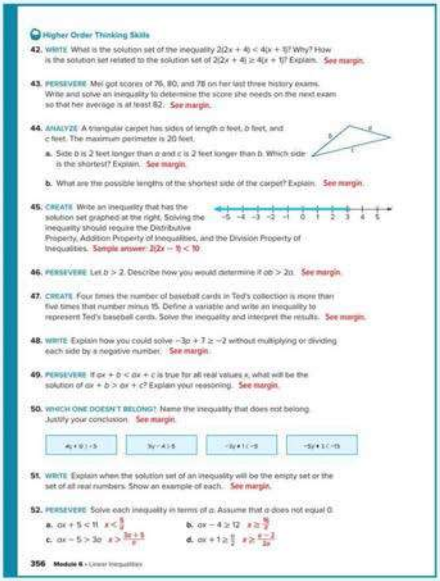
   - c. What is the maximum adult dosage in milliliters? 31.25 mL

39. **CONSTRUCT ARGUMENTS** Eric says that 15 more than 6 times the number of pencils he has is less than 20. What can you conclude about the number of pencils Eric has? Justify your argument. See margin.

40. **REASONING** The perimeter of a rectangular playground can be no greater than 120 meters. The width of the playground cannot exceed 22 meters. What are the possible lengths of the playground? 38 meters or less

41. **STRUCTURE** Solve $10n - 7(n + 2) > 5n - 12$. Explain each step in your solution. See margin.

Lesson 6-2 • Solving Multi-Step Inequalities 355



Higher Order Thinking Skills

42. **WRITE** What is the solution set of the inequality $2(2x + 4) < 4(x + 1)$? Why? How is the solution set related to the solution set of $2(2x + 4) \ge 4(x + 1)$? Explain. See margin.

43. **PERSEVERE** Mei got scores of 76, 80, and 78 on her last three history exams. Write and solve an inequality to determine the score she needs on the next exam so that her average is at least 82. See margin.

44. **ANALYZE** A triangular carpet has sides of length a feet, b feet, and c feet. The maximum perimeter is 20 feet.
   - a. Side b is 2 feet longer than a and c is 2 feet longer than b. Which side is the shortest? Explain. See margin.
   - b. What are the possible lengths of the shortest side of the carpet? Explain. See margin.

45. **CREATE** Write an inequality that has the solution set graphed at the right. Solving the inequality should require the Distributive Property, Addition Property of Inequalities, and the Division Property of Inequalities. Sample answer: $2(2x - 1) < 10$

46. **PERSEVERE** Let $b > 2$. Describe how you would determine if $ab > 2a$. See margin.

47. **CREATE** Four times the number of baseball cards in Ted's collection is more than five times that number minus 15. Define a variable and write an inequality to represent Ted's baseball cards. Solve the inequality and interpret the results. See margin.

48. **WRITE** Explain how you could solve $-3p + 7 \ge -2$ without multiplying or dividing each side by a negative number. See margin.

49. **PERSEVERE** If $ax + b < ax + c$ is true for all real values x, what will be the solution of $ax + b > ax + c$? Explain your reasoning. See margin.

50. **WHICH ONE DOESN'T BELONG?** Name the inequality that does not belong. Justify your conclusion. See margin.

$4y + 9 > -3$ | $3y - 4 > 5$ | $-2y + 1 < -5$ | $-5y + 3 < -12$

51. **WRITE** Explain when the solution set of an inequality will be the empty set or the set of all real numbers. Show an example of each. See margin.

52. **PERSEVERE** Solve each inequality in terms of a. Assume that a does not equal 0.
   - a. $ax + 5 < 11$ $x < \frac{6}{a}$
   - b. $ax - 4 \ge 12$ $x \ge \frac{16}{a}$
   - c. $ax - 5 > 3a$ $x > \frac{3a+5}{a}$
   - d. $ax + 1 \ge \frac{a}{2}$ $x \ge \frac{a-2}{2a}$

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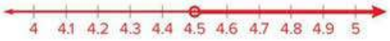
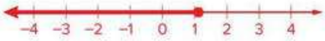
Answers

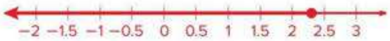
34. $\{g \mid g > 4.5\}$



35. $\left\{p \mid p \le 1\frac{1}{9}\right\}$



36. $r \le 2.3$



39. Eric does not have any pencils. Based on his statement, the inequality is $6p + 15 < 20$, where p is the number of pencils. The solution of the inequality is $p < \frac{5}{6}$. However, the number of pencils must be a whole number, so $p = 0$.

41.

$10n - 7(n + 2) > 5n - 12$	Original inequality
$10n - 7n - 14 > 5n - 12$	Distributive Property
$3n - 14 > 5n - 12$	Combine like terms.
$3n - 14 - 5n > 5n - 12 - 5n$	Subtract $5n$ from each side.
$-2n - 14 > -12$	Simplify.
$-2n - 14 + 14 > -12 + 14$	Add 14 to each side.
$-2n > 2$	Simplify.
$\frac{-2n}{-2} < \frac{2}{-2}$	Divide each side by -2. Change $>$ to $<$.
$n < -1$	Simplify.

The solution set is $\{n \mid n < -1\}$.

42. The solution set is the empty set because solving the inequality results in a false statement. When you replace the inequality symbol with $\ge$, the solution set becomes all real numbers since solving the inequality now results in a statement that is always true.

43. $\frac{76 + 80 + 78 + x}{4} \ge 82$; $x \ge 94$; Mei needs a score of at least 94 on the next exam.

44a. The shortest side must be a, since $b = a + 2$ shows that $b > a$ and $c = b + 2$ shows that $c > b$.

44b. Solve the inequality $a + b + c \le 20$ or $a + (a + 2) + (b + 2) \le 20$ or $a + (a + 2) + (a + 2 + 2) \le 20$. The solution is $a \le \frac{14}{3}$. So the shortest side is greater than 0 feet but less than or equal to $4\frac{2}{3}$ feet.

46. The inequality $ab > 2a$ can be determined to be true or false by considering the value of a. Since $b > 2$, by the Multiplication Property of Inequality $ab > 2a$ is true if a is a positive number.

47. Let c = the number of baseball cards Ted has; $4c > 5c - 15$; $15 > c$; Ted has fewer than 15 cards.

48. Add $3p$ and 2 to each side. The inequality becomes $9 \ge 3p$. Then divide each side by 3 to get $3 \ge p$.

49. $\varnothing$; If the inequality is always true, the opposite inequality will always be false.

50. $4y + 9 > -3$; It is the only inequality that does not have a solution set of $\{y \mid y > 3\}$.

51. Sample answer: The solution set for the inequality that results in a false statement is the empty set, as in $12 \ge 15$. The solution set for an inequality in which any value of x results in a true statement is all real numbers, as in $12 \le 12$.

Lesson 6-3

Solving Compound Inequalities

A.CED.1, A.CED.3

LESSON GOAL

Students write and solve combinations of two inequalities joined by “and” or “or.”

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Guess the Range

Develop:

Solving Compound Inequalities Using the Word *and*
- Solve and Graph an Intersection
- Apply Compound Inequalities

Solving Compound Inequalities Using the Word *or*
- Solve and Graph a Union
- Overlapping Intervals

Writing Compound Inequalities
- Write a Compound Inequality for an Intersection
- Write a Compound Inequality for a Union

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Represent Integers	●	●		●
Extension: Precision of Measurement		●	●	●

Language Development Handbook

Assign page 35 of the *Language Development Handbook* to help your students build mathematical language related to solving combinations of two inequalities.

ELL You can use the tips and suggestions on page T35 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Standards for Mathematical Practice:

4 Model with mathematics.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous
Students solved multi-step inequalities.
7.EE.4, A.CED.1, A.REI.3

Now
Students solve compound inequalities.
A.CED.1, A.CED.3

Next
Students will solve absolute value inequalities.
A.CED.1, A.CED.3

Rigor

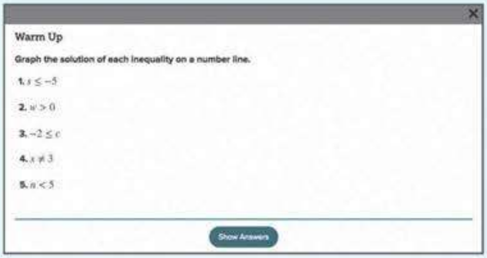
The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
Conceptual Bridge In this lesson, students draw on their understanding of solving multi-step inequalities and build fluency with solving and graphing compound inequalities. They apply their understanding of compound inequalities by solving real-world problems.		

Mathematical Background

A compound inequality is an open sentence in which an algebraic expression is constrained by two different inequality relationships. Compound inequalities can be solved by using algebraic methods similar to solving equations and standard inequalities.

Interactive Presentation

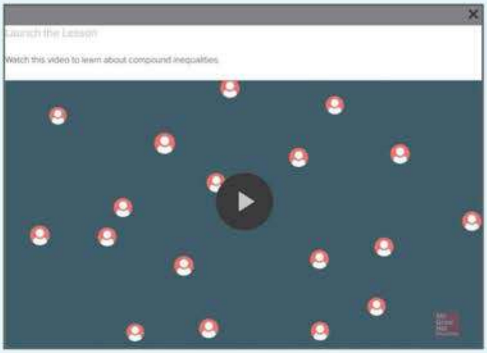


Warm Up

Graph the solution of each inequality on a number line.

1. $x \leq -5$
2. $w > 0$
3. $-2 \leq c$
4. $x \neq 3$
5. $n < 5$

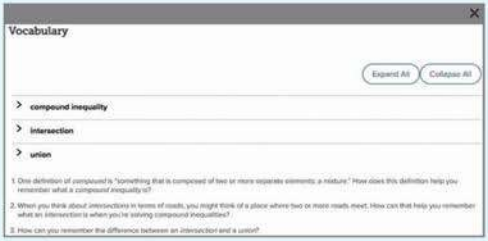
Warm Up



Launch the Lesson

Watch this video to learn about compound inequalities.

Launch the Lesson



Vocabulary

1. One definition of compound is "something that is composed of two or more separate elements; a mixture." How does this definition help you remember what a compound inequality is?
2. When you think about intersections in terms of roads, you might think of a place where two or more roads meet. How can that help you remember what an intersection is when you're solving compound inequalities?
3. How can you remember the difference between an intersection and a union?

Today's Vocabulary

Warm Up

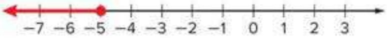
Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- graphing on a number line

Answers:

1.



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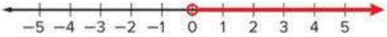
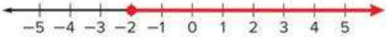


3.



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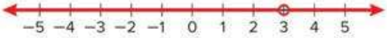

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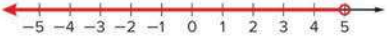


Launch the Lesson

Teaching the Mathematical Practices

4 Apply Mathematics In this Launch, students learn how to apply what they have learned about compound inequalities to a real-world situation about the process of becoming an FBI agent.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Guess the Range

Objective

Students use a sketch to explore writing and interpreting compound inequalities by using the word *and*.

Teaching the Mathematical Practices

4 Apply Mathematics In this Explore, students use what they have learned about inequalities and apply it to a real-world situation.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

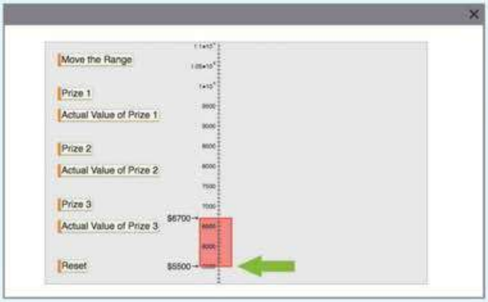
Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. The students will interact with a sketch about the values of prizes on a game show, then answer a series of questions about the price ranges shown and how they relate to writing compound inequalities using the word *and*. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

WEB SKETCHPAD

Students use a sketch to guess the range for each prize on a game show.

TYPE

Students answer questions to show they understand the compound inequalities generated from the sketch.

SELECT

Students select the correct answer to a question to show they understand compound inequalities.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Guess the Range (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Would it make sense to write the inequality $6700 \leq x \leq 5500$? Why or why not? Sample answer: No, because a value can't be both greater than 6700 and less than 5500.
- Do you think it would be easier or more difficult to win if the accepted range was $250? Sample answer: It would be more difficult to win because there are fewer values in a range of $250. You would have to be much closer to the actual value.

Inquiry

How can you tell if a value will satisfy a compound inequality that includes the word *and*? Sample answer: If a value falls between the lowest and greatest values of a compound inequality that uses the word *and*, then it satisfies the inequality.

Learn Solving Compound Inequalities Using the Word *and*

Objective

Students solve and graph linear inequalities containing the word *and* by applying properties of inequalities.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students are Learning

A compound inequality is two or more inequalities that are connected by *and* or *or*. The word *and* means all inequalities must be true, and the solution set is where the graph of the inequalities overlap. This is called the *intersection*.

Common Misconception

A common misconception some students may have is that the solution inequality must have different inequality symbols for an *and* compound inequality. Remind students the inequality symbols keep the solution set between two values and cannot have different symbols.

Example 1 Solve and Graph an Intersection

Teaching the Mathematical Practices

7 Interpret Complicated Expressions Mathematically proficient students can see complicated expressions as single objects or as being composed of several objects. In this example, guide students to see what information they can gather about the expression just from looking at it.

Questions for Mathematical Discourse

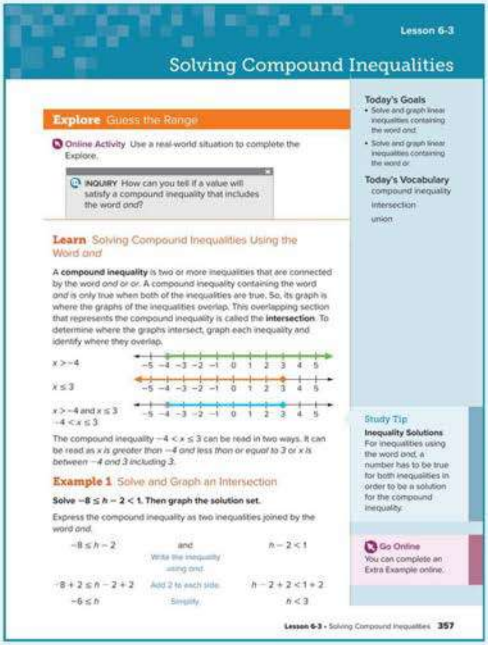
AL Why does it help to break the inequality into two pieces? Sample answer: It is easier to focus having the variable on one side of the inequality symbol and a constant on the other side.

OL How do you think we can graph the solution set? Sample answer: We can plot the endpoints and then draw a line connecting them.

BL If a value is a solution of one of the two inequalities, is it a solution to the compound inequality? Explain. No; sample answer: The intersection means the solutions have to be true for both inequalities. A value that is a solution to one inequality but not the other would not be a solution to the compound inequality.

Common Error

Students may assume that the solution set for $\{h \mid -6 \leq h < 3\}$ includes 3. It does not. Less than 3 does not include 3.



Lesson 6-3

Solving Compound Inequalities

Explore Guess the Range

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you tell if a value will satisfy a compound inequality that includes the word *and*?

Today's Goals
- Solve and graph linear inequalities containing the word *and*.
- Solve and graph linear inequalities containing the word *or*.

Today's Vocabulary
compound inequality
intersection
union

Learn Solving Compound Inequalities Using the Word *and*

A **compound inequality** is two or more inequalities that are connected by the word *and* or *or*. A compound inequality containing the word *and* is only true when both of the inequalities are true. So, its graph is where the graphs of the inequalities overlap. This overlapping section that represents the compound inequality is called the **intersection**. To determine where the graphs intersect, graph each inequality and identify where they overlap.

$x > -4$

$x \leq 3$

$x > -4$ and $x \leq 3$

$-4 < x \leq 3$

The compound inequality $-4 < x \leq 3$ can be read in two ways. It can be read as *x is greater than −4 and less than or equal to 3* or *x is between −4 and 3 including 3*.

Study Tip

Inequality Solutions For inequalities using the word *and*, a number has to be true for both inequalities in order to be a solution for the compound inequality.

Example 1 Solve and Graph an Intersection

Solve $-8 \leq h - 2 < 1$. Then graph the solution set.

Express the compound inequality as two inequalities joined by the word *and*.

$-8 \leq h - 2$	and Write the inequality using *and*.	$h - 2 < 1$
$-8 + 2 \leq h - 2 + 2$	Add 2 to each side.	$h - 2 + 2 < 1 + 2$
$-6 \leq h$	Simplify.	$h < 3$

Go Online You can complete an Extra Example online.

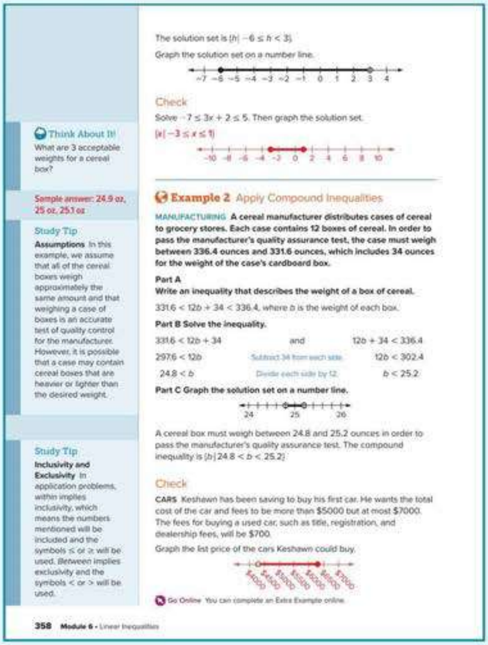
Lesson 6-3 • Solving Compound Inequalities 357

Interactive Presentation

Learn

TYPE

Students answer a question to show they understand if a value is a solution to a compound inequality.



The solution set is $\{h \mid -6 \leq h < 3\}$.

Graph the solution set on a number line.

Check

Solve $-7 \leq 3x + 2 \leq 5$. Then graph the solution set.

$\{x \mid -3 \leq x \leq 1\}$

Think About It!

What are 3 acceptable weights for a cereal box?

Sample answer: 24.9 oz, 25 oz, 25.1 oz

Study Tip

Assumptions In this example, we assume that all of the cereal boxes weigh approximately the same amount and that weighing a case of boxes is an accurate test of quality control for the manufacturer. However, it is possible that a case may contain cereal boxes that are heavier or lighter than the desired weight.

Example 2 Apply Compound Inequalities

MANUFACTURING **A cereal manufacturer distributes cases of cereal to grocery stores. Each case contains 12 boxes of cereal. In order to pass the manufacturer's quality assurance test, the case must weigh between 336.4 ounces and 331.6 ounces, which includes 34 ounces for the weight of the case's cardboard box.**

Part A

Write an inequality that describes the weight of a box of cereal.

$331.6 < 12b + 34 < 336.4$, where b is the weight of each box.

Part B Solve the inequality.

$331.6 < 12b + 34$ and $12b + 34 < 336.4$

$297.6 < 12b$ Subtract 34 from each side. $12b < 302.4$

$24.8 < b$ Divide each side by 12. $b < 25.2$

Part C Graph the solution set on a number line.

A cereal box must weigh between 24.8 and 25.2 ounces in order to pass the manufacturer's quality assurance test. The compound inequality is $\{b \mid 24.8 < b < 25.2\}$.

Study Tip

Inclusivity and Exclusivity In application problems, within implies inclusivity, which means the numbers mentioned will be included and the symbols ≤ or ≥ will be used. Between implies exclusivity and the symbols < or > will be used.

Check

CARS Keshawn has been saving to buy his first car. He wants the total cost of the car and fees to be more than \$5000 but at most \$7000. The fees for buying a used car, such as title, registration, and dealership fees, will be \$700.

Graph the list price of the cars Keshawn could buy.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Apply Compound Inequalities

MANUFACTURING **A cereal manufacturer distributes cases of cereal to grocery stores. Each case contains 12 boxes of cereal. In order to pass the manufacturer's quality assurance test, the case must weigh between 336.4 ounces and 331.6 ounces, which includes 34 ounces for the weight of the case's cardboard box.**

Example 2

TYPE

Students complete the statements to solve the compound inequality and give three acceptable weights.

TAP

Students move through the steps to graph the solution set.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Apply Compound Inequalities

MP Teaching the Mathematical Practices

4 Make Assumptions In the Study Tip, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL What is the maximum allowed weight for a box of cereal to pass the manufacturer's quality assurance test? less than 25.2 ounces

OL What is the unknown quantity in this problem? the weight of a box of cereal

BL How do you determine if this is an intersection or a union? Sample answer: The word "between" tells you that there is a value that is least and a value that is greatest. So, you know that it has to be an intersection.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Solving Compound Inequalities Using the Word *or*

Objective

Students solve and graph linear inequalities containing the word *or* by applying properties of inequalities.

MP Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of compound inequalities using the word *or* in this Learn.

Important to Know

A compound inequality containing the word *or* is true if at least one of the inequalities is true. The graph of the solution is the union of all solutions of the individual inequalities.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF some students are overwhelmed trying to discern whether word problems represent compound inequalities and whether the inequalities are inclusive or exclusive,

THEN pair these students with more advanced students to work the problems, encouraging both students to take an active role in solving the problems. If a student proposes a good way of thinking about compound inequalities, encourage that student to share their thinking with the rest of the class.

Example 3 Solve and Graph a Union

MP Teaching the Mathematical Practices

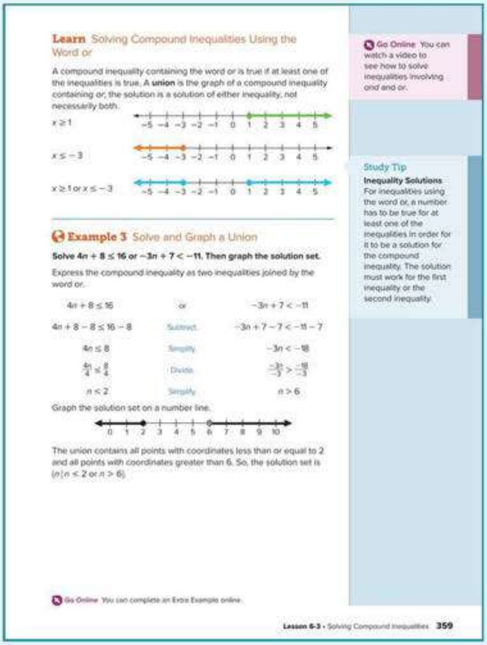
7 Use Structure Students will use the structure of the compound inequality to write two cases.

Questions for Mathematical Discourse

AL What do 2 and 6 represent? the endpoints of the two inequalities in the compound inequality

OL How can we graph the solution set? Sample answer: Plot a closed circle at 2 and shade to the left. Then plot an open circle at 6 and shade everything to the right.

BL If a value is a solution of one of the two inequalities, is it a solution to the compound inequality? Explain. Yes; sample answer: The union includes all values that are solutions to either of the inequalities. A value that is a solution to one inequality but not the other would still be a solution to this type of compound inequality.



Learn Solving Compound Inequalities Using the Word *or*

A compound inequality containing the word *or* is true if at least one of the inequalities is true. A **union** is the graph of a compound inequality containing *or*; the solution is a solution of either inequality, not necessarily both.

$x \geq 1$

$x \leq -3$

$x \geq 1$ or $x \leq -3$

Example 3 Solve and Graph a Union

Solve $4n + 8 \leq 16$ or $-3n + 7 < -11$. Then graph the solution set.

Express the compound inequality as two inequalities joined by the word *or*.

$4n + 8 \leq 16$	or	$-3n + 7 < -11$
$4n + 8 - 8 \leq 16 - 8$	Subtract.	$-3n + 7 - 7 < -11 - 7$
$4n \leq 8$	Simplify.	$-3n < -18$
$\frac{4n}{4} \leq \frac{8}{4}$	Divide.	$\frac{-3n}{-3} > \frac{-18}{-3}$
$n \leq 2$	Simplify.	$n > 6$

Graph the solution set on a number line.

The union contains all points with coordinates less than or equal to 2 and all points with coordinates greater than 6. So, the solution set is $\{n \mid n \leq 2 \text{ or } n > 6\}$.

Go Online You can complete an Extra Example online.

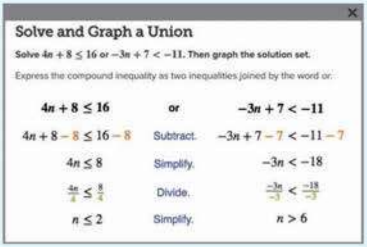
Go Online You can watch a video to see how to solve inequalities involving *and* and *or*.

Study Tip

Inequality Solutions For inequalities using the word *or*, a number has to be true for at least one of the inequalities in order for it to be a solution for the compound inequality. The solution must work for the first inequality or the second inequality.

Lesson 6-3 • Solving Compound Inequalities 359

Interactive Presentation



Solve and Graph a Union

Solve $4n + 8 \leq 16$ or $-3n + 7 < -11$. Then graph the solution set.

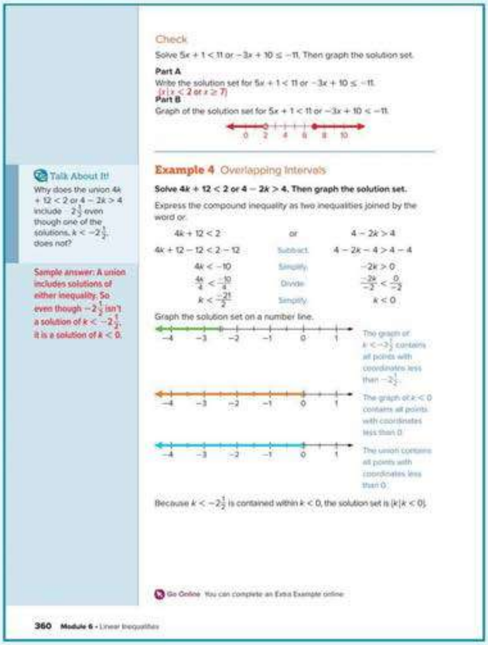
Express the compound inequality as two inequalities joined by the word *or*.

$4n + 8 \leq 16$	or	$-3n + 7 < -11$
$4n + 8 - 8 \leq 16 - 8$	Subtract.	$-3n + 7 - 7 < -11 - 7$
$4n \leq 8$	Simplify.	$-3n < -18$
$\frac{4n}{4} \leq \frac{8}{4}$	Divide.	$\frac{-3n}{-3} < \frac{-18}{-3}$
$n \leq 2$	Simplify.	$n > 6$

Example 3

TAP

Students move through the steps to graph the solution set on a number line.



Check

Solve $5x + 1 < 11$ or $-3x + 10 \leq -11$. Then graph the solution set.

Part A

Write the solution set for $5x + 1 < 11$ or $-3x + 10 \leq -11$.

$\{x \mid x < 2 \text{ or } x \geq 7\}$

Part B

Graph of the solution set for $5x + 1 < 11$ or $-3x + 10 \leq -11$.

Example 4 Overlapping Intervals

Solve $4k + 12 < 2$ or $4 - 2k > 4$. Then graph the solution set.

Express the compound inequality as two inequalities joined by the word *or*.

$4k + 12 < 2$	or	$4 - 2k > 4$
$4k + 12 - 12 < 2 - 12$	Subtract.	$4 - 2k - 4 > 4 - 4$
$4k < -10$	Simplify.	$-2k > 0$
$\frac{4k}{4} < \frac{-10}{4}$	Divide.	$\frac{-2k}{-2} < \frac{0}{-2}$
$k < -2\frac{1}{2}$	Simplify.	$k < 0$

Graph the solution set on a number line.

The graph of $k < -2\frac{1}{2}$ contains all points with coordinates less than $-2\frac{1}{2}$.

The graph of $k < 0$ contains all points with coordinates less than 0.

The union contains all points with coordinates less than 0.

Because $k < -2\frac{1}{2}$ is contained within $k < 0$, the solution set is $\{k \mid k < 0\}$.

Talk About It!

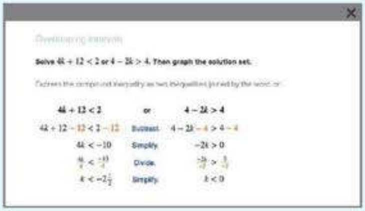
Why does the union $4k + 12 < 2$ or $4 - 2k > 4$ include $-2\frac{1}{2}$ even though one of the solutions, $k < -2\frac{1}{2}$, does not?

Sample answer: A union includes solutions of either inequality. So even though $-2\frac{1}{2}$ isn't a solution of $k < -2\frac{1}{2}$, it is a solution of $k < 0$.

Go Online You can complete an Extra Example online.

360 Module 6 • Linear Inequalities

Interactive Presentation



Overlapping Intervals

Solve $4k + 12 < 2$ or $4 - 2k > 4$. Then graph the solution set.

Express the compound inequality as two inequalities joined by the word or.

$4k + 12 < 2$	or	$4 - 2k > 4$
$4k + 12 - 12 < 2 - 12$	Subtract.	$4 - 2k - 4 > 4 - 4$
$4k < -10$	Simplify.	$-2k > 0$
$\frac{4k}{4} < \frac{-10}{4}$	Divide.	$\frac{-2k}{-2} > \frac{0}{-2}$
$k < -2\frac{1}{2}$	Simplify.	$k < 0$

Example 4

TAP

Students move through the steps to graph the solution set.

TYPE

Students answer a question to show they understand why the union of two inequalities includes a certain solution.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Overlapping Intervals

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What does *or* in the compound inequality mean? Any number that is a solution to one of the inequalities is also a solution to the compound inequality.

OL Would the compound inequality $x > 0$ or $x > 1$ include 1 as a solution? Explain. Yes; The value 1 makes the inequality $x > 0$ true, so it is a solution to the union shown in the compound inequality, even though it is not a solution to $x > 1$.

BL Write the solution in set-builder notation. $\{k \mid k < 0\}$

Common Error

Students may confuse the meaning of the words *intersection* and *union*. Have them compare the definitions of these two words in real-world situations. For intersection, students may suggest two roads overlapping at an intersection. For union, they might suggest the 50 states coming together to form a union.

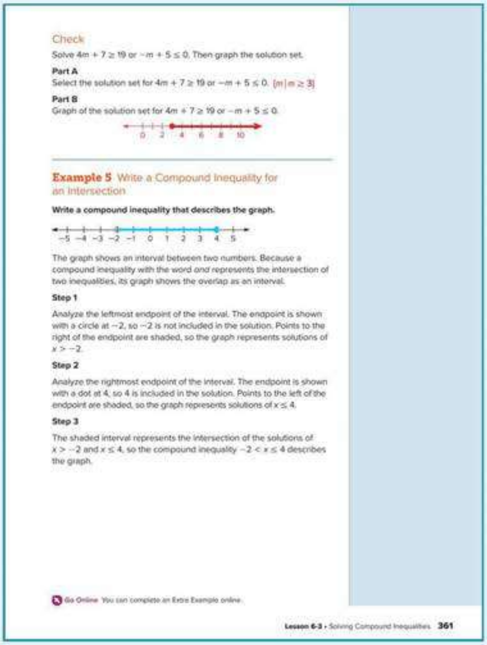
Example 5 Write a Compound Inequality for an Intersection

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the compound inequality and its graph used in this example.

Questions for Mathematical Discourse

- **AL** Is −2 a solution to the compound inequality shown on the graph? Explain. No; sample answer: It is represented by an open circle.
- **OL** Does this graph represent an intersection or a union? intersection
- **BL** Describe all of the solutions to this compound inequality. Sample answer: every value between −2 and 4, including 4



Check

Solve $4m + 7 \geq 19$ or $-m + 5 \leq 0$. Then graph the solution set.

Part A

Select the solution set for $4m + 7 \geq 19$ or $-m + 5 \leq 0$. $\{m \mid m \geq 3\}$

Part B

Graph of the solution set for $4m + 7 \geq 19$ or $-m + 5 \leq 0$.

Example 5 Write a Compound Inequality for an Intersection

Write a compound inequality that describes the graph.

The graph shows an interval between two numbers. Because a compound inequality with the word *and* represents the intersection of two inequalities, its graph shows the overlap as an interval.

Step 1

Analyze the leftmost endpoint of the interval. The endpoint is shown with a circle at −2, so −2 is not included in the solution. Points to the right of the endpoint are shaded, so the graph represents solutions of $x > -2$.

Step 2

Analyze the rightmost endpoint of the interval. The endpoint is shown with a dot at 4, so 4 is included in the solution. Points to the left of the endpoint are shaded, so the graph represents solutions of $x \leq 4$.

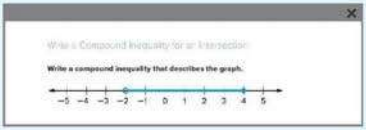
Step 3

The shaded interval represents the intersection of the solutions of $x > -2$ and $x \leq 4$, so the compound inequality $-2 < x \leq 4$ describes the graph.

Go Online You can complete an Extra Example online.

Lesson 6-3 • Solving Compound Inequalities 361

Interactive Presentation



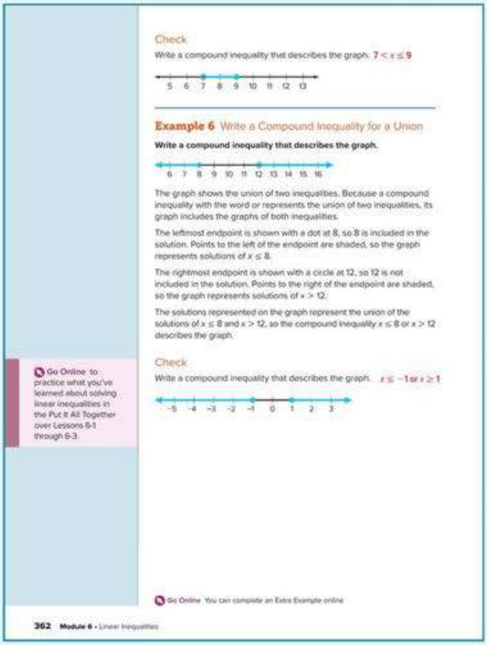
Write a Compound Inequality for an Intersection

Write a compound inequality that describes the graph.

Example 5

TAP

Students move through the slides to see how to write a compound inequality for the graph.



Check

Write a compound inequality that describes the graph. $7 < x \leq 9$

Example 6 Write a Compound Inequality for a Union

Write a compound inequality that describes the graph.

The graph shows the union of two inequalities. Because a compound inequality with the word or represents the union of two inequalities, its graph includes the graphs of both inequalities.

The leftmost endpoint is shown with a dot at 8, so 8 is included in the solution. Points to the left of the endpoint are shaded, so the graph represents solutions of $x \leq 8$.

The rightmost endpoint is shown with a circle at 12, so 12 is not included in the solution. Points to the right of the endpoint are shaded, so the graph represents solutions of $x > 12$.

The solutions represented on the graph represent the union of the solutions of $x \leq 8$ and $x > 12$, so the compound inequality $x \leq 8$ or $x > 12$ describes the graph.

Check

Write a compound inequality that describes the graph. $x \leq -1$ or $x \geq 1$

Go Online to practice what you've learned about solving linear inequalities in the Put It All Together over Lessons 6-1 through 6-3.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Write a Compound Inequality for a Union

Write a compound inequality that describes the graph.

Example 6

SELECT

Students select the correct words, phrases, or symbols to complete the sentence.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 6 Write a Compound Inequality for a Union

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the compound inequality and its graph used in this example.

Questions for Mathematical Discourse

AL What is a value that is not a solution to the inequality? Sample answer: 10

OL Is 8 a solution to the graphed compound inequality? Yes What does this tell you about the inequality? The inequality will include the $\leq$ symbol.

BL Based on the graph, how can you tell whether the inequality is a union or intersection? Because the graph represents all the solutions of two distinct inequalities, the compound inequality represents a union.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–43
3	exercises that emphasize higher-order and critical-thinking skills	44–54

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 1–43 odd, 44–54
- Extension: Precision of Measurement
- **ALEKS**® Writing and Graphing Inequalities; Linear Inequalities and Applications

IF students score 66%–89% on the Checks, **THEN** assign: OL

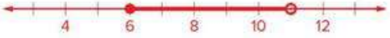
- Practice, Exercises 1–53 odd
- Remediation, Review Resources: Represent Integers
- Personal Tutors
- Extra Examples 1–6
- **ALEKS**® Plotting and Comparing Signed Numbers

IF students score 65% or less on the Checks, **THEN** assign: AL

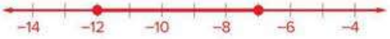
- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Represent Integers
- *Quick Review Math Handbook*: Solving Compound Inequalities
- Arrive**MATH** Take Another Look
- **ALEKS**® Plotting and Comparing Signed Numbers

Answers

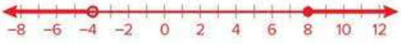
1. $\{f \mid 6 \le f < 11\}$



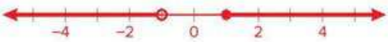
2. $\{n \mid -12 \le n \le -7\}$

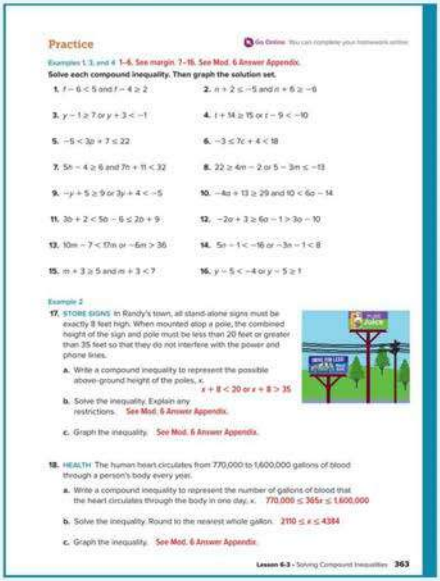


3. $\{y \mid y \ge 8 \text{ or } y < -4\}$



4. $\{t \mid t \ge 1 \text{ or } t < -1\}$





Practice

Go Online You can complete your homework online.

Examples 1, 3, and 4 1–6. See margin. 7–16. See Mod. 6 Answer Appendix.

Solve each compound inequality. Then graph the solution set.

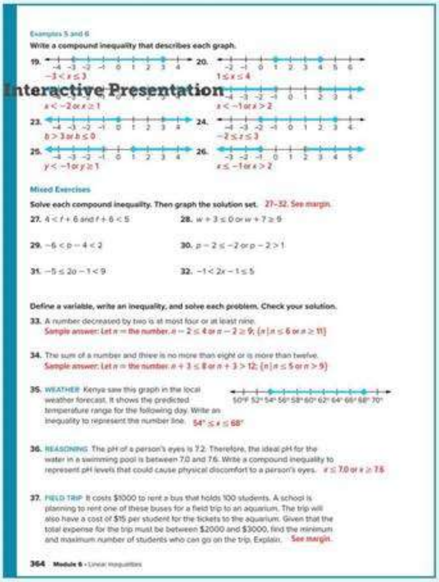
1. $f - 6 < 5$ and $f - 4 \ge 2$
2. $n + 2 \le -5$ and $n + 6 \ge -6$
3. $y - 1 \ge 7$ or $y + 3 < -1$
4. $t + 14 \ge 15$ or $t - 9 < -10$
5. $-5 < 3p + 7 \le 22$
6. $-3 \le 7c + 4 < 18$
7. $5h - 4 \ge 6$ and $7h + 11 < 32$
8. $22 \ge 4m - 2$ or $5 - 3m \le -13$
9. $-y + 5 \ge 9$ or $3y + 4 < -5$
10. $-4a + 13 \ge 29$ and $10 < 6a - 14$
11. $3b + 2 < 5b - 6 \le 2b + 9$
12. $-2a + 3 \ge 6a - 1 > 3a - 10$
13. $10m - 7 < 17m$ or $-6m > 36$
14. $5n - 1 < -16$ or $-3n - 1 < 8$
15. $m + 3 \ge 5$ and $m + 3 < 7$
16. $y - 5 < -4$ or $y - 5 \ge 1$

Example 2

17. STORE SIGNS In Randy's town, all stand-alone signs must be exactly 8 feet high. When mounted atop a pole, the combined height of the sign and pole must be less than 20 feet or greater than 35 feet so that they do not interfere with the power and phone lines.
    a. Write a compound inequality to represent the possible above-ground height of the poles, x. $x + 8 < 20$ or $x + 8 > 35$
    b. Solve the inequality. Explain any restrictions. See Mod. 6 Answer Appendix.
    c. Graph the inequality. See Mod. 6 Answer Appendix.

18. HEALTH The human heart circulates from 770,000 to 1,600,000 gallons of blood through a person's body every year.
    a. Write a compound inequality to represent the number of gallons of blood that the heart circulates through the body in one day, x. $770{,}000 \le 365x \le 1{,}600{,}000$
    b. Solve the inequality. Round to the nearest whole gallon. $2110 \le x \le 4384$
    c. Graph the inequality. See Mod. 6 Answer Appendix.

Lesson 6-3 • Solving Compound Inequalities 363



Examples 5 and 6

Write a compound inequality that describes each graph.

19. $-3 < x \le 3$
20. $1 \le x \le 4$

Interactive Presentation

21. $x < -2$ or $x \ge 1$
22. $x < -1$ or $x > 2$
23. $b > 3$ or $b \le 0$
24. $-2 \le t \le 3$
25. $y < -1$ or $y \ge 1$
26. $x \le -1$ or $x > 2$

Mixed Exercises

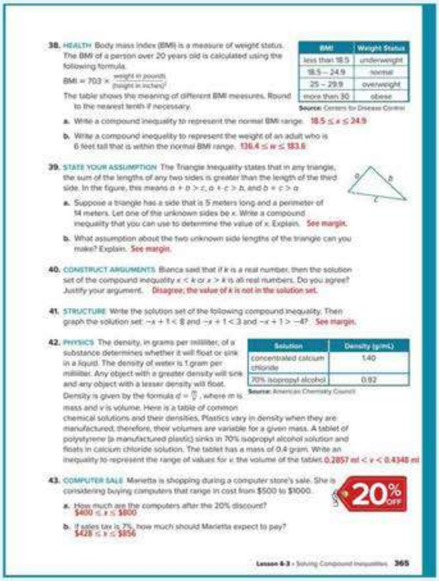
Solve each compound inequality. Then graph the solution set. 27–32. See margin.

27. $4 < f + 6$ and $f + 6 < 5$
28. $w + 3 \le 0$ or $w + 7 \ge 9$
29. $-6 < b - 4 < 2$
30. $p - 2 \le -2$ or $p - 2 > 1$
31. $-5 \le 2a - 1 < 9$
32. $-1 < 2x - 1 \le 5$

Define a variable, write an inequality, and solve each problem. Check your solution.

33. A number decreased by two is at most four or at least nine. Sample answer: Let n = the number. $n - 2 \le 4$ or $n - 2 \ge 9$; $\{n \mid n \le 6 \text{ or } n \ge 11\}$
34. The sum of a number and three is no more than eight or is more than twelve. Sample answer: Let n = the number. $n + 3 \le 8$ or $n + 3 > 12$; $\{n \mid n \le 5 \text{ or } n > 9\}$
35. WEATHER Kenya saw this graph in the local weather forecast. It shows the predicted temperature range for the following day. Write an inequality to represent the number line. $54° \le x \le 68°$
36. REASONING The pH of a person's eyes is 7.2. Therefore, the ideal pH for the water in a swimming pool is between 7.0 and 7.6. Write a compound inequality to represent pH levels that could cause physical discomfort to a person's eyes. $x \le 7.0$ or $x \ge 7.6$
37. FIELD TRIP It costs \$1000 to rent a bus that holds 100 students. A school is planning to rent one of these buses for a field trip to an aquarium. The trip will also have a cost of \$15 per student for the tickets to the aquarium. Given that the total expense for the trip must be between \$2000 and \$3000, find the minimum and maximum number of students who can go on the trip. Explain. See margin.

364 Module 6 • Linear Inequalities



38. HEALTH Body mass index (BMI) is a measure of weight status. The BMI of a person over 20 years old is calculated using the following formula.

$\text{BMI} = 703 \times \frac{\text{weight in pounds}}{(\text{height in inches})^2}$

The table shows the meaning of different BMI measures. Round to the nearest tenth if necessary.

BMI	Weight Status
less than 18.5	underweight
18.5 – 24.9	normal
25 – 29.9	overweight
more than 30	obese

Source: Centers for Disease Control

a. Write a compound inequality to represent the normal BMI range. $18.5 \le x \le 24.9$

b. Write a compound inequality to represent the weight of an adult who is 6 feet tall that is within the normal BMI range. $136.4 \le w \le 183.6$

39. STATE YOUR ASSUMPTION The Triangle Inequality states that in any triangle, the sum of the lengths of any two sides is greater than the length of the third side. In the figure, this means $a + b > c$, $a + c > b$, and $b + c > a$.

a. Suppose a triangle has a side that is 5 meters long and a perimeter of 14 meters. Let one of the unknown sides be x. Write a compound inequality that you can use to determine the value of x. Explain. See margin.

b. What assumption about the two unknown side lengths of the triangle can you make? Explain. See margin.

40. CONSTRUCT ARGUMENTS Bianca said that if k is a real number, then the solution set of the compound inequality $x < k$ or $x > k$ is all real numbers. Do you agree? Justify your argument. Disagree; the value of k is not in the solution set.

41. STRUCTURE Write the solution set of the following compound inequality. Then graph the solution set: $-x + 1 < 8$ and $-x + 1 < 3$ and $-x + 1 > -4$? See margin.

42. PHYSICS The density, in grams per milliliter, of a substance determines whether it will float or sink in a liquid. The density of water is 1 gram per milliliter. Any object with a greater density will sink and any object with a lesser density will float. Density is given by the formula $d = \frac{m}{v}$, where m is mass and v is volume. Here is a table of common chemical solutions and their densities. Plastics vary in density when they are manufactured; therefore, their volumes are variable for a given mass. A tablet of polystyrene (a manufactured plastic) sinks in 70% isopropyl alcohol solution and floats in calcium chloride solution. The tablet has a mass of 0.4 gram. Write an inequality to represent the range of values for v, the volume of the tablet. $0.2857 \text{ ml} < v < 0.4348 \text{ ml}$

Solution	Density (g/mL)
concentrated calcium chloride	1.40
70% isopropyl alcohol	0.92

Source: American Chemistry Council

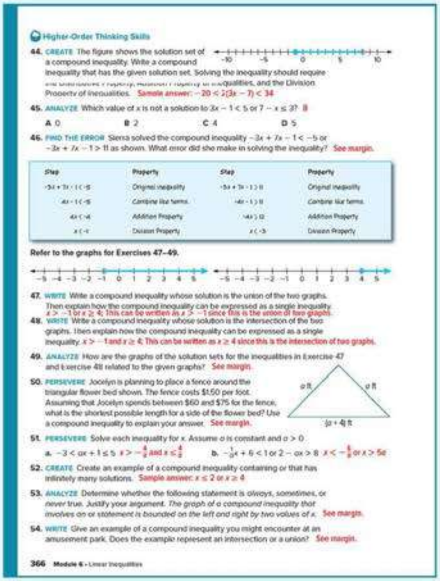
43. COMPUTER SALE Marietta is shopping during a computer store's sale. She is considering buying computers that range in cost from \$500 to \$1000.

20% OFF

a. How much are the computers after the 20% discount? $\$400 \le x \le \$800$

b. If sales tax is 7%, how much should Marietta expect to pay? $\$428 \le x \le \$856$

Lesson 6-3 • Solving Compound Inequalities 365



Higher-Order Thinking Skills

44. CREATE The figure shows the solution set of a compound inequality. Write a compound inequality that has the given solution set. Solving the inequality should require the Distributive Property, Addition Property of Inequalities, and the Division Property of Inequalities. Sample answer: $-20 < 2(3x - 7) < 34$

45. ANALYZE Which value of x is not a solution to $3x - 1 < 5$ or $7 - x \le 3$? B

A 0 B 2 C 4 D 5

46. FIND THE ERROR Sierra solved the compound inequality $-3x + 7x - 1 < -5$ or $-3x + 7x - 1 > 11$ as shown. What error did she make in solving the inequality? See margin.

Step	Property	Step	Property
$-3x + 7x - 1 < -5$	Original inequality	$-3x + 7x - 1 > 11$	Original inequality
$4x - 1 < -5$	Combine like terms.	$-4x - 1 > 11$	Combine like terms.
$4x < -4$	Addition Property	$-4x > 12$	Addition Property
$x < -1$	Division Property	$x < -3$	Division Property

Refer to the graphs for Exercises 47–49.

47. WRITE Write a compound inequality whose solution is the union of the two graphs. Then explain how the compound inequality can be expressed as a single inequality. $x > -1$ or $x \ge 4$; This can be written as $x > -1$ since this is the union of two graphs.

48. WRITE Write a compound inequality whose solution is the intersection of the two graphs. Then explain how the compound inequality can be expressed as a single inequality. $x > -1$ and $x \ge 4$; This can be written as $x \ge 4$ since this is the intersection of two graphs.

49. ANALYZE How are the graphs of the solution sets for the inequalities in Exercise 47 and Exercise 48 related to the given graphs? See margin.

50. PERSEVERE Jocelyn is planning to place a fence around the triangular flower bed shown. The fence costs \$1.50 per foot. Assuming that Jocelyn spends between \$60 and \$75 for the fence, what is the shortest possible length for a side of the flower bed? Use a compound inequality to explain your answer. See margin.

51. PERSEVERE Solve each inequality for x. Assume a is constant and $a > 0$.

a. $-3 < ax + 1 \le 5$ $x > -\frac{4}{a}$ and $x \le \frac{4}{a}$

b. $-\frac{1}{a}x + 6 < 1$ or $2 - ax > 8$ $x < -\frac{6}{a}$ or $x > 5a$

52. CREATE Create an example of a compound inequality containing or that has infinitely many solutions. Sample answer: $x \le 2$ or $x \ge 4$

53. ANALYZE Determine whether the following statement is always, sometimes, or never true. Justify your argument. *The graph of a compound inequality that involves an or statement is bounded on the left and right by two values of x.* See margin.

54. WRITE Give an example of a compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union? See margin.

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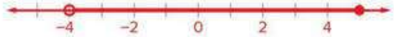
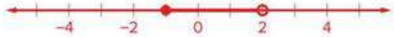
Answers

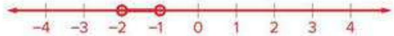
5. $\{p \mid -4 < p \le 5\}$



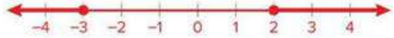
6. $\{c \mid -1 \le c < 2\}$



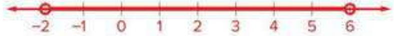
27. $\{f \mid -2 < f < -1\}$



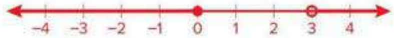
28. $\{w \mid w \le -3 \text{ or } w \ge 2\}$



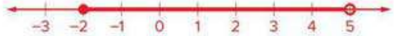
29. $\{b \mid -2 < b < 6\}$



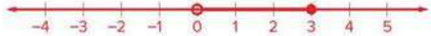
30. $\{p \mid p \le 0 \text{ or } p > 3\}$



31. $\{a \mid -2 \le a < 5\}$



32. $\{x \mid 0 < x \le 3\}$

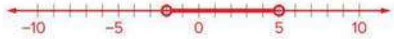


37. The minimum is 67, since the solution of the inequality $2000 \le 1000 + 15x$ is $66\frac{2}{3} \le x$, and the number of students must be a whole number. The maximum is 100, since the solution of the inequality $1000 + 15x \le 3000$ is $x \le 133\frac{1}{3}$, but the bus can only hold 100 students.

39a. The side lengths must be 5, x, and $9 - x$. Using the Triangle Inequality results in the compound inequality $x + 5 > 9 - x$ and $14 - x > x$.

39b. The solution of the compound inequality is $2 < x < 7$, so each of the lengths must be greater than 2 m but less than 7 m. The sum of the two lengths must be equal to 9 m.

41. $\{x \mid -2 < x < 5\}$



46. When solving $-3x + 7x - 1 > 11$, Sierra incorrectly added $-3x$ and $7x$, which caused her to divide by a negative coefficient and incorrectly switch the inequality symbol.

49. The union of the two graphs is the graph on the left, so the graph on the left is the graph of the solution set for Exercise 47. The intersection of the two graphs is the graph on the right, so the graph on the right is the graph of the solution set for Exercise 48.

50. 12 feet; The cost of the fence is represented by the compound inequality $60 \le 1.5(a + a + a + 4) \le 75$ or $60 \le 1.5(3a + 4) \le 75$. The solution is $12 \le a \le 15\frac{1}{3}$. The flower bed with the shortest possible sides occurs when $a = 12$, and the lengths in this case are 12 ft, 12 ft, and 16 ft.

53. Sometimes; The graph of $x > 2$ or $x < 5$ includes the entire number line.

54. Sample answer: The speed at which a roller coaster runs while on the track could represent a compound inequality that is an intersection.

Lesson 6-4

Solving Absolute Value Inequalities

A.CED.1, A.CED.3

LESSON GOAL

Students solve absolute value inequalities.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Solving Absolute Value Inequalities

Develop:

Solving Inequalities Involving < and Absolute Value
- Solve Absolute Value Inequalities (<)
- Absolute Value Inequalities (<) with No Solutions
- Use Absolute Value Inequalities

Solving Inequalities Involving > and Absolute Value
- Solve Absolute Value Inequalities (>)
- Absolute Value Inequalities (>) with Overlapping Case Solutions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Solving Equations Involving Absolute Value	●	●		●
Extension: Using Graphs to Solve Absolute Value Inequalities		●	●	●

Language Development Handbook

Assign page 36 of the *Language Development Handbook* to help your students build mathematical language related to absolute value inequalities.

ELL You can use the tips and suggestions on page T36 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

Coherence

Vertical Alignment

Previous
Students solved compound inequalities.
A.CED.1, A.CED.3

Now
Students solve absolute value inequalities.
A.CED.1, A.CED.3

Next
Students will solve and graph inequalities in two variables.
A.CED.1, A.REI.12

Rigor

The Three Pillars of Rigor

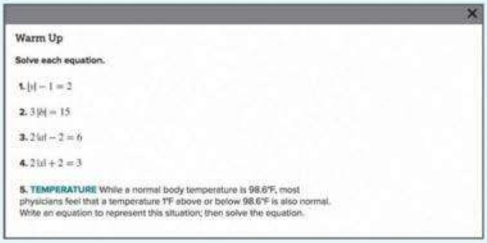
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of absolute value equations to build fluency with solving inequalities that involve absolute value. They apply their understanding of solving absolute value inequalities by solving real-world problems.

Mathematical Background

Inequalities involving absolute value can be solved by writing them as compound inequalities. These compound inequalities can then be solved using the algebraic methods previously explored.

Interactive Presentation



Warm Up

Solve each equation.

1. $|x| - 1 = 2$
2. $3|b| = 15$
3. $2|a| - 2 = 6$
4. $2|x| + 2 = 3$
5. TEMPERATURE While a normal body temperature is 98.6°F, most physicians feel that a temperature 1°F above or below 98.6°F is also normal. Write an equation to represent this situation; then solve the equation.

Warm Up

Launch the Lesson

Candy making requires precise temperature measurement. If the temperature is too hot, the candy may burn. If it is not hot enough, the candy may not set and just end up as a gooey mess.

To make chewy caramel, the temperature of the mixture must be within 3 degrees of 247°F. Harder candies like lollipops or peanut brittle require the temperature to be within 4 degrees of 300°F. These situations can be

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- solving absolute value equations

Answers:

1. 3 or −3
2. 5 or −5
3. 4 or −4
4. $\frac{1}{2}$ or $-\frac{1}{2}$
5. $|x - 98.6| = 1$; 97.6, 99.6

Launch the Lesson

Teaching the Mathematical Practices

6 Communicate Precisely In this Launch, students will learn the importance of communicating precisely when specifying the baking temperatures of candy. Baking temperatures can be modeled with absolute value inequalities.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Explore Solving Absolute Value Inequalities

Objective

Students explore how to use the process for solving absolute value equations to solve absolute value inequalities.

MP Teaching the Mathematical Practices

3 Make Conjectures In this Explore, students will make conjectures and then build a logical progression of statements to validate the conjectures. Once students have made their conjectures, guide the students to validate them.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

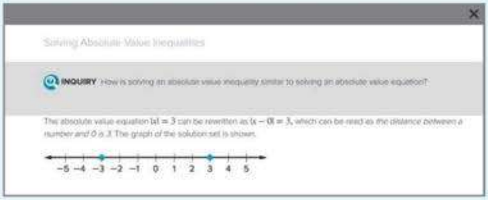
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students examine solution cases for a basic inequality involving an absolute value, relating the methods of solving them to solving absolute value equations. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

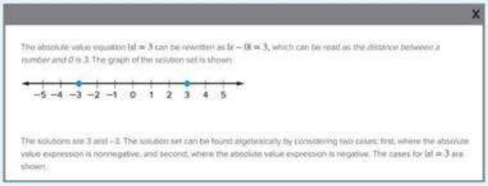


Solving Absolute Value Inequalities

INQUIRY How is solving an absolute value inequality similar to solving an absolute value equation?

The absolute value equation $|x| = 3$ can be rewritten as $|x - 0| = 3$, which can be read as *the distance between a number and 0 is 3*. The graph of the solution set is shown.

Explore



The absolute value equation $|x| = 3$ can be rewritten as $|x - 0| = 3$, which can be read as *the distance between a number and 0 is 3*. The graph of the solution set is shown.

The solutions are 3 and −3. The solution set can be found algebraically by considering two cases: first, where the absolute value expression is nonnegative, and second, where the absolute value expression is negative. The cases for $|x| = 3$ are shown.

Explore

TYPE

Students make a conjecture about how the graph of the inequality differs from the graph of the equation.

Select

Students select the correct inequality symbols and values to complete the statements.

Interactive Presentation

INQUIRY How is solving an absolute value inequality similar to solving an absolute value equation?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Solving Absolute Value Inequalities (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why do you have to solve two cases for the absolute value inequality? Sample answer: Similar to an absolute value equation, you have to take into account both distances from zero on the number line.
- How do you think the graph would change for $|x| \geq 3$? Sample answer: You are looking for distances that are greater than or equal to 3 from zero, so you would shade $x > 3$ and $x < -3$.

Inquiry

How is solving an absolute value inequality similar to solving an absolute value equation? Sample answer: To solve both an absolute value inequality and an absolute value equation, you must consider the case where *x* is nonnegative and the case where *x* is negative.

Learn Solving Inequalities Involving < and Absolute Value

Objective

Students solve absolute value inequalities involving a less than symbol by applying properties of inequalities.

MP Teaching the Mathematical Practices

3 Analyze Cases This Learn guides students to examine the cases arising when solving an absolute value inequality. Encourage students to familiarize themselves with both cases.

Important to Know

Discuss with students that absolute values should be thought of as a distance. Relate the symbolic representation of the absolute value with the graphical representation on the number line, and encourage students to use this representation as a way to confirm their solutions to absolute value inequalities as they move forward.

DIFFERENTIATE

Reteaching Activity AL

IF students do not understand why $|m + 5| < 3$ is rewritten as $m + 5 < 3$ and $m + 5 > -3$,
THEN have them rewrite the second inequality as $-(m + 5) < 3$ and multiply each side by -1 to yield $m + 5 > -3$. This method makes the switch of the direction of the inequality more obvious, as students must make the switch when they multiply each side by -1.

Example 1 Solve Absolute Value Inequalities (<)

MP Teaching the Mathematical Practices

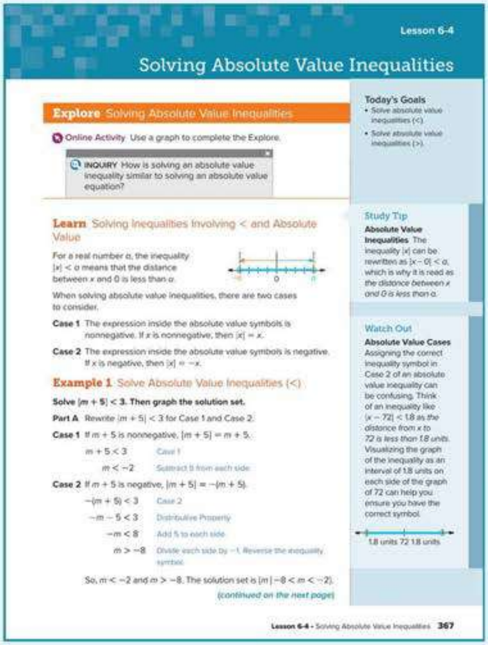
1 Understand the Approaches of Others Work with students to look at the Alternate Method. Ask students to compare and contrast the original method and the alternate method.

Questions for Mathematical Discourse

AL In Example 1, what are the two possible cases for $m + 5$? $m + 5$ is positive and $m + 5$ is negative.

OL Why are there two cases to consider in an absolute value inequality? Sample answer: Absolute value means distance away from zero on a number line. There is the distance in the positive direction from zero and the distance in the negative direction from zero.

BL For what values of b would $|m + 5| < b$ have no solutions? Explain. Sample answer: All values of b such that $b < 0$ would have no solutions, because the absolute value quantity must be nonnegative.



Lesson 6-4

Solving Absolute Value Inequalities

Explore Solving Absolute Value Inequalities

Online Activity Use a graph to complete the Explore.

INQUIRY How is solving an absolute value inequality similar to solving an absolute value equation?

Learn Solving Inequalities Involving < and Absolute Value

For a real number a, the inequality $|x| < a$ means that the distance between x and 0 is less than a.

When solving absolute value inequalities, there are two cases to consider.

Case 1 The expression inside the absolute value symbols is nonnegative. If x is nonnegative, then $|x| = x$.

Case 2 The expression inside the absolute value symbols is negative. If x is negative, then $|x| = -x$.

Example 1 Solve Absolute Value Inequalities (<)

Solve $|m + 5| < 3$. Then graph the solution set.

Part A Rewrite $|m + 5| < 3$ for Case 1 and Case 2.

Case 1 If $m + 5$ is nonnegative, $|m + 5| = m + 5$.

$m + 5 < 3$ Case 1

$m < -2$ Subtract 5 from each side.

Case 2 If $m + 5$ is negative, $|m + 5| = -(m + 5)$.

$-(m + 5) < 3$ Case 2

$-m - 5 < 3$ Distributive Property

$-m < 8$ Add 5 to each side.

$m > -8$ Divide each side by −1. Reverse the inequality symbol.

So, $m < -2$ and $m > -8$. The solution set is $\{m \mid -8 < m < -2\}$.

(continued on the next page)

Today's Goals
- Solve absolute value inequalities (<).
- Solve absolute value inequalities (>).

Study Tip

Absolute Value Inequalities The inequality $|x|$ can be rewritten as $|x - 0| < a$, which is why it is read as *the distance between x and 0 is less than a.*

Watch Out

Absolute Value Cases Assigning the correct inequality symbol in Case 2 of an absolute value inequality can be confusing. Think of an inequality like $|x - 72| < 1.8$ as *the distance from x to 72 is less than 1.8 units.* Visualizing the graph of the inequality as an interval of 1.8 units on each side of the graph of 72 can help you ensure you have the correct symbol.

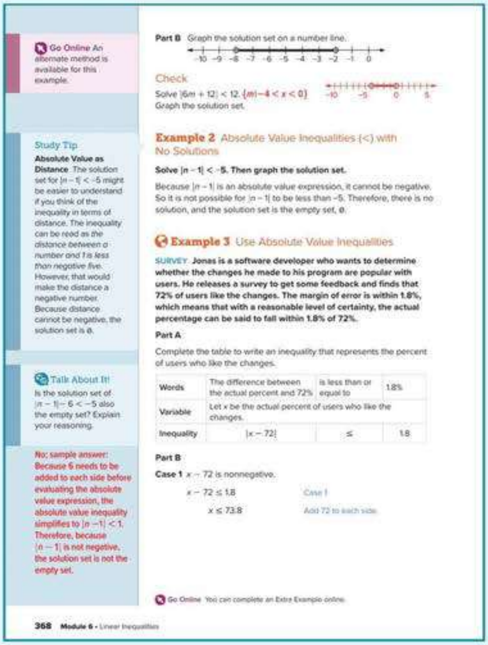
Lesson 6-4 • Solving Absolute Value Inequalities 367

Interactive Presentation

Learn

TAP

Students tap to see a Watch Out! feature about visualizing the graph of the inequality to help determine which inequality symbol to use.



Go Online An alternate method is available for this example.

Study Tip

Absolute Value as Distance The solution set for $|n - 1| < -5$ might be easier to understand if you think of the inequality in terms of distance. The inequality can be read as *the distance between a number and 1 is less than negative five.* However, that would make the distance a negative number. Because distance cannot be negative, the solution set is ∅.

Talk About It!

Is the solution set of $|n - 1| - 6 < -5$ also the empty set? Explain your reasoning.

No; sample answer: Because 6 needs to be added to each side before evaluating the absolute value expression, the absolute value inequality simplifies to $|n - 1| < 1$. Therefore, because $|n - 1|$ is not negetive, the solution set is not the empty set.

Part B Graph the solution set on a number line.

Check

Solve $|6m + 12| < 12$. $\{m \mid -4 < x < 0\}$
Graph the solution set.

Example 2 Absolute Value Inequalities (<) with No Solutions

Solve $|n - 1| < -5$. Then graph the solution set.

Because $|n - 1|$ is an absolute value expression, it cannot be negative. So it is not possible for $|n - 1|$ to be less than −5. Therefore, there is no solution, and the solution set is the empty set, ∅.

Example 3 Use Absolute Value Inequalities

SURVEY **Jonas is a software developer who wants to determine whether the changes he made to his program are popular with users. He releases a survey to get some feedback and finds that 72% of users like the changes. The margin of error is within 1.8%, which means that with a reasonable level of certainty, the actual percentage can be said to fall within 1.8% of 72%.**

Part A

Complete the table to write an inequality that represents the percent of users who like the changes.

Words	The difference between the actual percent and 72%	is less than or equal to	1.8%
Variable	Let x be the actual percent of users who like the changes.		
Inequality	$\lvert x - 72\rvert$	≤	1.8

Part B

Case 1 $x - 72$ is nonnegative.

$x - 72 \leq 1.8$ Case 1
$x \leq 73.8$ Add 72 to each side.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Solve $|n - 1| < -5$. Then graph the solution set.

Example 2

TYPE

Students answer a question to show they understand whether a given absolute value inequality has an empty solution set.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Absolute Value Inequalities (<) with No Solutions

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL The expression $|n - 1|$ represents the distance of n from which number? 1

OL What is the name of the solution set for an inequality that has no solutions? the empty set, or null set

BL How could you change the inequality so it has a solution of all real numbers? Sample answer: Reverse the inequality to $|n - 1| > -5$. Because the absolute value is always nonnegative for all values of n, all values of n would be solutions to the inequality.

Example 3 Use Absolute Value Inequalities

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about absolute value inequalities to solving a real-world problem.

Questions for Mathematical Discourse

AL What is meant by the phrase *margin of error*? Sample answer: The actual percentage can be within a range of 1.8 percentage points higher or lower than 72%.

OL What are the two possible cases? $x - 72 \leq 1.8$ and $-(x - 72) \leq 1.8$

BL How do you know this is an absolute value problem? Sample answer: I am finding distance away from the number in both directions.

Common Error

Students may be more comfortable using decimal notation to represent percent values, rather than leaving them in percent format. If a student brings up that 72% should be written as 0.72, for example, review the ways of writing a percent. Have two groups work out the solution method using different notations to verify that the results are consistent for both notations.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Solving Inequalities Involving > and Absolute Value

Objective

Students solve absolute value inequalities involving a greater than symbol by applying properties of inequalities.

MP Teaching the Mathematical Practices

3 Analyze Cases This Learn guides students to examine the cases arising when solving an absolute value inequality. Encourage students to familiarize themselves with both cases.

Important to Know

The inequality $|x| > a$ means the distance between x and 0 is greater than a. As with $<$ absolute value inequalities, there are two cases to consider.

DIFFERENTIATE

Enrichment Activity BL

Draw a number line on the board. Have a student use your number line to create the graph of an absolute value inequality. Ask the rest of the class to write the inequality that the graph models.

Example 4 Solve Absolute Value Inequalities (>)

MP Teaching the Mathematical Practices

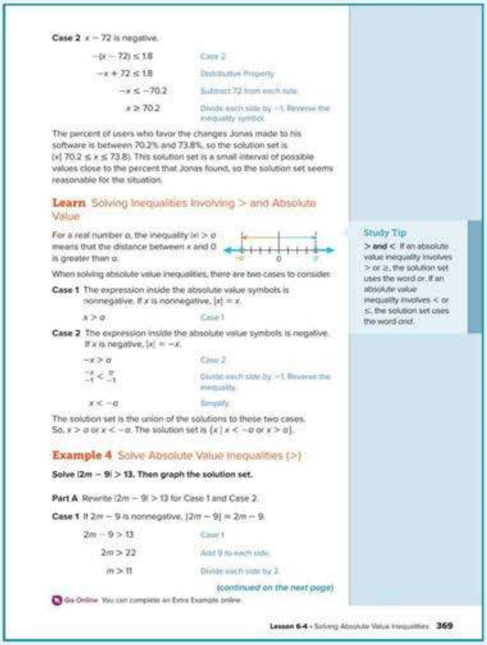
1 Explain Correspondences Encourage students to explain the relationships between the absolute value inequality and its graph used in this example.

Questions for Mathematical Discourse

AL What is the first step in solving this problem? Sample answer: Rewrite the inequality as two cases. One case is nonnegative and the other is negative.

OL Describe how to graph the solution set. Plot an open circle for the endpoint −2 and shade to the left. Then plot an open circle for the endpoint 11 and shade to the right.

BL Does this represent an intersection or a union? Explain. The absolute value inequality represents a union, because the solution set represents all the solutions of two distinct non-overlapping solution sets.



Case 2 $x - 72$ is negative.

$-(x - 72) \leq 1.8$ Case 2

$-x + 72 \leq 1.8$ Distributive Property

$-x \leq -70.2$ Subtract 72 from each side.

$x \geq 70.2$ Divide each side by −1. Reverse the inequality symbol.

The percent of users who favor the changes Jonas made to his software is between 70.2% and 73.8%, so the solution set is $\{x \mid 70.2 \leq x \leq 73.8\}$. This solution set is a small interval of possible values close to the percent that Jonas found, so the solution set seems reasonable for the situation.

Learn Solving Inequalities Involving > and Absolute Value

For a real number a, the inequality $|x| > a$ means that the distance between x and 0 is greater than a.

When solving absolute value inequalities, there are two cases to consider.

Case 1 The expression inside the absolute value symbols is nonnegative. If x is nonnegative, $|x| = x$.

$x > a$ Case 1

Case 2 The expression inside the absolute value symbols is negative. If x is negative, $|x| = -x$.

$-x > a$ Case 2

$\frac{-x}{-1} < \frac{a}{-1}$ Divide each side by −1. Reverse the inequality.

$x < -a$ Simplify.

The solution set is the union of the solutions to these two cases. So, $x > a$ or $x < -a$. The solution set is $\{x \mid x < -a \text{ or } x > a\}$.

Study Tip

> and < If an absolute value inequality involves > or ≥, the solution set uses the word *or*. If an absolute value inequality involves < or ≤, the solution set uses the word *and*.

Example 4 Solve Absolute Value Inequalities (>)

Solve $|2m - 9| > 13$. Then graph the solution set.

Part A Rewrite $|2m - 9| > 13$ for Case 1 and Case 2.

Case 1 If $2m - 9$ is nonnegative, $|2m - 9| = 2m - 9$.

$2m - 9 > 13$ Case 1

$2m > 22$ Add 9 to each side.

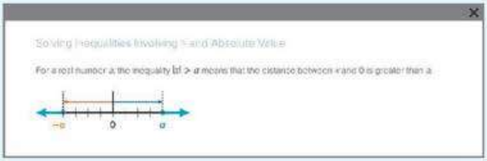
$m > 11$ Divide each side by 2.

(continued on the next page)

Go Online You can complete an Extra Example online.

Lesson 6-4 • Solving Absolute Value Inequalities 369

Interactive Presentation



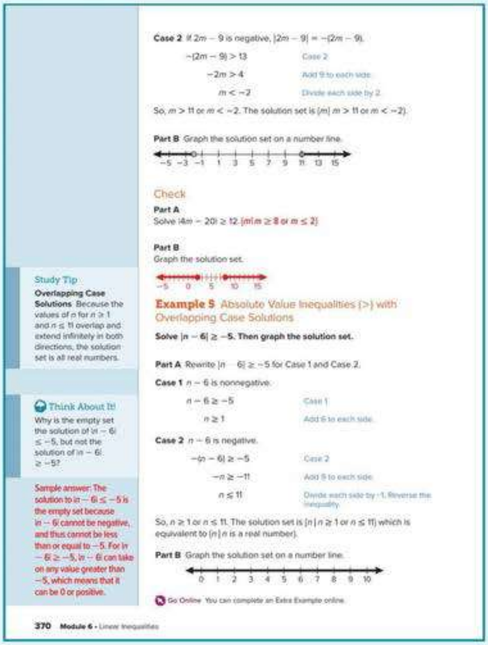
Solving Inequalities Involving > and Absolute Value

For a real number a, the inequality $|x| > a$ means that the distance between x and 0 is greater than a.

Learn

TAP

Students tap to see a Study Tip about the correspondence between inequality symbols with intersections and unions.



Case 2 If $2m - 9$ is negative, $|2m - 9| = -(2m - 9)$.

$-(2m - 9) > 13$ Case 2

$-2m > 4$ Add 9 to each side.

$m < -2$ Divide each side by 2.

So, $m > 11$ or $m < -2$. The solution set is $\{m \mid m > 11 \text{ or } m < -2\}$.

Part B Graph the solution set on a number line.

Check

Part A

Solve $|4m - 20| \geq 12$. $\{m \mid m \geq 8 \text{ or } m \leq 2\}$

Part B

Graph the solution set.

Study Tip

Overlapping Case Solutions Because the values of n for $n \geq 1$ and $n \leq 11$ overlap and extend infinitely in both directions, the solution set is all real numbers.

Example 5 Absolute Value Inequalities (>) with Overlapping Case Solutions

Solve $|n - 6| \geq -5$. Then graph the solution set.

Part A Rewrite $|n - 6| \geq -5$ for Case 1 and Case 2.

Case 1 $n - 6$ is nonnegative.

$n - 6 \geq -5$ Case 1

$n \geq 1$ Add 6 to each side.

Case 2 $n - 6$ is negative.

$-(n - 6) \geq -5$ Case 2

$-n \geq -11$ Add 6 to each side.

$n \leq 11$ Divide each side by -1. Reverse the inequality.

So, $n \geq 1$ or $n \leq 11$. The solution set is $\{n \mid n \geq 1 \text{ or } n \leq 11\}$ which is equivalent to $\{n \mid n \text{ is a real number}\}$.

Part B Graph the solution set on a number line.

Think About It!

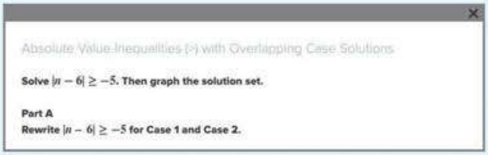
Why is the empty set the solution of $|n - 6| \leq -5$, but not the solution of $|n - 6| \geq -5$?

Sample answer: The solution to $|n - 6| \leq -5$ is the empty set because $|n - 6|$ cannot be negative, and thus cannot be less than or equal to -5. For $|n - 6| \geq -5$, $|n - 6|$ can take on any value greater than -5, which means that it can be 0 or positive.

Go Online You can complete an Extra Example online.

370 Module 6 • Linear Inequalities

Interactive Presentation



Absolute Value Inequalities (>) with Overlapping Case Solutions

Solve $|n - 6| \geq -5$. Then graph the solution set.

Part A

Rewrite $|n - 6| \geq -5$ for Case 1 and Case 2.

Example 5

TAP

Students move through the two cases to solve absolute value inequalities.

TYPE

Students answer a question to show they understand why one inequality has an empty solution set while a different one does not.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 5 Absolute Value Inequalities (>) with Overlapping Case Solutions

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL When you graph each piece of the solution, what is the significance of the overlapping section? Sample answer: It is the solution of the inequality.

OL What is always true about a distance? Sample answer: A distance cannot be a negative number.

BL What does it mean if the solution set for an inequality is all real numbers? Sample answer: Any value on the number line would be a solution to the inequality.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–25
2	exercises that use a variety of skills from this lesson	26–50
3	exercises that emphasize higher-order and critical-thinking skills	51–60

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–49 odd, 51–60
- Extension: Using Graphs to Solve Absolute Value Inequalities
- **ALEKS**® Absolute Value Inequalities

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–59 odd
- Remediation, Review Resources: Solving Equations Involving Absolute Value
- Personal Tutors
- Extra Examples 1–5
- **ALEKS**® Solving Absolute Value Equations

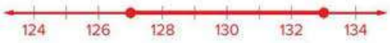
AL

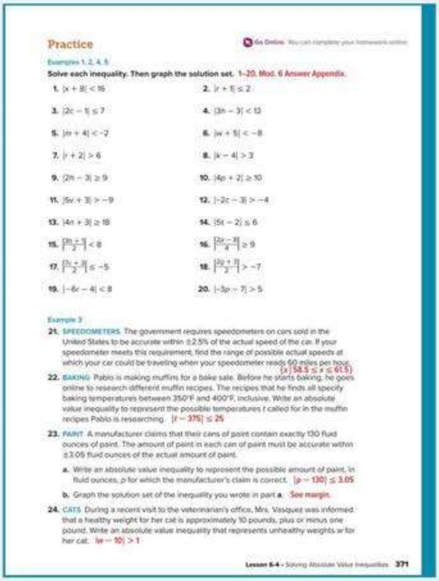
IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Solving Equations Involving Absolute Value
- *Quick Review Math Handbook*: Inequalities Involving Absolute Value
- Arrive**MATH** Take Another Look
- **ALEKS**® Solving Absolute Value Equations

Answer

23b.





Practice

Go Online You can complete your homework online.

Examples 1, 2, 4, 5

Solve each inequality. Then graph the solution set. 1–20. Mod. 6 Answer Appendix.

1. $|x+8|<16$
2. $|r+1|\le 2$
3. $|2c-1|\le 7$
4. $|3h-3|<12$
5. $|m+4|<-2$
6. $|w+5|<-8$
7. $|r+2|>6$
8. $|k-4|>3$
9. $|2h-3|\ge 9$
10. $|4p+2|\ge 10$
11. $|5v+3|>-9$
12. $|-2c-3|>-4$
13. $|4n+3|\ge 18$
14. $|5t-2|\le 6$
15. $\left|\frac{3n+1}{2}\right|<8$
16. $\left|\frac{2v-3}{4}\right|\ge 9$
17. $\left|\frac{2c+3}{2}\right|\le -5$
18. $\left|\frac{2y+3}{2}\right|>-7$
19. $|-6r-4|<8$
20. $|-3p-7|>5$

Example 3

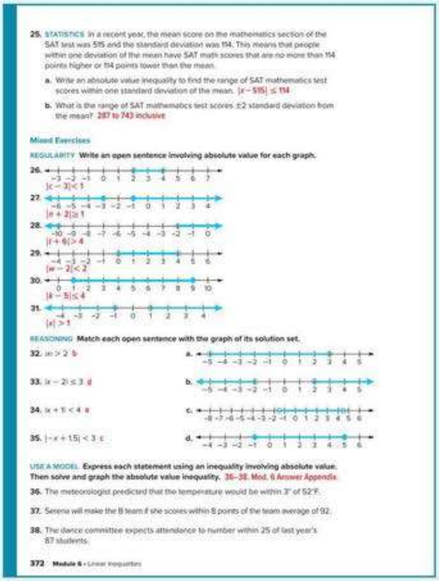
21. SPEEDOMETERS The government requires speedometers on cars sold in the United States to be accurate within ±2.5% of the actual speed of the car. If your speedometer meets this requirement, find the range of possible actual speeds at which your car could be traveling when your speedometer reads 60 miles per hour. $\{x \mid 58.5 \le x \le 61.5\}$

22. BAKING Pablo is making muffins for a bake sale. Before he starts baking, he goes online to research different muffin recipes. The recipes that he finds all specify baking temperatures between 350°F and 400°F, inclusive. Write an absolute value inequality to represent the possible temperatures t called for in the muffin recipes Pablo is researching. $|t-375|\le 25$

23. PAINT A manufacturer claims that their cans of paint contain exactly 130 fluid ounces of paint. The amount of paint in each can of paint must be accurate within ±3.05 fluid ounces of the actual amount of paint.
   a. Write an absolute value inequality to represent the possible amount of paint, in fluid ounces, p for which the manufacturer's claim is correct. $|p-130|\le 3.05$
   b. Graph the solution set of the inequality you wrote in part **a**. See margin.

24. CATS During a recent visit to the veterinarian's office, Mrs. Vasquez was informed that a healthy weight for her cat is approximately 10 pounds, plus or minus one pound. Write an absolute value inequality that represents unhealthy weights w for her cat. $|w-10|>1$

Lesson 6-4 • Solving Absolute Value Inequalities 371



25. STATISTICS In a recent year, the mean score on the mathematics section of the SAT test was 515 and the standard deviation was 114. This means that people within one deviation of the mean have SAT math scores that are no more than 114 points higher or 114 points lower than the mean.
   a. Write an absolute value inequality to find the range of SAT mathematics test scores within one standard deviation of the mean. $|x-515|\le 114$
   b. What is the range of SAT mathematics test scores ±2 standard deviation from the mean? 287 to 743 inclusive

Mixed Exercises

REGULARITY **Write an open sentence involving absolute value for each graph.**

26. $|c-3|<1$
27. $|n+2|\ge 1$
28. $|r+6|>4$
29. $|w-2|<2$
30. $|k-5|\le 4$
31. $|x|>1$

REASONING **Match each open sentence with the graph of its solution set.**

32. $|n|>2$ b
33. $|x-2|\le 3$ d
34. $|x+1|<4$ a
35. $|-x+1.5|<3$ c

USE A MODEL **Express each statement using an inequality involving absolute value. Then solve and graph the absolute value inequality.** 36–38. Mod. 6 Answer Appendix.

36. The meteorologist predicted that the temperature would be within 3° of 52°F.
37. Serena will make the B team if she scores within 8 points of the team average of 92.
38. The dance committee expects attendance to number within 25 of last year's 87 students.

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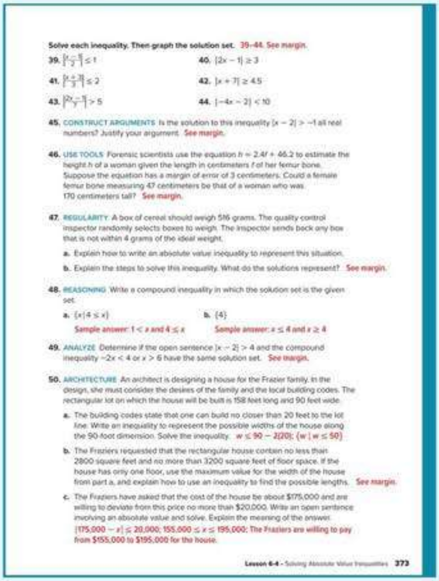


Solve each inequality. Then graph the solution set. 39–44. See margin.

39. $\left|\frac{x-1}{2}\right| \leq 1$

40. $|2x - 1| \geq 3$

41. $\left|\frac{x+3}{3}\right| \leq 2$

42. $|x + 7| \geq 4.5$

43. $\left|\frac{2x-1}{7}\right| > 5$

44. $|-4x - 2| < 10$

45. CONSTRUCT ARGUMENTS Is the solution to this inequality $|x - 2| > -1$ all real numbers? Justify your argument. See margin.

46. USE TOOLS Forensic scientists use the equation $h = 2.4f + 46.2$ to estimate the height h of a woman given the length in centimeters f of her femur bone. Suppose the equation has a margin of error of 3 centimeters. Could a female femur bone measuring 47 centimeters be that of a woman who was 170 centimeters tall? See margin.

47. REGULARITY A box of cereal should weigh 516 grams. The quality control inspector randomly selects boxes to weigh. The inspector sends back any box that is not within 4 grams of the ideal weight.

a. Explain how to write an absolute value inequality to represent this situation.

b. Explain the steps to solve this inequality. What do the solutions represent? See margin.

48. REASONING Write a compound inequality in which the solution set is the given set.

a. $\{x \mid 4 \leq x\}$ Sample answer: $1 < x$ and $4 \leq x$

b. $\{4\}$ Sample answer: $x \leq 4$ and $x \geq 4$

49. ANALYZE Determine if the open sentence $|x - 2| > 4$ and the compound inequality $-2x < 4$ or $x > 6$ have the same solution set. See margin.

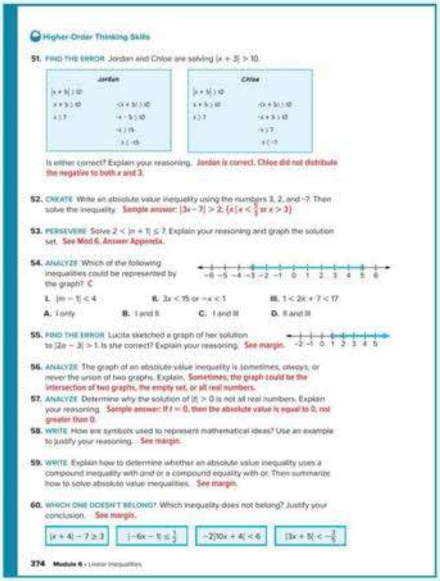
50. ARCHITECTURE An architect is designing a house for the Frazier family. In the design, she must consider the desires of the family and the local building codes. The rectangular lot on which the house will be built is 158 feet long and 90 feet wide.

a. The building codes state that one can build no closer than 20 feet to the lot line. Write an inequality to represent the possible widths of the house along the 90-foot dimension. Solve the inequality. $w \leq 90 - 2(20)$; $\{w \mid w \leq 50\}$

b. The Fraziers requested that the rectangular house contain no less than 2800 square feet and no more than 3200 square feet of floor space. If the house has only one floor, use the maximum value for the width of the house from part a, and explain how to use an inequality to find the possible lengths. See margin.

c. The Fraziers have asked that the cost of the house be about \$175,000 and are willing to deviate from this price no more than \$20,000. Write an open sentence involving an absolute value and solve. Explain the meaning of the answer. $|175{,}000 - x| \leq 20{,}000$; $155{,}000 \leq x \leq 195{,}000$; The Fraziers are willing to pay from \$155,000 to \$195,000 for the house.

Lesson 6-4 • Solving Absolute Value Inequalities 373



Higher-Order Thinking Skills

51. FIND THE ERROR Jordan and Chloe are solving $|x + 3| > 10$.

Jordan
$|x + 3| > 10$
$x + 3 > 10$ $\quad -(x + 3) > 10$
$x > 7$ $\quad -x - 3 > 10$
$\quad -x > 13$
$\quad x < -13$

Chloe
$|x + 3| > 10$
$x + 3 > 10$ $\quad -(x + 3) > 10$
$x > 7$ $\quad -x + 3 > 10$
$\quad -x > 7$
$\quad x < -7$

Is either correct? Explain your reasoning. Jordan is correct. Chloe did not distribute the negative to both x and 3.

52. CREATE Write an absolute value inequality using the numbers 3, 2, and −7. Then solve the inequality. Sample answer: $|3x - 7| > 2$; $\{x \mid x < \frac{5}{3}$ or $x > 3\}$

53. PERSEVERE Solve $2 < |n + 1| \leq 7$. Explain your reasoning and graph the solution set. See Mod 6. Answer Appendix.

54. ANALYZE Which of the following inequalities could be represented by the graph? C

I. $|m - 1| < 4$ II. $3x < 15$ or $-x < 1$ III. $1 < 2k + 7 < 17$

A. I only B. I and II C. I and III D. II and III

55. FIND THE ERROR Lucita sketched a graph of her solution to $|2a - 3| > 1$. Is she correct? Explain your reasoning. See margin.

56. ANALYZE The graph of an absolute value inequality is sometimes, always, or never the union of two graphs. Explain. Sometimes; the graph could be the intersection of two graphs, the empty set, or all real numbers.

57. ANALYZE Determine why the solution of $|t| > 0$ is not all real numbers. Explain your reasoning. Sample answer: If $t = 0$, then the absolute value is equal to 0, not greater than 0.

58. WRITE How are symbols used to represent mathematical ideas? Use an example to justify your reasoning. See margin.

59. WRITE Explain how to determine whether an absolute value inequality uses a compound inequality with *and* or a compound equality with *or*. Then summarize how to solve absolute value inequalities. See margin.

60. WHICH ONE DOESN'T BELONG? Which inequality does not belong? Justify your conclusion. See margin.

$|x + 4| - 7 \geq 3$ $\quad |-6x - 1| \leq \frac{1}{3}$ $\quad -2|10x + 4| < 6$ $\quad |3x + 5| < -\frac{3}{5}$

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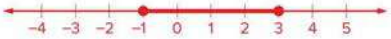
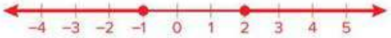
Answers

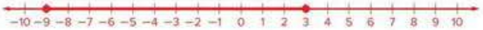
39. $\{x \mid -1 \leq x \leq 3\}$



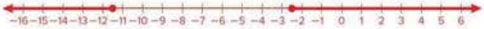
40. $\{x \mid x \geq 2$ or $x \leq -1\}$



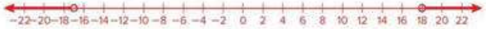
41. $\{x \mid -9 \leq x \leq 3\}$



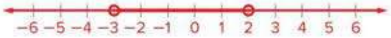
42. $\{x \mid x \geq -2.5$ or $x \leq -11.5\}$



43. $\{x \mid x > 18$ or $x < -17\}$



44. $\{x \mid -3 < x < 2\}$



45. By definition, the absolute value is always greater than a negative number. Therefore, no matter what number is chosen, it will always be greater than −1 when evaluated in the absolute value inequality given.

46. No; based on the equation for estimating height, a femur bone measuring 47 cm would come from a woman who was between 156 and 162 cm tall.

47a. Set the absolute value of an unknown variable, x, minus the recommended weight, 516, to be less than or equal to the variance of 4. So, the inequality $|x - 516| \leq 4$ represents the situation.

47b. Write two inequalities, one for each case: $x - 516 \leq 4$ and $-(x - 516) \leq 4$. For the first case, add 516 to both sides: $x \leq 520$. For the second case, distribute the negative on the left side, subtract 516 from both sides and divide by a negative 1 remembering to switch the inequality sign: $x \geq 512$. This means a box of cereal should have a minimum weight of 512 g and a maximum weight of 520 g.

49. The solution set for $|x - 2| > 4$ is $\{x \mid x < -2$ or $x > 6\}$. The solution set for $-2x < 4$ or $x > 6$ is $\{x \mid x > -2\}$. One includes numbers greater than −2, and the other includes numbers less than −2 or greater than 6. These solution sets are not the same.

50b. The formula for the area of a rectangle with 50 substituted for the width can be used to write the compound inequality $2800 \leq 50\ell \leq 3200$. The possible lengths are found by solving this compound inequality for ℓ. The solution set is $\{\ell \mid 56 \leq \ell \leq 64\}$.

55. No; Sample answer: Lucita forgot to change the direction of the inequality sign for the negative case of the absolute value.

58. Sample answer: Symbols can be used as a shorthand way to represent ideas such as operations, equality, absolute value, and the empty set. For example, instead of writing 5 minus the absolute value of $2x$ equals 10, you could write $5 - |2x| = 10$.

59. Sample answer: When an absolute value is on the left and the inequality symbol $<$ or $\leq$, the compound sentence uses *and*, and if the inequality symbol is $>$ or $\geq$, the compound sentence uses *or*. To solve, if $|x| < n$, then set up and solve the inequalities $x < n$ and $x > -n$, and if $|x| > n$, then set up and solve the inequalities $x > n$ or $x < -n$.

60. $|3x + 5| < -\frac{3}{5}$; The other three inequalities have real number solutions, but this inequality has a solution of $\varnothing$.

Lesson 6-5

Graphing Inequalities in Two Variables

A.CED.3, A.REI.12

LESSON GOAL

Students graph linear inequalities on the coordinate plane.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Graphing Linear Inequalities on the Coordinate Plane

Develop:

Graphing Linear Inequalities in Two Variables

- Graph an Inequality with an Open Half-Plane
- Graph an Inequality with a Closed Half-Plane
- Apply Graphing Inequalities in Two Variables
- Solve Linear Inequalities

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Write and Solve Two-Step Inequalities	●	●		●
Extension: Linear Programming		●	●	●

Language Development Handbook

Assign page 37 of the *Language Development Handbook* to help your students build mathematical language related to graphing linear inequalities on the coordinate plane.

ELL You can use the tips and suggestions on page T37 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Standards for Mathematical Practice:

4 Model with mathematics.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous

Students graphed equations in two variables, and graphed inequalities in one variable.

8.EE.5, 7.EE.4

Now

Students solve and graph inequalities in two variables.

A.CED.3, A.REI.12

Next

Students will solve systems of inequalities.

A.REI.12 (Course 1, Course 2)

Rigor

The Three Pillars of Rigor

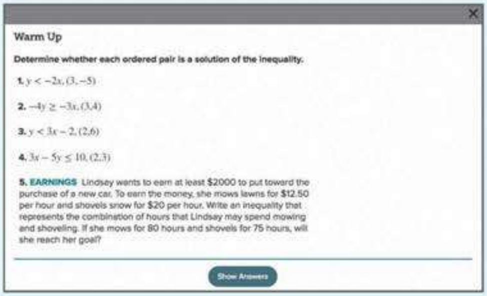
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of graphing inequalities on a number line to build fluency with graphing linear inequalities. They apply their understanding of graphing linear inequalities by solving real-world problems.

Mathematical Background

Inequalities in two variables are solved by graphing the inequality as if it were an equation, and then shading the half-plane that makes the inequality true.

Interactive Presentation



Warm Up

Determine whether each ordered pair is a solution of the inequality.

1. $y < -2x, (3, -5)$

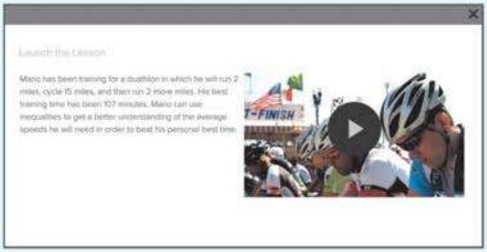
2. $-4y \geq -3x, (3, 4)$

3. $y < 3x - 2, (2, 6)$

4. $3x - 5y \leq 10, (2, 3)$

5. **EARNINGS** Lindsey wants to earn at least $2000 to put toward the purchase of a new car. To earn the money, she mows lawns for $12.50 per hour and shovels snow for $20 per hour. Write an inequality that represents the combination of hours that Lindsey may spend mowing and shoveling. If she mows for 80 hours and shovels for 75 hours, will she reach her goal?

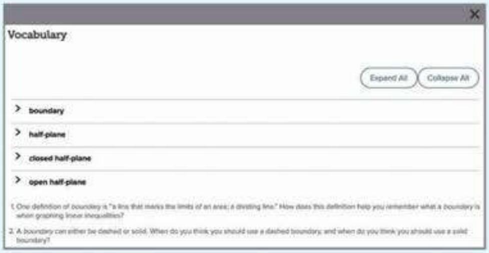
Warm Up



Launch the Lesson

Mario has been training for a duathlon in which he will run 2 miles, cycle 15 miles, and then run 2 more miles. His best training time has been 107 minutes. Mario can use inequalities to get a better understanding of the average speeds he will need in order to beat his personal best time.

Launch the Lesson



Vocabulary

- boundary
- half-plane
- closed half-plane
- open half-plane

1. One definition of boundary is "a line that marks the limits of an area; a dividing line." How does this definition help you remember what a boundary is when graphing linear inequalities?
2. A boundary can either be dashed or solid. When do you think you should use a dashed boundary, and when do you think you should use a solid boundary?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- determining whether a number is a solution to an inequality

Answers:

1. no
2. no
3. no
4. yes
5. $12.5x + 20y \geq 2000$; yes

Launch the Lesson

MP Teaching the Mathematical Practices

4 Apply Mathematics In this Launch, students learn how to apply what they have learned about graphing inequalities to a real-world situation about improving a marathon time.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions with the class.

Explore Graphing Linear Inequalities on the Coordinate Plane

Objective

Students use a sketch to explore graphing linear inequalities on the coordinate plane.

Teaching the Mathematical Practices

5 Decide When to Use Tools Mathematically proficient students can make sound decisions about when to use mathematical tools such as sketches. Help them see why using these tools will help to solve problems and what the limitations are of using the tools.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to graph linear inequalities in the coordinate plane, comparing this to their previous work with graphing equations. They will equate graphing on the coordinate plane with graphing on a number line. Then, students will answer the Inquiry Question.

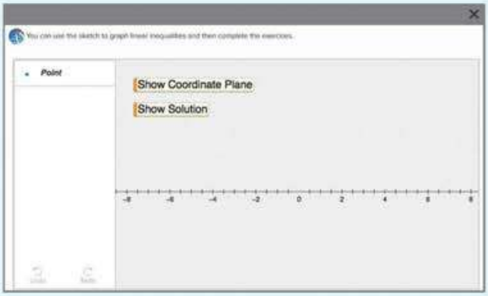
Interactive Presentation

Graphing Linear Inequalities on the Coordinate Plane

INQUIRY How is graphing a linear inequality on the coordinate plane similar to and different from graphing on the number line?

You can use the sketch to graph linear inequalities and then complete the exercises.

Explore



You can use the sketch to graph linear inequalities and then complete the exercises.

Explore

WEB SKETCHPAD

Students use a sketch to graph linear inequalities and complete the exercises.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Graphing Linear Inequalities on the Coordinate Plane (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Which side would be shaded for $x \geq 2$? Sample answer: The region on the coordinate plane to the right of the line $x = 2$, because this includes all x values greater than 2.
- What do you think the graph of $y \leq x$ would look like? Sample answer: The line $y = x$ would be used to divide the coordinate plane into two regions, and the side with coordinates that make the inequality true would be shaded.

Inquiry

How is graphing a linear inequality on the coordinate plane similar to and different from graphing on the number line? Sample answer: When graphing on the coordinate plane and on the number line, you graph $x = a$ and the points to the left or right represent the solution. However, when you graph on the coordinate plane, you graph a line and when you graph on the number line, you plot a point.

Learn Graphing Linear Inequalities in Two Variables

Objective

Students graph the solutions of linear inequalities in two variables as half-planes.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the inequality and graph for each example. Remind students that the shaded region represents all solutions of the inequality.

About the Key Concept

The graph of a linear inequality represents the set of all points that are solutions to the inequality. The boundary will be either solid or dashed and then the area above or below the boundary is shaded to indicate the solution region.

Example 1 Graph an Inequality with an Open Half-Plane

Teaching the Mathematical Practices

2 Attend to Quantities It is important to note the meaning of the quantities used in this problem. Remind students that points on the boundary are not solutions to the inequality.

Questions for Mathematical Discourse

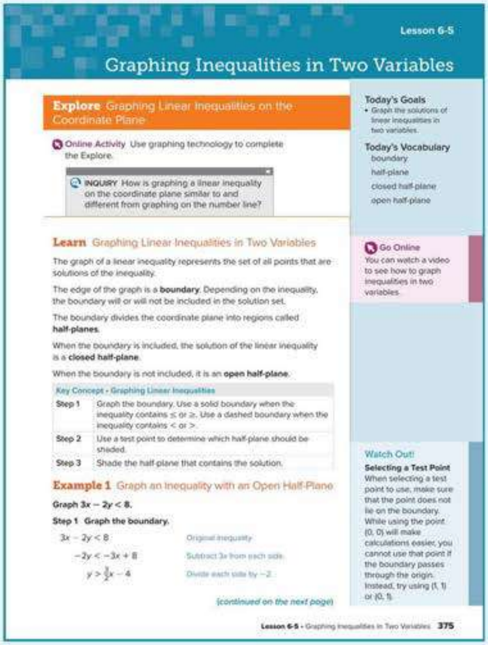
AL Why is the inequality symbol reversed in the fourth line? Because we divided by a negative number.

OL Is (−6, −6) a solution? Explain using both the graph and algebraic inequality. Yes; sample answer: Point (−6, −6) is in the shaded region. Substituting the value −6 for both *x* and *y* results in $3(-6) - 2(-6) < 8$, which is equivalent to the true inequality $-6 < 8$.

BL Does any point on the boundary make this inequality a true statement? Explain. No; sample answer: The boundary is not included in the solution. Those points on the boundary make the two sides of the inequality equal to each other.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 6-5

Graphing Inequalities in Two Variables

Explore Graphing Linear Inequalities on the Coordinate Plane

Online Activity Use graphing technology to complete the Explore.

INQUIRY How is graphing a linear inequality on the coordinate plane similar to and different from graphing on the number line?

Learn Graphing Linear Inequalities in Two Variables

The graph of a linear inequality represents the set of all points that are solutions of the inequality.

The edge of the graph is a **boundary**. Depending on the inequality, the boundary will or will not be included in the solution set.

The boundary divides the coordinate plane into regions called **half-planes**.

When the boundary is included, the solution of the linear inequality is a **closed half-plane**.

When the boundary is not included, it is an **open half-plane**.

Key Concept • Graphing Linear Inequalities

Step 1	Graph the boundary. Use a solid boundary when the inequality contains ≤ or ≥. Use a dashed boundary when the inequality contains < or >.
Step 2	Use a test point to determine which half-plane should be shaded.
Step 3	Shade the half-plane that contains the solution.

Example 1 Graph an Inequality with an Open Half-Plane

Graph $3x - 2y < 8$.

Step 1 Graph the boundary.

$3x - 2y < 8$ Original inequality

$-2y < -3x + 8$ Subtract 3x from each side.

$y > \frac{3}{2}x - 4$ Divide each side by −2.

(continued on the next page)

Today's Goals
- Graph the solutions of linear inequalities in two variables.

Today's Vocabulary
boundary
half-plane
closed half-plane
open half-plane

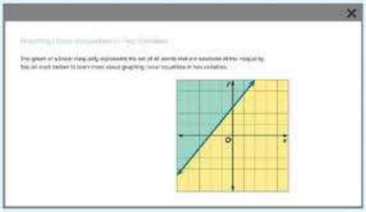
Go Online You can watch a video to see how to graph inequalities in two variables.

Watch Out!

Selecting a Test Point When selecting a test point to use, make sure that the point does not lie on the boundary. While using the point (0, 0) will make calculations easier, you cannot use that point if the boundary passes through the origin. Instead, try using (1, 1) or (0, 1).

Lesson 6-5 • Graphing Inequalities in Two Variables 375

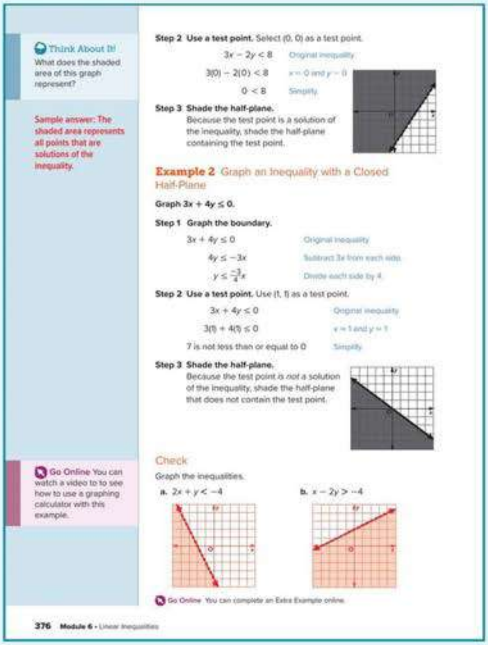
Interactive Presentation



Learn

TAP

Students tap to learn more about the parts of a graph of a linear inequality.



Think About It! What does the shaded area of this graph represent?

Sample answer: The shaded area represents all points that are solutions of the inequality.

Step 2 Use a test point. Select (0, 0) as a test point.

$3x - 2y < 8$ Original inequality

$3(0) - 2(0) < 8$ $x = 0$ and $y = 0$

$0 < 8$ Simplify.

Step 3 Shade the half-plane. Because the test point is a solution of the inequality, shade the half-plane containing the test point.

Example 2 Graph an Inequality with a Closed Half-Plane

Graph $3x + 4y \leq 0$.

Step 1 Graph the boundary.

$3x + 4y \leq 0$ Original inequality

$4y \leq -3x$ Subtract 3x from each side.

$y \leq \frac{-3}{4}x$ Divide each side by 4.

Step 2 Use a test point. Use (1, 1) as a test point.

$3x + 4y \leq 0$ Original inequality

$3(1) + 4(1) \leq 0$ $x = 1$ and $y = 1$

7 is not less than or equal to 0 Simplify.

Step 3 Shade the half-plane. Because the test point is *not* a solution of the inequality, shade the half-plane that does not contain the test point.

Go Online You can watch a video to to see how to use a graphing calculator with this example.

Check

Graph the inequalities.

a. $2x + y < -4$

b. $x - 2y > -4$

Go Online You can complete an Extra Example online.

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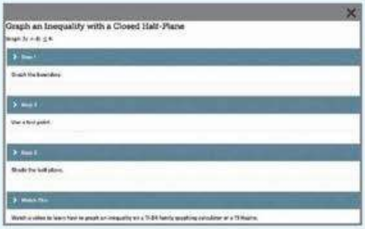
Interactive Presentation



Graph an Inequality with a Closed Half-Plane

Example 2

TAP

Students move through the steps to graph an inequality with a closed half plane.

Essential Question Follow-Up

Students have explored absolute value.

Ask:

How are graphs helpful when solving inequalities in two variables? Sample answer: Graphs help you visualize the solutions. I know all the points in the shaded region make the inequality true.

Example 2 Graph an Inequality with a Closed Half-Plane

Teaching the Mathematical Practices

1 Understand Different Approaches Point out to students that any point that is not on the boundary can be used as a test point.

Questions for Mathematical Discourse

AL Is the line solid or dashed? How do you know? Solid; sample answer: The inequality includes points that make the statement equal.

OL Why should you not use (0, 0) as the test point? (0, 0) is on the boundary and will make the inequality true, but does not give any information about which side of the boundary to shade.

BL How many solutions does this inequality have? The inequality has an infinite number of solutions.

Common Error

Though it is often easiest to use the origin as the test point to determine which side of the boundary to shade, this does not work if the related equation that defines the inequality passes through the origin. In this case, choose another convenient test point.

Example 3 Apply Graphing Inequalities in Two Variables

Teaching the Mathematical Practices

4 Use Tools In Example 3, students will need to identify important quantities in the problem and use a graph to find viable solutions.

Questions for Mathematical Discourse

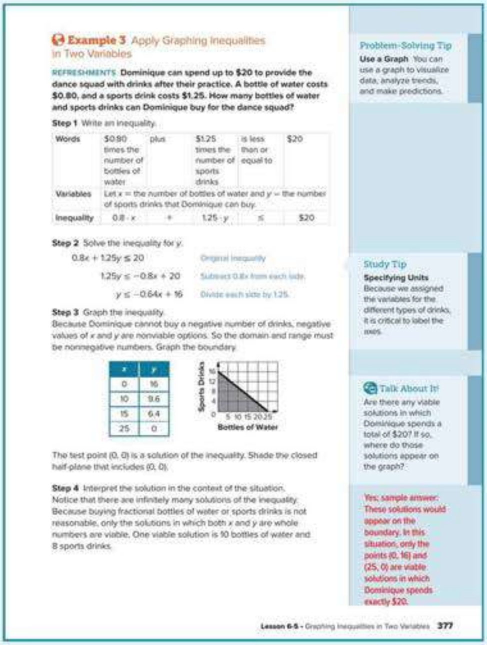
AL Suppose Dominique needs to buy at least 12 drinks for the squad. Name 2 viable solutions. Sample answer: 5 bottles of water and 8 sports drinks; 15 bottles of water and 0 sports drinks

OL Is it necessary to solve for *y* in Step 2? Explain. No; sample answer: The boundary could also be graphed by plotting the *x*- and *y*-intercepts.

BL Given the context of the problem, are there other boundaries? Explain. Yes; sample answer: The *x*- and *y*-axes are also boundaries because Dominique cannot buy a negative number of items.

Common Error

Many students expect a word problem to have an exact solution, but when the situation involves a linear inequality there may be many viable solutions. Remind students that linear inequalities create a region of potential answers, not an exact value.



Example 3 Apply Graphing Inequalities in Two Variables

REFRESHMENTS **Dominique can spend up to $20 to provide the dance squad with drinks after their practice. A bottle of water costs $0.80, and a sports drink costs $1.25. How many bottles of water and sports drinks can Dominique buy for the dance squad?**

Step 1 Write an inequality.

Words	$0.80 times the number of bottles of water	plus	$1.25 times the number of sports drinks	is less than or equal to	$20
Variables	Let x = the number of bottles of water and y = the number of sports drinks that Dominique can buy.				
Inequality	$0.8 \cdot x$	+	$1.25 \cdot y$	≤	$20

Step 2 Solve the inequality for y.

$0.8x + 1.25y \leq 20$ Original inequality

$1.25y \leq -0.8x + 20$ Subtract 0.8x from each side.

$y \leq -0.64x + 16$ Divide each side by 1.25.

Step 3 Graph the inequality.

Because Dominique cannot buy a negative number of drinks, negative values of x and y are nonviable options. So the domain and range must be nonnegative numbers. Graph the boundary.

x	y
0	16
10	9.6
15	6.4
25	0

The test point (0, 0) is a solution of the inequality. Shade the closed half-plane that includes (0, 0).

Step 4 Interpret the solution in the context of the situation.

Notice that there are infinitely many solutions of the inequality. Because buying fractional bottles of water or sports drinks is not reasonable, only the solutions in which both x and y are whole numbers are viable. One viable solution is 10 bottles of water and 8 sports drinks.

Problem-Solving Tip

Use a Graph You can use a graph to visualize data, analyze trends, and make predictions.

Study Tip

Specifying Units Because we assigned the variables for the different types of drinks, it is critical to label the axes.

Talk About It!

Are there any viable solutions in which Dominique spends a total of $20? If so, where do those solutions appear on the graph?

Yes; sample answer: These solutions would appear on the boundary. In this situation, only the points (0, 16) and (25, 0) are viable solutions in which Dominique spends exactly $20.

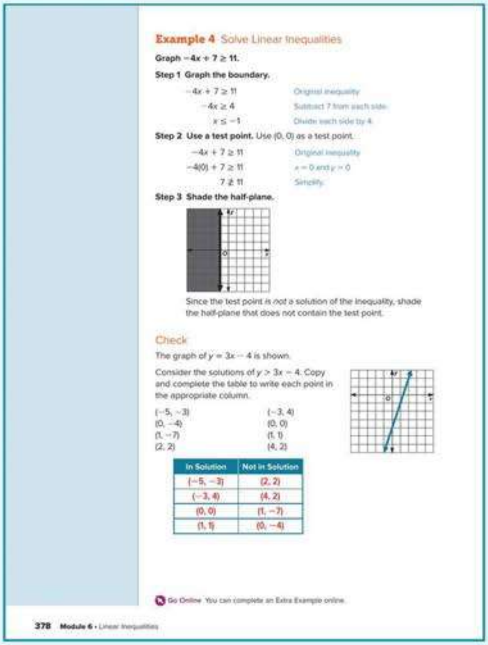
Lesson 6-5 - Graphing Inequalities in Two Variables 377

Interactive Presentation

Example 3

TYPE

Students answer a question about viable solutions for a given constraint.



Example 4 Solve Linear Inequalities

Graph $-4x + 7 \geq 11$.

Step 1 Graph the boundary.

$-4x + 7 \geq 11$ Original inequality
$-4x \geq 4$ Subtract 7 from each side.
$x \leq -1$ Divide each side by 4.

Step 2 Use a test point. Use (0, 0) as a test point.

$-4x + 7 \geq 11$ Original inequality
$-4(0) + 7 \geq 11$ $x = 0$ and $y = 0$
$7 \not\geq 11$ Simplify.

Step 3 Shade the half-plane.

Since the test point is *not* a solution of the inequality, shade the half-plane that does not contain the test point.

Check

The graph of $y = 3x - 4$ is shown.

Consider the solutions of $y > 3x - 4$. Copy and complete the table to write each point in the appropriate column.

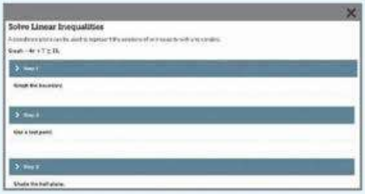
(−5, −3)
(0, −4)
(1, −7)
(2, 2)
(−3, 4)
(0, 0)
(1, 1)
(4, 2)

In Solution	Not in Solution
(−5, −3)	(2, 2)
(−3, 4)	(4, 2)
(0, 0)	(1, −7)
(1, 1)	(0, −4)

Go Online You can complete an Extra Example online.

378 Module 6 • Linear Inequalities

Interactive Presentation



Solve Linear Inequalities

Graph $-4x + 7 \geq 11$.

Step 1
Graph the boundary.

Step 2
Use a test point.

Step 3
Shade the half-plane.

Example 4

TAP

Students move through the steps to solve a linear inequality with one variable.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Solve Linear Inequalities

MP Teaching the Mathematical Practices

2 Make Sense of Quantities Mathematically proficient students need to be able to make sense of quantities and their relationships. In Example 4, note the relationship between an inequality in one variable and its corresponding half-plane.

Questions for Mathematical Discourse

AL How many unknown values are in this equation? one

OL What is the slope of the boundary? undefined

BL Suppose you graphed two linear inequalities and there was a region where the shading of the inequalities overlapped. What would this mean? The points in the region are valid solutions to both inequalities.

Common Error

Students might struggle with graphing a one-variable equation on the coordinate plane. Remind them that when the equation contains only one variable, the graph will be a line crossing directly through the axis of the variable present.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–17
2	exercises that use a variety of skills from this lesson	18–27
3	exercises that emphasize higher-order and critical-thinking skills	28–32

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice, Exercises 1–27 odd, 28–32
- Extension: Linear Programming
- **ALEKS** Graphing Linear Inequalities

OL

IF students score 66%–89% on the Checks,
THEN assign:

- Practice, Exercises 1–31 odd
- Remediation, Review Resources: Write and Solve Two-Step Inequalities
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** applications of inequalities

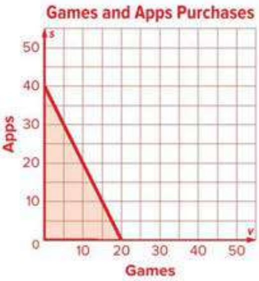
AL

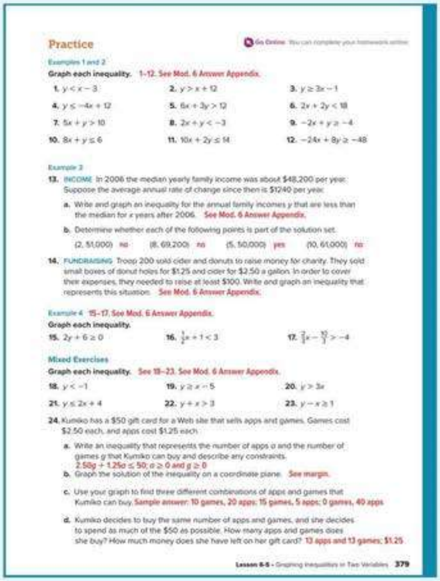
IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–17 odd
- Remediation, Review Resources: Write and Solve Two-Step Inequalities
- *Quick Review Math Handbook*: Graphing Inequalities in Two Variables
- Arrive**MATH** Take Another Look
- **ALEKS** applications of inequalities

Answer

24b.





Practice

Go Online You can complete your homework online.

Examples 1 and 2

Graph each inequality. 1–12. See Mod. 6 Answer Appendix.

1. $y < x - 3$
2. $y > x + 12$
3. $y \geq 3x - 1$
4. $y \leq -4x + 12$
5. $6x + 3y > 12$
6. $2x + 2y < 18$
7. $5x + y > 10$
8. $2x + y < -3$
9. $-2x + y \geq -4$
10. $8x + y \leq 6$
11. $10x + 2y \leq 14$
12. $-24x + 8y \geq -48$

Example 3

13. INCOME In 2006 the median yearly family income was about $48,200 per year. Suppose the average annual rate of change since then is $1240 per year.
    a. Write and graph an inequality for the annual family incomes y that are less than the median for x years after 2006. See Mod. 6 Answer Appendix.
    b. Determine whether each of the following points is part of the solution set.
    (2, 51,000) no (8, 69,200) no (5, 50,000) yes (10, 61,000) no

14. FUNDRAISING Troop 200 sold cider and donuts to raise money for charity. They sold small boxes of donut holes for $1.25 and cider for $2.50 a gallon. In order to cover their expenses, they needed to raise at least $100. Write and graph an inequality that represents this situation. See Mod. 6 Answer Appendix.

Example 4 15–17. See Mod. 6 Answer Appendix.

Graph each inequality.

15. $2y + 6 \geq 0$
16. [illegible] $x + 1 < 3$
17. [illegible] > -4

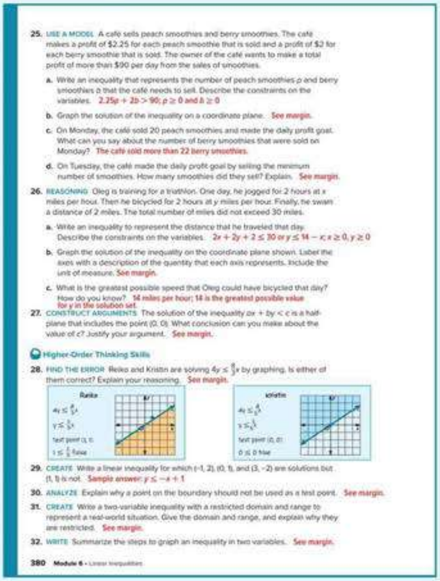
Mixed Exercises

Graph each inequality. See 18–23. See Mod. 6 Answer Appendix.

18. $y < -1$
19. $y \geq x - 5$
20. $y > 3x$
21. $y \leq 2x + 4$
22. $y + x > 3$
23. $y - x \geq 1$

24. Kumiko has a $50 gift card for a Web site that sells apps and games. Games cost $2.50 each, and apps cost $1.25 each.
    a. Write an inequality that represents the number of apps a and the number of games g that Kumiko can buy and describe any constraints.
    $2.50g + 1.25a \leq 50$; $a \geq 0$ and $g \geq 0$
    b. Graph the solution of the inequality on a coordinate plane. See margin.
    c. Use your graph to find three different combinations of apps and games that Kumiko can buy. Sample answer: 10 games, 20 apps; 15 games, 5 apps; 0 games, 40 apps
    d. Kumiko decides to buy the same number of apps and games, and she decides to spend as much of the $50 as possible. How many apps and games does she buy? How much money does she have left on her gift card? 13 apps and 13 games; $1.25

Lesson 6-5 • Graphing Inequalities in Two Variables 379



25. USE A MODEL A café sells peach smoothies and berry smoothies. The café makes a profit of $2.25 for each peach smoothie that is sold and a profit of $2 for each berry smoothie that is sold. The owner of the café wants to make a total profit of more than $90 per day from the sales of smoothies.
    - a. Write an inequality that represents the number of peach smoothies p and berry smoothies b that the café needs to sell. Describe the constraints on the variables. $2.25p + 2b > 90$; $p \geq 0$ and $b \geq 0$
    - b. Graph the solution of the inequality on a coordinate plane. See margin.
    - c. On Monday, the café sold 20 peach smoothies and made the daily profit goal. What can you say about the number of berry smoothies that were sold on Monday? The café sold more than 22 berry smoothies.
    - d. On Tuesday, the café made the daily profit goal by selling the minimum number of smoothies. How many smoothies did they sell? Explain. See margin.
26. REASONING Oleg is training for a triathlon. One day, he jogged for 2 hours at x miles per hour. Then he bicycled for 2 hours at y miles per hour. Finally, he swam a distance of 2 miles. The total number of miles did not exceed 30 miles.
    - a. Write an inequality to represent the distance that he traveled that day. Describe the constraints on the variables. $2x + 2y + 2 \leq 30$ or $y \leq 14 - x$; $x \geq 0$, $y \geq 0$
    - b. Graph the solution of the inequality on the coordinate plane shown. Label the axes with a description of the quantity that each axis represents. Include the unit of measure. See margin.
    - c. What is the greatest possible speed that Oleg could have bicycled that day? How do you know? 14 miles per hour; 14 is the greatest possible value for y in the solution set.
27. CONSTRUCT ARGUMENTS The solution of the inequality $ax + by < c$ is a half-plane that includes the point (0, 0). What conclusion can you make about the value of c? Justify your argument. See margin.

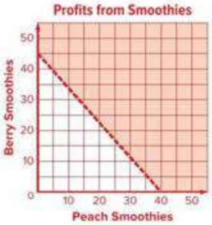
Higher-Order Thinking Skills

28. FIND THE ERROR Reiko and Kristin are solving $4y \leq \frac{8}{3}x$ by graphing. Is either of them correct? Explain your reasoning. See margin.
29. CREATE Write a linear inequality for which (−1, 2), (0, 1), and (3, −2) are solutions but (1, 1) is not. Sample answer: $y \leq -x + 1$
30. ANALYZE Explain why a point on the boundary should not be used as a test point. See margin.
31. CREATE Write a two-variable inequality with a restricted domain and range to represent a real-world situation. Give the domain and range, and explain why they are restricted. See margin.
32. WRITE Summarize the steps to graph an inequality in two variables. See margin.

380 Module 6 • Linear Inequalities

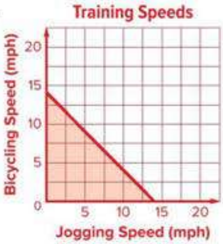
Answers

25b.



25d. 41; Sample explanation: 40 peach smoothies results in a profit of exactly $90, so to make a profit of more than $90, the café must have sold 41 smoothies.

26b.



27. The value of c must be positive. Because (0, 0) is a solution of the inequality, $a(0) + b(0) < c$ must be a true statement, so $0 < c$.

28. Reiko; Kristin used a test point located on the line and shaded in the incorrect half-plane.

30. A test point on the boundary does not show which half-plane contains the points that make the inequality true.

31. Sample answer: The inequality $y > 10x + 45$ represents the cost of a monthly smartphone data plan with a one-time fee of $45, plus $10 per GB of data used. Both the domain and range are nonnegative real numbers because the GB used and the total cost cannot be negative.

32. Sample answer: First solve the inequality for y. Then change the inequality sign to an equal sign and graph the boundary. If $<$ or $>$ is used, the boundary is not included in the graph and the line is dashed. Otherwise, the boundary is included and the line is solid. Then choose a test point not on the boundary. Substitute the coordinates of the test point into the original inequality. If the result makes the inequality true, then shade the half-plane that includes the test point. If the result makes the inequality false, shade the half-plane that does not include the test point. Lastly, check your solution by choosing a test point that is in the half-plane that is not shaded. This second test point should make the inequality false if the solution is correct.

Review

Rate Yourself

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it important to understand what the symbols in a mathematical sentence represent?
- How are symbols used to write expressions, equations, and inequalities?
- How are graphs helpful when solving inequalities in two variables?

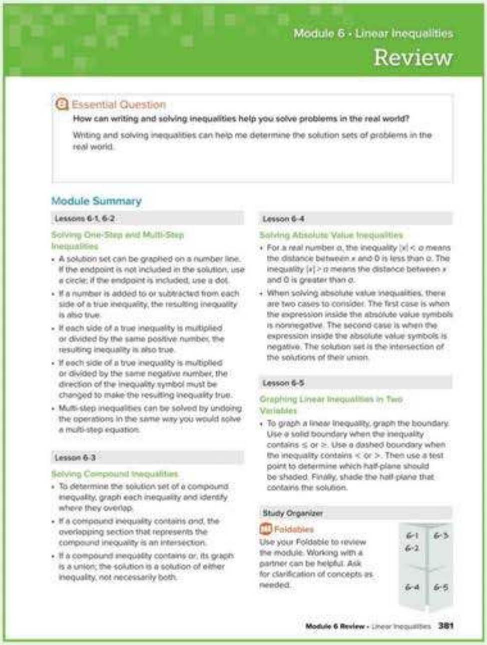
Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to solving and graphing inequalities.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Relationships Between Quantities and Reasoning with Equations** and **Linear and Exponential Relationships**.

- Create and Solve Linear Inequalities
- Represent and Solve Equations and Inequalities Graphically



Module 6 • Linear Inequalities

Review

Essential Question

How can writing and solving inequalities help you solve problems in the real world?

Writing and solving inequalities can help me determine the solution sets of problems in the real world.

Module Summary

Lessons 6-1, 6-2

Solving One-Step and Multi-Step Inequalities

- A solution set can be graphed on a number line. If the endpoint is not included in the solution, use a circle; if the endpoint is included, use a dot.
- If a number is added to or subtracted from each side of a true inequality, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be changed to make the resulting inequality true.
- Multi-step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.

Lesson 6-3

Solving Compound Inequalities

- To determine the solution set of a compound inequality, graph each inequality and identify where they overlap.
- If a compound inequality contains *and*, the overlapping section that represents the compound inequality is an intersection.
- If a compound inequality contains *or*, its graph is a union; the solution is a solution of either inequality, not necessarily both.

Lesson 6-4

Solving Absolute Value Inequalities

- For a real number a, the inequality $|x| < a$ means the distance between x and 0 is less than a. The inequality $|x| > a$ means the distance between x and 0 is greater than a.
- When solving absolute value inequalities, there are two cases to consider. The first case is when the expression inside the absolute value symbols is nonnegative. The second case is when the expression inside the absolute value symbols is negative. The solution set is the intersection of the solutions of their union.

Lesson 6-5

Graphing Linear Inequalities in Two Variables

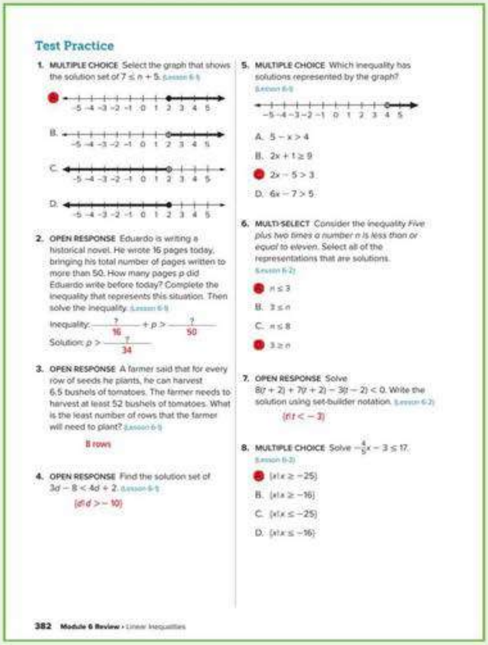
- To graph a linear inequality, graph the boundary. Use a solid boundary when the inequality contains $\leq$ or $\geq$. Use a dashed boundary when the inequality contains $<$ or $>$. Then use a test point to determine which half-plane should be shaded. Finally, shade the half-plane that contains the solution.

Study Organizer

Foldables

Use your Foldable to review the module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 6 Review • Linear Inequalities **381**



Test Practice

1. **MULTIPLE CHOICE** Select the graph that shows the solution set of $7 \le n + 5$. (Lesson 6-1)
   - ●
   - B.
   - C.
   - D.

2. **OPEN RESPONSE** Eduardo is writing a historical novel. He wrote 16 pages today, bringing his total number of pages written to more than 50. How many pages p did Eduardo write before today? Complete the inequality that represents this situation. Then solve the inequality. (Lesson 6-1)

   Inequality: $\underset{16}{?} + p > \underset{50}{?}$

   Solution: $p > \underset{34}{?}$

3. **OPEN RESPONSE** A farmer said that for every row of seeds he plants, he can harvest 6.5 bushels of tomatoes. The farmer needs to harvest at least 52 bushels of tomatoes. What is the least number of rows that the farmer will need to plant? (Lesson 6-1)

   8 rows

4. **OPEN RESPONSE** Find the solution set of $3d - 8 < 4d + 2$. (Lesson 6-1)

   $\{d \mid d > -10\}$

5. **MULTIPLE CHOICE** Which inequality has solutions represented by the graph? (Lesson 6-1)
   - A. $5 - x > 4$
   - B. $2x + 1 \ge 9$
   - ● $2x - 5 > 3$
   - D. $6x - 7 > 5$

6. **MULTI-SELECT** Consider the inequality *Five plus two times a number n is less than or equal to eleven.* Select all of the representations that are solutions. (Lesson 6-2)
   - ● $n \le 3$
   - B. $3 \le n$
   - C. $n \le 8$
   - ● $3 \ge n$

7. **OPEN RESPONSE** Solve $8(t + 2) + 7(t + 2) - 3(t - 2) < 0$. Write the solution using set-builder notation. (Lesson 6-2)

   $\{t \mid t < -3\}$

8. **MULTIPLE CHOICE** Solve $-\frac{4}{5}x - 3 \le 17$. (Lesson 6-2)
   - ● $\{x \mid x \ge -25\}$
   - B. $\{x \mid x \ge -16\}$
   - C. $\{x \mid x \le -25\}$
   - D. $\{x \mid x \le -16\}$

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 6-1 through 6-3
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

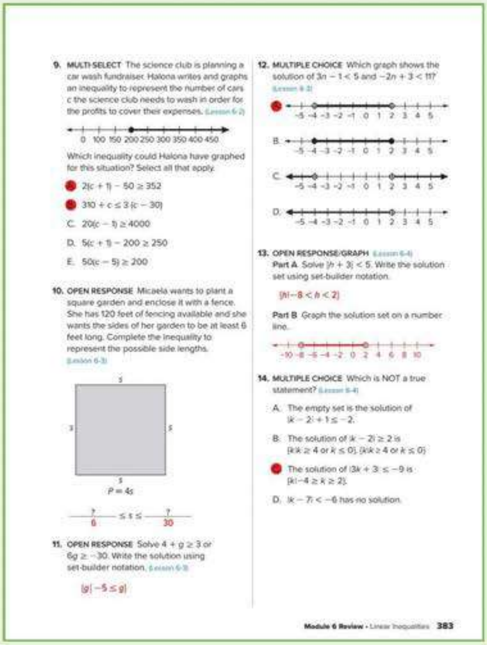
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–18 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 5, 8, 12, 14, 15
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	9
Table Item	Students complete a table by entering in the correct values.	6, 17
Graph	Students create a graph on an online coordinate plane.	18
Open Response	Students construct their own response.	2–4, 7, 10, 11, 13, 16

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
A.REI.3	6–1, 6–2	1, 4–8
A.CED.1	6–1, 6–2, 6–3, 6–4	2, 3, 9
A.CED.3	6–3, 6–4	10–15
A.REI.12	6–5	16–18



9. MULTI-SELECT The science club is planning a car wash fundraiser. Halona writes and graphs an inequality to represent the number of cars c the science club needs to wash in order for the profits to cover their expenses. (Lesson 6-2)

Which inequality could Halona have graphed for this situation? Select all that apply.

● $2(c + 1) - 50 \geq 352$

● $310 + c \leq 3(c - 30)$

C. $20(c - 1) \geq 4000$

D. $5(c + 1) - 200 \geq 250$

E. $50(c - 5) \geq 200$

10. OPEN RESPONSE Micaela wants to plant a square garden and enclose it with a fence. She has 120 feet of fencing available and she wants the sides of her garden to be at least 6 feet long. Complete the inequality to represent the possible side lengths. (Lesson 6-3)

$P = 4s$

$\overset{?}{6} \leq s \leq \overset{?}{30}$

11. OPEN RESPONSE Solve $4 + g \geq 3$ or $6g \geq -30$. Write the solution using set-builder notation. (Lesson 6-3)

$\{g \mid -5 \leq g\}$

12. MULTIPLE CHOICE Which graph shows the solution of $3n - 1 < 5$ and $-2n + 3 < 11$? (Lesson 6-3)

●

B.

C.

D.

13. OPEN RESPONSE/GRAPH (Lesson 6-4)

Part A Solve $|h + 3| < 5$. Write the solution set using set-builder notation.

$\{h \mid -8 < h < 2\}$

Part B Graph the solution set on a number line.

14. MULTIPLE CHOICE Which is NOT a true statement? (Lesson 6-4)

A. The empty set is the solution of $|k - 2| + 1 \leq -2$.

B. The solution of $|k - 2| \geq 2$ is $\{k \mid k \geq 4$ or $k \leq 0\}$. $\{k \mid k \geq 4$ or $k \leq 0\}$

● The solution of $|3k + 3| \leq -9$ is $\{k \mid -4 \geq k \geq 2\}$.

D. $|k - 7| < -6$ has no solution.

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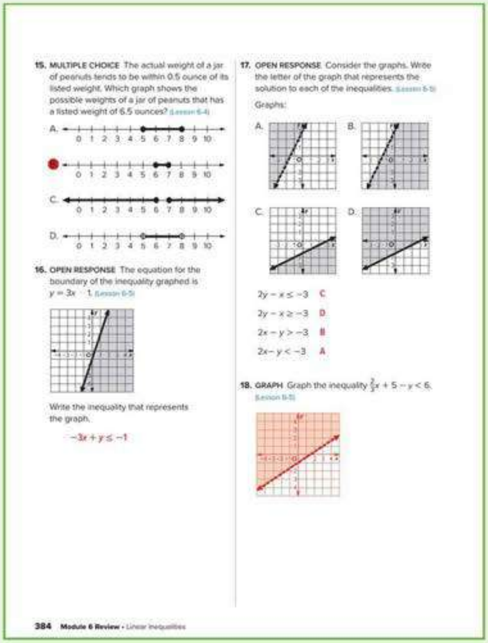

15. MULTIPLE CHOICE The actual weight of a jar of peanuts tends to be within 0.5 ounce of its listed weight. Which graph shows the possible weights of a jar of peanuts that has a listed weight of 6.5 ounces? (Lesson 6-4)

A.

B.

C.

D.

16. OPEN RESPONSE The equation for the boundary of the inequality graphed is $y = 3x - 1$. (Lesson 6-5)

Write the inequality that represents the graph.

$-3x + y \leq -1$

17. OPEN RESPONSE Consider the graphs. Write the letter of the graph that represents the solution to each of the inequalities. (Lesson 6-5)

Graphs:

A.

B.

C.

D.

$2y - x \leq -3$ C

$2y - x \geq -3$ D

$2x - y > -3$ B

$2x - y < -3$ A

18. GRAPH Graph the inequality $\frac{2}{3}x + 5 - y < 6$. (Lesson 6-5)

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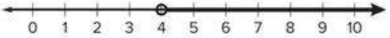
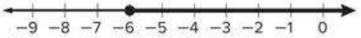
Lesson 6-1

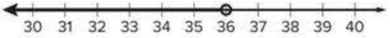
67. $\{c \mid c > 4\}$



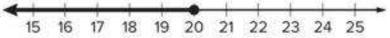
68. $\{b \mid b \geq -6\}$



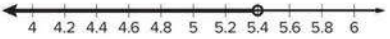
69. $\{x \mid x < 36\}$



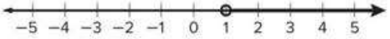
70. $\{x \mid x \leq 20\}$



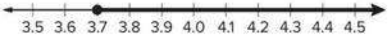
71. $\{m \mid m < 5.4\}$



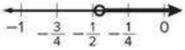
72. $\{n \mid n > 1\}$



73. $\{c \mid c \geq 3.7\}$

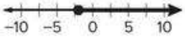


74. $\{k \mid k > -\frac{5}{12}\}$

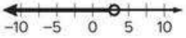


Lesson 6-2

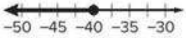
6. $5x - 1 \geq -11; x \geq -2$



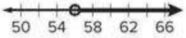
7. $21 > 15 + 2x; x < 3$



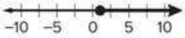
8. $-9 \geq \frac{2}{5}x + 7; x \leq -40$



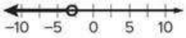
9. $\frac{x}{8} - 13 > -6; x > 56$



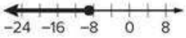
10. $-x + 6 \leq 5; x \geq 1$



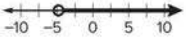
11. $37 < 7 - 10x; x < -3$



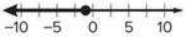
12. $8 - \frac{x}{3} \geq 11; x \leq -9$



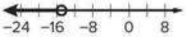
13. $-\frac{5}{4}x + 6 < 12; x > -\frac{24}{5}$



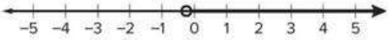
14. $3x - 6 \geq 15 + 24x; x \leq -1$



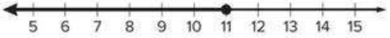
15. $15x + 30 < 10x - 45; x < -15$



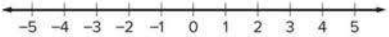
16. $\{n \mid n > -\frac{1}{3}\}$



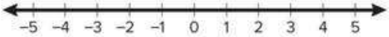
17. $\{a \mid a \leq 11\}$



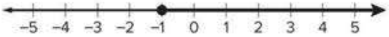
18. $\{\varnothing\}$



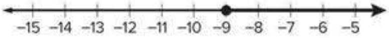
19. $\{b \mid b$ is a real number.$\}$



20. $\{t \mid t \geq -1\}$

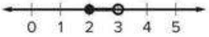


21. $\{a \mid a \geq -9\}$

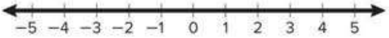


Lesson 6-3

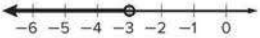
7. $\{h \mid 2 \leq h < 3\}$



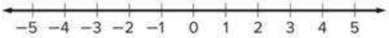
8. $\{m \mid m$ is a real number.$\}$



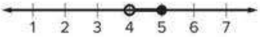
9. $\{y \mid y < -3\}$



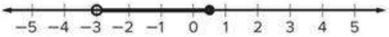
10. $\{\varnothing\}$



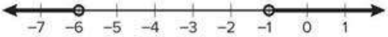
11. $\{b \mid 4 < b \leq 5\}$



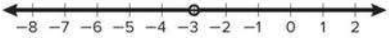
12. $\{a \mid -3 < a \leq \frac{1}{2}\}$



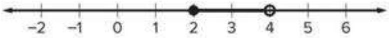
13. $\{m \mid m < -6$ or $m > -1\}$



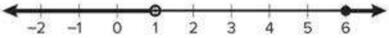
14. $\{n \mid n < -3$ or $n > -3\}$



15. $\{m \mid 2 \leq m < 4\}$

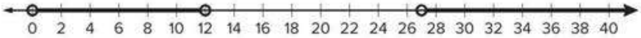


16. $\{y \mid y < 1$ or $y \geq 6\}$

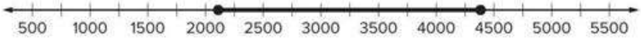


17b. $0 < x < 12$ or $x > 27$; Because the combined height of the sign and pole cannot be negative, the value of x must be greater than 0.

17c.

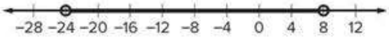


18c.

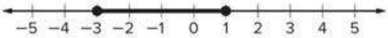


Lesson 6-4

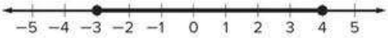
1. $\{x \mid -24 < x < 8\}$



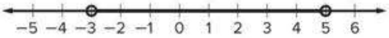
2. $\{r \mid -3 \leq r \leq 1\}$



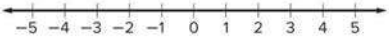
3. $\{c \mid -3 \leq c \leq 4\}$



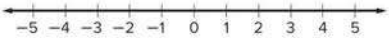
4. $\{h \mid -3 < h < 5\}$



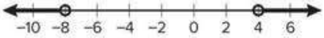
5. $\{\emptyset\}$



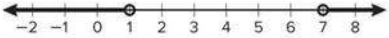
6. $\{\emptyset\}$



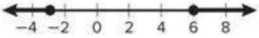
7. $\{r \mid r < -8 \text{ or } r > 4\}$



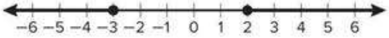
8. $\{k \mid k < 1 \text{ or } k > 7\}$



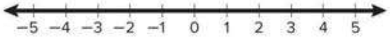
9. $\{h \mid h \leq -3 \text{ or } h \geq 6\}$



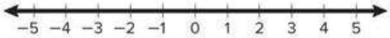
10. $\{p \mid p \leq -3 \text{ or } p \geq 2\}$



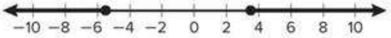
11. $\{v \mid v \text{ is a real number.}\}$



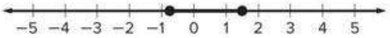
12. $\{c \mid c \text{ is a real number.}\}$



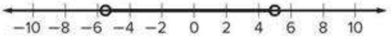
13. $\{n \mid n \leq -5\frac{1}{4} \text{ or } n \geq 3\frac{3}{4}\}$



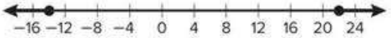
14. $\{t \mid -\frac{4}{5} \leq t \leq 1\frac{3}{5}\}$



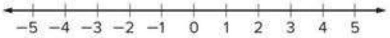
15. $\{h \mid -5\frac{2}{3} < h < 5\}$



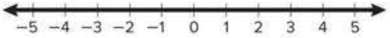
16. $\{p \mid p \leq -14 \text{ or } p \geq 22\}$



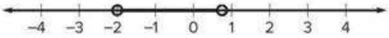
17. $\{\emptyset\}$



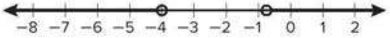
18. $\{g \mid g \text{ is a real number.}\}$



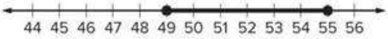
19. $\{r \mid -2 < r < \frac{2}{3}\}$



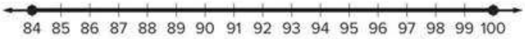
20. $\{p \mid p < -4 \text{ or } p > -\frac{2}{3}\}$



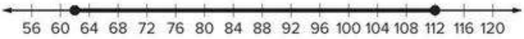
36. $|x - 52| \leq 3$; $\{x \mid 49 \leq x \leq 55\}$



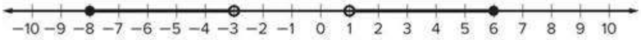
37. $|x - 92| \leq 8$; $\{x \mid 84 \leq x \leq 100\}$



38. $|x - 87| \leq 25$; $\{x \mid 62 \leq x \leq 112\}$

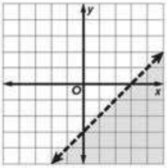


53. $(-8 \le n < -3)$ or $(1 < n \le 6)$. To solve this compound inequality, split it into two inequalities. The first one to solve is $|n + 1| > 2$ and the second one is $|n + 1| \le 7$. The solution set of the entire problem is the overlap of the individual solutions.

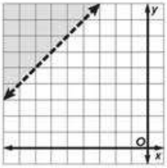


Lesson 6-5

1.



2.



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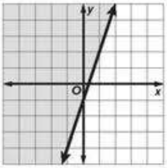

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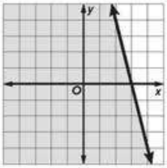

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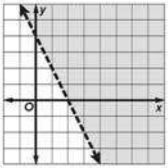

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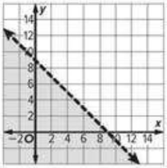

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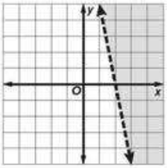

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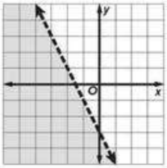

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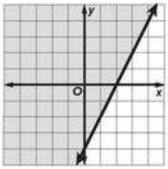

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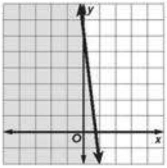

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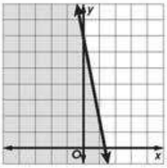

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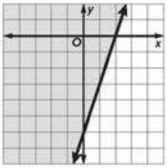
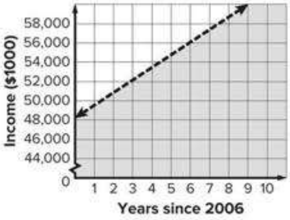
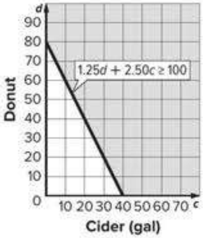


13a. $y < 1240x + 48{,}200$



14. $1.25d + 2.5c \ge 100$



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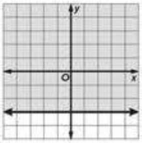

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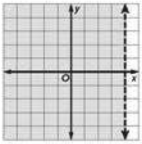

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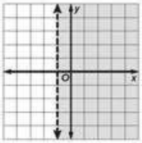

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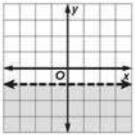

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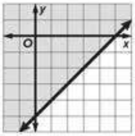

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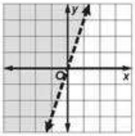

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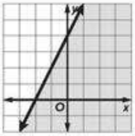

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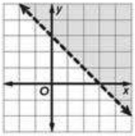

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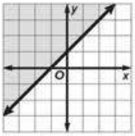


Module 7

Systems of Linear Equations and Inequalities

Module Goals

- Students solve systems of equations using a variety of methods.
- Students solve systems of equations using graphing technology.
- Students graph the solution sets of systems of linear inequalities.

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Also addresses A.REI.5 and A.REI.11.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Coherence

Vertical Alignment

Previous

Students analyzed and solved simultaneous linear equations.

8.EE.8

Now

Students write and solve systems of two equations in two variables and solve systems of two inequalities in two variables.

A.CED.3, A.REI.6, A.REI.12

Next

Students will graph exponential functions, showing intercepts and end behavior, and interpret the parameters of the function in terms of a context.

F.IF.7e, F.LE.2

Rigor

The Three Pillars of Rigor

Students will use the three pillars of rigor to help them meet the standards. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

EXPLORE → **LEARN** → **EXAMPLE & PRACTICE**

Suggested Pacing

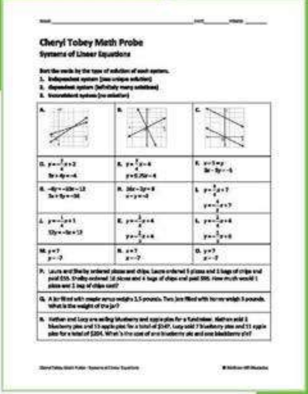
Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
7-1 Graphing Systems of Equations	A.REI.6, A.REI.11	2	1
7-2 Substitution	A.CED.3, A.REI.6	1	0.5
7-3 Elimination Using Addition and Subtraction	A.CED.3, A.REI.6	1	0.5
7-4 Elimination Using Multiplication	A.REI.5, A.REI.6	1	0.5
Put It All Together: Lessons 7-1 through 7-4		1	0.5
7-5 Systems of Inequalities	A.CED.3, A.REI.12	2	1
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	11	5.5

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Systems of Linear Equations

Module Resource



Correct Answers:
1. A, B, G, H, I, O, R
2. E, J, K, Q
3. C, D, F, L, M, N, P

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which graphs fall into three categories and explain their choices.

Targeted Concepts Systems of equations can be analyzed to distinguish between systems with

- one unique solution (an independent system),
- no solution (a dependent system, parallel lines), or
- infinitely many solutions (an inconsistent system, coinciding lines).

Targeted Misconceptions Students may not be able to analyze the slope and *y*-intercept to categorize systems of equations. They may:

- have to solve the systems, using one of the algebraic/graphical methods for solving to analyze solutions,
- have difficulty working with equations when both are not written in slope-intercept form ($y = mx + b$), or
- have difficulty analyzing word problems.
- confuse horizontal/vertical lines and/or have difficulty analyzing systems with them.
- multiply quantities without consideration of all of the numbers and symbols.

Use the Probe after Lesson 7-4

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
incorrect placement of one or more cards	has one or more of the targeted difficulties. To learn about student difficulties, give individual students, groups of students, and/or the whole class opportunities to discuss how they categorized each system. For example: • An incorrect placement of D, F, G, H, and/or J indicates difficulty with categorizing equations when not written in $y = mx + b$ form. • An incorrect placement of K and/or L indicates difficulty with analyzing the *y*-intercept. • An incorrect placement of M, N, and/or O indicates difficulty with analyzing and/or categorizing horizontal and vertical lines. • An incorrect placement of P , Q, and/or R indicates difficulty with representing written descriptions with equations.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Systems of Linear Equations
- Lessons 7-1 through 7-4, all Learns, all Examples

Revisit the Probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

How are systems of equations useful in the real world? Sample answer: Writing and solving systems of equations can help you find unknown values in real-world situations.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

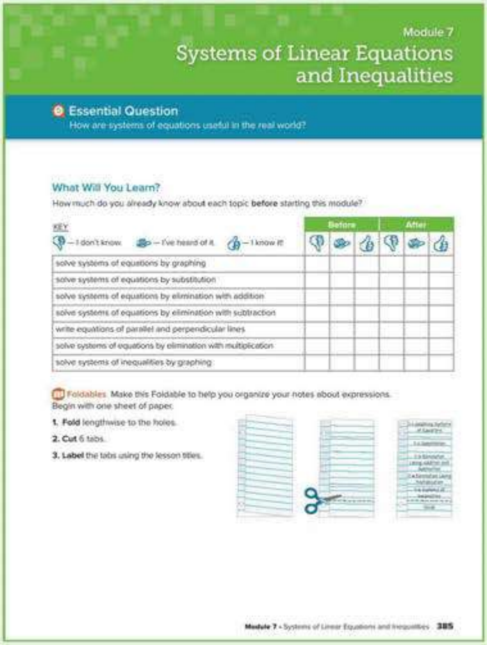
Focus Students write notes about solving systems of linear equations and inequalities throughout the lessons of this module.

Teach Have students make and label their Foldable as illustrated. Have students write a word or concept from the lesson on the back of each lesson's tab. Under the word or concept, ask students to include a definition and an example.

When to Use It Encourage students to add to their Foldable as they work through the module and to use them to review for the module test.

Launch the Module

The Launch the Module video uses a trip to the zoo to show real-world applications of systems of equations. Students learn about using systems to determine the number of child and adult tickets sold on a given day, the most cost-efficient mix of foods to feed the animals, and the number of snacks that can be purchased for a given amount of money.



Module 7

Systems of Linear Equations and Inequalities

Essential Question

How are systems of equations useful in the real world?

What Will You Learn?

How much do you already know about each topic **before** starting this module?

KEY

👎 — I don't know. ✋ — I've heard of it. 👍 — I know it!

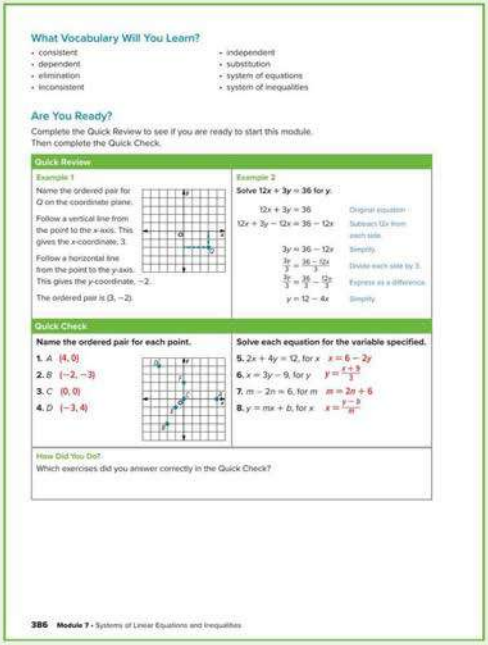
	Before			After		
	👎	✋	👍	👎	✋	👍
solve systems of equations by graphing						
solve systems of equations by substitution						
solve systems of equations by elimination with addition						
solve systems of equations by elimination with subtraction						
write equations of parallel and perpendicular lines						
solve systems of equations by elimination with multiplication						
solve systems of inequalities by graphing						

Foldables Make this Foldable to help you organize your notes about expressions. Begin with one sheet of paper.

1. **Fold** lengthwise to the holes.
2. **Cut** 6 tabs.
3. **Label** the tabs using the lesson titles.

Module 7 • Systems of Linear Equations and Inequalities **385**

Interactive Presentation



What Vocabulary Will You Learn?

- consistent
- dependent
- elimination
- inconsistent
- independent
- substitution
- system of equations
- system of inequalities

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Name the ordered pair for Q on the coordinate plane.

Follow a vertical line from the point to the x-axis. This gives the x-coordinate, 3.

Follow a horizontal line from the point to the y-axis. This gives the y-coordinate, -2.

The ordered pair is $(3, -2)$.

Example 2

Solve $12x + 3y = 36$ for y.

$12x + 3y = 36$ Original equation

$12x + 3y - 12x = 36 - 12x$ Subtract $12x$ from each side.

$3y = 36 - 12x$ Simplify.

$\frac{3y}{3} = \frac{36 - 12x}{3}$ Divide each side by 3.

$\frac{3y}{3} = \frac{36}{3} - \frac{12x}{3}$ Express as a difference.

$y = 12 - 4x$ Simplify.

Quick Check

Name the ordered pair for each point.

1. A $(4, 0)$
2. B $(-2, -3)$
3. C $(0, 0)$
4. D $(-3, 4)$

Solve each equation for the variable specified.

5. $2x + 4y = 12$, for x $x = 6 - 2y$
6. $x = 3y - 9$, for y $y = \frac{x + 9}{3}$
7. $m - 2n = 6$, for m $m = 2n + 6$
8. $y = mx + b$, for x $x = \frac{y - b}{m}$

How Did You Do?

Which exercises did you answer correctly in the Quick Check?

386 **Module 7** • Systems of Linear Equations and Inequalities

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define A system of equations is a set of equations with the same variables.

Example $y = 342 - 14.9x$ and $y = 3.3 + 4.7x$

Ask What variables do both equations have in common? x and y

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- identifying the solution of a system of equations
- solving for a specific variable
- simplifying expressions by using the Distributive Property
- solving systems of equations by using substitution
- testing half-planes

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You may want to use the **Systems** section to ensure student success in this module.

Mindset Matters

Reward Effort, Not Talent

When adults praise students for their hard work toward a solution, rather than praising them for being smart or talented, it supports students' development of a growth mindset. Reward *actions* like hard work, determination, and perseverance instead of *traits* like inherent skill or talent.

How Can I Apply It?

Have students complete the **Performance Task** for the module. Allow students a forum to discuss their process or the strategy that they used, and give them positive feedback on their diligence in completing the task.

Lesson 7-1

Graphing Systems of Equations

A.REI.6, A.REI.11

LESSON GOAL

Students solve systems of equations by graphing.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Intersections of Graphs

Develop:

Graphs of Systems of Equations
- Consistent Systems
- Inconsistent Systems
- Number of Solutions, Equations in Slope-Intercept Form
- Number of Solutions, Equations in Standard Form

Solve Systems of Equations by Graphing
- Solve a System by Graphing
- Graph and Solve a System of Equations
- Write a System of Equations

Using Systems to Solve Linear Equations
- Use a System to Solve a Linear Equation

Solving Systems of Equations by Using Graphing Technology
- Solve a System of Equations
- Write and Solve a System of Equations

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Solve Systems of Equations by Graphing	●	●		●
Extension: Systems with Three Equations			●	●

Language Development Handbook

Assign page 38 of the *Language Development Handbook* to help your students build mathematical language related to solving systems of equations by graphing.

ELL You can use the tips and suggestions on page T38 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Standards for Mathematical Practice:

5 Use appropriate tools strategically.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous
Students understood the solution to a system of linear equations in two variables corresponds to the point(s) of intersection of their graphs.
8.EE.8a

Now
Students solve systems of equations by graphing and use systems to solve linear equations. **A.REI.6, A.REI.11**

Next
Students will solve systems of equations by using substitution. **A.CED.3, A.REI.6**

Rigor

The Three Pillars of Rigor

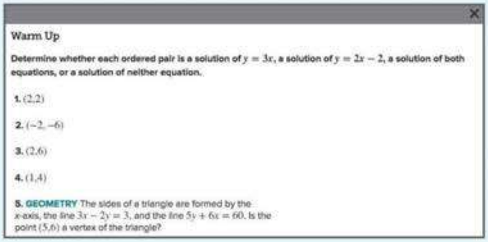
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of graphing linear equations to build fluency with graphing systems of linear equations. They apply their understanding of solving systems of linear equations by solving real-world problems.

Mathematical Background

A solution of a system of two linear equations is an ordered pair that satisfies both equations in the system. A system of equations can be solved by graphing the equations on the same coordinate plane, which can intersect at one point (exactly one solution), be parallel (no solution), or be the same line (infinitely many solutions).

Interactive Presentation

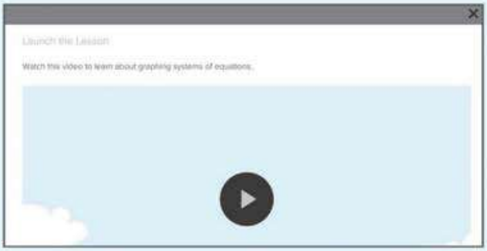


Warm Up

Determine whether each ordered pair is a solution of $y = 3x$, a solution of $y = 2x - 2$, a solution of both equations, or a solution of neither equation.

1. $(2,2)$
2. $(-2,-6)$
3. $(2,6)$
4. $(1,4)$
5. GEOMETRY The sides of a triangle are formed by the x-axis, the line $3x - 2y = 3$, and the line $5y + 6x = 60$. Is the point $(5,6)$ a vertex of the triangle?

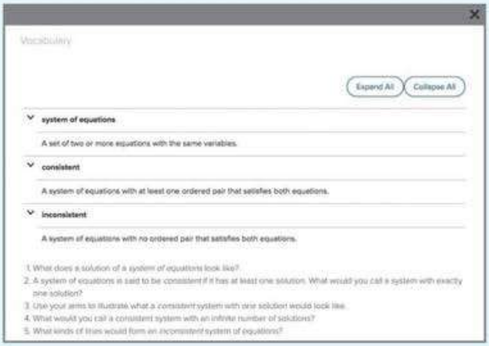
Warm Up



Launch the Lesson

Watch this video to learn about graphing systems of equations.

Launch the Lesson



system of equations

A set of two or more equations with the same variables.

consistent

A system of equations with at least one ordered pair that satisfies both equations.

inconsistent

A system of equations with no ordered pair that satisfies both equations.

1. What does a solution of a system of equations look like?
2. A system of equations is said to be consistent if it has at least one solution. What would you call a system with exactly one solution?
3. Use your arms to illustrate what a consistent system with one solution would look like.
4. What would you call a consistent system with an infinite number of solutions?
5. What kinds of lines would form an inconsistent system of equations?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying the solution of a system of equations

Answers:

1. $y = 2x - 2$
2. both
3. $y = 3x$
4. neither
5. yes

Launch the Lesson

Teaching the Mathematical Practices

5 Analyze Graphs Encourage students to analyze the graph and describe how it can be used to find the number of lawns Laila needs to mow before making a profit.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Intersections of Graphs

Objective

Students use a sketch to explore solving linear equations graphically.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use the sketch. Work with students to explore and deepen their understanding of solving linear equations graphically.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

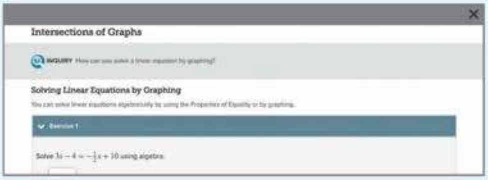
What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to solve an algebraic equation. They will graph the functions defined by the expressions on each side of the equal sign and find their point of intersection. They will compare the coordinates of this point with the solution they find algebraically. They will repeat the process for a second equation. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Intersections of Graphs

INQUIRY How can you solve a linear equation by graphing?

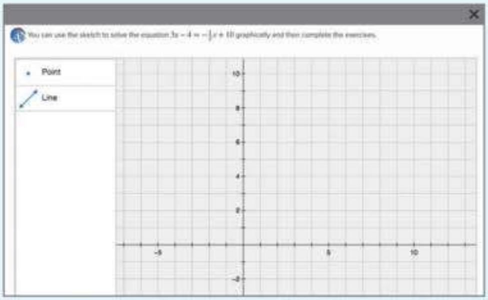
Solving Linear Equations by Graphing

You can solve linear equations algebraically by using the Properties of Equality or by graphing.

Exercise 1

Solve $3x - 4 = -\frac{1}{2}x + 10$ using algebra.

Explore



You can use the sketch to solve the equation $3x - 4 = -\frac{1}{2}x + 10$ graphically and then complete the exercises.

Explore

WEB SKETCHPAD

Students use a sketch to solve an equation graphically.

TYPE

Students answer the exercises about solving equations by graphing.

Interactive Presentation

Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Intersections of Graphs (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is the *x*-value of the point of intersection the solution to the equation? Sample answer: because it is the value of *x* that makes both sides of the equation equal to each other
- What does the *y*-value of the point of intersection represent? the value when each side of the original equation is evaluated at the *x*-value of the point of intersection

Inquiry

How can you solve a linear equation by graphing? Sample answer: Graph each side of the equation as a linear function set equal to *y*. Then, the *x*-coordinate of the point of intersection is the solution of the equation.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphs of Systems of Equations

Objective

Students determine the number of solutions of a system of linear equations by examining the equations and their graphs.

MP Teaching the Mathematical Practices

7 Use Structure Help students explore the structure of systems of equation and their graphs in this Learn.

What Students Are Learning

Students will consider the possible relationships between the graphs of two linear equations. The lines may intersect at a point, may be parallel, or may be the same line. These three different cases produce the three different types of solutions of a system of two linear equations. Also important is the understanding that the solutions of a system are represented by the points that the two graphs have in common. There may be one such point, no points, or infinitely many points.

Example 1 Consistent Systems

MP Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and the system of equations used in this example.

Questions for Mathematical Discourse

AL How do you use the graph to solve the problem? Sample answer: I look to see how many points the two lines have in common.

OL How do you know there is one solution? Sample answer: The graphs intersect in one point.

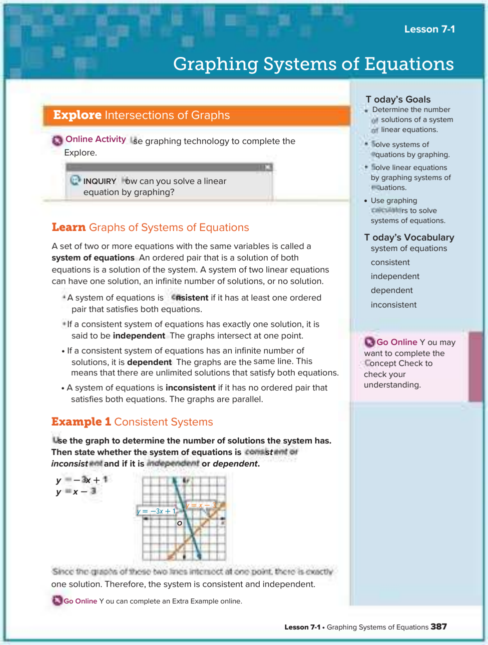
BL Can two lines ever intersect in exactly two points? Explain. No; sample answer: To intersect in exactly two points, one of the lines would not be straight. It would cross the other line once, then turn and cross back through the line again.

Common Error

To support the new terminology introduced in the lesson, encourage students to review the vocabulary for the lesson and to write their own descriptions of each type of system, using words that they find helpful.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 7-1

Graphing Systems of Equations

Explore Intersections of Graphs

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you solve a linear equation by graphing?

Learn Graphs of Systems of Equations

A set of two or more equations with the same variables is called a **system of equations**. An ordered pair that is a solution of both equations is a solution of the system. A system of two linear equations can have one solution, an infinite number of solutions, or no solution.

- A system of equations is **consistent** if it has at least one ordered pair that satisfies both equations.
- If a consistent system of equations has exactly one solution, it is said to be **independent**. The graphs intersect at one point.
- If a consistent system of equations has an infinite number of solutions, it is **dependent**. The graphs are the same line. This means that there are unlimited solutions that satisfy both equations.
- A system of equations is **inconsistent** if it has no ordered pair that satisfies both equations. The graphs are parallel.

Example 1 Consistent Systems

Use the graph to determine the number of solutions the system has. Then state whether the system of equations is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

$y = -3x + 1$
$y = x - 3$

Since the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

Go Online You can complete an Extra Example online.

Today's Goals

- Determine the number of solutions of a system of linear equations.
- Solve systems of equations by graphing.
- Solve linear equations by graphing systems of equations.
- Use graphing calculators to solve systems of equations.

Today's Vocabulary

system of equations
consistent
independent
dependent
inconsistent

Go Online You may want to complete the Concept Check to check your understanding.

Lesson 7-1 • Graphing Systems of Equations 387

Interactive Presentation

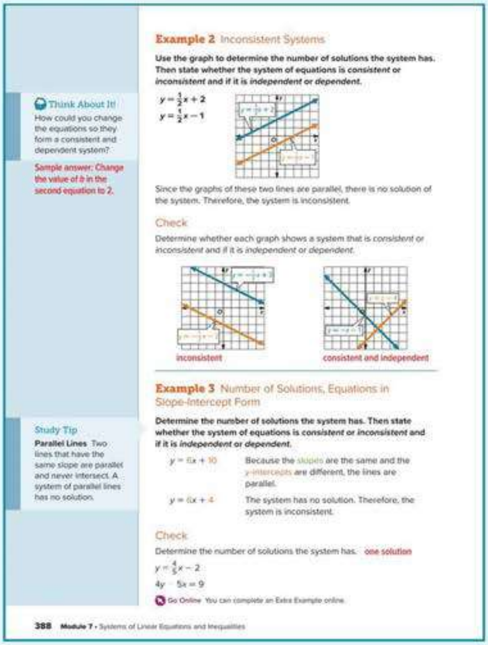
Learn

TAP

Students move through the categories to compare systems of linear equations.

TAP

Students select the slope and y-intercept values to create systems with one, an infinite number, or no solutions.



Example 2 Inconsistent Systems

Use the graph to determine the number of solutions the system has. Then state whether the system of equations is consistent or *inconsistent* and if it is *independent* or *dependent*.

$y = \frac{1}{2}x + 2$
$y = \frac{1}{2}x - 1$

Since the graphs of these two lines are parallel, there is no solution of the system. Therefore, the system is inconsistent.

Check

Determine whether each graph shows a system that is consistent or *inconsistent* and if it is *independent* or *dependent*.

inconsistent

consistent and independent

Example 3 Number of Solutions, Equations in Slope-Intercept Form

Determine the number of solutions the system has. Then state whether the system of equations is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

$y = 6x + 10$ Because the slopes are the same and the y-intercepts are different, the lines are parallel.

$y = 6x + 4$ The system has no solution. Therefore, the system is inconsistent.

Check

Determine the number of solutions the system has. one solution

$y = \frac{4}{5}x - 2$
$4y - 5x = 9$

Go Online You can complete an Extra Example online.

Think About It!

How could you change the equations so they form a consistent and dependent system?

Sample answer: Change the value of *b* in the second equation to 2.

Study Tip

Parallel Lines Two lines that have the same slope are parallel and never intersect. A system of parallel lines has no solution.

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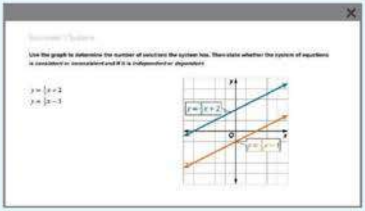
Interactive Presentation



Use the graph to determine the number of solutions the system has. Then state whether the system of equations is consistent or inconsistent and if it is independent or dependent.

Example 2

TAP

Students select the correct category for the system of equations.

TYPE

Students explain how the equations could be changed to form a consistent and dependent system.

Example 2 Inconsistent Systems

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph, system of equations, and solution in this example.

Questions for Mathematical Discourse

AL Explain why the system has no solutions. Sample answer: The solutions of a system are the points the lines have in common, and because these lines are parallel, they will have no points in common. So, there are no solutions.

OL How can you solve the problem without graphing? Sample answer: Because the equations have the same slope but different *y*-intercepts, I know that the lines are parallel and will never intersect. Therefore, I know that the system is inconsistent.

BL Is it possible for a system to be inconsistent and independent? Why? No; sample answer: In order for a system to be independent, it must have exactly one solution. If a system is inconsistent, then it has no solutions.

Example 3 Number of Solutions, Equations in Slope-Intercept Form

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL What do you notice about the slopes of the lines? They are the same.

OL Why is it helpful to examine the equations in slope-intercept form? you can see the slope and the *y*-intercept of each line

BL What types of lines are these? parallel How could you change the second equation to make a system that has infinitely many solutions? set the *y*-intercept at 10

Example 4 Number of Solutions, Equations in Standard Form

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What is slope-intercept form? $y = mx + b$

OL Why is it useful to have equations in slope-intercept form when determining the number of solutions? Sample answer: It allows you to easily see the slope and y-intercept, which can help you to determine if a system has one, none, or infinitely many solutions.

BL Explain how you could determine that these equations are equivalent without solving for y. Sample answer: Divide the first equation by 2 and the second equation by -3, and you will get the same equation.

Common Error

Some students may state that the system is inconsistent, thinking that because the two equations are equivalent, there is “only one equation” and, therefore, no point of intersection. Correct this line of reasoning, explaining that all points on the line satisfy both equations and, for that reason, there are infinitely many solutions, and the system is consistent and dependent.

Learn Solve Systems of Equations by Graphing

Objective

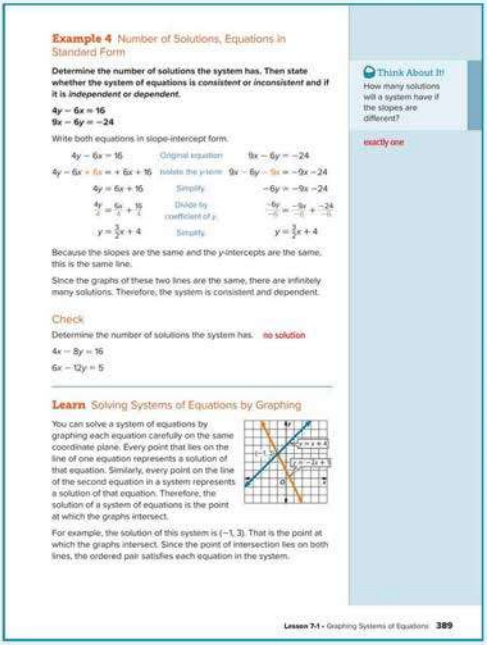
Students solve systems of equations by graphing.

Teaching the Mathematical Practices

5 Analyze Graphs In this Learn, students learn how to solve a system of equations by analyzing a graph.

What Students Are Learning

Students are learning that they can find the solution of a system of linear equations by graphing the equations in the system and identifying the point where the lines intersect. Because this point represents the solution of each equation, its coordinates satisfy both equations, which is shown by the algebraic check of the solution.



Example 4 Number of Solutions, Equations in Standard Form

Determine the number of solutions the system has. Then state whether the system of equations is consistent or inconsistent and if it is *independent* or *dependent*.

$4y - 6x = 16$
$9x - 6y = -24$

Write both equations in slope-intercept form.

$4y - 6x = 16$	Original equation	$9x - 6y = -24$
$4y - 6x + 6x = + 6x + 16$	Isolate the y-term.	$9x - 6y - 9x = -9x - 24$
$4y = 6x + 16$	Simplify.	$-6y = -9x - 24$
$\frac{4y}{4} = \frac{6x}{4} + \frac{16}{4}$	Divide by coefficient of y.	$\frac{-6y}{-6} = \frac{-9x}{-6} + \frac{-24}{-6}$
$y = \frac{3}{2}x + 4$	Simplify.	$y = \frac{3}{2}x + 4$

Because the slopes are the same and the y-intercepts are the same, this is the same line.

Since the graphs of these two lines are the same, there are infinitely many solutions. Therefore, the system is consistent and dependent.

Think About It! How many solutions will a system have if the slopes are different? exactly one

Check

Determine the number of solutions the system has. no solution

$4x - 8y = 16$
$6x - 12y = 5$

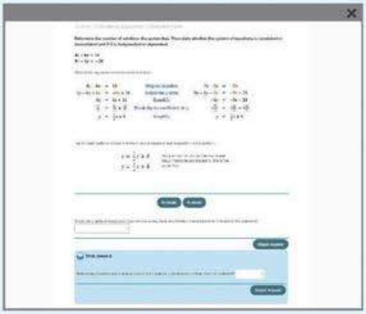
Learn Solving Systems of Equations by Graphing

You can solve a system of equations by graphing each equation carefully on the same coordinate plane. Every point that lies on the line of one equation represents a solution of that equation. Similarly, every point on the line of the second equation in a system represents a solution of that equation. Therefore, the solution of a system of equations is the point at which the graphs intersect.

For example, the solution of this system is $(-1, 3)$. That is the point at which the graphs intersect. Since the point of intersection lies on both lines, the ordered pair satisfies each equation in the system.

Lesson 7-1 • Graphing Systems of Equations 389

Interactive Presentation



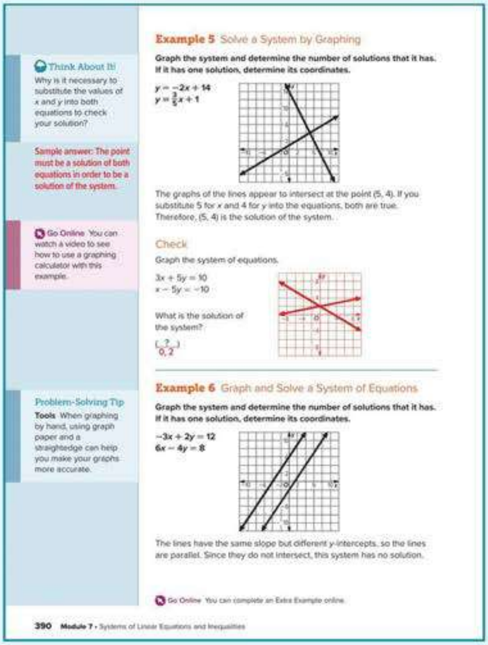
Example 4

TAP

Students select the correct category and number of solutions for the system of equations.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Think About It! Why is it necessary to substitute the values of x and y into both equations to check your solution?

Sample answer: The point must be a solution of both equations in order to be a solution of the system.

Go Online You can watch a video to see how to use a graphing calculator with this example.

Problem-Solving Tip

Tools When graphing by hand, using graph paper and a straightedge can help you make your graphs more accurate.

Example 5 Solve a System by Graphing

Graph the system and determine the number of solutions that it has. If it has one solution, determine its coordinates.

$y = -2x + 14$
$y = \frac{3}{5}x + 1$

The graphs of the lines appear to intersect at the point (5, 4). If you substitute 5 for x and 4 for y into the equations, both are true. Therefore, (5, 4) is the solution of the system.

Check

Graph the system of equations.

$3x + 5y = 10$
$x - 5y = -10$

What is the solution of the system?

$(\underline{\ ?\ })$ 0, 2

Example 6 Graph and Solve a System of Equations

Graph the system and determine the number of solutions that it has. If it has one solution, determine its coordinates.

$-3x + 2y = 12$
$6x - 4y = 8$

The lines have the same slope but different y-intercepts, so the lines are parallel. Since they do not intersect, this system has no solution.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Graph the system and determine the number of solutions that it has. If it has one solution, name it.

$y = -2x + 14$
$y = \frac{3}{5}x + 1$

Example 5

WEB SKETCHPAD

Students use a sketch to graph a system of equations and find the intersection.

TYPE

Students enter the coordinates for the point of intersection and explain why it is necessary to check the point in both equations.

Example 5 Solve a System by Graphing

Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students need to check their answer. Point out that they should ask themselves whether their answer makes sense and whether they have answered the problem question.

Questions for Mathematical Discourse

AL What are the slopes of each equation? $-2, \frac{3}{5}$

OL How do you know by looking at the equations that there is one solution or no solutions to this system? The slopes are different.

BL Explain how to check to make sure the solution is correct. Sample answer: Substitute 5 for x and substitute 4 for y in both equations. Simplify, and make sure both resulting equations are true.

Example 6 Graph and Solve a System of Equations

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this example, students will need to use a sketch. Work with students to explore and deepen their understanding of graphing and solving systems of linear equations.

Questions for Mathematical Discourse

AL Why does it help to write the equations in slope-intercept form before graphing? Sample answer: Using slope-intercept form, you can easily see the slopes and y-intercepts of the equations.

OL What do you notice about the equations once they are written in slope-intercept form? They have the same slope, but different y-intercepts.

BL How could this system be changed so that it has infinitely many solutions? Sample answer: Change 8 in the second equation to -24 so that the y-intercepts are the same.

Common Error

If students make an error when writing the equations in slope-intercept form but check their answer in the slope-intercept forms of the equations, they will not recognize that their answer is incorrect. To help students avoid making this error, encourage them to use the original equations for the check.

Apply Example 7 Write a System of Equations

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- At what *x* value and at what *y* value do the two lines intersect?
- W hat does the intersection of the two lines represent in the context of the situation?

Write About It!

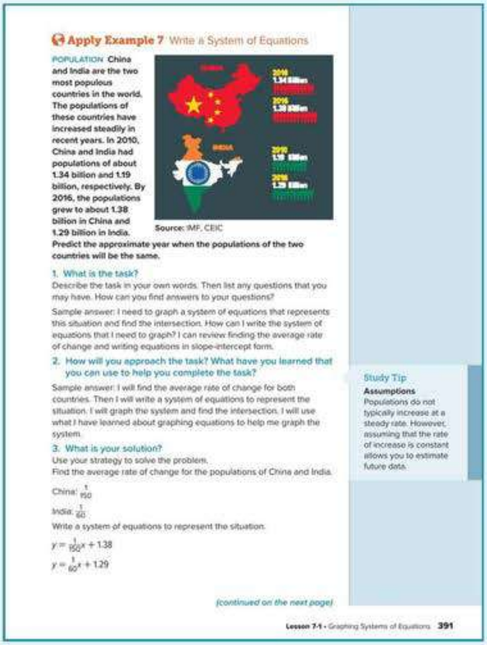
Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

DIFFERENTIATE

Reteaching Activity AL

IF students are having difficulty solving problems involving real-world applications of systems of equations,

THEN pair them with students who seem to have a better grasp of the concept, and have them work together to prepare a presentation of their solution. Require that the presentation include a clear explanation of how the mathematics relates to the situation.



Apply Example 7 Write a System of Equations

POPULATION **China and India are the two most populous countries in the world. The populations of these countries have increased steadily in recent years. In 2010, China and India had populations of about 1.34 billion and 1.19 billion, respectively. By 2016, the populations grew to about 1.38 billion in China and 1.29 billion in India.**

Source: IMF, CEIC

Predict the approximate year when the populations of the two countries will be the same.

1. What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to graph a system of equations that represents this situation and find the intersection. How can I write the system of equations that I need to graph? I can review finding the average rate of change and writing equations in slope-intercept form.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will find the average rate of change for both countries. Then I will write a system of equations to represent the situation. I will graph the system and find the intersection. I will use what I have learned about graphing equations to help me graph the system.

3. What is your solution?

Use your strategy to solve the problem.

Find the average rate of change for the populations of China and India.

China: $\frac{1}{150}$

India: $\frac{1}{60}$

Write a system of equations to represent the situation.

$y = \frac{1}{150}x + 1.38$

$y = \frac{1}{60}x + 1.29$

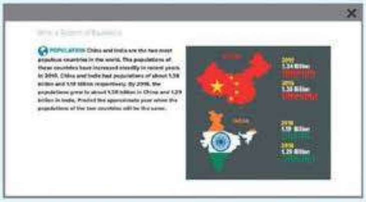
(continued on the next page)

Study Tip

Assumptions
Populations do not typically increase at a steady rate. However, assuming that the rate of increase is constant allows you to estimate future data.

Lesson 7-1 • Graphing Systems of Equations 391

Interactive Presentation



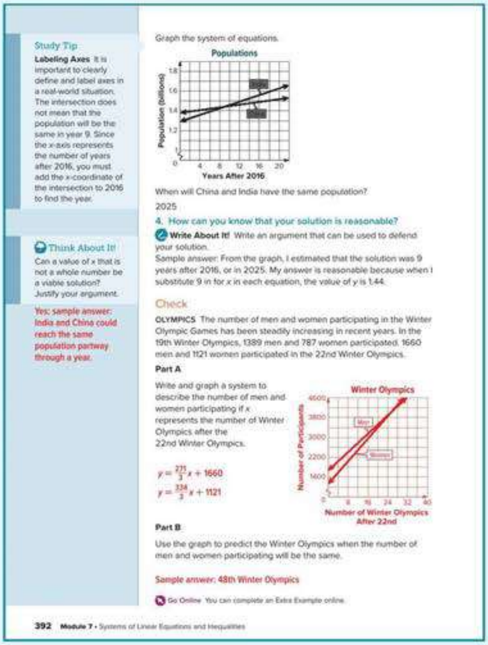
Apply Example 7

TYPE

Students determine if a value of *x* that is not a whole number can be a viable solution.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Study Tip

Labeling Axes It is important to clearly define and label axes in a real-world situation. The intersection does not mean that the population will be the same in year 9. Since the x-axis represents the number of years after 2016, you must add the x-coordinate of the intersection to 2016 to find the year.

Graph the system of equations.

When will China and India have the same population?

2025

4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: From the graph, I estimated that the solution was 9 years after 2016, or in 2025. My answer is reasonable because when I substitute 9 in for x in each equation, the value of y is 1.44.

Think About It!

Can a value of x that is not a whole number be a viable solution? Justify your argument.

Yes; sample answer: India and China could reach the same population partway through a year.

Check

OLYMPICS The number of men and women participating in the Winter Olympic Games has been steadily increasing in recent years. In the 19th Winter Olympics, 1389 men and 787 women participated. 1660 men and 1121 women participated in the 22nd Winter Olympics.

Part A

Write and graph a system to describe the number of men and women participating if x represents the number of Winter Olympics after the 22nd Winter Olympics.

$y = \frac{271}{3}x + 1660$

$y = \frac{334}{3}x + 1121$

Part B

Use the graph to predict the Winter Olympics when the number of men and women participating will be the same.

Sample answer: 48th Winter Olympics

Go Online You can complete an Extra Example online.

392 **Module 7** • Systems of Linear Equations and Inequalities

Interactive Presentation

Question 2

Part A

$y = 1389x + 1660$
$y = 787x + 1121$

$y = \frac{271}{3}x + 1121$
$y = \frac{334}{3}x + 1660$

Check

MULTIPLE CHOICE

Students select the system that best describes the situation.

Essential Question Follow-Up

Students have used systems of equations to solve problems involving real-world situations.

Ask:

How can a system of equations help a business owner make decisions? Sample answer: It can help him or her see how the costs of running the business compare to the money the business takes in. This could be helpful when making decisions about pricing and expenditures.

Common Error

Some students may have difficulty graphing the equations because they contain slopes that are decimals. Encourage students to convert the slopes to fractions in lowest terms before graphing.

DIFFERENTIATE

Enrichment Activity BL

Write a system of three equations in two variables on the board. Have the students determine if the system has one solution, no solution, or infinitely many solutions. If it has one solution, have students name it. For example, the following system has no solution because the three lines do not intersect at one point.

$x + y = 2$
$x - y = 0$
$y = -2$

Learn Using Systems to Solve Linear Equations

Objective

Students solve linear equations by graphing systems of equations.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

About the Key Concept

Rewriting an equation as a system of equations allows for the solution to be found by graphing the resulting equations and finding the x-value of their point of intersection. This is the value that makes the expressions on the two sides of the original equation equal in value. When those expressions are evaluated for the x-value, the result is the corresponding y-value of the point of intersection.

Common Misconception

Some students may believe that the solution that is found using this procedure is somehow different in meaning than the solution found by solving algebraically. Address this misconception by solving an equation using both methods, showing that the solutions are the same, and emphasizing what the solution represents in terms of the original equation.

Example 8 Use a System to Solve a Linear Equation

Teaching the Mathematical Practices

5 Analyze Graphs Help students analyze the graph they have generated using graphing calculators. Point out that to see the entire graph, students may need to adjust the viewing window.

Questions for Mathematical Discourse

AL What type of line is $y = -4$? horizontal

OL What do you expect the y-value of your solution to be? -4 Why? Sample answer: The y-value has to be -4 because the solution has to satisfy both equations, and the second equation will be satisfied only when $y = -4$.

BL How can you check your solution? Sample answer: Replace x with 2 in the original equation and see if the equation balances.

Common Error

Some students may state that the answer is the ordered pair $(2, -4)$. These students may not understand that the system of equations is being used as a tool to find the value of x that makes the original equation true. Although $(2, -4)$ is the point of intersection of the two equations in the system, the solution of the original equation is $x = 2$.

Learn Solving Systems of Equations by Using Graphing Technology

Objective

Students solve systems of equations by using graphing technology.

Teaching the Mathematical Practices

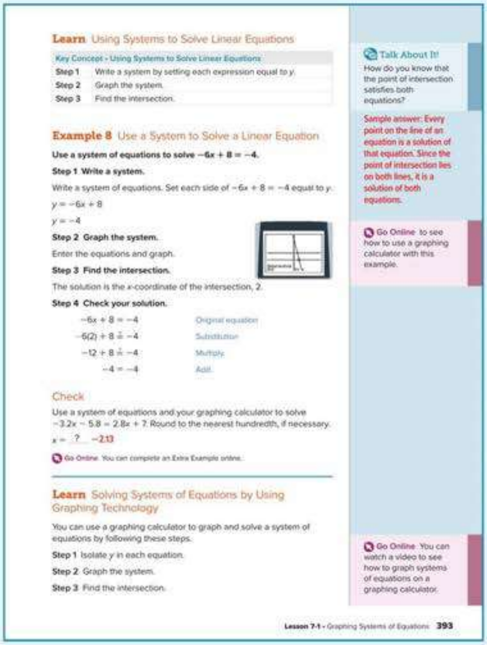
5 Use Mathematical Tools Students will use graphing technology to find an approximation of the solution of a system of linear equations.

Things to Remember

Students may need to be reminded to change the viewing window on their calculators in order to find the point of intersection of the lines.

Common Misconception

Some students may believe that the calculator will give an exact answer in all situations. Remind them of the fact that, in some instances, a number given on the display may have been rounded due to the number of digits in the decimal.



Learn Using Systems to Solve Linear Equations

Key Concept • Using Systems to Solve Linear Equations	
Step 1	Write a system by setting each expression equal to y.
Step 2	Graph the system.
Step 3	Find the intersection.

Example 8 Use a System to Solve a Linear Equation

Use a system of equations to solve $-6x + 8 = -4$.

Step 1 Write a system.

Write a system of equations. Set each side of $-6x + 8 = -4$ equal to y.

$y = -6x + 8$

$y = -4$

Step 2 Graph the system.

Enter the equations and graph.

Step 3 Find the intersection.

The solution is the x-coordinate of the intersection, 2.

Step 4 Check your solution.

$-6x + 8 = -4$ Original equation

$-6(2) + 8 \stackrel{?}{=} -4$ Substitution

$-12 + 8 \stackrel{?}{=} -4$ Multiply.

$-4 = -4$ Add.

Check

Use a system of equations and your graphing calculator to solve $-3.2x - 5.8 = 2.8x + 7$. Round to the nearest hundredth, if necessary.

$x =$? -2.13

Go Online You can complete an Extra Example online.

Learn Solving Systems of Equations by Using Graphing Technology

You can use a graphing calculator to graph and solve a system of equations by following these steps.

Step 1 Isolate y in each equation.

Step 2 Graph the system.

Step 3 Find the intersection.

Talk About It! How do you know that the point of intersection satisfies both equations?

Sample answer: Every point on the line of an equation is a solution of that equation. Since the point of intersection lies on both lines, it is a solution of both equations.

Go Online to see how to use a graphing calculator with this example.

Go Online You can watch a video to see how to graph systems of equations on a graphing calculator.

Lesson 7-1 • Graphing Systems of Equations 393

Interactive Presentation

Use a System to Solve a Linear Equation

Use a system of equations to solve $-6x + 8 = -4$.

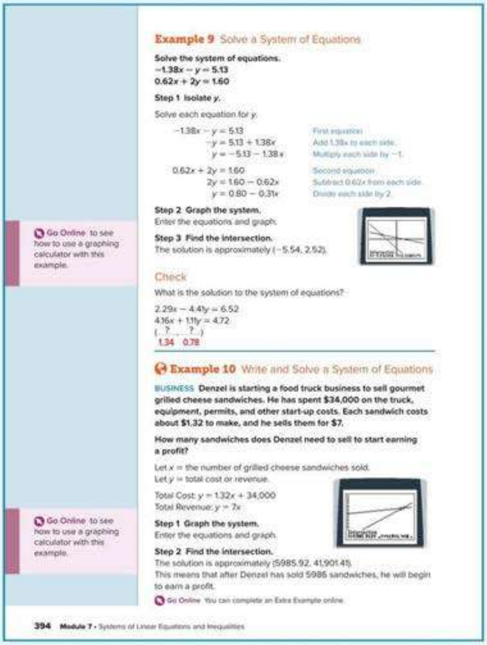
Example 8

TAP

Students move through the steps of solving an equation using a graphing calculator.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Example 9 Solve a System of Equations

Solve the system of equations.
$-1.38x - y = 5.13$
$0.62x + 2y = 1.60$

Step 1 Isolate *y*.

Solve each equation for *y*.

$$\begin{aligned} -1.38x - y &= 5.13 && \text{First equation} \\ -y &= 5.13 + 1.38x && \text{Add } 1.38x \text{ to each side.} \\ y &= -5.13 - 1.38x && \text{Multiply each side by } -1. \end{aligned}$$

$$\begin{aligned} 0.62x + 2y &= 1.60 && \text{Second equation} \\ 2y &= 1.60 - 0.62x && \text{Subtract } 0.62x \text{ from each side.} \\ y &= 0.80 - 0.31x && \text{Divide each side by 2.} \end{aligned}$$

Step 2 Graph the system.
Enter the equations and graph.

Step 3 Find the intersection.
The solution is approximately (−5.54, 2.52).

Go Online to see how to use a graphing calculator with this example.

Check

What is the solution to the system of equations?

$2.29x - 4.41y = 6.52$
$4.16x + 1.11y = 4.72$
(? , ?)
1.34 0.78

Example 10 Write and Solve a System of Equations

BUSINESS **Denzel is starting a food truck business to sell gourmet grilled cheese sandwiches. He has spent $34,000 on the truck, equipment, permits, and other start-up costs. Each sandwich costs about $1.32 to make, and he sells them for $7.**

How many sandwiches does Denzel need to sell to start earning a profit?

Let *x* = the number of grilled cheese sandwiches sold.
Let *y* = total cost or revenue.

Total Cost: $y = 1.32x + 34{,}000$
Total Revenue: $y = 7x$

Step 1 Graph the system.
Enter the equations and graph.

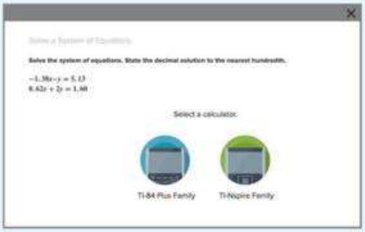
Step 2 Find the intersection.
The solution is approximately (5985.92, 41,901.45).
This means that after Denzel has sold 5986 sandwiches, he will begin to earn a profit.

Go Online to see how to use a graphing calculator with this example.

Go Online You can complete an Extra Example online.

394 Module 7 • Systems of Linear Equations and Inequalities

Interactive Presentation



Solve the system of equations. State the decimal solution to the nearest hundredth.

$-1.38x - y = 5.13$
$0.62x + 2y = 1.60$

Select a calculator.

Example 9

TAP

Students move through the steps of solving a system of equations using a graphing calculator.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 9 Solve a System of Equations

Teaching the Mathematical Practices

6 Use Precision In this example, students learn how to calculate accurately and efficiently and to express numerical answers with a degree of precision appropriate to the problem context.

Questions for Mathematical Discourse

AL What does it mean to solve the system of equations? Sample answer: It means to find the ordered pair that satisfies both equations.

OL Why do you need to solve each equation for *y*? Sample answer: The calculator graphing functionality needs equations to be solved for *y*.

BL How can you check your answer? Sample answer: I can substitute the values of *x* and *y* into the original equations and see if they satisfy the equations, accounting for the fact that the values of *x* and *y* are approximations.

Example 10 Write and Solve a System of Equations

Teaching the Mathematical Practices

2 Consider Units Point out that it is important to note the units involved in this problem.

Questions for Mathematical Discourse

AL Why is the total cost represented by the equation $y = 1.32x + 34{,}000$? Sample answer: Denzel spent $34,000 for the truck and spends $1.32 to make each sandwich.

OL What does the point of intersection of the graphs represent? Sample answer: It represents the point at which Denzel’s costs equal his revenue.

BL How does the graph show the relationship between cost and revenue? Sample answer: When the graph of the cost function lies above the graph of the revenue function, Denzel’s costs are more than his revenue. To the right of the point of intersection, the graph of the revenue function lies above the graph of the cost function, so that means he is making a profit.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

A.REI.6, A.REI.11

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–29
2	exercises that use a variety of skills from this lesson	30–39
2	exercises that extend concepts learned in this lesson to new contexts	40–44
3	exercises that emphasize higher-order and critical-thinking skills	45–51

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice Exercises 1–43 odd, 45–51
- Extension: Systems with Three Equations
- ALEKS Systems of Linear Equations

IF students score 66%–89% on the Checks, **THEN** assign: OL

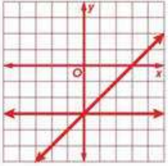
- Practice Exercises 1–51 odd
- Remediation, Review Resources: Solve Systems of Equations by Graphing
- Personal Tutors
- Extra Examples 1–10
- ALEKS Systems of Equations

IF students score 65% or less on the Checks, **THEN** assign: AL

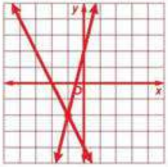
- Practice, Exercises 1–29 odd
- Remediation, Review Resources: Solve Systems of Equations by Graphing
- *Quick Review Math Handbook*: Graphing Systems of Equations
- ArriveMATH Take Another Look
- ALEKS Systems of Equations

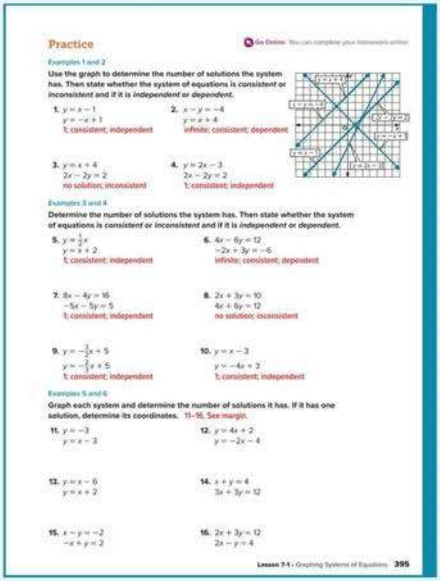
Answers

11. 1 solution; (0, −3)



12. 1 solution; (−1, −2)





Practice

Go Online You can complete your homework online.

Examples 1 and 2

Use the graph to determine the number of solutions the system has. Then state whether the system of equations is consistent or inconsistent and if it is *independent* or *dependent*.

1. $y = x - 1$
$y = -x + 1$
1; consistent; independent

2. $x - y = -4$
$y = x + 4$
infinite; consistent; dependent

3. $y = x + 4$
$2x - 2y = 2$
no solution; inconsistent

4. $y = 2x - 3$
$2x - 2y = 2$
1; consistent; independent

Examples 3 and 4

Determine the number of solutions the system has. Then state whether the system of equations is consistent or inconsistent and if it is *independent* or *dependent*.

5. $y = \frac{1}{2}x$
$y = x + 2$
1; consistent; independent

6. $4x - 6y = 12$
$-2x + 3y = -6$
infinite; consistent; dependent

7. $8x - 4y = 16$
$-5x - 5y = 5$
1; consistent; independent

8. $2x + 3y = 10$
$4x + 6y = 12$
no solution; inconsistent

9. $y = -\frac{3}{2}x + 5$
$y = -\frac{2}{3}x + 5$
1; consistent; independent

10. $y = x - 3$
$y = -4x + 3$
1; consistent; independent

Examples 5 and 6

Graph each system and determine the number of solutions it has. If it has one solution, determine its coordinates. 11–16. See margin.

11. $y = -3$
$y = x - 3$

12. $y = 4x + 2$
$y = -2x - 4$

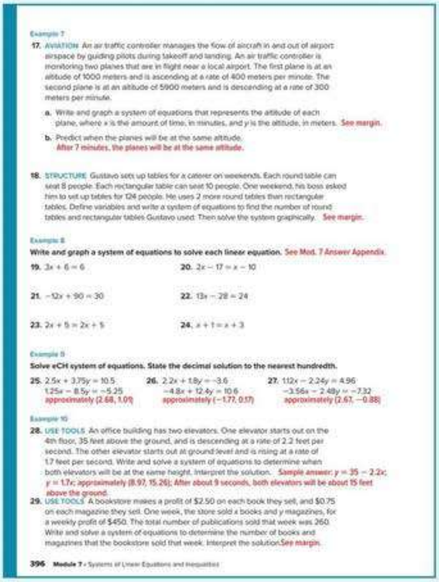
13. $y = x - 6$
$y = x + 2$

14. $x + y = 4$
$3x + 3y = 12$

15. $x - y = -2$
$-x + y = 2$

16. $2x + 3y = 12$
$2x - y = 4$

Lesson 7-1 • Graphing Systems of Equations **395**



Example 7

17. AVIATION An air traffic controller manages the flow of aircraft in and out of airport airspace by guiding pilots during takeoff and landing. An air traffic controller is monitoring two planes that are in flight near a local airport. The first plane is at an altitude of 1000 meters and is ascending at a rate of 400 meters per minute. The second plane is at an altitude of 5900 meters and is descending at a rate of 300 meters per minute.

a. Write and graph a system of equations that represents the altitude of each plane, where x is the amount of time, in minutes, and y is the altitude, in meters. See margin.

b. Predict when the planes will be at the same altitude.
After 7 minutes, the planes will be at the same altitude.

18. STRUCTURE Gustavo sets up tables for a caterer on weekends. Each round table can seat 8 people. Each rectangular table can seat 10 people. One weekend, his boss asked him to set up tables for 124 people. He uses 2 more round tables than rectangular tables. Define variables and write a system of equations to find the number of round tables and rectangular tables Gustavo used. Then solve the system graphically. See margin.

Example 8

Write and graph a system of equations to solve each linear equation. See Mod. 7 Answer Appendix.

19. $3x + 6 = 6$

20. $2x - 17 = x - 10$

21. $-12x + 90 = 30$

22. $13x - 28 = 24$

23. $2x + 5 = 2x + 5$

24. $x + 1 = x + 3$

Example 9

Solve eCH system of equations. State the decimal solution to the nearest hundredth.

25. $2.5x + 3.75y = 10.5$
$1.25x - 8.5y = -5.25$
approximately (2.68, 1.01)

26. $2.2x + 1.8y = -3.6$
$-4.8x + 12.4y = 10.6$
approximately (−1.77, 0.17)

27. $1.12x - 2.24y = 4.96$
$-3.56x - 2.48y = -7.32$
approximately (2.67, −0.88)

Example 10

28. USE TOOLS An office building has two elevators. One elevator starts out on the 4th floor, 35 feet above the ground, and is descending at a rate of 2.2 feet per second. The other elevator starts out at ground level and is rising at a rate of 1.7 feet per second. Write and solve a system of equations to determine when both elevators will be at the same height. Interpret the solution. Sample answer: $y = 35 - 2.2x$; $y = 1.7x$; approximately (8.97, 15.26); After about 9 seconds, both elevators will be about 15 feet above the ground.

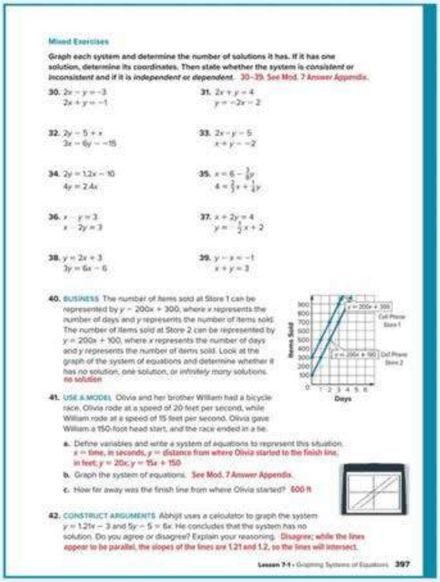
29. USE TOOLS A bookstore makes a profit of \$2.50 on each book they sell, and \$0.75 on each magazine they sell. One week, the store sold x books and y magazines, for a weekly profit of \$450. The total number of publications sold that week was 260. Write and solve a system of equations to determine the number of books and magazines that the bookstore sold that week. Interpret the solution. See margin.

396 **Module 7** • Systems of Linear Equations and Inequalities

3 REFLECT AND PRACTICE

A.REI.6, A.REI.11

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**



Mixed Exercises

Graph each system and determine the number of solutions it has. If it has one solution, determine its coordinates. Then state whether the system is *consistent* or *inconsistent* and if it is *independent* or *dependent*. 30–39. See Mod. 7 Answer Appendix.

30. $2x - y = -3$
$2x + y = -1$

31. $2x + y = 4$
$y = -2x - 2$

32. $2y = 5 + x$
$3x - 6y = -15$

33. $2x - y = 5$
$x + y = -2$

34. $2y = 1.2x - 10$
$4y = 2.4x$

35. $x = 6 - \frac{3}{8}y$
$4 = \frac{2}{3}x + \frac{1}{4}y$

36. $x - y = 3$
$x - 2y = 3$

37. $x + 2y = 4$
$y = -\frac{1}{2}x + 2$

38. $y = 2x + 3$
$3y = 6x - 6$

39. $y - x = -1$
$x + y = 3$

40. BUSINESS The number of items sold at Store 1 can be represented by $y = 200x + 300$, where x represents the number of days and y represents the number of items sold. The number of items sold at Store 2 can be represented by $y = 200x + 100$, where x represents the number of days and y represents the number of items sold. Look at the graph of the system of equations and determine whether it has no solution, one solution, or infinitely many solutions. no solution

41. USE A MODEL Olivia and her brother William had a bicycle race. Olivia rode at a speed of 20 feet per second, while William rode at a speed of 15 feet per second. Olivia gave William a 150-foot head start, and the race ended in a tie.

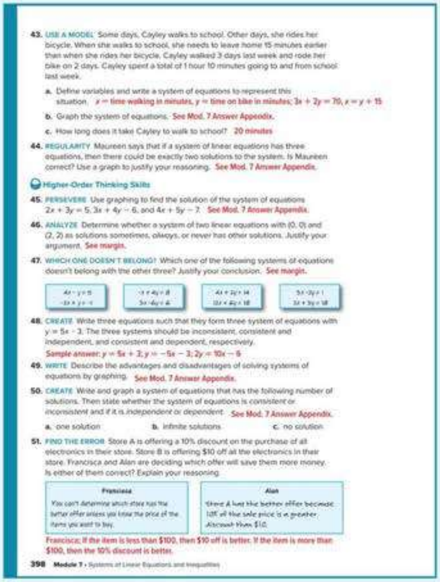
a. Define variables and write a system of equations to represent this situation. x = time, in seconds, y = distance from where Olivia started to the finish line, in feet; $y = 20x$; $y = 15x + 150$

b. Graph the system of equations. See Mod. 7 Answer Appendix.

c. How far away was the finish line from where Olivia started? 600 ft

42. CONSTRUCT ARGUMENTS Abhijit uses a calculator to graph the system $y = 1.21x - 3$ and $5y - 5 = 6x$. He concludes that the system has no solution. Do you agree or disagree? Explain your reasoning. Disagree; while the lines appear to be parallel, the slopes of the lines are 1.21 and 1.2, so the lines will intersect.

Lesson 7-1 • Graphing Systems of Equations **397**



43. USE A MODEL Some days, Cayley walks to school. Other days, she rides her bicycle. When she walks to school, she needs to leave home 15 minutes earlier than when she rides her bicycle. Cayley walked 3 days last week and rode her bike on 2 days. Cayley spent a total of 1 hour 10 minutes going to and from school last week.

a. Define variables and write a system of equations to represent this situation. x = time walking in minutes, y = time on bike in minutes; $3x + 2y = 70$, $x = y + 15$

b. Graph the system of equations. See Mod. 7 Answer Appendix.

c. How long does it take Cayley to walk to school? 20 minutes

44. REGULARITY Maureen says that if a system of linear equations has three equations, then there could be exactly two solutions to the system. Is Maureen correct? Use a graph to justify your reasoning. See Mod. 7 Answer Appendix.

Higher-Order Thinking Skills

45. PERSEVERE Use graphing to find the solution of the system of equations $2x + 3y = 5$, $3x + 4y = 6$, and $4x + 5y = 7$. See Mod. 7 Answer Appendix.

46. ANALYZE Determine whether a system of two linear equations with (0, 0) and (2, 2) as solutions sometimes, always, or never has other solutions. Justify your argument. See margin.

47. WHICH ONE DOESN'T BELONG? Which one of the following systems of equations doesn't belong with the other three? Justify your conclusion. See margin.

$4x - y = 5$ $-2x + y = -1$	$-x + 4y = 8$ $3x - 6y = 6$	$4x + 2y = 14$ $12x + 6y = 18$	$5x - 3y = 1$ $3x + 3y = 18$

48. CREATE Write three equations such that they form three system of equations with $y = 5x - 3$. The three systems should be inconsistent, consistent and independent, and consistent and dependent, respectively.
Sample answer: $y = 5x + 3$; $y = -5x - 3$; $2y = 10x - 6$

49. WRITE Describe the advantages and disadvantages of solving systems of equations by graphing. See Mod. 7 Answer Appendix.

50. CREATE Write and graph a system of equations that has the following number of solutions. Then state whether the system of equations is consistent or inconsistent and if it is independent or dependent. See Mod. 7 Answer Appendix.

a. one solution
b. infinite solutions
c. no solution

51. FIND THE ERROR Store A is offering a 10% discount on the purchase of all electronics in their store. Store B is offering \$10 off all the electronics in their store. Francisca and Alan are deciding which offer will save them more money. Is either of them correct? Explain your reasoning.

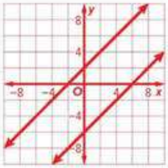
Francisca	Alan
You can't determine which store has the better offer unless you know the price of the items you want to buy.	Store A has the better offer because 10% of the sale price is a greater discount than \$10.

Francisca; If the item is less than \$100, then \$10 off is better. If the item is more than \$100, then the 10% discount is better.

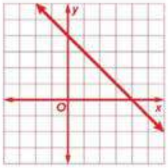
398 **Module 7** • Systems of Linear Equations and Inequalities

Answers

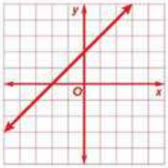
13. no solution



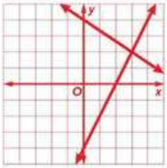
14. infinitely many solutions



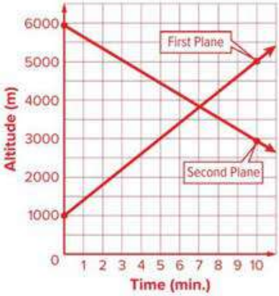
15. infinitely many solutions



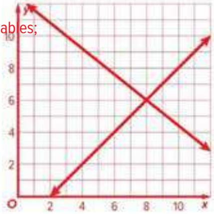
16. 1 solution; (3, 2)



17a. $y = 400x + 1000$; $y = 5900 - 300x$



18. x = the number of round tables, y = the number of rectangular tables; $8x + 10y = 124$; $y = x - 2$; 8 round tables and 6 rectangular tables



29. Sample answer: $x + y = 260$; $2.5x + 0.75y = 450$; approximately (145.71, 114.29); The bookstore will make a weekly profit of \$450 with total weekly sales of 260 publications when about 146 books and about 114 magazines are sold.

46. Always; if the equations are linear and have more than one common solution, they must be consistent and dependent, which means that they have an infinite number of solutions in common.

47. Sample answer: $4x + 2y = 14$, $12x + 6y = 18$; This system is inconsistent, while the others are consistent and independent.

Lesson 7-2

Substitution

A.CED.3, A.REI.6

LESSON GOAL

Students solve systems of equations by using substitution.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Using Substitution

Develop:

Solving Systems of Equations by Substitution

- Solve a System by Substitution
- Solve and Then Substitute
- Use Substitution When There Are No or Many Solutions
- Write and Solve a System of Equations

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Using Formulas	●	●		●
Extension: Intersection of Two Parabolas		●	●	●

Language Development Handbook

Assign page 39 of the *Language Development Handbook* to help your students build mathematical language related to solving systems of equations by using substitution.

ELL You can use the tips and suggestions on page T39 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

6 Attend to precision.

Coherence

Vertical Alignment

Previous

Students solved systems of equations by graphing and used systems to solve linear equations.

8.EE.8, A.REI.6, A.REI.11

Now

Students solve systems of equations by using substitution.

A.CED.3, A.REI.6

Next

Students will solve systems of equations by using elimination with addition or subtraction.

A.CED.3, A.REI.6

Rigor

The Three Pillars of Rigor

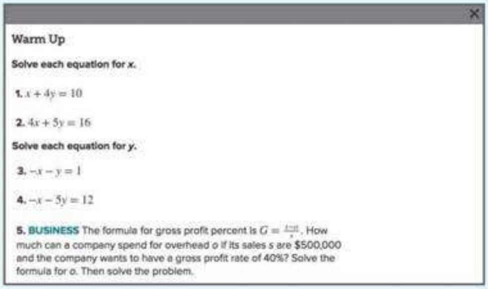
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of using algebraic methods to solve systems of linear equations. They build fluency by using substitution to solve systems of equations, and they apply their understanding by solving real-world problems.

Mathematical Background

Solving a system by substitution involves solving one equation for a specific variable and then substituting the resulting expression in for the variable in the other equation.

Interactive Presentation



Warm Up

Solve each equation for *x*.

1. $x + 4y = 10$

2. $4x + 5y = 16$

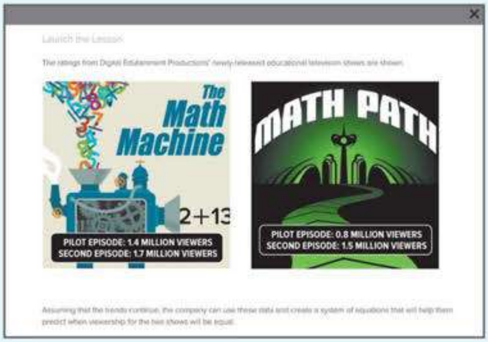
Solve each equation for *y*.

3. $-x - y = 1$

4. $-x - 5y = 12$

5. **BUSINESS** The formula for gross profit percent is $G = \frac{s-o}{s}$. How much can a company spend for overhead *o* if its sales *s* are $500,000 and the company wants to have a gross profit rate of 40%? Solve the formula for *o*. Then solve the problem.

Warm Up



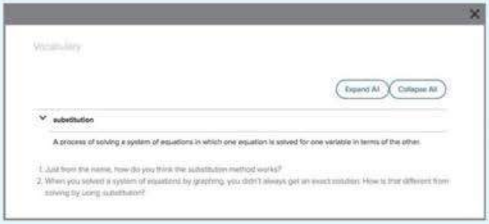
Launch the Lesson

The ratings from Digital Edutainment Productions' newly released educational television shows are shown.

The Math Machine
2+13
PILOT EPISODE: 1.4 MILLION VIEWERS
SECOND EPISODE: 1.7 MILLION VIEWERS
MATH PATH
PILOT EPISODE: 0.8 MILLION VIEWERS
SECOND EPISODE: 1.5 MILLION VIEWERS

Assuming that the trends continue, the company can use these data and create a system of equations that will help them predict when viewership for the two shows will be equal.

Launch the Lesson



Vocabulary

substitution

A process of solving a system of equations in which one equation is solved for one variable in terms of the other.

1. Just from the name, how do you think the substitution method works?
2. When you solved a system of equations by graphing, you didn't always get an exact solution. How is that different from solving by using substitution?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- solving for a specific variable

Answers:

1. $x = -4y + 10$
2. $x = -\frac{5}{4}y + 4$
3. $y = -x - 1$
4. $y = -\frac{1}{5}x - \frac{12}{5}$
5. $s - Gs = o$; $300,000

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Guide students to use the information given in the Launch about two television shows to create a system of equations modeling the solution.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Explore Using Substitution

Objective

Students explore solving a system of equations by using substitution.

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

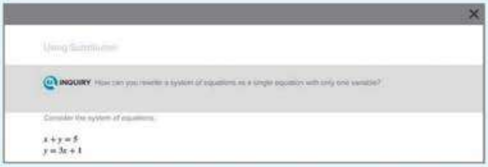
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will work through a series of guiding exercises requiring that they analyze the meaning of an equation that is solved for one variable. Students will then consider how that equation can be used to make a substitution into a second equation containing the same variable. They then analyze the resulting equation and describe how it can be solved. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



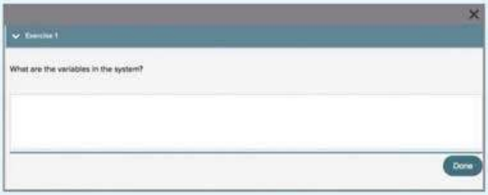
Using Substitution

INQUIRY How can you rewrite a system of equations as a single equation with only one variable?

Consider the system of equations.

$x + y = 5$
$y = 3x + 1$

Explore



What are the variables in the system?

Explore

TYPE

Students move through the slides answering questions about the system of equations.

Interactive Presentation

INQUIRY How can you rewrite a system of equations as a single equation with only one variable?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Using Substitution (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does performing the substitution make it possible to solve the system of equations? Sample answer: It produces an equation that contains only one variable, which can then be solved for that variable.
- How is the value that you find for x related to the graph of the two equations? It is the x-coordinate of the point of intersection of the two lines.

Inquiry

How can you rewrite a system of equations as a single equation with only one variable? Sample answer: If one equation is solved for a variable, then you know the value of that variable. Then, you can replace that variable in the other equation.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Systems of Equations by Substitution

Objective

Students solve systems of equations by using the substitution method.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students Are Learning

Students previously found the solution of a system of equations by graphing. They also used graphing technology to approximate a solution. In this lesson, students learn that the exact solution of a system can be found using an algebraic method, which in this lesson, is the substitution method.

Example 1 Solve a System by Substitution

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the equations in this example to solve the system.

Questions for Mathematical Discourse

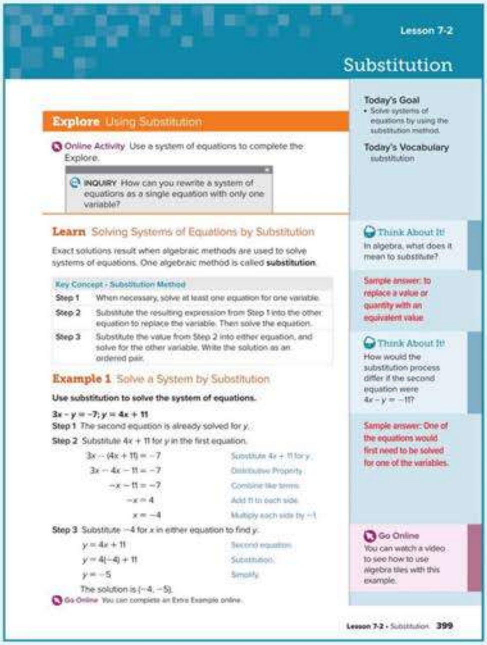
AL Do you have at least one equation solved for one variable? Yes; sample answer: y is equal to $4x + 11$

OL How does making the substitution allow you to solve the system? Sample answer: it creates an equation with only one variable, so I can solve for that variable, and then use that value to find the value of the other variable

BL What is an advantage of solving a system of equations by substitution instead of by graphing? Sample answer: When using a graph, the solution sometimes has to be estimated, but substitution gives an exact answer.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 7-2

Substitution

Today's Goal
- Solve systems of equations by using the substitution method.

Today's Vocabulary
substitution

Explore Using Substitution

Online Activity Use a system of equations to complete the Explore.

INQUIRY How can you rewrite a system of equations as a single equation with only one variable?

Learn Solving Systems of Equations by Substitution

Exact solutions result when algebraic methods are used to solve systems of equations. One algebraic method is called **substitution**.

Key Concept • Substitution Method

Step 1	When necessary, solve at least one equation for one variable.
Step 2	Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.
Step 3	Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Think About It! In algebra, what does it mean to substitute?

Sample answer: to replace a value or quantity with an equivalent value.

Example 1 Solve a System by Substitution

Use substitution to solve the system of equations.

$3x - y = -7$; $y = 4x + 11$

Step 1 The second equation is already solved for y.

Step 2 Substitute $4x + 11$ for y in the first equation.

$3x - (4x + 11) = -7$ Substitute $4x + 11$ for y.

$3x - 4x - 11 = -7$ Distributive Property

$-x - 11 = -7$ Combine like terms.

$-x = 4$ Add 11 to each side.

$x = -4$ Multiply each side by -1.

Step 3 Substitute -4 for x in either equation to find y.

$y = 4x + 11$ Second equation

$y = 4(-4) + 11$ Substitution

$y = -5$ Simplify.

The solution is $(-4, -5)$.

Go Online You can complete an Extra Example online.

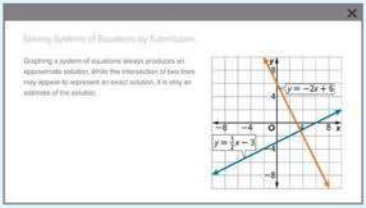
Think About It! How would the substitution process differ if the second equation were $4x - y = -11$?

Sample answer: One of the equations would first need to be solved for one of the variables.

Go Online You can watch a video to see how to use algebra tiles with this example.

Lesson 7-2 • Substitution 399

Interactive Presentation



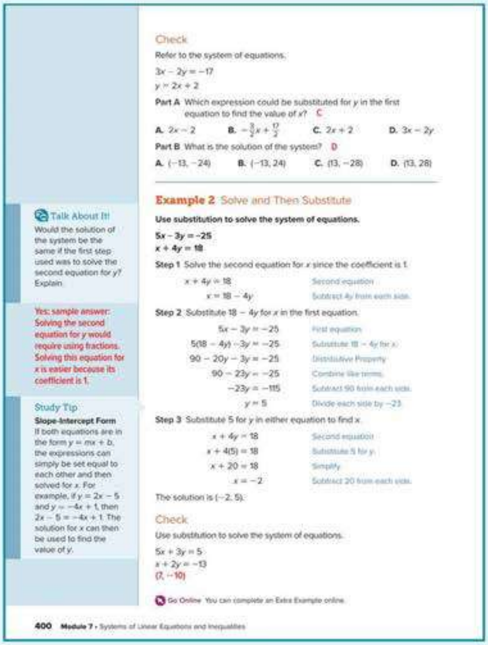
Solving Systems of Equations by Substitution

Graphing a system of equations always produces an approximate solution. While the intersection of two lines may appear to represent an exact solution, it is only an estimate of the solution.

Learn

TYPE

Students explain what it means to substitute in algebra.



Check

Refer to the system of equations.

$3x - 2y = -17$
$y = 2x + 2$

Part A Which expression could be substituted for y in the first equation to find the value of x? C

A. $2x - 2$ **B.** $-\frac{3}{2}x + \frac{17}{2}$ **C.** $2x + 2$ **D.** $3x - 2y$

Part B What is the solution of the system? D

A. $(-13, -24)$ **B.** $(-13, 24)$ **C.** $(13, -28)$ **D.** $(13, 28)$

Example 2 Solve and Then Substitute

Use substitution to solve the system of equations.

$5x - 3y = -25$
$x + 4y = 18$

Step 1 Solve the second equation for x since the coefficient is 1.

$x + 4y = 18$	Second equation
$x = 18 - 4y$	Subtract $4y$ from each side.

Step 2 Substitute $18 - 4y$ for x in the first equation.

$5x - 3y = -25$	First equation
$5(18 - 4y) - 3y = -25$	Substitute $18 - 4y$ for x.
$90 - 20y - 3y = -25$	Distributive Property
$90 - 23y = -25$	Combine like terms.
$-23y = -115$	Subtract 90 from each side.
$y = 5$	Divide each side by -23.

Step 3 Substitute 5 for y in either equation to find x.

$x + 4y = 18$	Second equation
$x + 4(5) = 18$	Substitute 5 for y.
$x + 20 = 18$	Simplify.
$x = -2$	Subtract 20 from each side.

The solution is $(-2, 5)$.

Check

Use substitution to solve the system of equations.

$5x + 3y = 5$
$x + 2y = -13$
$(7, -10)$

Go Online You can complete an Extra Example online.

Talk About It!

Would the solution of the system be the same if the first step used was to solve the second equation for y? Explain.

Yes; sample answer: Solving the second equation for y would require using fractions. Solving this equation for x is easier because its coefficient is 1.

Study Tip

Slope-Intercept Form If both equations are in the form $y = mx + b$, the expressions can simply be set equal to each other and then solved for x. For example, if $y = 2x - 5$ and $y = -4x + 1$, then $2x - 5 = -4x + 1$. The solution for x can then be used to find the value of y.

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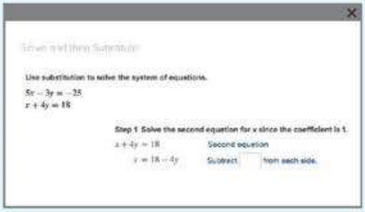
Interactive Presentation



Solve and then Substitute

Use substitution to solve the system of equations.

$5x - 3y = -25$
$x + 4y = 18$

Step 1 Solve the second equation for x since the coefficient is 1.

$x + 4y = 18$ Second equation
$x = 18 - 4y$ Subtract ☐ from each side.

Example 2

TAP

Students move through the steps to solve a system using substitution.

TYPE

Students enter the correct values while solving using substitution, and then discuss if the solution would be the same if the first step was used to solve the second equation.

Example 2 Solve and then Substitute

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress and change course to find the ultimate solution.

Questions for Mathematical Discourse

AL How can you tell which variable to solve for the substitution? Sample answer: It is easiest to look for a variable that has a coefficient of 1. Then, all you have to do is add or subtract the other term to solve.

OL When the quantity $18 - 4y$ is substituted for x, why is it necessary to use parentheses? Sample answer: The entire quantity needs to be multiplied by 5. If parentheses are not used, only the 18 will be multiplied by 5.

BL What would the graph of this system of equations look like? Sample answer: two lines that intersect at the point $(-2, 5)$

Common Error

Some students may forget to isolate x before making the substitution and substitute $4y + 18$ for x by mistake. Reinforce that the first step is to isolate a variable in one of the equations.

Common Misconception

A common misconception some students may have is that the substitution method can be used only when one of the equations contains a variable with a coefficient of 1. Explain that this is not the case, as any equation can be solved for any of its variables. Explain further that students will learn another algebraic method of solution and that for any system of equations, one method may be easier to use than another.

Example 3 Use Substitution When There Are No or Many Solutions

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and the system of equations used in this example.

Questions for Mathematical Discourse

AL Why is the first step to substitute $-2x - 4$ for y in the first equation? Sample answer: The second equation states that y is equal to $-2x - 4$, so you can replace y in the first equation with $-2x - 4$.

OL What does it mean to say that $-8 = -8$ is an identity? It means that the statement is always true. What does this mean about the solutions of the system? Sample answer: It means that all of the solutions of one of the equations are also solutions of the other equation.

BL Name three solutions of the system. Sample answer: $(0, -4)$, $(-2, 0)$, and $(1, -6)$

Common Error

Some students may state that the solution is $(-8, -8)$. Help them to see that the variable has been eliminated from the equation, so the resulting equation does not give any information about the values of x or y.

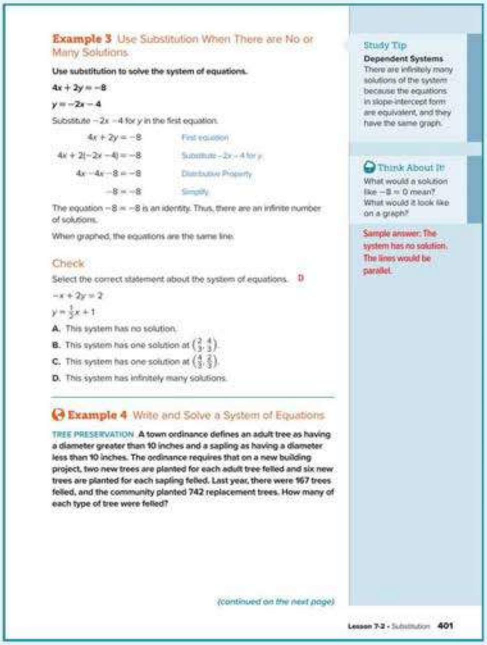
DIFFERENTIATE

Language Development Activity **ELL**

Beginning Ask questions about the lesson content to elicit yes/no answers: "Look at Example 1. Is one of the equations solved for one of the variables?" yes "Is the first step of solving the system complete?" yes

Intermediate/Advanced Ask questions about the lesson content to elicit short answers: "Look at Example 1. Which equation is solved for a variable?" the second "What should be substituted for y in the first equation?" $4x + 11$

Advanced-High Ask questions about the lesson content to elicit complete sentences: "How does Example 1 compare to Example 2?" In Example 1, Step 1 is already completed. "How would you choose the variable to solve for to solve Example 2 by substitution?" Solving the second equation for x makes sense because its coefficient is 1.



Example 3 Use Substitution When There are No or Many Solutions

Use substitution to solve the system of equations.

$4x + 2y = -8$

$y = -2x - 4$

Substitute $-2x - 4$ for y in the first equation.

$4x + 2y = -8$	First equation
$4x + 2(-2x - 4) = -8$	Substitute $-2x - 4$ for y.
$4x - 4x - 8 = -8$	Distributive Property
$-8 = -8$	Simplify.

The equation $-8 = -8$ is an identity. Thus, there are an infinite number of solutions.

When graphed, the equations are the same line.

Check

Select the correct statement about the system of equations. D

$-x + 2y = 2$

$y = \frac{1}{2}x + 1$

A. This system has no solution.

B. This system has one solution at $\left(\frac{2}{3}, \frac{4}{3}\right)$.

C. This system has one solution at $\left(\frac{4}{3}, \frac{2}{3}\right)$.

D. This system has infinitely many solutions.

Example 4 Write and Solve a System of Equations

TREE PRESERVATION **A town ordinance defines an adult tree as having a diameter greater than 10 inches and a sapling as having a diameter less than 10 inches. The ordinance requires that on a new building project, two new trees are planted for each adult tree felled and six new trees are planted for each sapling felled. Last year, there were 167 trees felled, and the community planted 742 replacement trees. How many of each type of tree were felled?**

Study Tip

Dependent Systems There are infinitely many solutions of the system because the equations in slope-intercept form are equivalent, and they have the same graph.

Think About It!

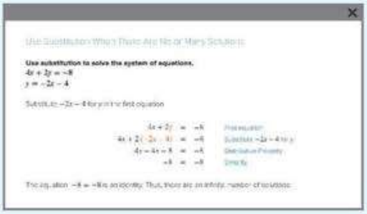
What would a solution like $-8 = 0$ mean? What would it look like on a graph?

Sample answer: The system has no solution. The lines would be parallel.

(continued on the next page)

Lesson 7-2 • Substitution 401

Interactive Presentation



Use substitution to solve the system of equations.

$4x + 2y = -8$

$y = -2x - 4$

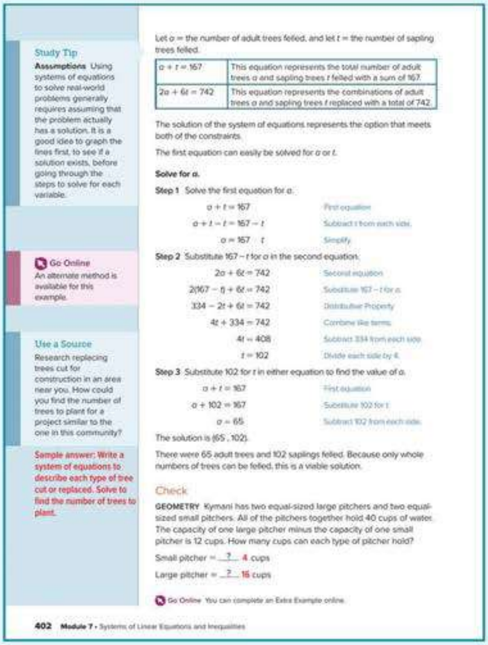
Example 3

TAP

Students tap on the graph to observe that the graphs of both equations are the same line.

TYPE

Students explain what certain solutions would look like on a graph.



Study Tip

Assumptions Using systems of equations to solve real-world problems generally requires assuming that the problem actually has a solution. It is a good idea to graph the lines first, to see if a solution exists, before going through the steps to solve for each variable.

Go Online An alternate method is available for this example.

Use a Source

Research replacing trees cut for construction in an area near you. How could you find the number of trees to plant for a project similar to the one in this community?

Sample answer: Write a system of equations to describe each type of tree cut or replaced. Solve to find the number of trees to plant.

Let a = the number of adult trees felled, and let t = the number of sapling trees felled.

$a + t = 167$	This equation represents the total number of adult trees a and sapling trees t felled with a sum of 167.
$2a + 6t = 742$	This equation represents the combinations of adult trees a and sapling trees t replaced with a total of 742.

The solution of the system of equations represents the option that meets both of the constraints.

The first equation can easily be solved for a or t.

Solve for a.

Step 1 Solve the first equation for a.

$a + t = 167$ First equation

$a + t - t = 167 - t$ Subtract t from each side.

$a = 167 - t$ Simplify.

Step 2 Substitute $167 - t$ for a in the second equation.

$2a + 6t = 742$ Second equation

$2(167 - t) + 6t = 742$ Substitute $167 - t$ for a.

$334 - 2t + 6t = 742$ Distributive Property

$4t + 334 = 742$ Combine like terms.

$4t = 408$ Subtract 334 from each side.

$t = 102$ Divide each side by 4.

Step 3 Substitute 102 for t in either equation to find the value of a.

$a + t = 167$ First equation

$a + 102 = 167$ Substitute 102 for t.

$a = 65$ Subtract 102 from each side.

The solution is (65, 102).

There were 65 adult trees and 102 saplings felled. Because only whole numbers of trees can be felled, this is a viable solution.

Check

GEOMETRY Kymani has two equal-sized large pitchers and two equal-sized small pitchers. All of the pitchers together hold 40 cups of water. The capacity of one large pitcher minus the capacity of one small pitcher is 12 cups. How many cups can each type of pitcher hold?

Small pitcher = __?__ 4 cups

Large pitcher = __?__ 16 cups

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4

TAP

Students tap each marker to explore the equations and two solution pathways.

TYPE

Students compare and contrast the solution methods and then research replacing trees cut for construction.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Write and Solve a System of Equations

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL What do the terms $2a$ and $6t$ represent? Sample answer: $2a$ represents the number of trees planted to replace the adult trees that were felled; $6t$ represents the number of trees planted to replace the saplings that were felled.

OL What steps are needed to solve the system? Solve for one of the variables in the first equation, then substitute the quantity into the second equation and solve for the variable. Finally, substitute the value back into the first equation and solve for the other variable.

BL How would the answer have changed if you had solved for t in Step 1? Sample answer: The final answer wouldn't change. If you solved for t in Step 1, you would then substitute $167 - a$ for t in Step 2 to get $a = 65$. In Step 3, you would solve for t to get 102.

Essential Question Follow-Up

Students are writing and solving systems of equations in two variables.

Ask:

Why is it necessary to use a system of equations to solve a real-world problem that involves two unknowns? Sample answer: When there are two unknowns, you need two equations to find the values of the variables.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–17
2	exercises that use a variety of skills from this lesson	18–20
2	exercises that extend concepts learned in this lesson to new contexts	21–24
3	exercises that emphasize higher-order and critical-thinking skills	25–29

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

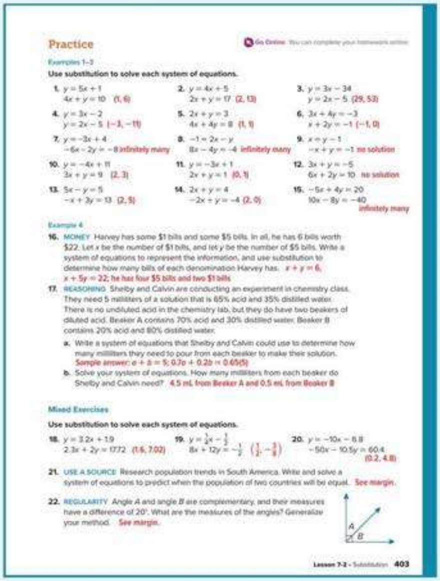
- Practice Exercises 1–23 odd, 25–29
- Extension: Intersection of Two Parabolas
- **ALEKS** Systems of Linear Equations

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice Exercises 1–29 odd
- Remediation, Review Resources: Using Formulas
- Personal Tutors
- Extra Examples 1–4
- **ALEKS** Solving for Variable and Dimensional Analysis

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice Exercises 1–17 odd
- Remediation, Review Resources: Using Formulas
- *Quick Review Math Handbook*: Substitution
- Arrive**MATH** Take Another Look
- **ALEKS** Solving for Variable and Dimensional Analysis



Practice

Go Online You can complete your homework online.

Examples 1–3

Use substitution to solve each system of equations.

1. $y = 5x + 1$
$4x + y = 10$ (1, 6)

2. $y = 4x + 5$
$2x + y = 17$ (2, 13)

3. $y = 3x - 34$
$y = 2x - 5$ (29, 53)

4. $y = 3x - 2$
$y = 2x - 5$ (−3, −11)

5. $2x + y = 3$
$4x + 4y = 8$ (1, 1)

6. $3x + 4y = -3$
$x + 2y = -1$ (−1, 0)

7. $y = -3x + 4$
$-6x - 2y = -8$ infinitely many

8. $-1 = 2x - y$
$8x - 4y = -4$ infinitely many

9. $x = y - 1$
$-x + y = -1$ no solution

10. $y = -4x + 11$
$3x + y = 9$ (2, 3)

11. $y = -3x + 1$
$2x + y = 1$ (0, 1)

12. $3x + y = -5$
$6x + 2y = 10$ no solution

13. $5x - y = 5$
$-x + 3y = 13$ (2, 5)

14. $2x + y = 4$
$-2x + y = -4$ (2, 0)

15. $-5x + 4y = 20$
$10x - 8y = -40$ infinitely many

Example 4

16. MONEY Harvey has some \$1 bills and some \$5 bills. In all, he has 6 bills worth \$22. Let x be the number of \$1 bills, and let y be the number of \$5 bills. Write a system of equations to represent the information, and use substitution to determine how many bills of each denomination Harvey has. $x + y = 6$, $x + 5y = 22$; he has four \$5 bills and two \$1 bills

17. REASONING Shelby and Calvin are conducting an experiment in chemistry class. They need 5 milliliters of a solution that is 65% acid and 35% distilled water. There is no undiluted acid in the chemistry lab, but they do have two beakers of diluted acid. Beaker A contains 70% acid and 30% distilled water. Beaker B contains 20% acid and 80% distilled water.
a. Write a system of equations that Shelby and Calvin could use to determine how many milliliters they need to pour from each beaker to make their solution. Sample answer: $a + b = 5$; $0.7a + 0.2b = 0.65(5)$
b. Solve your system of equations. How many milliliters from each beaker do Shelby and Calvin need? 4.5 mL from Beaker A and 0.5 mL from Beaker B

Mixed Exercises

Use substitution to solve each system of equations.

18. $y = 3.2x + 1.9$
$2.3x + 2y = 17.72$ (1.6, 7.02)

19. $y = \frac{1}{4}x - \frac{1}{2}$
$8x + 12y = -\frac{1}{2}$ $\left(\frac{1}{2}, -\frac{3}{8}\right)$

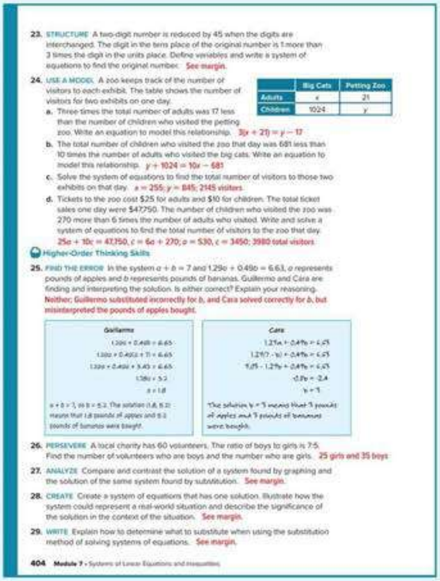
20. $y = -10x - 8.8$
$-50x - 10.5y = 60.4$ (0.2, 4.8)

21. USE A SOURCE Research population trends in South America. Write and solve a system of equations to predict when the population of two countries will be equal. See margin.

22. REGULARITY Angle A and angle B are complementary, and their measures have a difference of 20°. What are the measures of the angles? Generalize your method. See margin.

Lesson 7-2 • Substitution 403

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



23. STRUCTURE A two-digit number is reduced by 45 when the digits are interchanged. The digit in the tens place of the original number is 1 more than 3 times the digit in the units place. Define variables and write a system of equations to find the original number. See margin.

24. USE A MODEL A zoo keeps track of the number of visitors to each exhibit. The table shows the number of visitors for two exhibits on one day.

	Big Cats	Petting Zoo
Adults	x	21
Children	1024	y

a. Three times the total number of adults was 17 less than the number of children who visited the petting zoo. Write an equation to model this relationship. $3(x + 21) = y - 17$

b. The total number of children who visited the zoo that day was 681 less than 10 times the number of adults who visited the big cats. Write an equation to model this relationship. $y + 1024 = 10x - 681$

c. Solve the system of equations to find the total number of visitors to those two exhibits on that day. $x = 255$; $y = 845$; 2145 visitors

d. Tickets to the zoo cost $25 for adults and $10 for children. The total ticket sales one day were $47,750. The number of children who visited the zoo was 270 more than 6 times the number of adults who visited. Write and solve a system of equations to find the total number of visitors to the zoo that day. $25a + 10c = 47{,}750$, $c = 6a + 270$; $a = 530$, $c = 3450$; 3980 total visitors

Higher-Order Thinking Skills

25. FIND THE ERROR In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents pounds of apples and b represents pounds of bananas. Guillermo and Cara are finding and interpreting the solution. Is either correct? Explain your reasoning. Neither; Guillermo substituted incorrectly for b, and Cara solved correctly for b, but misinterpreted the pounds of apples bought.

Guillermo

$1.29a + 0.49b = 6.63$

$1.29a + 0.49(a + 7) = 6.63$

$1.29a + 0.49a + 3.43 = 6.63$

$1.78a = 3.2$

$a = 1.8$

$a + b = 7$, so $b = 5.2$. The solution (1.8, 5.2) means that 1.8 pounds of apples and 5.2 pounds of bananas were bought.

Cara

$1.29a + 0.49b = 6.63$

$1.29(7 - b) + 0.49b = 6.63$

$9.03 - 1.29b + 0.49b = 6.63$

$-0.8b = -2.4$

$b = 3$

The solution $b = 3$ means that 3 pounds of apples and 3 pounds of bananas were bought.

26. PERSEVERE A local charity has 60 volunteers. The ratio of boys to girls is 7:5. Find the number of volunteers who are boys and the number who are girls. 25 girls and 35 boys

27. ANALYZE Compare and contrast the solution of a system found by graphing and the solution of the same system found by substitution. See margin.

28. CREATE Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation. See margin.

29. WRITE Explain how to determine what to substitute when using the substitution method of solving systems of equations. See margin.

404 Module 7 • Systems of Linear Equations and Inequalities

Answers

21. Sample answer: In 2011, the population of Ecuador was about 15,180,000 and the population of Chile was about 17,150,000. The population of Ecuador increased by 1,210,000 and the population of Chile increased by 760,000 from 2011 to 2016. Let x = the number of 5-yr periods and y = population. The system is $y = 15{,}180{,}000 + 1{,}210{,}000x$ and $y = 17{,}150{,}000 + 760{,}000x$. Solve by substitution to find that $x \approx 4.4$, or $4.4 \times 5 = 22$ years. So, the population of Ecuador and Chile will be equal in about $2011 + 22 = 2033$. (Source: World Bank)

22. 35° and 55°; Let x = the measure of angle A, then $x - 20$ = the measure of angle B. Add the angle measures: $x + (x - 20)$. Set the expression equal to 90° because the measures of complementary angles have a sum of 90°: $x + (x - 20) = 90$. Solve for x: $x = 55°$ and $x - 20 = 35°$.

23. Let x = tens digit and y = units digit of the original number; $10y + x = 10x + y - 45$; $x = 3y + 1$; (7, 2); The original number is 72.

27. Sample answer: The solutions found by each of these methods should be the same. However, it may be necessary to estimate when using a graph. So, when a precise solution is needed, you should use substitution.

28. Sample answer: $a + b = 25$, $24a + 16b = 464$; Let a = the number of bags of dog food, and let b = the number of bags of cat food a service organization donated to the local animal shelter. They donated a total of 25 bags of food. Each bag of dog food costs $24, and each bag of cat food costs $16. They spent a total of $464. How many bags of each type of food did the service organization buy? They purchased 8 bags of dog food and 17 bags of cat food.

29. An equation containing a variable with a coefficient of 1 can easily be solved for the variable. That expression can then be substituted into the second equation for the variable.

Lesson 7-3

A.CED.3, A.REI.6

Elimination Using Addition and Subtraction

LESSON GOAL

Students solve systems of equations by using elimination with addition or subtraction.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Solving Systems of Equations by Elimination with Addition
- Elimination Using Addition
- Write and Solve a System Using Addition

Solving Systems of Equations by Elimination with Subtraction
- Elimination Using Subtraction
- Write and Solve a System Using Subtraction

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation:	●	●		●
Extension: Translating Symbols into Equations		●	●	●

Language Development Handbook

Assign page 40 of the *Language Development Handbook* to help your students build mathematical language related to solving systems of equations by using elimination with addition or subtraction.

ELL You can use the tips and suggestions on page T40 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous

Students solved systems of equations by using substitution.

8.EE.8b, A.CED.3, A.REI.6

Now

Students solve systems of equations by using elimination with addition or subtraction.

A.CED.3, A.REI.6

Next

Students will solve systems of equations by using elimination with multiplication.

A.REI.5, A.REI.6

Rigor

The Three Pillars of Rigor

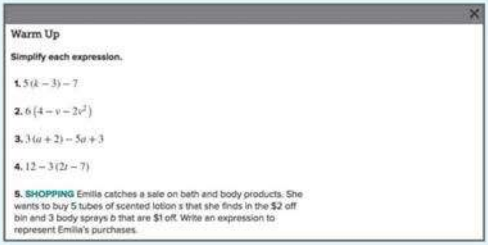
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop understanding of using algebraic methods to solve systems of linear equations. They build fluency by using elimination to solve systems of equations, and they apply their understanding by solving real-world problems.

Mathematical Background

Elimination using addition or subtraction involves manipulating one or both equations so that one variable is eliminated when the equations are added or subtracted. The solution is the ordered pair consisting of the two values. This is the point of intersection of the graphs of the two equations.

Interactive Presentation



Warm Up

Simplify each expression.

1. $5(k-3)-7$

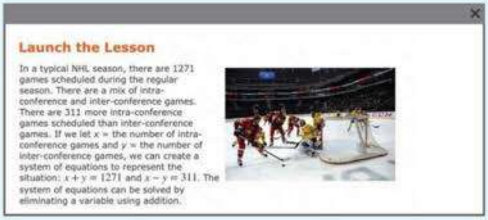
2. $6(4-v-2v^2)$

3. $3(a+2)-5a+3$

4. $12-3(2t-7)$

5. SHOPPING Emilia catches a sale on bath and body products. She wants to buy 5 tubes of scented lotion *s* that she finds in the \$2 off bin and 3 body sprays *b* that are \$1 off. Write an expression to represent Emilia's purchases.

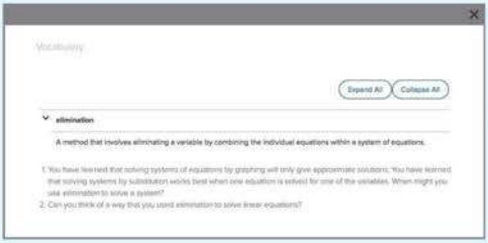
Warm Up



Launch the Lesson

In a typical NHL season, there are 1271 games scheduled during the regular season. There are a mix of intra-conference and inter-conference games. There are 311 more intra-conference games scheduled than inter-conference games. If we let x = the number of intra-conference games and y = the number of inter-conference games, we can create a system of equations to represent the situation: $x+y=1271$ and $x-y=311$. The system of equations can be solved by eliminating a variable using addition.

Launch the Lesson



Vocabulary

elimination

A method that involves eliminating a variable by combining the individual equations within a system of equations.

1. You have learned that solving systems of equations by graphing will only give approximate solutions. You have learned that solving systems by substitution works best when one equation is solved for one of the variables. When might you use elimination to solve a system?
2. Can you think of a way that you used elimination to solve linear equations?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- simplifying expressions by using the distributive property

Answers:

1. $5k - 22$
2. $24 - 6v - 12v^2$
3. $-2a + 9$
4. $33 - 6t$
5. $5(s - 2) + 3(b - 1)$

Launch the Lesson

MP Teaching the Mathematical Practices

2 Make Sense of Quantities Mathematically proficient students need to be able to make sense of quantities and their relationships. In this Launch, notice the relationship between the variables *r* and *t*.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Learn Solving Systems of Equations by Elimination with Addition

Objective

Students solve systems of equations by eliminating a variable using addition.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in solving systems of equations by elimination with addition.

What Students Are Learning

In the previous lesson, students learned how to use substitution to solve a system of equations. In that process, students worked with one equation at a time to eliminate a variable and solved for the other variable. In this lesson, students learn a different algebraic method for solving systems: elimination. In the elimination with addition method, the equations are added together, causing one of the variables to be eliminated. This results in an equation with only one unknown. The remaining steps of the solution are the same as those when using the substitution method.

Example 1 Elimination Using Addition

Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" Point out that in this example, students need to check their answer. Point out that they should ask themselves whether their answer makes sense and whether they have answered the problem question.

Questions for Mathematical Discourse

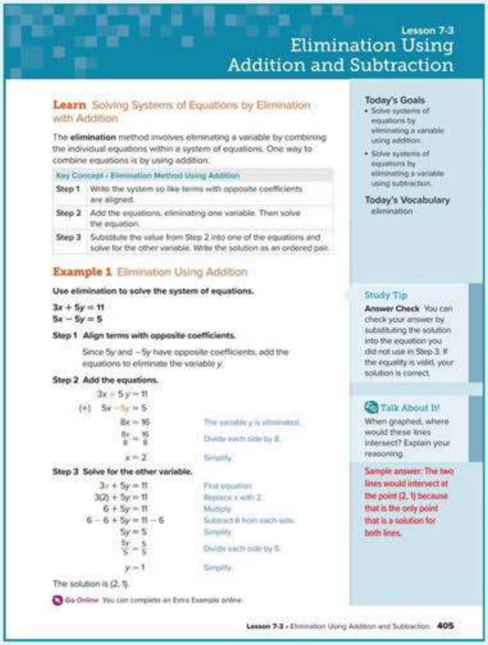
AL Why do you want to eliminate a variable from the equations? Sample answer: When solving for an unknown variable, you need to only have one unknown at a time.

OL Why do you add the two equations? Because the coefficients of the y-terms in the equations are opposites, you can add the equations together and the y-terms will be eliminated. You will be left with an equation with only one variable.

BL In this problem, why might it be easier to use elimination instead of substitution? Sample answer: By using elimination, the system can be solved for a variable in fewer steps. To use substitution, you would first need to isolate a variable in one of the equations.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 7-3

Elimination Using Addition and Subtraction

Learn Solving Systems of Equations by Elimination with Addition

The **elimination** method involves eliminating a variable by combining the individual equations within a system of equations. One way to combine equations is by using addition.

Key Concept • Elimination Method Using Addition

Step 1	Write the system so like terms with opposite coefficients are aligned.
Step 2	Add the equations, eliminating one variable. Then solve the equation.
Step 3	Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Elimination Using Addition

Use elimination to solve the system of equations.

$3x + 5y = 11$
$5x - 5y = 5$

Step 1 Align terms with opposite coefficients.

Since $5y$ and $-5y$ have opposite coefficients, add the equations to eliminate the variable y.

Step 2 Add the equations.

$$\begin{aligned} 3x + 5y &= 11 \\ (+)\quad 5x - 5y &= 5 \\ 8x &= 16 && \text{The variable } y \text{ is eliminated.} \\ \frac{8x}{8} &= \frac{16}{8} && \text{Divide each side by 8.} \\ x &= 2 && \text{Simplify.} \end{aligned}$$

Step 3 Solve for the other variable.

$$\begin{aligned} 3x + 5y &= 11 && \text{First equation} \\ 3(2) + 5y &= 11 && \text{Replace } x \text{ with 2.} \\ 6 + 5y &= 11 && \text{Multiply.} \\ 6 - 6 + 5y &= 11 - 6 && \text{Subtract 6 from each side.} \\ 5y &= 5 && \text{Simplify.} \\ \frac{5y}{5} &= \frac{5}{5} && \text{Divide each side by 5.} \\ y &= 1 && \text{Simplify.} \end{aligned}$$

The solution is (2, 1).

Go Online You can complete an Extra Example online.

Today's Goals
- Solve systems of equations by eliminating a variable using addition.
- Solve systems of equations by eliminating a variable using subtraction.

Today's Vocabulary
elimination

Study Tip

Answer Check You can check your answer by substituting the solution into the equation you did not use in Step 3. If the equality is valid, your solution is correct.

Talk About It!

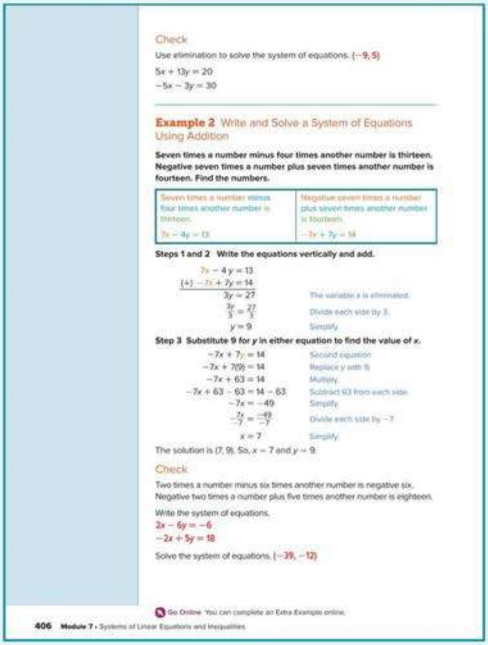
When graphed, where would these lines intersect? Explain your reasoning.

Sample answer: The two lines would intersect at the point (2, 1) because that is the only point that is a solution for both lines.

Lesson 7-3 • Elimination Using Addition and Subtraction **405**

Interactive Presentation

Learn



Check

Use elimination to solve the system of equations. (−9, 5)

$5x + 13y = 20$

$-5x - 3y = 30$

Example 2 Write and Solve a System of Equations Using Addition

Seven times a number minus four times another number is thirteen. Negative seven times a number plus seven times another number is fourteen. Find the numbers.

Seven times a number minus four times another number is thirteen. $7x - 4y = 13$	Negative seven times a number plus seven times another number is fourteen. $-7x + 7y = 14$

Steps 1 and 2 **Write the equations vertically and add.**

$$\begin{array}{rrl} & 7x - 4y = 13 & \\ (+) & -7x + 7y = 14 & \\ \hline & 3y = 27 & \text{The variable } x \text{ is eliminated.} \\ & \frac{3y}{3} = \frac{27}{3} & \text{Divide each side by 3.} \\ & y = 9 & \text{Simplify.} \end{array}$$

Step 3 **Substitute 9 for y in either equation to find the value of x.**

$$\begin{array}{rl} -7x + 7y = 14 & \text{Second equation} \\ -7x + 7(9) = 14 & \text{Replace } y \text{ with 9.} \\ -7x + 63 = 14 & \text{Multiply.} \\ -7x + 63 - 63 = 14 - 63 & \text{Subtract 63 from each side.} \\ -7x = -49 & \text{Simplify.} \\ \frac{-7x}{-7} = \frac{-49}{-7} & \text{Divide each side by } -7. \\ x = 7 & \text{Simplify.} \end{array}$$

The solution is (7, 9). So, $x = 7$ and $y = 9$.

Check

Two times a number minus six times another number is negative six. Negative two times a number plus five times another number is eighteen.

Write the system of equations.

$2x - 6y = -6$

$-2x + 5y = 18$

Solve the system of equations. (−39, −17)

Go Online You can complete an Extra Example online.

406 Module 7 • Systems of Linear Equations and Inequalities

Interactive Presentation

Example 2

TAP

Students tap each card to see the equation represented by each description and then move through the steps to solve the system using elimination.

TYPE

Students enter the values required when solving the system and then identify the point of intersection on the graph.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Write and Solve a System of Equations Using Addition

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL What words indicate the operations to perform in the equations? times, minus, and plus

OL What terms will be eliminated when using elimination with addition? Why? the x-terms; Sample answer: The coefficients of the x-terms are additive inverses and will cancel.

BL Does it matter which equation you substitute the 9 back into? Explain. No; sample answer: You will get the same value for x no matter which equation you use.

Common Error

Some students may need to review translating from words to symbols.

DIFFERENTIATE

Enrichment Activity **BL**

Write two equations on the board with the constants missing. Have students find the missing constants that produce the given solution. For example, write the system

$3x + 2y = ?$

$5x - 2y = ?$

and tell students that the solution is (1.5, 0.25). Then have them find the missing constants.

$3x + 2y = 5$, $5x - 2y = 7$

Common Misconception

A common misconception some students may have is that the use of different methods of solution may lead to different solutions. Explain that whether found by using a graphing method or one of the algebraic methods, the solution of a system will be the same. In every case, the solution represents the coordinates of the point of intersection of the graphs of the equations, which is the solution(s) that the two equations have in common.

Learn Solving Systems of Equations by Elimination with Subtraction

Objective

Students solve systems of equations by eliminating a variable using subtraction.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in solving systems of equations by elimination with subtraction.

Example 3 Elimination Using Subtraction

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress and change course to find the solution.

Questions for Mathematical Discourse

AL Why would you subtract these equations instead of adding them? Sample answer: Because the *y*-terms have the same sign, none of the variables will cancel out when added.

OL What are two different ways that you could check the solution? Sample answer: Substitute the values back into the equations and see that they satisfy both equations, or graph the equations and find the point of intersection.

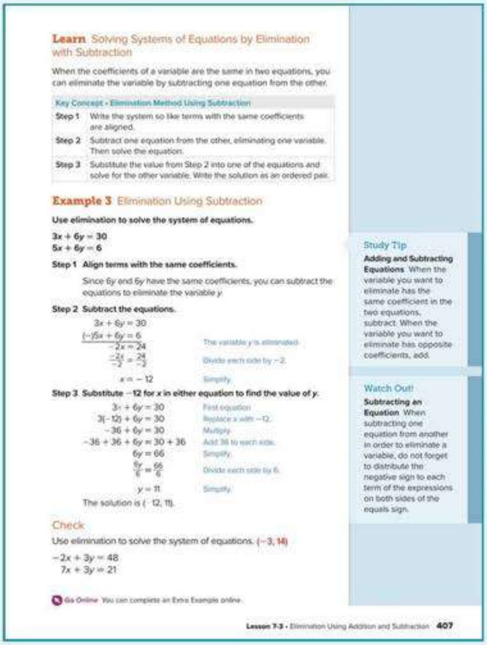
BL Why is adding the two equations not helpful in solving the system? Sample answer: Neither variable is eliminated when you add the equations, so you are left with another equation that contains two variables. You need to produce an equation with only one variable.

Essential Question Follow-Up

Students have been using systems of equations to solve real-world problems.

Ask:

Why is it helpful to know how to solve systems of equations when solving real-world problems? Sample answer: Some real-world problems involve two variables, so you need to know how to write and solve a system of two equations to solve the problem.



Learn Solving Systems of Equations by Elimination with Subtraction

When the coefficients of a variable are the same in two equations, you can eliminate the variable by subtracting one equation from the other.

Key Concept • Elimination Method Using Subtraction	
Step 1	Write the system so like terms with the same coefficients are aligned.
Step 2	Subtract one equation from the other, eliminating one variable. Then solve the equation.
Step 3	Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 3 Elimination Using Subtraction

Use elimination to solve the system of equations.

$3x + 6y = 30$
$5x + 6y = 6$

Step 1 Align terms with the same coefficients.

Since 6*y* and 6*y* have the same coefficients, you can subtract the equations to eliminate the variable *y*.

Step 2 Subtract the equations.

$$\begin{aligned} 3x + 6y &= 30 \\ \underline{(-)\,5x + 6y} &\underline{= 6} \\ -2x &= 24 && \text{The variable } y \text{ is eliminated.} \\ \frac{-2x}{-2} &= \frac{24}{-2} && \text{Divide each side by } -2. \\ x &= -12 && \text{Simplify.} \end{aligned}$$

Step 3 Substitute −12 for *x* in either equation to find the value of *y*.

$$\begin{aligned} 3x + 6y &= 30 && \text{First equation} \\ 3(-12) + 6y &= 30 && \text{Replace } x \text{ with } -12. \\ -36 + 6y &= 30 && \text{Multiply.} \\ -36 + 36 + 6y &= 30 + 36 && \text{Add 36 to each side.} \\ 6y &= 66 && \text{Simplify.} \\ \frac{6y}{6} &= \frac{66}{6} && \text{Divide each side by 6.} \\ y &= 11 && \text{Simplify.} \end{aligned}$$

The solution is (−12, 11).

Check

Use elimination to solve the system of equations. (−3, 14)

$-2x + 3y = 48$
$7x + 3y = 21$

Go Online You can complete an Extra Example online.

Study Tip

Adding and Subtracting Equations When the variable you want to eliminate has the same coefficient in the two equations, subtract. When the variable you want to eliminate has opposite coefficients, add.

Watch Out!

Subtracting an Equation When subtracting one equation from another in order to eliminate a variable, do not forget to distribute the negative sign to each term of the expressions on both sides of the equals sign.

Lesson 7-3 • Elimination Using Addition and Subtraction 407

Interactive Presentation

Elimination Using Subtraction

Use elimination to solve the system of equations.

$3x + 6y = 30$
$5x + 6y = 6$

Step 1 Align terms with the same coefficients.

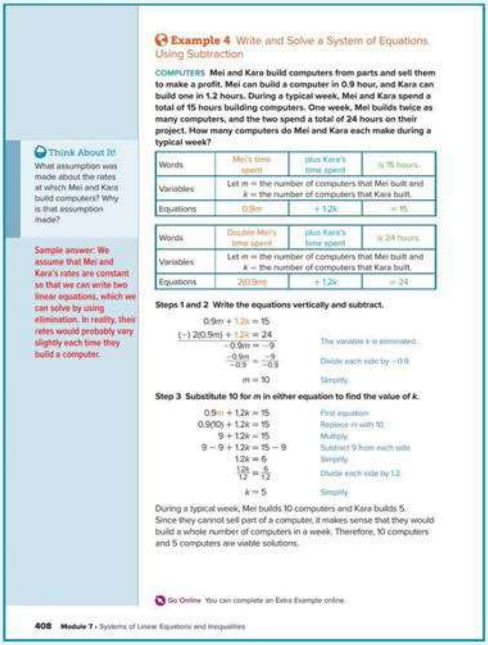
Example 3

TAP

Students move through the slides to solve a system using elimination.

TYPE

Students enter the values required when solving the system.



Example 4 Write and Solve a System of Equations Using Subtraction

COMPUTERS **Mei and Kara build computers from parts and sell them to make a profit. Mei can build a computer in 0.9 hour, and Kara can build one in 1.2 hours. During a typical week, Mei and Kara spend a total of 15 hours building computers. One week, Mei builds twice as many computers, and the two spend a total of 24 hours on their project. How many computers do Mei and Kara each make during a typical week?**

Words	Mei's time spent	plus Kara's time spent	is 15 hours.
Variables	Let m = the number of computers that Mei built and k = the number of computers that Kara built.		
Equations	$0.9m$	$+ 1.2k$	$= 15$

Words	Double Mei's time spent	plus Kara's time spent	is 24 hours.
Variables	Let m = the number of computers that Mei built and k = the number of computers that Kara built.		
Equations	$2(0.9m)$	$+ 1.2k$	$= 24$

Steps 1 and 2 Write the equations vertically and subtract.

$$\begin{array}{rll} 0.9m + 1.2k = 15 & & \\ (-)\ 2(0.9m) + 1.2k = 24 & & \\ \hline -0.9m = -9 & & \text{The variable } k \text{ is eliminated.} \\ \frac{-0.9m}{-0.9} = \frac{-9}{-0.9} & & \text{Divide each side by } -0.9. \\ m = 10 & & \text{Simplify.} \end{array}$$

Step 3 Substitute 10 for m in either equation to find the value of k.

$$\begin{array}{rll} 0.9m + 1.2k = 15 & & \text{First equation} \\ 0.9(10) + 1.2k = 15 & & \text{Replace } m \text{ with 10.} \\ 9 + 1.2k = 15 & & \text{Multiply.} \\ 9 - 9 + 1.2k = 15 - 9 & & \text{Subtract 9 from each side.} \\ 1.2k = 6 & & \text{Simplify.} \\ \frac{1.2k}{1.2} = \frac{6}{1.2} & & \text{Divide each side by 1.2.} \\ k = 5 & & \text{Simplify.} \end{array}$$

During a typical week, Mei builds 10 computers and Kara builds 5. Since they cannot sell part of a computer, it makes sense that they would build a whole number of computers in a week. Therefore, 10 computers and 5 computers are viable solutions.

Think About It! What assumption was made about the rates at which Mei and Kara build computers? Why is that assumption made?

Sample answer: We assume that Mei and Kara's rates are constant so that we can write two linear equations, which we can solve by using elimination. In reality, their rates would probably vary slightly each time they build a computer.

Go Online You can complete an Extra Example online.

408 Module 7 • Systems of Linear Equations and Inequalities

Interactive Presentation

COMPUTERS Mei and Kara build computers from parts and sell them to make a profit. Mei can build a computer in 0.9 hour, and Kara can build one in 1.2 hours. During a typical week, Mei and Kara spend a total of 15 hours building computers. One week, Mei builds twice as many computers, and the two spend a total of 24 hours on their project. How many computers do Mei and Kara each make during a typical week?

Example 4

TYPE

Students determine the assumption made, and also if elimination could be used if the number of computers were doubled.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Write and Solve a System of Equations Using Subtraction

Teaching the Mathematical Practices

4 Make Assumptions Have students explain an assumption or approximation that was made to solve the problem.

Questions for Mathematical Discourse

AL Which variable is it easiest to solve for first? Why? m; Sample answer: Because both equations contain $1.2k$, subtract them to eliminate k and solve for m.

OL Why does $1.2k$ represent the amount of time Kara spent building computers? Sample answer: Because it takes Kara 1.2 hours to build one computer, 1.2 times k represents the amount of times it takes her to build k computers.

BL In Step 3, when you substitute 10 for m in the first equation, you find that the value of k is 5. Assuming that you have performed this step correctly, can you be sure that the solution to the system is $m = 10$ and $k = 5$? Explain your reasoning. No; sample answer: All you know is that this is a solution of the first equation. If you made an error in Step 1 or 2, and the value of m is not actually 10, then $m = 10$ and $k = 5$ is not the solution of the problem.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

3 REFLECT AND PRACTICE

A.CED.3, A.REI.6

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–32
2	exercises that use a variety of skills from this lesson	33–38
2	exercises that extend concepts learned in this lesson to new contexts	39–43
3	exercises that emphasize higher-order and critical-thinking skills	44–49

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–43 odd, 44–49
- Extension: Translating Symbols into Equations
- ALEKS Systems of Linear Equations

OL

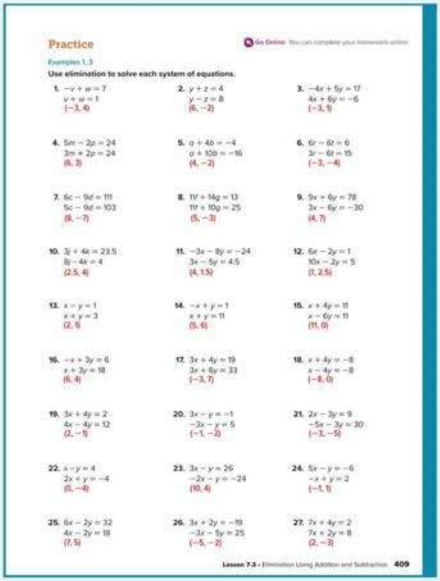
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–49 odd
- Remediation, Review Resources: Simplify Algebraic Expressions
- Personal Tutors
- Extra Examples 1–4
- ALEKS Simplifying Algebraic Expressions

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice Exercises 1–31 odd
- Remediation, Review Resources: Simplify Algebraic Expressions
- *Quick Review Math Handbook*: Elimination Using Addition and Subtraction
- Arrive**MATH** Take Another Look
- ALEKS Simplifying Algebraic Expressions



Practice

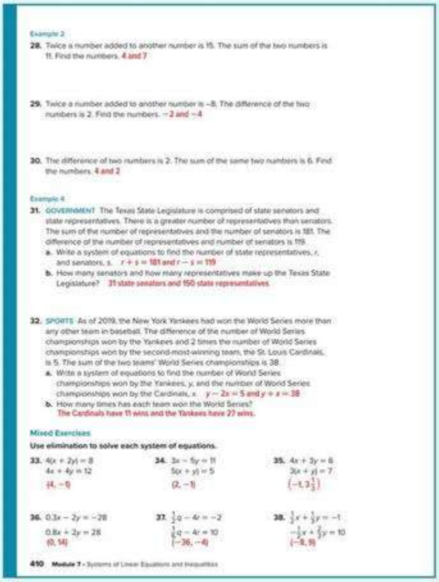
Go Online You can complete your homework online.

Examples 1, 3

Use elimination to solve each system of equations.

1. $-v + w = 7$
   $v + w = 1$
   $(-3, 4)$
2. $y + z = 4$
   $y - z = 8$
   $(6, -2)$
3. $-4x + 5y = 17$
   $4x + 6y = -6$
   $(-3, 1)$
4. $5m - 2p = 24$
   $3m + 2p = 24$
   $(6, 3)$
5. $a + 4b = -4$
   $a + 10b = -16$
   $(4, -2)$
6. $6r - 6t = 6$
   $3r - 6t = 15$
   $(-3, -4)$
7. $6c - 9d = 111$
   $5c - 9d = 103$
   $(8, -7)$
8. $11f + 14g = 13$
   $11f + 10g = 25$
   $(5, -3)$
9. $9x + 6y = 78$
   $3x - 6y = -30$
   $(4, 7)$
10. $3j + 4k = 23.5$
    $8j - 4k = 4$
    $(2.5, 4)$
11. $-3x - 8y = -24$
    $3x - 5y = 4.5$
    $(4, 1.5)$
12. $6x - 2y = 1$
    $10x - 2y = 5$
    $(1, 2.5)$
13. $x - y = 1$
    $x + y = 3$
    $(2, 1)$
14. $-x + y = 1$
    $x + y = 11$
    $(5, 6)$
15. $x + 4y = 11$
    $x - 6y = 11$
    $(11, 0)$
16. $-x + 3y = 6$
    $x + 3y = 18$
    $(6, 4)$
17. $3x + 4y = 19$
    $3x + 6y = 33$
    $(-3, 7)$
18. $x + 4y = -8$
    $x - 4y = -8$
    $(-8, 0)$
19. $3x + 4y = 2$
    $4x - 4y = 12$
    $(2, -1)$
20. $3x - y = -1$
    $-3x - y = 5$
    $(-1, -2)$
21. $2x - 3y = 9$
    $-5x - 3y = 30$
    $(-3, -5)$
22. $x - y = 4$
    $2x + y = -4$
    $(0, -4)$
23. $3x - y = 26$
    $-2x - y = -24$
    $(10, 4)$
24. $5x - y = -6$
    $-x + y = 2$
    $(-1, 1)$
25. $6x - 2y = 32$
    $4x - 2y = 18$
    $(7, 5)$
26. $3x + 2y = -19$
    $-3x - 5y = 25$
    $(-5, -2)$
27. $7x + 4y = 2$
    $7x + 2y = 8$
    $(2, -3)$

Lesson 7-3 • Elimination Using Addition and Subtraction **409**



Example 2

28. Twice a number added to another number is 15. The sum of the two numbers is 11. Find the numbers. 4 and 7

29. Twice a number added to another number is −8. The difference of the two numbers is 2. Find the numbers. −2 and −4

30. The difference of two numbers is 2. The sum of the same two numbers is 6. Find the numbers. 4 and 2

Example 4

31. GOVERNMENT The Texas State Legislature is comprised of state senators and state representatives. There is a greater number of representatives than senators. The sum of the number of representatives and the number of senators is 181. The difference of the number of representatives and number of senators is 119.
    a. Write a system of equations to find the number of state representatives, r, and senators, s. $r + s = 181$ and $r - s = 119$
    b. How many senators and how many representatives make up the Texas State Legislature? 31 state senators and 150 state representatives

32. SPORTS As of 2019, the New York Yankees had won the World Series more than any other team in baseball. The difference of the number of World Series championships won by the Yankees and 2 times the number of World Series championships won by the second-most-winning team, the St. Louis Cardinals, is 5. The sum of the two teams' World Series championships is 38.
    a. Write a system of equations to find the number of World Series championships won by the Yankees, y, and the number of World Series championships won by the Cardinals, x. $y - 2x = 5$ and $y + x = 38$
    b. How many times has each team won the World Series? The Cardinals have 11 wins and the Yankees have 27 wins.

Mixed Exercises

Use elimination to solve each system of equations.

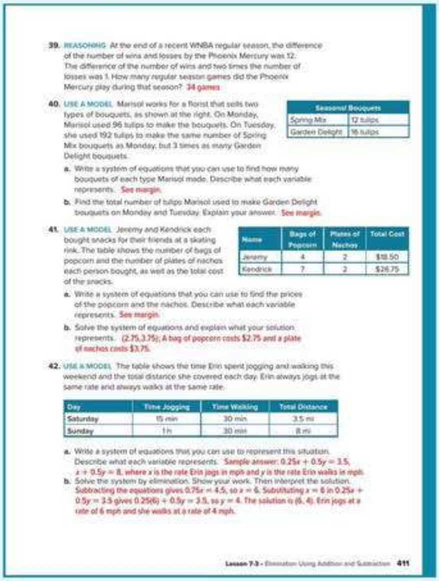
33. $4(x + 2y) = 8$
    $4x + 4y = 12$
    $(4, -1)$
34. $3x - 5y = 11$
    $5(x + y) = 5$
    $(2, -1)$
35. $4x + 3y = 6$
    $3(x + y) = 7$
    $\left(-1, 3\frac{1}{3}\right)$
36. $0.3x - 2y = -28$
    $0.8x + 2y = 28$
    $(0, 14)$
37. $\frac{1}{2}q - 4r = -2$
    $\frac{1}{6}q - 4r = 10$
    $(-36, -4)$
38. $\frac{1}{2}x + \frac{1}{3}y = -1$
    $-\frac{1}{2}x + \frac{2}{3}y = 10$
    $(-8, 9)$

410 **Module 7** • Systems of Linear Equations and Inequalities

3 REFLECT AND PRACTICE

A.CED.3, A.REI.6

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



39. REASONING At the end of a recent WNBA regular season, the difference of the number of wins and losses by the Phoenix Mercury was 12. The difference of the number of wins and two times the number of losses was 1. How many regular season games did the Phoenix Mercury play during that season? 34 games

40. USE A MODEL Marisol works for a florist that sells two types of bouquets, as shown at the right. On Monday, Marisol used 96 tulips to make the bouquets. On Tuesday, she used 192 tulips to make the same number of Spring Mix bouquets as Monday, but 3 times as many Garden Delight bouquets.

Seasonal Bouquets	
Spring Mix	12 tulips
Garden Delight	16 tulips

a. Write a system of equations that you can use to find how many bouquets of each type Marisol made. Describe what each variable represents. See margin.

b. Find the total number of tulips Marisol used to make Garden Delight bouquets on Monday and Tuesday. Explain your answer. See margin.

41. USE A MODEL Jeremy and Kendrick each bought snacks for their friends at a skating rink. The table shows the number of bags of popcorn and the number of plates of nachos each person bought, as well as the total cost of the snacks.

Name	Bags of Popcorn	Plates of Nachos	Total Cost
Jeremy	4	2	\$18.50
Kendrick	7	2	\$26.75

a. Write a system of equations that you can use to find the prices of the popcorn and the nachos. Describe what each variable represents. See margin.

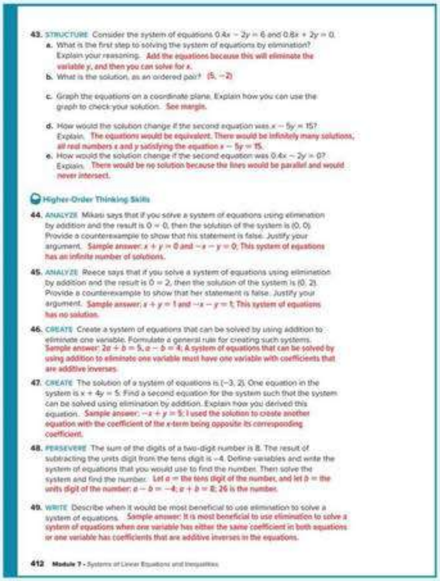
b. Solve the system of equations and explain what your solution represents. (2.75, 3.75); A bag of popcorn costs \$2.75 and a plate of nachos costs \$3.75.

42. USE A MODEL The table shows the time Erin spent jogging and walking this weekend and the total distance she covered each day. Erin always jogs at the same rate and always walks at the same rate.

Day	Time Jogging	Time Walking	Total Distance
Saturday	15 min	30 min	3.5 mi
Sunday	1 h	30 min	8 mi

a. Write a system of equations that you can use to represent this situation. Describe what each variable represents. Sample answer: $0.25x + 0.5y = 3.5$, $x + 0.5y = 8$, where x is the rate Erin jogs in mph and y is the rate Erin walks in mph.

b. Solve the system by elimination. Show your work. Then interpret the solution. Subtracting the equations gives $0.75x = 4.5$, so $x = 6$. Substituting $x = 6$ in $0.25x + 0.5y = 3.5$ gives $0.25(6) + 0.5y = 3.5$, so $y = 4$. The solution is (6, 4). Erin jogs at a rate of 6 mph and she walks at a rate of 4 mph.



43. STRUCTURE Consider the system of equations $0.4x - 2y = 6$ and $0.8x + 2y = 0$.

a. What is the first step to solving the system of equations by elimination? Explain your reasoning. Add the equations because this will eliminate the variable y, and then you can solve for x.

b. What is the solution, as an ordered pair? (5, −2)

c. Graph the equations on a coordinate plane. Explain how you can use the graph to check your solution. See margin.

d. How would the solution change if the second equation was $x - 5y = 15$? Explain. The equations would be equivalent. There would be infinitely many solutions, all real numbers x and y satisfying the equation $x - 5y = 15$.

e. How would the solution change if the second equation was $0.4x - 2y = 0$? Explain. There would be no solution because the lines would be parallel and would never intersect.

Higher-Order Thinking Skills

44. ANALYZE Mikasi says that if you solve a system of equations using elimination by addition and the result is $0 = 0$, then the solution of the system is (0, 0). Provide a counterexample to show that his statement is false. Justify your argument. Sample answer: $x + y = 0$ and $-x - y = 0$; This system of equations has an infinite number of solutions.

45. ANALYZE Reece says that if you solve a system of equations using elimination by addition and the result is $0 = 2$, then the solution of the system is (0, 2). Provide a counterexample to show that her statement is false. Justify your argument. Sample answer: $x + y = 1$ and $-x - y = 1$; This system of equations has no solution.

46. CREATE Create a system of equations that can be solved by using addition to eliminate one variable. Formulate a general rule for creating such systems. Sample answer: $2a + b = 5$, $a - b = 4$; A system of equations that can be solved by using addition to eliminate one variable must have one variable with coefficients that are additive inverses.

47. CREATE The solution of a system of equations is (−3, 2). One equation in the system is $x + 4y = 5$. Find a second equation for the system such that the system can be solved using elimination by addition. Explain how you derived this equation. Sample answer: $-x + y = 5$; I used the solution to create another equation with the coefficient of the x-term being opposite its corresponding coefficient.

48. PERSEVERE The sum of the digits of a two-digit number is 8. The result of subtracting the units digit from the tens digit is −4. Define variables and write the system of equations that you would use to find the number. Then solve the system and find the number. Let a = the tens digit of the number, and let b = the units digit of the number; $a - b = -4$; $a + b = 8$; 26 is the number.

49. WRITE Describe when it would be most beneficial to use elimination to solve a system of equations. Sample answer: It is most beneficial to use elimination to solve a system of equations when one variable has either the same coefficient in both equations or one variable has coefficients that are additive inverses in the equations.

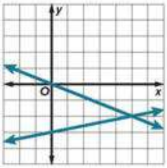
Answers

40a. Sample answer: $12x + 16y = 96$, $12x + 48y = 192$, where x is the number of Spring Mix bouquets and y is the number of Garden Delight bouquets Marisol made on Monday.

40b. 192; The solution of the system is (4, 3), so Marisol made 3 Garden Delight bouquets on Monday and 9 Garden Delight bouquets on Tuesday, for a total of 12 Garden Delight bouquets; 12 × 16 = 192.

41a. Sample answer: $4p + 2n = 18.50$, $7p + 2n = 26.75$, where p is the price of a bag of popcorn and n is the price of a plate of nachos.

43c. Sample answer: The point of intersection on the graph will match the solution (5, −2).



Lesson 7-4

Elimination Using Multiplication

A.REI.5, A.REI.6

LESSON GOAL

Students solve systems of equations by using elimination with multiplication.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Graphing and Elimination Using Multiplication

Develop:

Solving Systems of Equations by Elimination with Multiplication
- Elimination Using Multiplication
- Multiply Both Equations to Eliminate a Variable
- Write and Solve a System Using Multiplication

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Solve Systems of Equations by Substitution	●	●		●
Extension: Solving Systems of Equations in Three Variables		●	●	●

Language Development Handbook

Assign page 41 of the Language Development Handbook to help your students build mathematical language related to solving systems of equations by using elimination with multiplication.

ELL You can use the tips and suggestions on page T41 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra

Standards for Mathematical Content:

A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students solved systems of equations by using elimination with addition or subtraction. **8.EE.8b, A.CED.3, A.REI.6**

Now
Students solve systems of equations by using elimination with multiplication. **A.REI.5, A.REI.6**

Next
Students will solve systems of inequalities by graphing. **A.CED.3, A.REI.12**

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of using elimination to solve systems of linear equations and build fluency with using elimination and multiplication to solve systems. They apply their understanding of the elimination method by solving real-world problems.

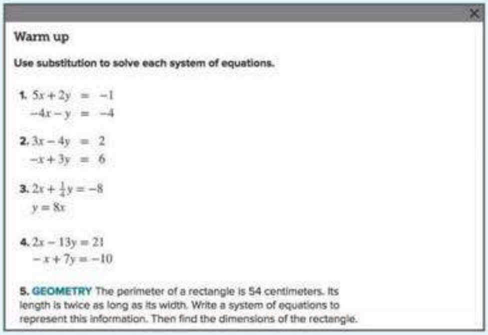
Mathematical Background

When the coefficients of like variable terms are neither the same nor additive inverses, elimination using multiplication can be used to solve the system of equations. This method requires that either one or both of the equations be multiplied by a number so that when the equations are added or subtracted, a variable is eliminated. The system can then be solved using elimination by addition or subtraction.

1 LAUNCH

A.REI.5, A.REI.6

Interactive Presentation



Warm up

Use substitution to solve each system of equations.

1. $5x + 2y = -1$
$-4x - y = -4$

2. $3x - 4y = 2$
$-x + 3y = 6$

3. $2x + \frac{1}{4}y = -8$
$y = 8x$

4. $2x - 13y = 21$
$-x + 7y = -10$

5. GEOMETRY The perimeter of a rectangle is 54 centimeters. Its length is twice as long as its width. Write a system of equations to represent this information. Then find the dimensions of the rectangle.

Warm Up

Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- solving systems of equations by using substitution

Answers:

1. (3, −8)
2. (6, 4)
3. (−2, −16)
4. (17, 1)
5. $2\ell + 2w = 54$ and $w = 2\ell$; 9 cm by 18 cm

Launch the Lesson

MP Teaching the Mathematical Practices

2 Create Representations Guide students to write a system of equations that models the number of recreational visits to national parks. The system of equations can be solved by elimination using multiplication and addition.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Explore Graphing and Elimination Using Multiplication

Objective

Students use a graph to explore how to solve a system of equations by eliminating a variable using multiplication.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

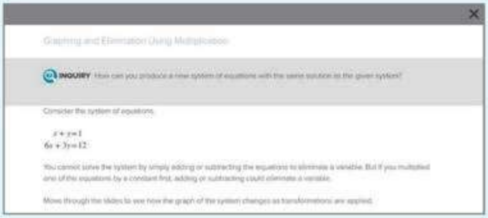
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will progress through a series of slides that show how the graph of a system of equations changes as the equations are manipulated. They will observe how the solution to the system can be found by transforming the equations and applying algebraic methods that they learned earlier in the module. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Graphing and Elimination Using Multiplication

INQUIRY How can you produce a new system of equations with the same solution as the given system?

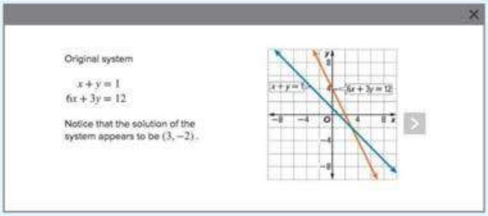
Consider the system of equations.

$$\begin{aligned} x + y &= 1 \\ 6x + 3y &= 12 \end{aligned}$$

You cannot solve the system by simply adding or subtracting the equations to eliminate a variable. But if you multiplied one of the equations by a constant first, adding or subtracting could eliminate a variable.

Move through the slides to see how the graph of the system changes as transformations are applied.

Explore



Original system

$$\begin{aligned} x + y &= 1 \\ 6x + 3y &= 12 \end{aligned}$$

Notice that the solution of the system appears to be $(3, -2)$.

Explore

TAP

Students move through the slides to see how the graph of the system changes as transformations are applied.

TYPE

Students explain whether eliminating different variables changes the solution.

Interactive Presentation

INQUIRY How can you produce a new system of equations with the same solution as the given system?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Graphing and Elimination Using Multiplication (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why does multiplying the equation by −3 not change its graph? Sample answer: The Multiplication Property of Equality states that if you multiply both sides of an equation by the same number, you maintain the same equality.
- How do you know what number to multiply the equation by? Y ou need to multiply by a number that will then cause one of the variables to be eliminated when the new equation is added to the other equation.

Inquiry

How can you produce a new system of equations with the same solution as the given system? Sample answer: Change the equations in the original system using multiplication and addition. When the equations are manipulated, the point of intersection of the graphs remains the same.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Systems of Equations by Elimination with Multiplication

Objective

Students solve systems of equations by eliminating a variable using multiplication and addition.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students Are Learning

Students may have recognized that not all systems of equations can be solved by simply adding or subtracting the equations. In this lesson, students learn that by applying the Multiplication Property of Equality, they can multiply one or both of the equations by a number (or numbers) of their choosing to create an equivalent system that can then be solved by addition or subtraction.

Common Misconception

A common misconception some students may have is that there is only one correct way to use multiplication to solve a system of equations. Explain that this is not the case and that there are many ways to transform a system to an equivalent system that can then be solved by elimination. Provide a few examples, and then have students offer additional methods of solution.

Example 1 Elimination Using Multiplication

Teaching the Mathematical Practices

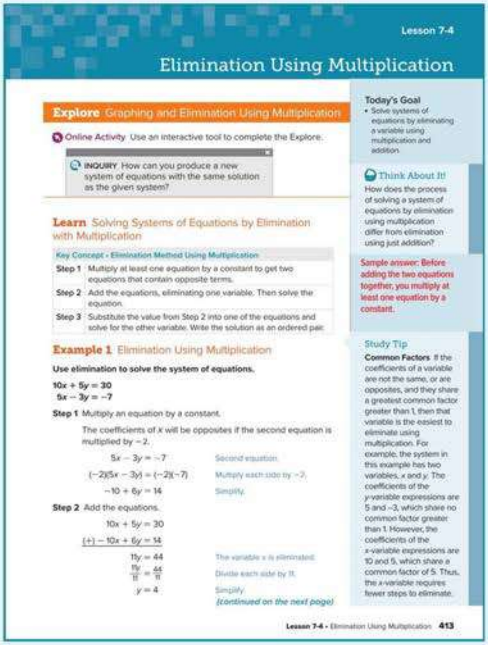
3 Make Conjectures In this example, students will make conjectures and then build a logical progression of statements to validate the conjectures. Once students have made their conjectures, guide the students to validate them.

Questions for Mathematical Discourse

AL Which variable terms have coefficients with common multiples? $10x$ and $5x$

OL Why can you eliminate the variable after multiplying by −2? Sample answer: If you multiply the second equation by −2, then the coefficients of the x-terms are additive inverses and you can add the equations to eliminate x.

BL How can you check your solution? Sample answer: Substitute 1 for x and 4 for y in the original equations and determine if both of the resulting equations are true.



Lesson 7-4

Elimination Using Multiplication

Explore Graphing and Elimination Using Multiplication

Online Activity Use an interactive tool to complete the Explore.

INQUIRY How can you produce a new system of equations with the same solution as the given system?

Today's Goal
- Solve systems of equations by eliminating a variable using multiplication and addition.

Think About It!
How does the process of solving a system of equations by elimination using multiplication differ from elimination using just addition?

Sample answer: Before adding the two equations together, you multiply at least one equation by a constant.

Learn Solving Systems of Equations by Elimination with Multiplication

Key Concept • Elimination Method Using Multiplication	
Step 1	Multiply at least one equation by a constant to get two equations that contain opposite terms.
Step 2	Add the equations, eliminating one variable. Then solve the equation.
Step 3	Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Elimination Using Multiplication

Use elimination to solve the system of equations.

$$10x + 5y = 30$$
$$5x - 3y = -7$$

Step 1 Multiply an equation by a constant.

The coefficients of x will be opposites if the second equation is multiplied by −2.

$$5x - 3y = -7 \quad \text{Second equation}$$
$$(-2)(5x - 3y) = (-2)(-7) \quad \text{Multiply each side by } -2.$$
$$-10 + 6y = 14 \quad \text{Simplify.}$$

Step 2 Add the equations.

$$\begin{array}{r} 10x + 5y = 30 \\ \underline{(+)\ -10x + 6y = 14} \\ 11y = 44 \end{array} \quad \text{The variable } x \text{ is eliminated.}$$
$$\frac{11y}{11} = \frac{44}{11} \quad \text{Divide each side by 11.}$$
$$y = 4 \quad \text{Simplify.}$$

(continued on the next page)

Study Tip
Common Factors If the coefficients of a variable are not the same, or are opposites, and they share a greatest common factor greater than 1, then that variable is the easiest to eliminate using multiplication. For example, the system in this example has two variables, x and y. The coefficients of the y-variable expressions are 5 and −3, which share no common factor greater than 1. However, the coefficients of the x-variable expressions are 10 and 5, which share a common factor of 5. Thus, the x-variable requires fewer steps to eliminate.

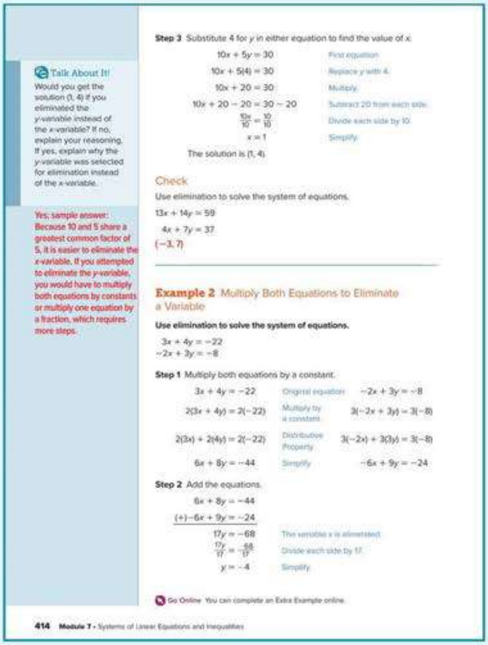
Lesson 7-4 • Elimination Using Multiplication 413

Interactive Presentation

Learn

TYPE

Students explain how the process of solving a system of equations by elimination using multiplication differs from just addition.



Talk About It! Would you get the solution (1, 4) if you eliminated the y-variable instead of the x-variable? If no, explain your reasoning. If yes, explain why the y-variable was selected for elimination instead of the x-variable.

Yes; sample answer: Because 10 and 5 share a greatest common factor of 5, it is easier to eliminate the x-variable. If you attempted to eliminate the y-variable, you would have to multiply both equations by constants or multiply one equation by a fraction, which requires more steps.

Step 3 Substitute 4 for y in either equation to find the value of x.

$$\begin{aligned} 10x + 5y &= 30 && \text{First equation} \\ 10x + 5(4) &= 30 && \text{Replace } y \text{ with 4.} \\ 10x + 20 &= 30 && \text{Multiply.} \\ 10x + 20 - 20 &= 30 - 20 && \text{Subtract 20 from each side.} \\ \frac{10x}{10} &= \frac{10}{10} && \text{Divide each side by 10.} \\ x &= 1 && \text{Simplify.} \end{aligned}$$

The solution is (1, 4).

Check

Use elimination to solve the system of equations.

$13x + 14y = 59$

$4x + 7y = 37$

$(-3, 7)$

Example 2 Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$3x + 4y = -22$

$-2x + 3y = -8$

Step 1 Multiply both equations by a constant.

$$\begin{array}{rcr} 3x + 4y = -22 & \text{Original equation} & -2x + 3y = -8 \\ 2(3x + 4y) = 2(-22) & \text{Multiply by a constant.} & 3(-2x + 3y) = 3(-8) \\ 2(3x) + 2(4y) = 2(-22) & \text{Distributive Property} & 3(-2x) + 3(3y) = 3(-8) \\ 6x + 8y = -44 & \text{Simplify.} & -6x + 9y = -24 \end{array}$$

Step 2 Add the equations.

$$\begin{aligned} 6x + 8y &= -44 && \\ \underline{(+)\,-6x + 9y} &\underline{= -24} && \\ 17y &= -68 && \text{The variable } x \text{ is eliminated.} \\ \frac{17y}{17} &= \frac{-68}{17} && \text{Divide each side by 17.} \\ y &= -4 && \text{Simplify.} \end{aligned}$$

Go Online You can complete an Extra Example online.

414 **Module 7** • Systems of Linear Equations and Inequalities

Interactive Presentation

Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$3x + 4y = -22$

$-2x + 3y = -8$

Step 1

Multiply both equations by a constant.

Step 2

Add the equations.

Step 3

Use substitution to find the value of x.

Example 2

TAP

Students tap on each step to see how to solve a system by multiplying both equations.

Example 2 Multiply Both Equations to Eliminate a Variable

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress and change course to find the solution.

Common Error

A common error some students may make is forgetting to multiply the constant on the right side of the equal sign by the number used to multiply the terms on the left side. Encourage students to write the multiplier on both sides of the equal sign before they actually perform the multiplication.

Questions for Mathematical Discourse

AL Do either of the variables have coefficients with common multiples? no

OL How do you determine what numbers to multiply by? Sample answer: You need to find a common multiple of the coefficients and then figure out what multipliers will make the coefficients that number and its opposite.

BL Why is it easier to eliminate *x* first? Sample answer: Because the *x*-terms have opposite signs, you can just multiply by 2 and 3 and then add, without having to worry about multiplying by a negative number.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Write and Solve a System Using Multiplication

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

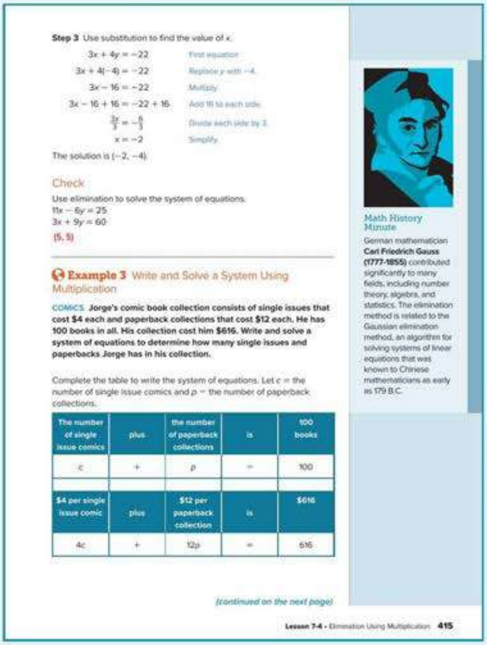
AL Why is the first equation multiplied by −4? The coefficient of x in the second equation is 4, so you need the coefficient in the first equation to be −4 in order to cancel.

OL What would you do if you wanted to eliminate p instead of c? Multiply the first equation by −12 and add the equations.

BL What number could you multiply the second equation by if you wanted to eliminate c using addition? $-\frac{1}{4}$

Common Error

Some students may have difficulty writing the equations for the system in these types of problems. Tell them to think about writing one equation that models "how many" and another equation that models "value."



Step 3 Use substitution to find the value of x.

$$\begin{aligned} 3x + 4y &= -22 && \text{First equation} \\ 3x + 4(-4) &= -22 && \text{Replace } y \text{ with } -4. \\ 3x - 16 &= -22 && \text{Multiply.} \\ 3x - 16 + 16 &= -22 + 16 && \text{Add 16 to each side.} \\ \frac{3x}{3} &= -\frac{6}{3} && \text{Divide each side by 3.} \\ x &= -2 && \text{Simplify.} \end{aligned}$$

The solution is $(-2, -4)$.

Check

Use elimination to solve the system of equations.

$11x - 6y = 25$

$3x + 9y = 60$

(5, 5)

Example 3 Write and Solve a System Using Multiplication

COMICS **Jorge's comic book collection consists of single issues that cost $4 each and paperback collections that cost $12 each. He has 100 books in all. His collection cost him $616. Write and solve a system of equations to determine how many single issues and paperbacks Jorge has in his collection.**

Complete the table to write the system of equations. Let c = the number of single issue comics and p = the number of paperback collections.

The number of single issue comics	plus	the number of paperback collections	is	100 books
c	+	p	=	100
$4 per single issue comic	**plus**	**$12 per paperback collection**	**is**	**$616**
$4c$	+	$12p$	=	616

(continued on the next page)

Math History Minute

German mathematician **Carl Friedrich Gauss (1777–1855)** contributed significantly to many fields, including number theory, algebra, and statistics. The elimination method is related to the Gaussian elimination method, an algorithm for solving systems of linear equations that was known to Chinese mathematicians as early as 179 B.C.

Lesson 7-4 • Elimination Using Multiplication **415**

Interactive Presentation

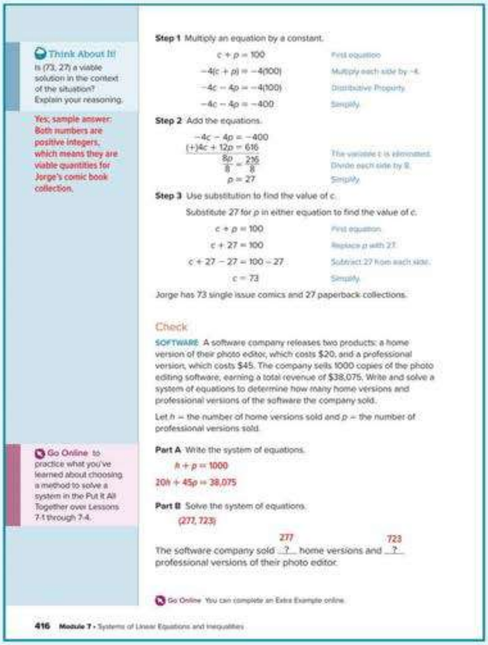
Example 3

TAP

Students select the correct variables to complete the equation and move through the steps to solve using multiplication.

CHECK

Students complete the Check online to determine whether they are ready to move on.



Think About It!
Is (73, 27) a viable solution in the context of the situation? Explain your reasoning.

Yes; sample answer: Both numbers are positive integers, which means they are viable quantities for Jorge's comic book collection.

Step 1 Multiply an equation by a constant.

$c + p = 100$	First equation
$-4(c + p) = -4(100)$	Multiply each side by −4.
$-4c - 4p = -4(100)$	Distributive Property
$-4c - 4p = -400$	Simplify.

Step 2 Add the equations.

$-4c - 4p = -400$	
$(+)4c + 12p = 616$	
$\frac{8p}{8} = \frac{216}{8}$	The variable c is eliminated. Divide each side by 8.
$p = 27$	Simplify.

Step 3 Use substitution to find the value of c.

Substitute 27 for p in either equation to find the value of c.

$c + p = 100$	First equation
$c + 27 = 100$	Replace p with 27.
$c + 27 - 27 = 100 - 27$	Subtract 27 from each side.
$c = 73$	Simplify.

Jorge has 73 single issue comics and 27 paperback collections.

Check

SOFTWARE A software company releases two products: a home version of their photo editor, which costs $20, and a professional version, which costs $45. The company sells 1000 copies of the photo editing software, earning a total revenue of $38,075. Write and solve a system of equations to determine how many home versions and professional versions of the software the company sold.

Let h = the number of home versions sold and p = the number of professional versions sold.

Part A Write the system of equations.

$h + p = 1000$

$20h + 45p = 38,075$

Part B Solve the system of equations.

(277, 723)

The software company sold $\overset{277}{\underline{\ ?\ }}$ home versions and $\overset{723}{\underline{\ ?\ }}$ professional versions of their photo editor.

Go Online to practice what you've learned about choosing a method to solve a system in the Put It All Together over Lessons 7-1 through 7-4.

Go Online You can complete an Extra Example online.

416 **Module 7** • Systems of Linear Equations and Inequalities

Interactive Presentation

Question 2

Check

TYPE

Students solve a system of equations.

Common Error

Some students may have difficulty writing the equations that represent the system. It may be helpful to pair these students with students who have a better grasp of the concept, and have them discuss the translation from words to symbols.

DIFFERENTIATE

Enrichment Activity BL

Ask: What are the benefits of having different strategies for solving systems of equations? Sample answer: You can use the strategy that is most efficient. For example, if the variable terms have coefficients that are opposites, elimination would be a good choice. If one of the equations is solved for a variable, substitution may be an efficient way to find the solution. If the equations are easily graphed, graphing might be a good method to use. Also, one method may be a good way to check a solution found using a different method.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–14
2	exercises that use a variety of skills from this lesson	15, 16
2	exercises that extend concepts learned in this lesson to new contexts	17–20
3	exercises that emphasize higher-order and critical-thinking skills	21–25

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or more on the Checks,
THEN assign:

- Practice Exercises 1–19 odd, 21–25
- Extension: Solving Systems of Equations in Three Variables
- **ALEKS** Systems of Linear Equations

OL

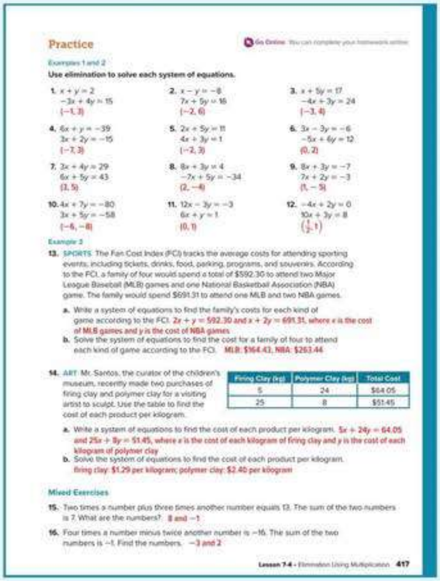
IF students score 66%–89% on the Checks,
THEN assign:

- Practice Exercises 1–25 odd
- Remediation, Review Resources: Solve Systems of Equations by Substitution
- Personal Tutors
- Extra Examples 1–3
- **ALEKS** Systems of Equations

AL

IF students score 65% or less on the Checks,
THEN assign:

- Practice, Exercises 1–13 odd
- Remediation, Review Resources: Solve Systems of Equations by Substitution
- *Quick Review Math Handbook*: Elimination Using Multiplication
- Arrive**MATH** Take Another Look
- **ALEKS** Systems of Equations



Practice

Examples 1 and 2

Use elimination to solve each system of equations.

1. $x + y = 2$
$-3x + 4y = 15$
$(-1, 3)$

2. $x - y = -8$
$7x + 5y = 16$
$(-2, 6)$

3. $x + 5y = 17$
$-4x + 3y = 24$
$(-3, 4)$

4. $6x + y = -39$
$3x + 2y = -15$
$(-7, 3)$

5. $2x + 5y = 11$
$4x + 3y = 1$
$(-2, 3)$

6. $3x - 3y = -6$
$-5x + 6y = 12$
$(0, 2)$

7. $3x + 4y = 29$
$6x + 5y = 43$
$(3, 5)$

8. $8x + 3y = 4$
$-7x + 5y = -34$
$(2, -4)$

9. $8x + 3y = -7$
$7x + 2y = -3$
$(1, -5)$

10. $4x + 7y = -80$
$3x + 5y = -58$
$(-6, -8)$

11. $12x - 3y = -3$
$6x + y = 1$
$(0, 1)$

12. $-4x + 2y = 0$
$10x + 3y = 8$
$\left(\frac{1}{2}, 1\right)$

Example 3

13. SPORTS The Fan Cost Index (FCI) tracks the average costs for attending sporting events, including tickets, drinks, food, parking, programs, and souvenirs. According to the FCI, a family of four would spend a total of $592.30 to attend two Major League Baseball (MLB) games and one National Basketball Association (NBA) game. The family would spend $691.31 to attend one MLB and two NBA games.
   a. Write a system of equations to find the family's costs for each kind of game according to the FCI. $2x + y = 592.30$ and $x + 2y = 691.31$, where x is the cost of MLB games and y is the cost of NBA games
   b. Solve the system of equations to find the cost for a family of four to attend each kind of game according to the FCI. MLB: $164.43, NBA: $263.44

14. ART Mr. Santos, the curator of the children's museum, recently made two purchases of firing clay and polymer clay for a visiting artist to sculpt. Use the table to find the cost of each product per kilogram.

Firing Clay (kg)	Polymer Clay (kg)	Total Cost
5	24	$64.05
25	8	$51.45

   a. Write a system of equations to find the cost of each product per kilogram. $5x + 24y = 64.05$ and $25x + 8y = 51.45$, where x is the cost of each kilogram of firing clay and y is the cost of each kilogram of polymer clay
   b. Solve the system of equations to find the cost of each product per kilogram. firing clay: $1.29 per kilogram; polymer clay: $2.40 per kilogram

Mixed Exercises

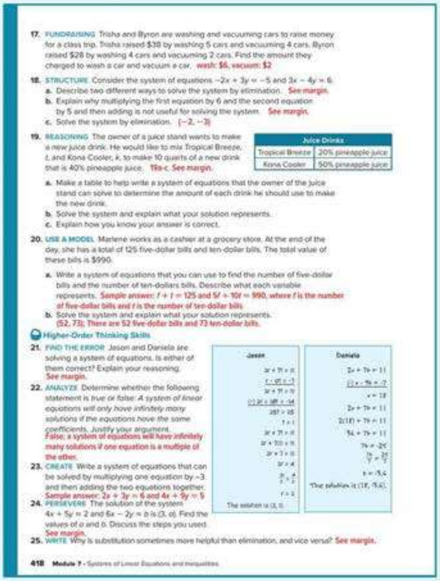
15. Two times a number plus three times another number equals 13. The sum of the two numbers is 7. What are the numbers? 8 and −1

16. Four times a number minus twice another number is −16. The sum of the two numbers is −1. Find the numbers. −3 and 2

Lesson 7-4 • Elimination Using Multiplication 417

3 REFLECT AND PRACTICE

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



17. FUNDRAISING Trisha and Byron are washing and vacuuming cars to raise money for a class trip. Trisha raised \$38 by washing 5 cars and vacuuming 4 cars. Byron raised \$28 by washing 4 cars and vacuuming 2 cars. Find the amount they charged to wash a car and vacuum a car. wash: \$6, vacuum: \$2

18. STRUCTURE Consider the system of equations $-2x + 3y = -5$ and $3x - 4y = 6$.
   a. Describe two different ways to solve the system by elimination. See margin.
   b. Explain why multiplying the first equation by 6 and the second equation by 5 and then adding is not useful for solving the system. See margin.
   c. Solve the system by elimination. $(-2, -3)$

19. REASONING The owner of a juice stand wants to make a new juice drink. He would like to mix Tropical Breeze, t, and Kona Cooler, k, to make 10 quarts of a new drink that is 40% pineapple juice. 19a–c. See margin.

Juice Drinks	
Tropical Breeze	20% pineapple juice
Kona Cooler	50% pineapple juice

   a. Make a table to help write a system of equations that the owner of the juice stand can solve to determine the amount of each drink he should use to make the new drink.
   b. Solve the system and explain what your solution represents.
   c. Explain how you know your answer is correct.

20. USE A MODEL Marlene works as a cashier at a grocery store. At the end of the day, she has a total of 125 five-dollar bills and ten-dollar bills. The total value of these bills is \$990.
   a. Write a system of equations that you can use to find the number of five-dollar bills and the number of ten-dollars bills. Describe what each variable represents. Sample answer: $f + t = 125$ and $5f + 10t = 990$, where f is the number of five-dollar bills and t is the number of ten-dollar bills.
   b. Solve the system and explain what your solution represents. (52, 73); There are 52 five-dollar bills and 73 ten-dollar bills.

Higher-Order Thinking Skills

21. FIND THE ERROR Jason and Daniela are solving a system of equations. Is either of them correct? Explain your reasoning. See margin.

22. ANALYZE Determine whether the following statement is true or false: *A system of linear equations will only have infinitely many solutions if the equations have the same coefficients.* Justify your argument. False; a system of equations will have infinitely many solutions if one equation is a multiple of the other.

23. CREATE Write a system of equations that can be solved by multiplying one equation by -3 and then adding the two equations together. Sample answer: $2x + 3y = 6$ and $4x + 9y = 5$

24. PERSEVERE The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b. Discuss the steps you used. See margin.

25. WRITE Why is substitution sometimes more helpful than elimination, and vice versa? See margin.

418 Module 7 • Systems of Linear Equations and Inequalities

Answers

18a. (1) Multiply the first equation by 3, the second equation by 2, and add to eliminate x; or (2) Multiply the first equation by 4, the second equation by 3, and add to eliminate y.

18b. This would allow for the elimination of the constant term on the right side, but it would still leave two equations with two variables. No variable would be eliminated.

19a.

	Tropical Breeze	Kona Cooler	Total
Amount of Juice (qt)	t	k	10
Amount of Pineapple Juice (qt)	$0.2t$	$0.5k$	4

19b. $\left(3\frac{1}{3}, 6\frac{2}{3}\right)$ The owner should mix $3\frac{1}{3}$ qt of Tropical Breeze and $6\frac{2}{3}$ qt of Kona Cooler.

19c. $3\frac{1}{3}\text{ qt} + 6\frac{2}{3}\text{ qt} = 10\text{ qt}$, so the total amount is correct, and $0.2\left(3\frac{1}{3}\text{qt}\right) + 0.5\left(6\frac{2}{3}\text{qt}\right) = 4\text{ qt}$, so the amount of pineapple juice in the new drink is correct.

21. Jason; In order to eliminate the t-terms, you can multiply the second equation by 2 and then subtract, or multiply the equation by -2 and then add. Daniela did not subtract the equations correctly.

24. $a = -2$, $b = 22$; Substitute 3 for x and a for y in the first equation and then solve for a to get $a = -2$. Then substitute 3 for x and -2 for y in the second equation and simplify to get $b = 22$.

25. Sample answer: It is more helpful to use substitution when one of the variables has a coefficient of 1 or if a coefficient can be reduced to 1 without turning other coefficients into fractions. Otherwise, elimination is more helpful because it will avoid the use of fractions when solving the system.

Lesson 7-5

Systems of Inequalities

A.CED.3, A.REI.12

LESSON GOAL

Students solve systems of inequalities by graphing.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Solutions of Systems of Inequalities

Develop:

Solving Systems of Inequalities by Graphing

- Solve by Graphing
- Solve by Graphing, No Solution
- Apply Systems of Inequalities

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Graphing Inequalities in Two Variables	●	●		●
Extension: Describing Regions		●	●	●

Language Development Handbook

Assign page 42 of the *Language Development Handbook* to help your students build mathematical language related to solving systems of inequalities.

ELL You can use the tips and suggestions on page T42 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra

Standards for Mathematical Content:

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Standards for Mathematical Practice:

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous

Students solved systems of equations by using elimination with multiplication.
A.REI.5, A.REI.6

Now

Students solve systems of inequalities by graphing.
A.CED.3, A.REI.12

Next

Students will use linear programming to find maximum or minimum values of a function. **A.CED.3 (Course 2)**

Rigor

The Three Pillars of Rigor

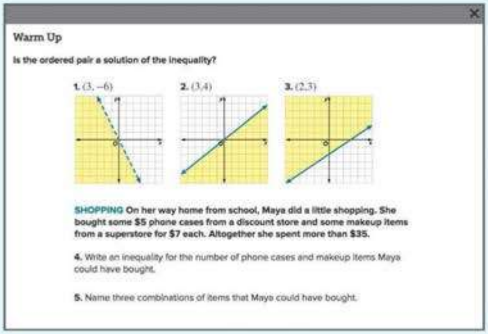
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their understanding of graphing linear inequalities to build fluency with graphing systems of linear inequalities. They apply their understanding of graphing systems of linear inequalities by solving real-world problems.

Mathematical Background

A solution of a system of inequalities is the set of all points that satisfy both inequalities. To solve the system, graph each inequality and shade the region where the graphs overlap, or intersect. If the boundary lines are parallel, and the shaded regions have no points in common, then there is no solution to the system. Otherwise, the system has infinitely many solutions.

Interactive Presentation



Warm Up

Is the ordered pair a solution of the inequality?

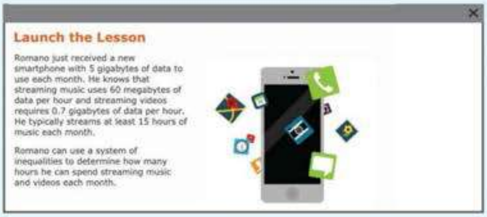
1. $(3, -6)$
2. $(3, 4)$
3. $(2, 3)$

SHOPPING On her way home from school, Maya did a little shopping. She bought some $5 phone cases from a discount store and some makeup items from a superstore for $7 each. Altogether she spent more than $35.

4. Write an inequality for the number of phone cases and makeup items Maya could have bought.

5. Name three combinations of items that Maya could have bought.

Warm Up

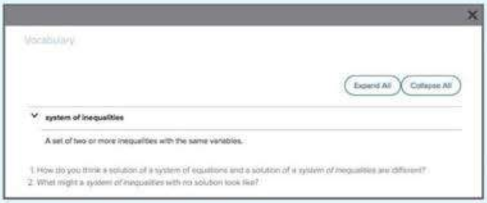


Launch the Lesson

Romano just received a new smartphone with 5 gigabytes of data to use each month. He knows that streaming music uses 60 megabytes of data per hour and streaming videos requires 0.7 gigabytes of data per hour. He typically streams at least 15 hours of music each month.

Romano can use a system of inequalities to determine how many hours he can spend streaming music and videos each month.

Launch the Lesson



system of inequalities

A set of two or more inequalities with the same variables.

1. How do you think a solution of a system of equations and a solution of a system of inequalities are different?
2. What might a system of inequalities with no solution look like?

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- testing half-planes

Answers:

1. no
2. no
3. yes
4. $5x + 7y > 35$
5. Sample answers: 3 phone cases and 4 makeup items; 5 phone cases and 5 makeup items; 6 phone cases and 2 makeup items

Launch the Lesson

MP Teaching the Mathematical Practices

6 State the Meaning of Symbols Guide students to define variables to represent the situation in this Launch. Then, help students write a system of inequalities to determine the amount of time Romano can spend streaming music and videos.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Explore Solutions of Systems of Inequalities

Objective

Students use a graph to explore the solutions of a system of inequalities.

MP Teaching the Mathematical Practices

3 Construct Arguments In this Explore, students will use stated assumptions, definitions, and previously established results to construct an argument.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

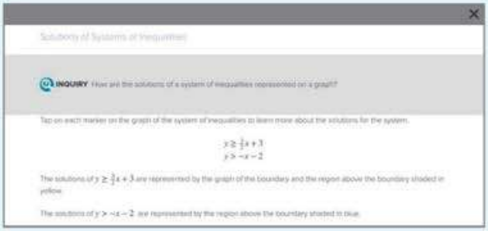
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will explore how a system of linear inequalities is represented on a graph. They will use a number of marked points to understand what the different shaded areas of the graph represent. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Solutions of Systems of Inequalities

INQUIRY How are the solutions of a system of inequalities represented on a graph?

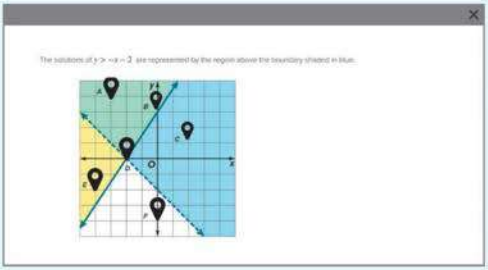
Tap on each marker on the graph of the system of inequalities to learn more about the solutions for the system.

$$y \geq \frac{3}{2}x + 3$$
$$y > -x - 2$$

The solutions of $y \geq \frac{3}{2}x + 3$ are represented by the graph of the boundary and the region above the boundary shaded in yellow.

The solutions of $y > -x - 2$ are represented by the region above the boundary shaded in blue.

Explore



The solutions of $y > -x - 2$ are represented by the region above the boundary shaded in blue.

Explore

TAP

Students tap on each marker on the graph to learn about the solutions of the systems of inequalities.

TYPE

Students complete the exercises about graphing systems of inequalities.

Interactive Presentation

INQUIRY How are the solutions of a system of inequalities represented on a graph?

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Solutions of Systems of Inequalities (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- What must be true about a point that lies in a region that is shaded in one color but not in both colors? That point is a solution of one of the inequalities but not of both.
- Is the point of intersection of the two boundary lines always a solution of the system of inequalities? Explain. No; It is a solution only if both boundary lines consist of points that are solutions of their respective inequalities.

Inquiry

How are the solutions of a system of inequalities represented on a graph? Sample answer: The solutions of the system are represented by the region where the solutions of the individual inequalities intersect.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Solving Systems of Inequalities by Graphing

Objective

Students graph the solution sets of systems of linear inequalities in two variables as the intersections of the corresponding half-planes.

MP Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

What Students Are Learning

Students will learn that a graph can be used to find the ordered pairs that satisfy both inequalities in a system. Students have already learned that the solution set of an inequality can be represented by a half-plane. Here they learn that the solution set of a system of inequalities is the region that is the overlap of the two half-planes from the system.

Example 1 Solve by Graphing

MP Teaching the Mathematical Practices

7 Interpret Complicated Expressions Mathematically proficient students can see complicated expressions as single objects or as being composed of several objects. In this example, guide students to see what information they can gather about the expression just from looking at it.

Questions for Mathematical Discourse

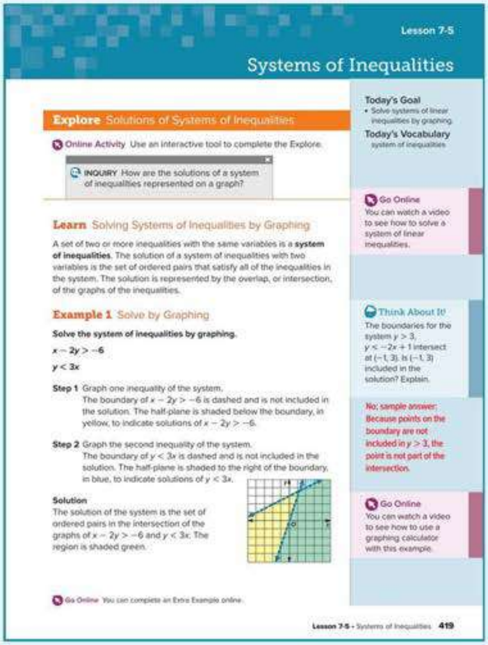
AL How do you know whether a boundary should be drawn as a solid line or a dashed line? It should be drawn as a solid line if the inequality is $\leq$ or $\geq$. It should be drawn as a dashed line if the inequality is $<$ or $>$.

OL Describe how to determine which side of the line to shade. Sample answer: Choose a point that is not on the boundary, and substitute its coordinates into the inequality. If the resulting inequality is true, then shade the half-plane in which the point lies. If it is not true, shade the other half-plane.

BL How do you know which shaded region represents the solution of the system? Sample answer: Because you want the solutions to be true for both inequalities, you are looking for the region that is shaded by both. Choose a point that lies in the region that you shaded. Substitute the coordinates of the point into each of the inequalities. If the resulting inequalities are both true, then you know you shaded correctly.

Go Online

- F ind additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 7-5

Systems of Inequalities

Explore Solutions of Systems of Inequalities

Online Activity Use an interactive tool to complete the Explore.

INQUIRY How are the solutions of a system of inequalities represented on a graph?

Learn Solving Systems of Inequalities by Graphing

A set of two or more inequalities with the same variables is a **system of inequalities**. The solution of a system of inequalities with two variables is the set of ordered pairs that satisfy all of the inequalities in the system. The solution is represented by the overlap, or intersection, of the graphs of the inequalities.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

$x - 2y > -6$

$y < 3x$

Step 1 Graph one inequality of the system. The boundary of $x - 2y > -6$ is dashed and is not included in the solution. The half-plane is shaded below the boundary, in yellow, to indicate solutions of $x - 2y > -6$.

Step 2 Graph the second inequality of the system. The boundary of $y < 3x$ is dashed and is not included in the solution. The half-plane is shaded to the right of the boundary, in blue, to indicate solutions of $y < 3x$.

Solution

The solution of the system is the set of ordered pairs in the intersection of the graphs of $x - 2y > -6$ and $y < 3x$. The region is shaded green.

Go Online You can complete an Extra Example online.

Today's Goal
- Solve systems of linear inequalities by graphing.

Today's Vocabulary
system of inequalities

Go Online You can watch a video to see how to solve a system of linear inequalities.

Think About It! The boundaries for the system $y > 3$, $y < -2x + 1$ intersect at $(-1, 3)$. Is $(-1, 3)$ included in the solution? Explain.

No; sample answer: Because points on the boundary are not included in $y > 3$, the point is not part of the intersection.

Go Online You can watch a video to see how to use a graphing calculator with this example.

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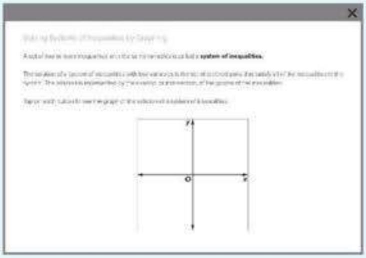
Interactive Presentation



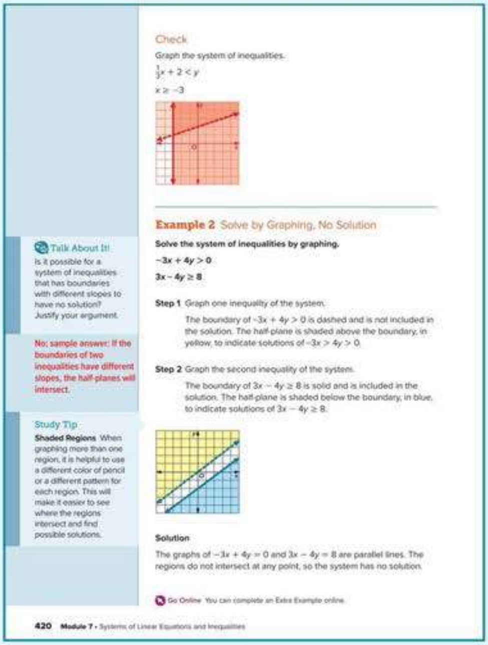
Learn

TAP

Students tap each button to see the graph of the solution of a system of inequalities.

TYPE

Students determine if a point on the boundary intersection is included in the solution.



Check

Graph the system of inequalities.

$\frac{1}{3}x + 2 < y$

$x \geq -3$

Example 2 Solve by Graphing, No Solution

Solve the system of inequalities by graphing.

$-3x + 4y > 0$

$3x - 4y \geq 8$

Step 1 Graph one inequality of the system.

The boundary of $-3x + 4y > 0$ is dashed and is not included in the solution. The half-plane is shaded above the boundary, in yellow, to indicate solutions of $-3x > 4y > 0$.

Step 2 Graph the second inequality of the system.

The boundary of $3x - 4y \geq 8$ is solid and is included in the solution. The half-plane is shaded below the boundary, in blue, to indicate solutions of $3x - 4y \geq 8$.

Solution

The graphs of $-3x + 4y = 0$ and $3x - 4y = 8$ are parallel lines. The regions do not intersect at any point, so the system has no solution.

Go Online You can complete an Extra Example online.

Talk About It!

Is it possible for a system of inequalities that has boundaries with different slopes to have no solution? Justify your argument.

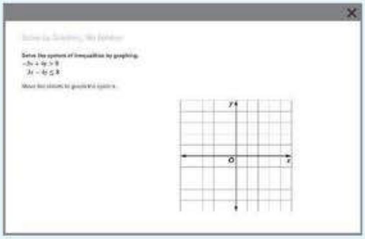
No; sample answer: If the boundaries of two inequalities have different slopes, the half-planes will intersect.

Study Tip

Shaded Regions When graphing more than one region, it is helpful to use a different color of pencil or a different pattern for each region. This will make it easier to see where the regions intersect and find possible solutions.

420 **Module 7** • Systems of Linear Equations and Inequalities

Interactive Presentation



Example 2

SWIPE

Students move the sliders to graph the system of inequalities.

TYPE

Students discuss if it is possible for a system of inequalities whose boundaries have different slopes to have no solution.

Common Error

A common error that some students may make is demonstrated in the Avoid a Common Error feature online. The graph shows that the student did not shade the half-planes separately, and therefore could not identify the overlap of the half-planes.

Example 2 Solve by Graphing, No Solution

MP Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL What does the $>$ symbol in the first equation tell you about the boundary of that half-pane? It is a dashed line. What does the $\leq$ symbol in the second equation tell you about the boundary of that half-pane? It is a solid line.

OL Why does it not work to use the intercepts in order to graph the system? Sample answer: The intercepts for the first inequality are both 0, so you only know that the graph will go through the origin. The *x*-intercept of the second inequality is a fraction that is not easily graphed.

BL How would the solution be different if the inequality sign in the first inequality was $<$ instead of $>$? Sample answer: The solution would be the half-plane that represents the second inequality.

Common Error

Some students may correctly state that the system has no solution, but do so based on their recognition of the fact that the boundaries are parallel lines. Point out that it is not the boundaries that indicate that this system has no solution, it is the fact that the half-planes do not overlap. Lead students to see that it is possible to have overlapping half-planes with parallel boundary lines.

Example 3 Apply Systems of Inequalities

Teaching the Mathematical Practices

2 Represent a Situation Symbolically Guide students to define variables to solve the problem in this example. Help students to identify the independent and dependent variables. Then, work with them to find the other relationships in the problem.

Questions for Mathematical Discourse

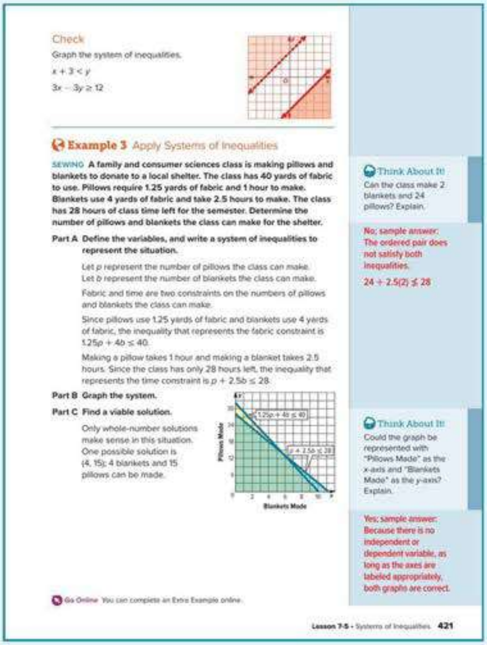
AL Why is this system of linear inequalities graphed only in the first quadrant? Sample answer: The number of pillows and blankets must each be nonnegative, so only first-quadrant values and values on the axes make sense.

OL Name three possible solutions and interpret them in the context of the situation. Sample answer: The class could make 2 blankets and 18 pillows, 4 blankets and 9 pillows, or 8 blankets and 3 pillows.

BL Is the point (10, 0) in the solution set of the system of inequalities? Yes What does the point represent in the context of the situation? The class can make 10 blankets and 0 pillows.

Common Error

Some students may give an estimate of the point of intersection of the boundary lines as the solution, as they would when solving a system of equations. Explain why real-world problems of this type have many solutions and how this is indicated by the graph.



Check

Graph the system of inequalities.

$x + 3 < y$

$3x - 3y \geq 12$

Example 3 Apply Systems of Inequalities

SEWING **A family and consumer sciences class is making pillows and blankets to donate to a local shelter. The class has 40 yards of fabric to use. Pillows require 1.25 yards of fabric and 1 hour to make. Blankets use 4 yards of fabric and take 2.5 hours to make. The class has 28 hours of class time left for the semester. Determine the number of pillows and blankets the class can make for the shelter.**

Part A Define the variables, and write a system of inequalities to represent the situation.

Let p represent the number of pillows the class can make.
Let b represent the number of blankets the class can make.

Fabric and time are two constraints on the numbers of pillows and blankets the class can make.

Since pillows use 1.25 yards of fabric and blankets use 4 yards of fabric, the inequality that represents the fabric constraint is $1.25p + 4b \leq 40$.

Making a pillow takes 1 hour and making a blanket takes 2.5 hours. Since the class has only 28 hours left, the inequality that represents the time constraint is $p + 2.5b \leq 28$.

Part B Graph the system.

Part C Find a viable solution.

Only whole-number solutions make sense in this situation. One possible solution is (4, 15); 4 blankets and 15 pillows can be made.

Think About It! Can the class make 2 blankets and 24 pillows? Explain.

No; sample answer: The ordered pair does not satisfy both inequalities.

$24 + 2.5(2) \nleq 28$

Think About It! Could the graph be represented with "Pillows Made" as the x-axis and "Blankets Made" as the y-axis? Explain.

Yes; sample answer: Because there is no independent or dependent variable, as long as the axes are labeled appropriately, both graphs are correct.

Go Online You can complete an Extra Example online.

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Interactive Presentation

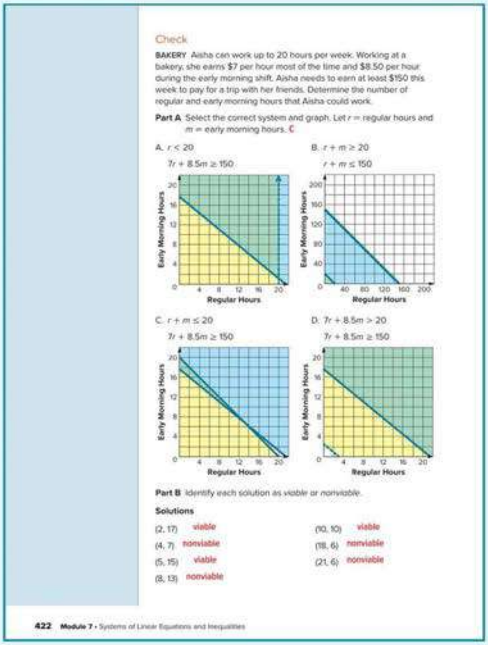
Example 3

TAP

Students select the correct variable to complete each sentence.

TYPE

Students enter the values necessary to complete each inequality, and then analyze possible solutions.



Check

BAKERY Aisha can work up to 20 hours per week. Working at a bakery, she earns \$7 per hour most of the time and \$8.50 per hour during the early morning shift. Aisha needs to earn at least \$150 this week to pay for a trip with her friends. Determine the number of regular and early morning hours that Aisha could work.

Part A Select the correct system and graph. Let r = regular hours and m = early morning hours. C

A. $r \le 20$
$7r + 8.5m \ge 150$

B. $r + m \ge 20$
$r + m \le 150$

C. $r + m \le 20$
$7r + 8.5m \ge 150$

D. $7r + 8.5m > 20$
$7r + 8.5m \ge 150$

Part B Identify each solution as *viable* or *nonviable*.

Solutions

(2, 17)	viable	(10, 10)	viable
(4, 7)	nonviable	(18, 6)	nonviable
(5, 15)	viable	(21, 6)	nonviable
(8, 13)	nonviable		

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Interactive Presentation

Check

MULTIPLE CHOICE

Students select the graph of the system of inequalities.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Essential Question Follow-Up

Students are learning about how a system of inequalities can be used to model the constraints of a real-world situation.

Ask:

How are systems of inequalities useful in the real world?

Sample answer: They can be used to model and solve situations that involve constraints on different quantities.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–14
2	exercises that use a variety of skills from this lesson	15–17
2	exercises that extend concepts learned in this lesson to new contexts	18–23
3	exercises that emphasize higher-order and critical-thinking skills	24–28

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: OL BL

- Practice Exercises 1–23 odd, 24–28
- Extension: Describing Regions
- **ALEKS** Systems of Linear Inequalities

IF students score 66%–89% on the Checks, **THEN** assign: AL OL

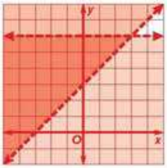
- Practice Exercises 1–27 odd
- Remediation, Review Resources: Graphing Inequalities in Two Variables
- Personal Tutors
- Extra Examples 1–3
- **ALEKS** Linear Inequalities ith Two Variables

IF students score 65% or less on the Checks, **THEN** assign: AL

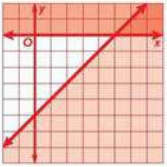
- Practice Exercises 1–13 odd
- Remediation, Review Resources: Graphing Inequalities in Two Variables
- *Quick Review Math Handbook*: Systems of Inequalities
- Arrive**MATH** Take Another Look
- **ALEKS** Linear Inequalities with Two Variables

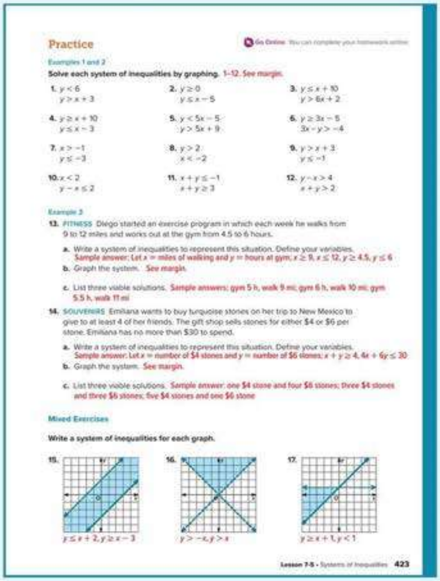
Answers

1.



2.





Practice

Go Online You can complete your homework online.

Examples 1 and 2

Solve each system of inequalities by graphing. 1–12. See margin.

1. $y < 6$
$y > x + 3$

2. $y \geq 0$
$y \leq x - 5$

3. $y \leq x + 10$
$y > 6x + 2$

4. $y \geq x + 10$
$y \leq x - 3$

5. $y < 5x - 5$
$y > 5x + 9$

6. $y \geq 3x - 5$
$3x - y > -4$

7. $x > -1$
$y \leq -3$

8. $y > 2$
$x < -2$

9. $y > x + 3$
$y \leq -1$

10. $x < 2$
$y - x \leq 2$

11. $x + y \leq -1$
$x + y \geq 3$

12. $y - x > 4$
$x + y > 2$

Example 3

13. FITNESS Diego started an exercise program in which each week he walks from 9 to 12 miles and works out at the gym from 4.5 to 6 hours.

a. Write a system of inequalities to represent this situation. Define your variables.
Sample answer: Let x = miles of walking and y = hours at gym; $x \geq 9$, $x \leq 12$, $y \geq 4.5$, $y \leq 6$

b. Graph the system. See margin.

c. List three viable solutions. Sample answers: gym 5 h, walk 9 mi; gym 6 h, walk 10 mi; gym 5.5 h, walk 11 mi

14. SOUVENIRS Emiliana wants to buy turquoise stones on her trip to New Mexico to give to at least 4 of her friends. The gift shop sells stones for either \$4 or \$6 per stone. Emiliana has no more than \$30 to spend.

a. Write a system of inequalities to represent this situation. Define your variables.
Sample answer: Let x = number of \$4 stones and y = number of \$6 stones; $x + y \geq 4$, $4x + 6y \leq 30$

b. Graph the system. See margin.

c. List three viable solutions. Sample answer: one \$4 stone and four \$6 stones; three \$4 stones and three \$6 stones; five \$4 stones and one \$6 stone

Mixed Exercises

Write a system of inequalities for each graph.

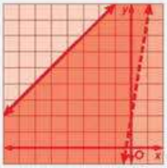
15. $y \leq x + 2$, $y \geq x - 3$

16. $y > -x$, $y > x$

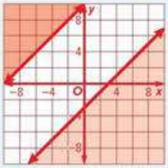
17. $y \geq x + 1$, $y < 1$

Lesson 7-5 • Systems of Inequalities 423

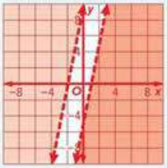
3.



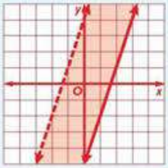
4. no solution

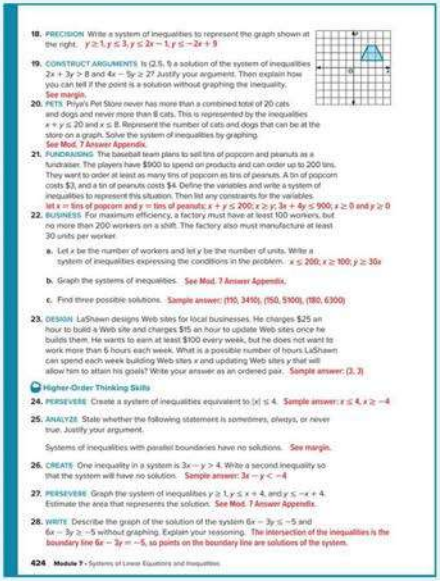


5. no solution



6. no solution





18. PRECISION Write a system of inequalities to represent the graph shown at the right. $y \geq 1, y \leq 3, y \leq 2x - 1, y \leq -2x + 9$

19. CONSTRUCT ARGUMENTS Is (2.5, 1) a solution of the system of inequalities $2x + 3y > 8$ and $4x - 5y \geq 2$? Justify your argument. Then explain how you can tell if the point is a solution without graphing the inequality. See margin.

20. PETS Priya's Pet Store never has more than a combined total of 20 cats and dogs and never more than 8 cats. This is represented by the inequalities $x + y \leq 20$ and $x \leq 8$. Represent the number of cats and dogs that can be at the store on a graph. Solve the system of inequalities by graphing. See Mod. 7 Answer Appendix.

21. FUNDRAISING The baseball team plans to sell tins of popcorn and peanuts as a fundraiser. The players have \$900 to spend on products and can order up to 200 tins. They want to order at least as many tins of popcorn as tins of peanuts. A tin of popcorn costs \$3, and a tin of peanuts costs \$4. Define the variables and write a system of inequalities to represent this situation. Then list any constraints for the variables. let x = tins of popcorn and y = tins of peanuts; $x + y \leq 200$; $x \geq y$; $3x + 4y \leq 900$; $x \geq 0$ and $y \geq 0$

22. BUSINESS For maximum efficiency, a factory must have at least 100 workers, but no more than 200 workers on a shift. The factory also must manufacture at least 30 units per worker.
    a. Let x be the number of workers and let y be the number of units. Write a system of inequalities expressing the conditions in the problem. $x \leq 200$; $x \geq 100$; $y \geq 30x$
    b. Graph the systems of inequalities. See Mod. 7 Answer Appendix.
    c. Find three possible solutions. Sample answer: (110, 3410), (150, 5100), (180, 6300)

23. DESIGN LaShawn designs Web sites for local businesses. He charges \$25 an hour to build a Web site and charges \$15 an hour to update Web sites once he builds them. He wants to earn at least \$100 every week, but he does not want to work more than 6 hours each week. What is a possible number of hours LaShawn can spend each week building Web sites x and updating Web sites y that will allow him to attain his goals? Write your answer as an ordered pair. Sample answer: (3, 3)

Higher-Order Thinking Skills

24. PERSEVERE Create a system of inequalities equivalent to $|x| \leq 4$. Sample answer: $x \leq 4$, $x \geq -4$

25. ANALYZE State whether the following statement is sometimes, always, or never true. Justify your argument.

Systems of inequalities with parallel boundaries have no solutions. See margin.

26. CREATE One inequality in a system is $3x - y > 4$. Write a second inequality so that the system will have no solution. Sample answer: $3x - y < -4$

27. PERSEVERE Graph the system of inequalities $y \geq 1$, $y \leq x + 4$, and $y \leq -x + 4$. Estimate the area that represents the solution. See Mod. 7 Answer Appendix.

28. WRITE Describe the graph of the solution of the system $6x - 3y \leq -5$ and $6x - 3y \geq -5$ without graphing. Explain your reasoning. The intersection of the inequalities is the boundary line $6x - 3y = -5$, so points on the boundary line are solutions of the system.

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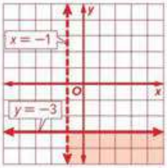
Answers

7.



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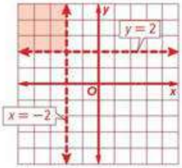

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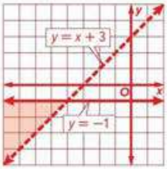

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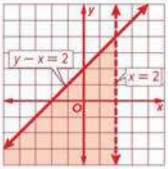
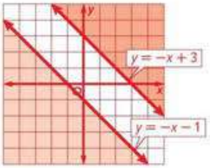


11. No solution



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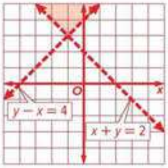

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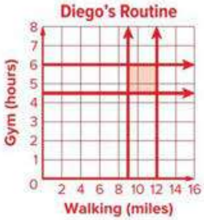

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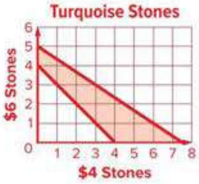


19. The solution set is the region where the graphs of the inequalities overlap. The point (2.5, 1) is not in the overlapping region, so it is not a solution. A solution must make all of the inequalities in the system true statements: $4x - 5y \geq 2 \rightarrow 4(2.5) - 5(1) \geq 2 \rightarrow 10 - 5 \geq 2 \rightarrow 5 \geq 2$; $2x + 3y > 8 \rightarrow 2(2.5) + 3(1) > 8 \rightarrow 5 + 3 > 8 \rightarrow 8 > 8$; The first inequality is true, but the second inequality is false. So, (2.5, 1) is not a solution.

25. Sometimes; sample answer: $y > 3$, $y < -3$ will have no solution, but $y < -3$, $y < 3$ will have solutions.

Module 7 • Systems of Linear Equations and Inequalities

Review

Rate Yourself

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- How can a system of equations help a business owner make decisions?
- Why is it necessary to use a system of equations to solve a real-world problem that involves two unknowns?
- Why is it helpful to know how to solve systems of equations when solving real-world problems?
- How are systems of inequalities useful in the real world?

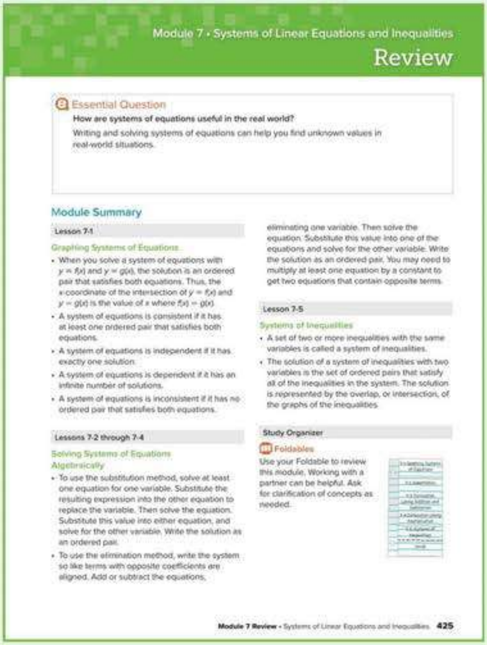
Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to systems of linear equations and inequalities.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Linear and Exponential Relationships.**

- Solve Systems of Equations



Module 7 • Systems of Linear Equations and Inequalities

Review

Essential Question

How are systems of equations useful in the real world?

Writing and solving systems of equations can help you find unknown values in real-world situations.

Module Summary

Lesson 7-1

Graphing Systems of Equations

- When you solve a system of equations with $y = f(x)$ and $y = g(x)$, the solution is an ordered pair that satisfies both equations. Thus, the x-coordinate of the intersection of $y = f(x)$ and $y = g(x)$ is the value of x where $f(x) = g(x)$.
- A system of equations is consistent if it has at least one ordered pair that satisfies both equations.
- A system of equations is independent if it has exactly one solution.
- A system of equations is dependent if it has an infinite number of solutions.
- A system of equations is inconsistent if it has no ordered pair that satisfies both equations.

Lessons 7-2 through 7-4

Solving Systems of Equations Algebraically

- To use the substitution method, solve at least one equation for one variable. Substitute the resulting expression into the other equation to replace the variable. Then solve the equation. Substitute this value into either equation, and solve for the other variable. Write the solution as an ordered pair.
- To use the elimination method, write the system so like terms with opposite coefficients are aligned. Add or subtract the equations, eliminating one variable. Then solve the equation. Substitute this value into one of the equations and solve for the other variable. Write the solution as an ordered pair. You may need to multiply at least one equation by a constant to get two equations that contain opposite terms.

Lesson 7-5

Systems of Inequalities

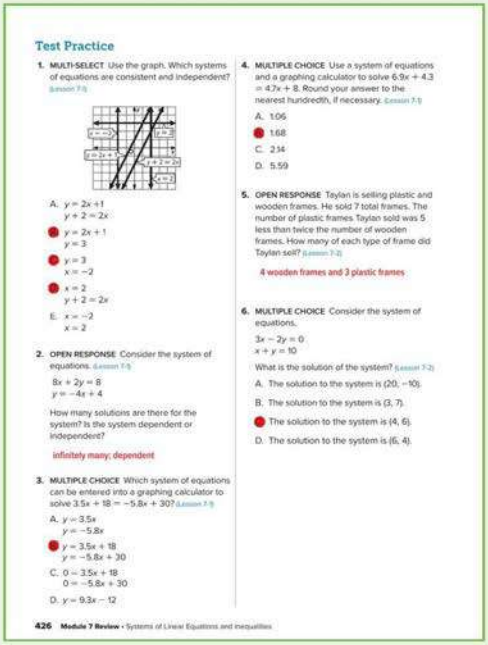
- A set of two or more inequalities with the same variables is called a system of inequalities.
- The solution of a system of inequalities with two variables is the set of ordered pairs that satisfy all of the inequalities in the system. The solution is represented by the overlap, or intersection, of the graphs of the inequalities.

Study Organizer

Foldables

Use your Foldable to review this module. Working with a partner can be helpful. Ask for clarification of concepts as needed.

Module 7 Review • Systems of Linear Equations and Inequalities **425**



Test Practice

1. **MULTI-SELECT** Use the graph. Which systems of equations are consistent and independent? (Lesson 7-1)

A. $y = 2x + 1$
$y + 2 = 2x$

● $y = 2x + 1$
$y = 3$

● $y = 3$
$x = -2$

● $x = 2$
$y + 2 = 2x$

E. $x = -2$
$x = 2$

2. **OPEN RESPONSE** Consider the system of equations. (Lesson 7-1)

$8x + 2y = 8$
$y = -4x + 4$

How many solutions are there for the system? Is the system dependent or independent?

infinitely many; dependent

3. **MULTIPLE CHOICE** Which system of equations can be entered into a graphing calculator to solve $3.5x + 18 = -5.8x + 30$? (Lesson 7-1)

A. $y = 3.5x$
$y = -5.8x$

● $y = 3.5x + 18$
$y = -5.8x + 30$

C. $0 = 3.5x + 18$
$0 = -5.8x + 30$

D. $y = 9.3x - 12$

4. **MULTIPLE CHOICE** Use a system of equations and a graphing calculator to solve $6.9x + 4.3 = 4.7x + 8$. Round your answer to the nearest hundredth, if necessary. (Lesson 7-1)

A. 1.06

● 1.68

C. 2.14

D. 5.59

5. **OPEN RESPONSE** Taylan is selling plastic and wooden frames. He sold 7 total frames. The number of plastic frames Taylan sold was 5 less than twice the number of wooden frames. How many of each type of frame did Taylan sell? (Lesson 7-2)

4 wooden frames and 3 plastic frames

6. **MULTIPLE CHOICE** Consider the system of equations.

$3x - 2y = 0$
$x + y = 10$

What is the solution of the system? (Lesson 7-2)

A. The solution to the system is (20, −10).

B. The solution to the system is (3, 7).

● The solution to the system is (4, 6).

D. The solution to the system is (6, 4).

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Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 7-1 through 7-4
Vocabulary Activity
Module Review

Assessment Resources

Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

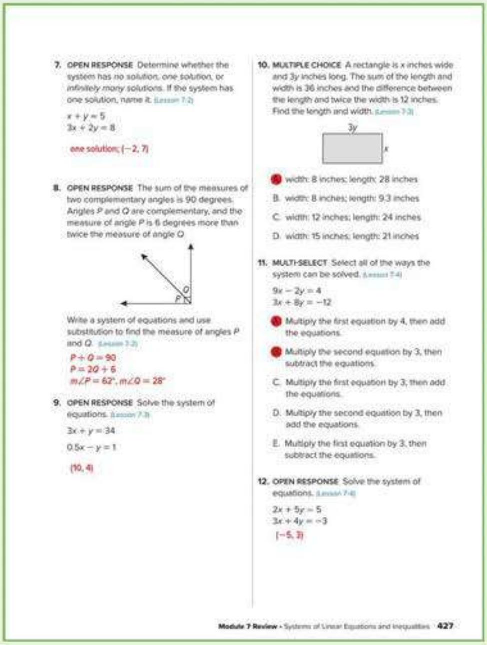
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–17 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	3, 4, 6, 10, 15–17
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	1, 11
Open Response	Students construct their own response.	2, 5, 7–9, 12–14

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
A.CED.3	7-2, 7-3, 7-5	5, 8, 10, 17
A.REI.5	7-4	11, 12
A.REI.6	7-1 – 7-4	1, 2, 4, 6, 7, 9, 13, 14
A.REI.11	7-1	3
A.REI.12	7-5	15, 16



7. **OPEN RESPONSE** Determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. (Lesson 7-2)

$x + y = 5$
$3x + 2y = 8$

one solution; $(-2, 7)$

8. **OPEN RESPONSE** The sum of the measures of two complementary angles is 90 degrees. Angles P and Q are complementary, and the measure of angle P is 6 degrees more than twice the measure of angle Q.

Write a system of equations and use substitution to find the measure of angles P and Q. (Lesson 7-2)

$P + Q = 90$
$P = 2Q + 6$
$m\angle P = 62°, m\angle Q = 28°$

9. **OPEN RESPONSE** Solve the system of equations. (Lesson 7-3)

$3x + y = 34$

$0.5x - y = 1$

$(10, 4)$

10. **MULTIPLE CHOICE** A rectangle is x inches wide and $3y$ inches long. The sum of the length and width is 36 inches and the difference between the length and twice the width is 12 inches. Find the length and width. (Lesson 7-3)

● width: 8 inches; length: 28 inches
B. width: 8 inches; length: 9.3 inches
C. width: 12 inches; length: 24 inches
D. width: 15 inches; length: 21 inches

11. **MULTI-SELECT** Select all of the ways the system can be solved. (Lesson 7-4)

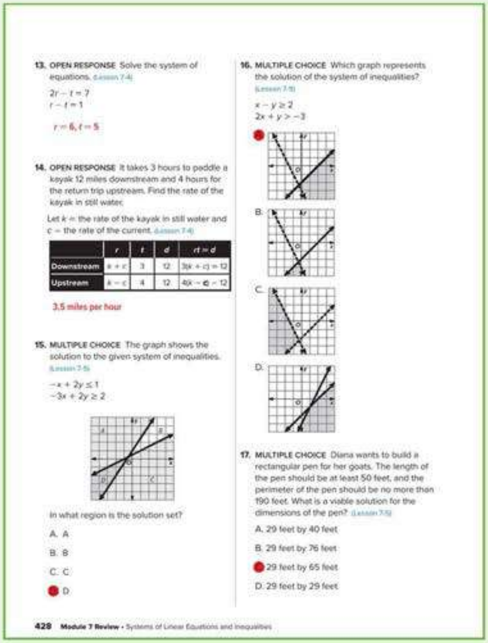
$9x - 2y = 4$
$3x + 8y = -12$

● Multiply the first equation by 4, then add the equations.
● Multiply the second equation by 3, then subtract the equations.
C. Multiply the first equation by 3, then add the equations.
D. Multiply the second equation by 3, then add the equations.
E. Multiply the first equation by 3, then subtract the equations.

12. **OPEN RESPONSE** Solve the system of equations. (Lesson 7-4)

$2x + 5y = 5$
$3x + 4y = -3$
$(-5, 3)$

Module 7 Review • Systems of Linear Equations and Inequalities 427



13. OPEN RESPONSE Solve the system of equations. (Lesson 7-4)

$2r - t = 7$
$r - t = 1$

$r = 6, t = 5$

14. OPEN RESPONSE It takes 3 hours to paddle a kayak 12 miles downstream and 4 hours for the return trip upstream. Find the rate of the kayak in still water.

Let k = the rate of the kayak in still water and c = the rate of the current. (Lesson 7-4)

	r	t	d	$rt = d$
Downstream	$k + c$	3	12	$3(k + c) = 12$
Upstream	$k - c$	4	12	$4(k - c) = 12$

3.5 miles per hour

15. MULTIPLE CHOICE The graph shows the solution to the given system of inequalities. (Lesson 7-5)

$-x + 2y \leq 1$
$-3x + 2y \geq 2$

In what region is the solution set?

A. A

B. B

C. C

● D

16. MULTIPLE CHOICE Which graph represents the solution of the system of inequalities? (Lesson 7-5)

$x - y \geq 2$
$2x + y > -3$

17. MULTIPLE CHOICE Diana wants to build a rectangular pen for her goats. The length of the pen should be at least 50 feet, and the perimeter of the pen should be no more than 190 feet. What is a viable solution for the dimensions of the pen? (Lesson 7-5)

A. 29 feet by 40 feet

B. 29 feet by 76 feet

● 29 feet by 65 feet

D. 29 feet by 29 feet

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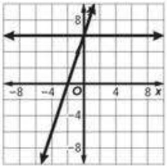
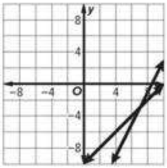
Lesson 7-1

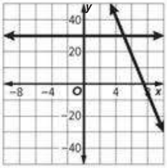
19. $y = 3x + 6$ and $y = 6$; (0, 6)



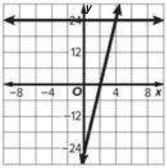
20. $y = 2x - 17$ and $y = x - 10$ (7, −3)



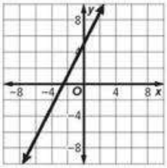
21. $y = -12x + 90$ and $y = 30$; (5, 30)



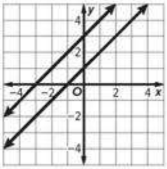
22. $y = 13x - 28$ and $y = 24$; (4, 24)



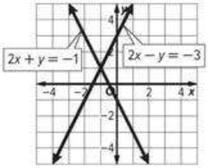
23. $y = 2x + 5$ and $y = 2x + 5$; infinitely many solutions



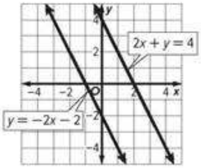
24. $y = x + 1$ and $y = x + 3$; no solution



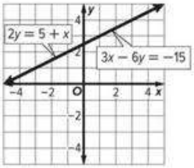
30. 1 solution; (−1, 1); consistent; independent



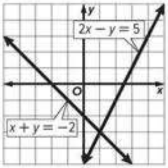
31. no solution; inconsistent



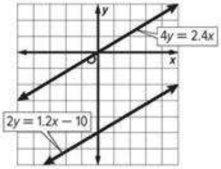
32. infinitely many solutions; consistent; dependent



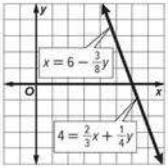
33. 1 solution; (1, −3); consistent; independent



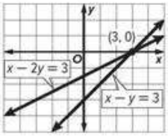
34. no solution; inconsistent



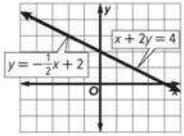
35. infinitely many solutions; consistent; dependent



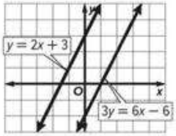
36. 1 solution; (3, 0); consistent; independent



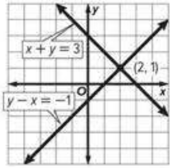
37. infinitely many solutions; consistent; dependent



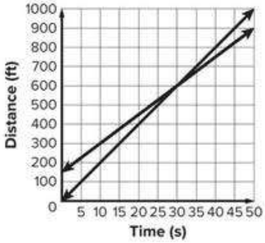
38. no solution; inconsistent



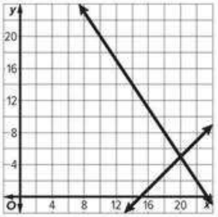
39. 1 solution; (2, 1); consistent; independent



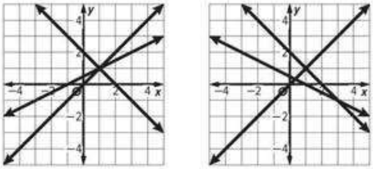
41b.



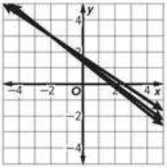
43b.



44. Maureen is incorrect. If all three equations have the same graph, there are infinitely many solutions. If two equations have the same graph, the third could be parallel to them (no solutions) or intersect them once. If the equations represent parallel lines, there are no solutions. If none of the lines are identical or parallel to each other, then they could intersect in one point (one solution) or three points (no solutions).

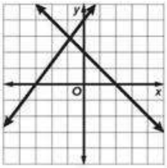


45. (−2, 3)

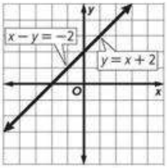


49. Graphing clearly shows whether a system of equations has one solution, no solution, or infinitely many solutions. However, finding the exact value of *x* and *y* from a graph can be difficult.

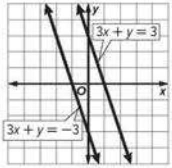
50. Sample answer: $-4x + 3y = 12$ and $x + y = 2$; consistent; independent



$y = x + 2$ and $x - y = -2$; consistent; dependent

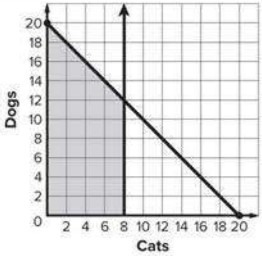


$3x + y = -3$ and $3x + y = 3$; inconsistent

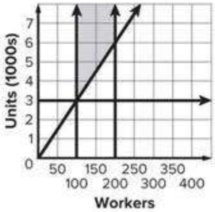


Lesson 7-5

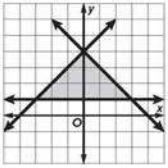
20.



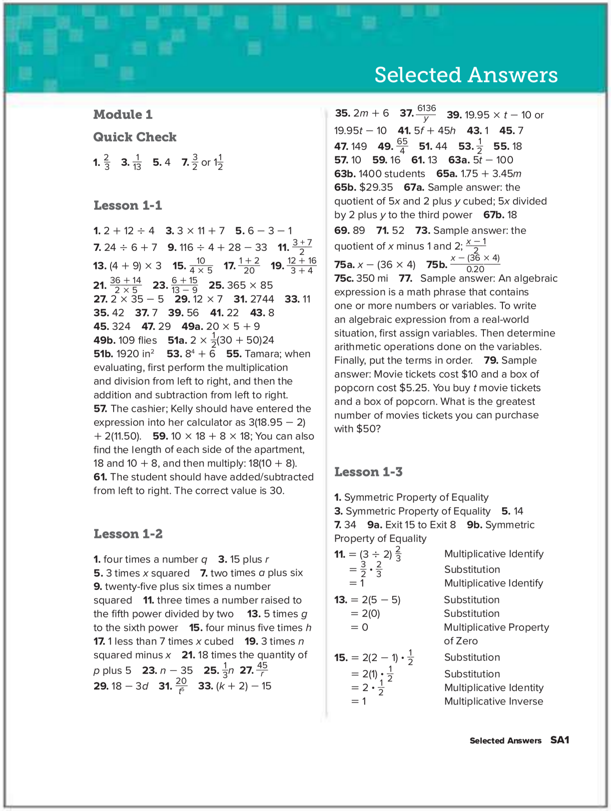
22b.



27. 9 units^2



Selected Answers



Selected Answers

Module 1

Quick Check

1. $\frac{2}{3}$ **3.** $\frac{1}{13}$ **5.** 4 **7.** $\frac{3}{2}$ or $1\frac{1}{2}$

Lesson 1-1

1. $2 + 12 \div 4$ **3.** $3 \times 11 + 7$ **5.** $6 - 3 - 1$ **7.** $24 \div 6 + 7$ **9.** $116 \div 4 + 28 - 33$ **11.** $\frac{3+7}{2}$ **13.** $(4 + 9) \times 3$ **15.** $\frac{10}{4 \times 5}$ **17.** $\frac{1+2}{20}$ **19.** $\frac{12+16}{3+4}$ **21.** $\frac{36+14}{2 \times 5}$ **23.** $\frac{6+15}{13-9}$ **25.** 365×85 **27.** $2 \times 35 - 5$ **29.** 12×7 **31.** 2744 **33.** 11 **35.** 42 **37.** 7 **39.** 56 **41.** 22 **43.** 8 **45.** 324 **47.** 29 **49a.** $20 \times 5 + 9$ **49b.** 109 flies **51a.** $2 \times \frac{1}{2}(30 + 50)24$ **51b.** 1920 in^2 **53.** $8^4 + 6$ **55.** Tamara; when evaluating, first perform the multiplication and division from left to right, and then the addition and subtraction from left to right. **57.** The cashier; Kelly should have entered the expression into her calculator as $3(18.95 - 2) + 2(11.50)$. **59.** $10 \times 18 + 8 \times 18$; You can also find the length of each side of the apartment, 18 and 10 + 8, and then multiply: $18(10 + 8)$. **61.** The student should have added/subtracted from left to right. The correct value is 30.

Lesson 1-2

1. four times a number *q* **3.** 15 plus *r* **5.** 3 times *x* squared **7.** two times *a* plus six **9.** twenty-five plus six times a number squared **11.** three times a number raised to the fifth power divided by two **13.** 5 times *g* to the sixth power **15.** four minus five times *h* **17.** 1 less than 7 times *x* cubed **19.** 3 times *n* squared minus *x* **21.** 18 times the quantity of *p* plus 5 **23.** $n - 35$ **25.** $\frac{1}{3}n$ **27.** $\frac{45}{r}$ **29.** $18 - 3d$ **31.** $\frac{20}{t^5}$ **33.** $(k + 2) - 15$

35. $2m + 6$ **37.** $\frac{6136}{y}$ **39.** $19.95 \times t - 10$ or $19.95t - 10$ **41.** $5f + 45h$ **43.** 1 **45.** 7 **47.** 149 **49.** $\frac{65}{4}$ **51.** 44 **53.** $\frac{1}{2}$ **55.** 18 **57.** 10 **59.** 16 **61.** 13 **63a.** $5t - 100$ **63b.** 1400 students **65a.** $1.75 + 3.45m$ **65b.** \$29.35 **67a.** Sample answer: the quotient of 5*x* and 2 plus *y* cubed; 5*x* divided by 2 plus *y* to the third power **67b.** 18

69. 89 **71.** 52 **73.** Sample answer: the quotient of *x* minus 1 and 2; $\frac{x-1}{2}$

75a. $x - (36 \times 4)$ **75b.** $\frac{x - (36 \times 4)}{0.20}$ **75c.** 350 mi **77.** Sample answer: An algebraic expression is a math phrase that contains one or more numbers or variables. To write an algebraic expression from a real-world situation, first assign variables. Then determine arithmetic operations done on the variables. Finally, put the terms in order. **79.** Sample answer: Movie tickets cost \$10 and a box of popcorn cost \$5.25. You buy *t* movie tickets and a box of popcorn. What is the greatest number of movies tickets you can purchase with \$50?

Lesson 1-3

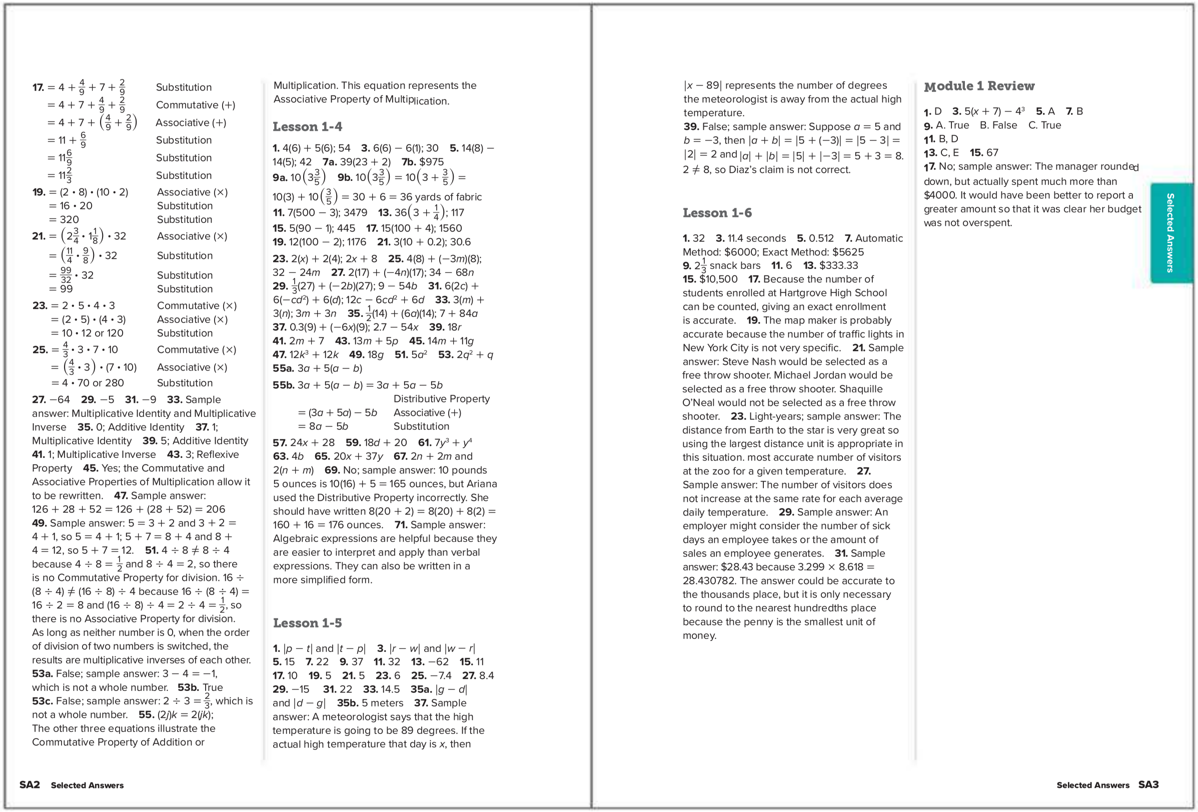
1. Symmetric Property of Equality
3. Symmetric Property of Equality **5.** 14
7. 34 **9a.** Exit 15 to Exit 8 **9b.** Symmetric Property of Equality

11. $= (3 \div 2)\frac{2}{3}$ Multiplicative Identify
$= \frac{3}{2} \cdot \frac{2}{3}$ Substitution
$= 1$ Multiplicative Identify

13. $= 2(5 - 5)$ Substitution
$= 2(0)$ Substitution
$= 0$ Multiplicative Property of Zero

15. $= 2(2 - 1) \cdot \frac{1}{2}$ Substitution
$= 2(1) \cdot \frac{1}{2}$ Substitution
$= 2 \cdot \frac{1}{2}$ Multiplicative Identity
$= 1$ Multiplicative Inverse

Selected Answers **SA1**



17. $= 4 + \frac{4}{9} + 7 + \frac{2}{9}$ Substitution
$= 4 + 7 + \frac{4}{9} + \frac{2}{9}$ Commutative (+)
$= 4 + 7 + \left(\frac{4}{9} + \frac{2}{9}\right)$ Associative (+)
$= 11 + \frac{6}{9}$ Substitution
$= 11\frac{6}{9}$ Substitution
$= 11\frac{2}{3}$ Substitution

19. $= (2 \cdot 8) \cdot (10 \cdot 2)$ Associative (×)
$= 16 \cdot 20$ Substitution
$= 320$ Substitution

21. $= \left(2\frac{3}{4} \cdot 1\frac{1}{8}\right) \cdot 32$ Associative (×)
$= \left(\frac{11}{4} \cdot \frac{9}{8}\right) \cdot 32$ Substitution
$= \frac{99}{32} \cdot 32$ Substitution
$= 99$ Substitution

23. $= 2 \cdot 5 \cdot 4 \cdot 3$ Commutative (×)
$= (2 \cdot 5) \cdot (4 \cdot 3)$ Associative (×)
$= 10 \cdot 12$ or 120 Substitution

25. $= \frac{4}{3} \cdot 3 \cdot 7 \cdot 10$ Commutative (×)
$= \left(\frac{4}{3} \cdot 3\right) \cdot (7 \cdot 10)$ Associative (×)
$= 4 \cdot 70$ or 280 Substitution

27. −64 **29.** −5 **31.** −9 **33.** Sample answer: Multiplicative Identity and Multiplicative Inverse **35.** 0; Additive Identity **37.** 1; Multiplicative Identity **39.** 5; Additive Identity **41.** 1; Multiplicative Inverse **43.** 3; Reflexive Property **45.** Yes; the Commutative and Associative Properties of Multiplication allow it to be rewritten. **47.** Sample answer: $126 + 28 + 52 = 126 + (28 + 52) = 206$ **49.** Sample answer: $5 = 3 + 2$ and $3 + 2 = 4 + 1$, so $5 = 4 + 1$; $5 + 7 = 8 + 4$ and $8 + 4 = 12$, so $5 + 7 = 12$. **51.** $4 \div 8 \neq 8 \div 4$ because $4 \div 8 = \frac{1}{2}$ and $8 \div 4 = 2$, so there is no Commutative Property for division. $16 \div (8 \div 4) \neq (16 \div 8) \div 4$ because $16 \div (8 \div 4) = 16 \div 2 = 8$ and $(16 \div 8) \div 4 = 2 \div 4 = \frac{1}{2}$, so there is no Associative Property for division. As long as neither number is 0, when the order of division of two numbers is switched, the results are multiplicative inverses of each other. **53a.** False; sample answer: $3 - 4 = -1$, which is not a whole number. **53b.** True **53c.** False; sample answer: $2 \div 3 = \frac{2}{3}$, which is not a whole number. **55.** $(2j)k = 2(jk)$; The other three equations illustrate the Commutative Property of Addition or Multiplication. This equation represents the Associative Property of Multiplication.

Lesson 1-4

1. 4(6) + 5(6); 54 **3.** 6(6) − 6(1); 30 **5.** 14(8) − 14(5); 42 **7a.** 39(23 + 2) **7b.** $975
9a. $10\left(3\frac{3}{5}\right)$ **9b.** $10\left(3\frac{3}{5}\right) = 10\left(3 + \frac{3}{5}\right) = 10(3) + 10\left(\frac{3}{5}\right) = 30 + 6 = 36$ yards of fabric
11. 7(500 − 3); 3479 **13.** $36\left(3 + \frac{1}{4}\right)$; 117
15. 5(90 − 1); 445 **17.** 15(100 + 4); 1560
19. 12(100 − 2); 1176 **21.** 3(10 + 0.2); 30.6
23. $2(x) + 2(4)$; $2x + 8$ **25.** $4(8) + (-3m)(8)$; $32 - 24m$ **27.** $2(17) + (-4n)(17)$; $34 - 68n$ **29.** $\frac{1}{3}(27) + (-2b)(27)$; $9 - 54b$ **31.** $6(2c) + 6(-cd^2) + 6(d)$; $12c - 6cd^2 + 6d$ **33.** $3(m) + 3(n)$; $3m + 3n$ **35.** $\frac{1}{2}(14) + (6a)(14)$; $7 + 84a$ **37.** $0.3(9) + (-6x)(9)$; $2.7 - 54x$ **39.** $18r$ **41.** $2m + 7$ **43.** $13m + 5p$ **45.** $14m + 11g$ **47.** $12k^3 + 12k$ **49.** $18g$ **51.** $5a^2$ **53.** $2q^2 + q$
55a. $3a + 5(a - b)$

55b. $3a + 5(a - b) = 3a + 5a - 5b$ Distributive Property
$= (3a + 5a) - 5b$ Associative (+)
$= 8a - 5b$ Substitution

57. $24x + 28$ **59.** $18d + 20$ **61.** $7y^3 + y^4$ **63.** $4b$ **65.** $20x + 37y$ **67.** $2n + 2m$ and $2(n + m)$ **69.** No; sample answer: 10 pounds 5 ounces is 10(16) + 5 = 165 ounces, but Ariana used the Distributive Property incorrectly. She should have written 8(20 + 2) = 8(20) + 8(2) = 160 + 16 = 176 ounces. **71.** Sample answer: Algebraic expressions are helpful because they are easier to interpret and apply than verbal expressions. They can also be written in a more simplified form.

Lesson 1-5

1. $|p - t|$ and $|t - p|$ **3.** $|r - w|$ and $|w - r|$ **5.** 15 **7.** 22 **9.** 37 **11.** 32 **13.** −62 **15.** 11 **17.** 10 **19.** 5 **21.** 5 **23.** 6 **25.** −7.4 **27.** 8.4 **29.** −15 **31.** 22 **33.** 14.5 **35a.** $|g - d|$ and $|d - g|$ **35b.** 5 meters **37.** Sample answer: A meteorologist says that the high temperature is going to be 89 degrees. If the actual high temperature that day is x, then $|x - 89|$ represents the number of degrees the meteorologist is away from the actual high temperature.
39. False; sample answer: Suppose $a = 5$ and $b = -3$, then $|a + b| = |5 + (-3)| = |5 - 3| = |2| = 2$ and $|a| + |b| = |5| + |-3| = 5 + 3 = 8$. $2 \neq 8$, so Diaz's claim is not correct.

Lesson 1-6

1. 32 **3.** 11.4 seconds **5.** 0.512 **7.** Automatic Method: $6000; Exact Method: $5625
9. $2\frac{1}{3}$ snack bars **11.** 6 **13.** $333.33
15. $10,500 **17.** Because the number of students enrolled at Hartgrove High School can be counted, giving an exact enrollment is accurate. **19.** The map maker is probably accurate because the number of traffic lights in New York City is not very specific. **21.** Sample answer: Steve Nash would be selected as a free throw shooter. Michael Jordan would be selected as a free throw shooter. Shaquille O'Neal would not be selected as a free throw shooter. **23.** Light-years; sample answer: The distance from Earth to the star is very great so using the largest distance unit is appropriate in this situation. most accurate number of visitors at the zoo for a given temperature. **27.** Sample answer: The number of visitors does not increase at the same rate for each average daily temperature. **29.** Sample answer: An employer might consider the number of sick days an employee takes or the amount of sales an employee generates. **31.** Sample answer: $28.43 because 3.299 × 8.618 = 28.430782. The answer could be accurate to the thousands place, but it is only necessary to round to the nearest hundredths place because the penny is the smallest unit of money.

Module 1 Review

1. D **3.** $5(x + 7) - 4^3$ **5.** A **7.** B
9. A. True B. False C. True
11. B, D
13. C, E **15.** 67
17. No; sample answer: The manager rounded down, but actually spent much more than $4000. It would have been better to report a greater amount so that it was clear her budget was not overspent.

Module 2

Quick Check

1. $6n + 2$ **3.** $4b + 9$ **5.** 8 **7.** 32 **9.** 36

Lesson 2-1

1. $3m + 2 = 18$ **3.** $\frac{24}{x} = 14 - 2x$ **5.** $2 + 3h = 6$ **7.** $(48 + 33) + n = 107$ **9.** $2a + a^3 = b$ **11.** $x + x^2 = yz$ **13.** $A = \ell^2$ **15.** $P = 2\ell + 2w$ **17.** $I = prt$ **19.** The sum of j and sixteen is thirty-five. **21.** Seven times the sum of p and twenty-three is the same as one hundred two. **23.** Two-fifths of v plus three-fourths is identical to two-thirds of x squared. **25.** g plus 10 is the same as 3 times g. **27.** 4 times the sum of a and b is 9 times a. **29.** Half of the sum of f and y is f minus 5. **31.** Sample answer: The volume equals π times the radius squared times the height. The base is a circle so the expression πr^2 represents the area of the base. **33.** Sample answer: The interest equals the product of the principal, the rate, and the time. **35.** Sample answer: Force equals mass times acceleration. The expression ma represents the force on an object with mass m that is accelerating. **37.** B **39.** A **41.** $y^2 - 12 = 5x$ **43.** $100 - 3b = 6b$ **45.** Four times n equals x times the difference of five and n. **47.** The sum of y and the product of 3 and the square of x is 5 times x. **49.** $V = \ell wh$ **51.** $m + 2m = 24$ or $3m = 24$ **53.** $c = 10w + 0.1(10w)$ or $c = 11w$ **55a.** It is correct. The product is squared, so parentheses are needed. **55b.** It is not correct. One-half of a number means to multiply, not divide, by one-half. It should be $\frac{1}{2}n + 3 = n - 2$. **57.** Sample answer: A teacher ordered 188 math books. The algebra books were packed in boxes of 12. The geometry books were packed in boxes of 10. He ordered one more box of algebra books than geometry books. How many books of each type book did he order? Let a = number of algebra books. **59.** $S = 6\ell^2$ **61.** Sample answer: First, you should identify the unknown quantity or quantities for which you are trying to solve, and assign variables. Then, you should look for key words or phrases that can help you to determine operations that are being used. You can then write the equation using the numbers that you are given and the variables and operations that you assigned.

Lesson 2-2

1. 23 **3.** −43 **5.** −12 **7.** 73 **9.** −15 **11.** −54 **13.** $\frac{7}{20}$ **15.** $-\frac{7}{15}$ **17.** −937 **19.** −147 **21.** −25 **23.** $-\frac{9}{2}$ **25.** 15 **27.** 10 **29.** 64 **31.** 28 **33.** 18 **35.** 24 **37.** 27 **39.** 39 **41.** 64 **43.** 9 **45.** −12 **47.** 7 **49.** 64 **51.** −252 **53.** −52 **55.** $x + 33 = 2005$; $x = 1972$ **57.** $x - 21 = -9$; $x = 12$°C **59a.** Let p = the number of players who signed up for the soccer league. If 13% of the players who signed up for the soccer league dropped out, then 100% −13%, or 87% of the players finished the season. So, $0.87p$ represents the number of players who finished the season. **59b.** $0.87p = 174$ **59c.** $p = 200$; 200 players signed up for the soccer league **61.** $\frac{2}{3} = -8n$; $-\frac{1}{12}$ **63.** $\frac{4}{5} = \frac{10}{16}n$; $\frac{32}{25}$ **65.** $4\frac{4}{5}n = 1\frac{1}{5}$; $\frac{1}{4}$ **67.** −77 **69.** $\frac{16}{3}$ **71.** −10 **73.** $-\frac{10}{7}$ or $-1\frac{3}{7}$ **75.** 18 **77.** 225 **79.** −14 **81.** 4 **83.** −49 **85.** 40 **87.** −15 **89.** $-\frac{8}{15}$ **91a.** $12x = 780$; $x = 65$ **91b.** \$20 **93.** $x = 216$; Multiplication Property of Equality **95.** $y = -224$; Subtraction Property of Equality **97.** $15 = b$; Division Property of Equality **99.** $n - 16 = 29$ does not belong because for the other three, $n = 13$, and for this one $n = 45$. **101.** Sample answer: $x - 4 = 10$ **103.** Sample answer: To solve $5x = 35$, I would divide each side by 5 to get $x = 7$. To solve $5 + x = 35$, I would subtract 5 from each side to get $x = 30$. In both equations I used properties of equality to isolate the variable. In the first equation I used the Division Property of Equality and I used the Subtraction Property of Equality in the second equation.

Lesson 2-3

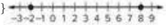
1. −5 **3.** −5 **5.** 70 **7.** 27 **9.** 16 **11.** −61 **13.** $\frac{1}{2}a - 5.25 = 22.50$; \$55.50 **15.** $\frac{t - 10}{15} = 4$; 70 treats **17.** $71 = 2h - 1$; 36 inches **19.** $\frac{18}{a}$ **21.** $\frac{-35}{a}$ **23.** $\frac{-24}{a}$ **25.** $\frac{-14}{a}$ **27.** 7 **29.** 10 **31.** −16 **33.** −2 **35.** 18 **37.** $(n - 2) \div 3 = 30$; 92 **39.** Sample answer: Both are correct. Dividing by a number and multiplying by that number's reciprocal are equivalent operations. **41a.** $x = \frac{-2}{a}$ **41b.** $x = 13a$ **41c.** $x = \frac{10}{a}$ **43.** Never; whenever three odd integers are added together, the sum is always odd.

Lesson 2-4

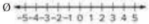
1. 6 **3.** 1 **5.** −2 **7.** −2 **9.** 14 **11.** 4 **13.** −5 **15.** 0 **17.** $7 + F = 4F + 1$; France won 2 gold medals and the U.S won 9 gold medals. **19.** $38 + 4x = 45.5 + 2.5x$; 5 years **21.** $180 - x = 10 + 2(90 - x)$; 10° **23.** $9(5 + x) = 15\frac{3}{7}x$; 7 **25.** no solution **27.** identity **29.** no solution **31.** one solution **33.** identity **35.** no solution **37.** no solution **39.** all numbers **41.** −25 **43.** 3 **45.** −2 **47.** 15 **49a.** Let n = the first odd integer; $2(n + 2) = 3n - 13$ **49b.** 17 and 19 **51a.** Let k = the number; $4k - 3 = 2k + 5$ **51b.** $k = 4$ **51c.** Substitute 4 for k in the expression for the perimeter of Figure 2, $2k + 5$. So the perimeter of Figure 2 is $2(4) + 5 = 8 + 5 = 13$. **51d.** Substitute 4 for k in the expression for the perimeter of Figure 1, $4k - 3$. So the perimeter of Figure 1 is $4(4) - 3 = 16 - 3 = 13$. The perimeter for Figure 1 and Figure 2 is the same, so the value of k is correct. **53.** Anthony is correct. When Patty added m to each side, she subtracted the terms instead of adding them. **55.** Sample answer: $2(3x + 6) = 3(2x + 5)$ **57a.** Incorrect; the 2 must be distributed over both g and 5, then 10 must be subtracted from each side; 6. **57b.** Correct; the Subtraction Property was used to combine the variable terms on the left side of the equation. The Division Property was used to isolate the variable on one side. **57c.** Incorrect; to eliminate $-6z$ on the left side of the equal sign, $6z$ must be added to each side of the equation; 1. **59.** Sample answer: $2x + 1 = x + 9$

Lesson 2-5

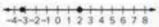
1. $\{-2, 8\}$



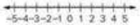
3. ∅



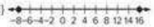
5. $\{-3.25, 2\}$



7. ∅



9. $\{-8, 16\}$



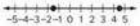
11. $\{2, 5\}$ **13.** $\{-6, 4\}$ **15.** $\{0, 4\}$ **17.** $\{-5, -1\}$ **19.** $|t - 400| = 15$; min = 385°F; max = 415°F **21a.** $|t - 20.9| = 5.3$ **21b.** 15.6 to 26.2; 10.3 to 31.5

23.

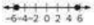
$\lvert x - 35\rvert = 0.5$	Absolute value equation
Case 1: $x - 35 = 0.5$	Definition of absolute value.
$x = 35.5$	Simplify
Case 2: $x - 35 = -0.5$	Definition of absolute value.
$x = 34.5$	Simplify

The bags of rock salt weigh no less than 34.5 pounds and no more than 35.5 pounds. **25.** $|x| = 6$ **27.** $|x + 2| = 4$ **29.** $|x + 3| = 2$ **31.** $|x| = 4$

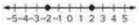
33. $\left\{-\frac{3}{2}, \frac{9}{2}\right\}$



35. $\{5.5, -5.5\}$



37. $\{2, -2\}$



39. $|x| = 1\frac{1}{2}$ **41.** $|x - \frac{1}{4}| = \frac{1}{4}$ **43.** $|x + \frac{1}{3}| = 1$ **45a.** $|x - 38| = 2$ **45b.** 40°F, 36°F **47.** $|x - 3| = 1$ **49.** $|a - b| + |b - c| = |a - c|$ **51.** Cami; The absolute value of a number cannot be negative. **53.** Sample answer: Let x = the temperature at night. Then the temperature is 4 ± 10 degrees.

Lesson 2-6

1. 40 **3.** 29.25 **5.** 9.8 **7.** 1.32 **9.** 0.84 **11.** 0.57 **13.** 6 **15.** 11 **17.** 18 **19.** 0.8 **21.** 11 **23.** −2 **25.** 1.44 **27.** −2.29 **29.** −2.2 **31.** 10 **33.** 3 **35.** −8.4 **37.** 12.5 gal **39.** $46.27 **41a.** 60 free throws **41b.** Sample answer: I assumed that Brent continues to make free throws at the same rate. **43.** 22.5 in. **45.** $3333.33 **47.** 204.55 mL **49.** 6 **51.** 10 **53.** 21 **55.** 8 **57.** 42 **59.** 27 **61.** 3 **63.** 15 **65.** −3 **67.** −0.4 **69.** −6 **71a.** Sample answer: 2.2 cm **71b.** Sample answer: about 6.6 miles **71c.** Sample answer: about 453.7 mi² **73a.** $4.50; because 8 potatoes cost $1.50, multiply by 3 to get a cost of $4.50 for 24 potatoes. **73b.** $4.13; Sample answer: $4.13 is slightly less than $4.50, which aligns with my estimate. **73c.** 37 **73d.** $0.19 **75.** $\frac{2}{4}$ or $\frac{1}{2}$ **77.** Ratios and rates each compare two numbers by using division. However, rates compare two measurements that involve different units of measure. **79.** $\frac{x}{100} = \frac{z}{y}$

Lesson 2-7

1. $y = \frac{x-1}{2}$ **3.** $f = \frac{5-g}{7}$ **5.** $t = \frac{x}{7}$ **7.** $r = \frac{q}{2}$ **9.** $a = -\frac{b}{8}$ **11.** $v = \frac{u-z}{w}$ **13.** $g = \frac{10j+9h}{f}$ **15.** $t = \frac{3}{2}(r - v)$ **17.** $a = \frac{-33+x}{10c}$ **19a.** $\ell = \frac{P-2w}{2}$ **19b.** 14 m **21a.** $g = \frac{c-p}{13.50}$ **21b.** 6 games **25.** ≈ 12.96 trillion pounds **27.** 0.44 ft **29.** 24 miles **31.** 90.2 gallons **33.** 82 students **35.** $c = \frac{2k-3g}{b}$ **37.** $c = \frac{5p-6j}{8}$ **39.** $c = x - 2d$ **41.** $t = \frac{w-11v}{31}$ **43.** $c = \frac{-13+f}{10-d}$ **45.** $r = \frac{A}{P} - 1 = \frac{2182.25}{2150} - 1 = 1.015 - 1 = 0.015$; The interest rate is 1.5%. **47a.** $y = \frac{mx+mt-z}{r}$ **47b.** Division by 0 is undefined, so in the original equation $m \neq 0$, and in the final equation $r \neq 0$. **49.** No; Sasha does not have a correct solution. When she multiplied F by $\frac{5}{9}$, she should have multiplied 32 by $\frac{5}{9}$.

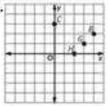
Module 2 Review

1. A **3.** D **5.** D **7.** C **9.** B, D **11.** C **13.** Let d = age of dogwood tree (in years); $3d - 2 = \frac{1}{2}(d + 8 + 8)$; The gingko tree is 10 years old, and the dogwood tree is 4 years old. **15.** A **17.** $b = 8$ **19.** B **21.** D **23A.** $h = 3\frac{V}{B}$ **23B.** 12

Module 3

Quick Check

1, 3.

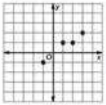


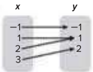
5. 20 **7.** −3

Lesson 3-1

1.

x	y
−1	−1
1	1
2	1
3	2

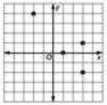


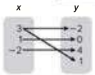


D = {−1, 1, 2, 3}; R = {−1, 1, 2}

3.

x	y
3	−2
1	0
−2	4
3	1

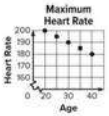




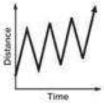
D = {−2, 1, 3}; R = {−2, 0, 1, 4}

5a. independent: price of item, dependent: number of items purchased **5b.** As the price of an item increases, the number of items purchased decreases.

7.



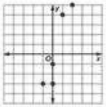
9. The x-axis represents the time in seconds. The y-axis represents the height of the elevator in feet. The x-axis has a scale of 1 mark = 1 second. The y-axis has a scale of 1 mark = 10 feet. The origin (0, 0) represents a height of 0 feet in 0 seconds. **11.** {(1, 7), (3, 45), (5, 11), (13, 15)} **13.** {(2, 5), (5, 0), (7, 8), (7, 10), (10, 2)} **15.** {(2, 80), (3, 120), (6, 240), (8, 320)}; The x-axis represents the number of gallons of syrup. The y-axis represents the number of gallons of sap. The x-axis has a scale of 1 mark = 1 gallon of syrup. The y-axis has a scale of 1 mark = 40 gallons of sap. The origin (0, 0) represents 0 gallons of sap makes 0 gallons of syrup. **17a.** {(0, 12), (1, 8), (2, 23), (3, 28), (4, 11), (5, 11)} **17b.** {0, 1, 2, 3, 4, 5} **17c.** {8, 11, 12, 23, 28} **19.** sample answer:

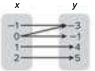


21. Tim drives away from the pizzeria, stops to make a delivery, continues to drive away from the pizzeria, stops to make another delivery, and then returns to the pizzeria. **23.** Disagree; The intersection point represents a time when Tim and Lauren were both at the same distance from the pizzeria. **25.** Disagree; sample counterexample: In the relation {(1, 2), (1, 3)}, the domain is {1}, so it has one element, while the range is {2, 3}, which has two elements. **27a.** Sample answer: {(−1, −3), (0, −3), (0, −1), (1, 4), (2, 5)}

27b. sample answer:

x	y
−1	−3
0	−1
0	−3
1	4
2	5





29. Sample answer: A dependent variable is determined by the independent variable for a given relation.

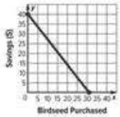
Lesson 3-2

1. Yes; for each element of the domain, there is only one element of the range. **3.** No; the element 4 in the domain is paired with both 2 and 5 in the range. **5.** No; the element 5 in the domain is paired with both −3 and 2 in the range. **7.** Yes; for each element of the domain, there is only one element of the range.
9. Yes; for any value x, the vertical line passes through no more than one point on the graph.
11. Yes; for any value x, the vertical line passes through no more than one point on the graph.
13. No; for $x > 0$, the vertical line passes through more than one point on the graph.
15a.

Year	2014	2015	2016	2017
Value ($)	254,000	293,000	338,000	372,000

15b. Domain: {2014, 2105, 2016, 2017}; Range: {254,000; 293,000; 338,000; 372,000}
15c. For each element of the domain, there is only one element of the range. So, this relation is a function. **17.** 26 **19.** 2 **21.** 42 **23.** 4
25. 6 **27.** $9b^2 - 3b$ **29.** $f(3.5) = 12.25$, which means the area of a square with a side of length 3.5 units is 12.25 square units.
31. $f(12) = \$435$, which is the cost of a gym membership for 12 months, or 1 year. **33.** −1
35. 14 **37.** −4 **39.** $-8y - 3$ **41.** $-2c - 8$
43. $-10d - 15$

45a.

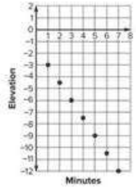


45b. Yes; for any value x, the vertical line passes through no more than one point on the graph. **45c.** $f(3) = 36.25$, which means if Aisha buys 3 pounds of birdseed, she saves \$36.25; $f(18) = 17.50$, which means if Aisha buys 18 pounds of birdseed, she saves \$17.50; $f(36) = -5$, which means if Aisha wants to buy 36 pounds of birdseed, she needs \$5 extra. **45d.** 8 pounds **47a.** $h(20) = 46$; The height of the balloon 20 seconds after it is released is 46 feet. **47b.** 2 minutes is $2(60) = 120$ seconds, so calculate $h(120)$ by substituting $t = 120$ in the equation; $h(120) = 2(120) + 6 = 246$; The height of the balloon is 246 feet. **47c.** 6 feet; $t = 0$ before the balloon is released, and $h(0) = 6$. **47d.** Sample answer: The values of t must be greater than or equal to zero because a negative value for the time does not make sense for the given situation. The graph would start at the vertical axis and go only to the right. **49.** Sample answer: You can determine whether each element of the domain is paired with exactly one element of the range. For example, if given a graph, you could use the vertical line test; if a vertical line intersects the graph more than once, then the relation that the graph represents is not a function. **51.** $f(g + 3.5) = -4.3g - 17.05$
53. Sample answer: $f(x) = 3x + 2$

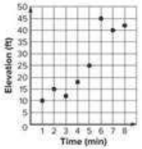
Lesson 3-3

1. Neither; because the function has continuous sections but is not a single line or curve, it is neither continuous or discrete **3.** Discrete; because the function is made up entirely of individual points, it is discrete. **5.** Continuous; because the function is graphed with a single line, it is continuous. **7.** Discrete; because the function is made up entirely of individual points, it is discrete. **9.** discrete **11.** discrete

13. linear **15.** nonlinear **17.** nonlinear
19. nonlinear **21.** nonlinear **23.** linear
25a. linear
25b.



27a. n
27b.

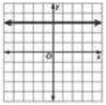


29. continuous; nonlinear **31.** neither; nonlinear **33.** discrete; nonlinear
35. Sample answer: A studio charges musicians to use the space and recording equipment by the hour, rounding a fraction of an hour up. So, for up to 1 hour, the studio charges \$100, but for up to 2 hours, the studio charges \$200, and so on. The function that models this situation is neither discrete nor continuous.

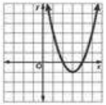
Lesson 3-4

1. x-intercept: (−0.75, 0) y-intercept: (0, 2) positive: when $x > -0.75$ negative: when $x < -0.75$ **3.** x−intercepts: (0, 0) and (2, 0) y-intercept: (0, 0) positive: when $x < 0$ and when $x > 2$ negative: $0 < x < 2$
5. x-intercepts: (−2, 0) y-intercept: (0, 4) positive: when $x > -2$ negative: when $x < -2$

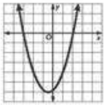
7. x-intercepts: (−5, 0) and (3, 0) y-intercept: (0, 3) positive: $-5 < x < 3$ negative: $x < -5$ and when $x > 3$
9. x-intercepts: none y-intercept: (0, −3) positive: never negative: always
11. The x-intercept is 0. The y-intercept is 0. This means that Ryan earns \$0 for working 0 hours. The function is positive when x is greater than 0, which means that Ryan earns money for working. No portion of the graph shows that the function is negative.
13a. The x-intercept is 6. The y-intercept is 1950. **13b.** The x-intercept means that after 6 months, Javier's remaining balance will be \$0, or it will take Javier 6 months to repay his parents. The y-intercept means that Javier owes his parents \$1950 after 0 months, or Javier initially borrowed \$1950 from his parents.
15. Sample graph; no solution



17. 2, 4



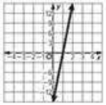
19. −3, 2



21. The zero of the function is at 32. This represents that after tying ribbon on 32 gift bags, Juanita will have no ribbon left.

23. *x*-intercepts: (−2, 0) and (2, 0) *y*-intercept: (0, 4) positive: $-2 < x < 2$ negative: when $x < -2$ and when $x > 2$ **25.** The *x*-intercepts are 3 and 7. That means that the bird will be at sea level at 3 seconds and at 7 seconds. The *y*-intercept is 4.5. This means that at time 0, the bird was at a height of 4.5 feet. The function is positive when *x* is less than 3 and when *x* is greater than 7, which means that the bird is above sea level from 0 to 3 seconds and after 7 seconds. The function is negative when *x* is between 3 and 7, which means that the bird is below sea, or under water, for 4 seconds.

27. 1; *x*-int: 1; *y*-int: −15



29. To find the *x*-intercept in a graph, find the place where the function crosses the *x*-axis. To find the *y*-intercept in a graph, find the place where the function crosses the *y*-axis. To find the *x*-intercept in a table, find the *x*-value when the *y*-value is 0. To find the *y*-intercept in a table, find the *y*-value when the *x*-value is 0. **31.** Find the related function. Subtract 16 from each side: $0 = x + 4 + (2^4 - 6) - 16$. Evaluate the exponent: $0 = x + 4 + (16 - 6) - 16$. Evaluate the expression in parentheses: $0 = x + 4 + 10 - 16$. Add and subtract: $0 = x - 2$. Replace 0 for *f*(*x*). The related function is $f(x) = x - 2$. The graph of the related function intersects the *x*-axis at 2. This is the *x*-intercept, or zero. So the solution of the equation is 2. Check the solution by solving the equation algebraically. Evaluate the exponent: $16 = x + 4 + (16 - 6)$. Evaluate the expression in parentheses: $16 = x + 4 + 10$. Add: $16 = x + 14$. Subtract 14 from each side: $2 = x$.

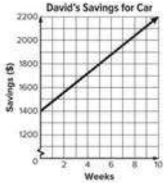
Lesson 3-5

1. This function is symmetric in the line $x = -1$.
3. This function is symmetric in the line $x = 2.5$.

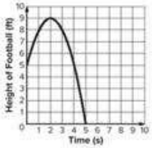
5. The graph is symmetric in the line $x = 5$. In the context of the situation, the symmetry of the graph tells you that the area is the same when the width is a number less than or greater than 5. **7.** always decreasing **9.** decreasing: $x < 1.5$; increasing: $x > 1.5$ **11.** extrema: *B* and *D*; rel min: *D*; rel max: *B* **13.** extrema: *B* and *D*; rel min: *B*; rel max: *D* **15.** Point *A* is a relative maximum. Point *A* represents the greatest height of the golf ball given the distance from the tee. **17.** As *x* decreases, *y* increases. As *x* increases, *y* increases. **19.** As *x* decreases, *y* decreases. As *x* increases, *y* increases. **21.** no line symmetry; always decreasing; extrema: none; As *x* decreases, *y* increases. As *x* increases, *y* decreases. **23.** The approximate point (2.5, 114) is a relative maximum. This represents the greatest height of the rock given the time. **25.** The graph has one relative minimum at about (−2.25, −16); This statement is not true because there are two relative minimums: one at about (−2.25, −16) and one at about (2.25, −16).

Lesson 3-6

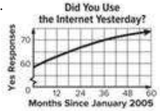
1.



3.

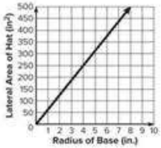


5.



7. Sample answer: Internet use at home initially has a higher number of users than Internet use away from home. Both Internet use at home and Internet use away from home increase after 36 months since March 2004. Neither Internet use at home nor Internet use away from home reaches 0 users.

9.



11. Sample answer: The graph on the calculator and the graph I sketched are both linear, increasing, and have an *x*- and *y*-intercept at 0. **13.** $P(x) = 28x - 840$; *P*(*x*) is Aidan's profit from fixing and selling *x* bicycles. **15.** The *x*-intercept; To find the *x*-intercept, locate the point on the graph when $P(x) = 0$, which is 30. So, when 30 bicycles are bought and sold, Aidan makes a profit of $0.

Module 3 Review

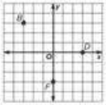
1. B, C, F
3. C

5. Sample answer: The element −4 in the domain is paired with both 8 and 13 in the range. This relation is not a function. **7.** B **9.** (−1, 0) and (0, −1) **11.** 40; 60 **13.** Sample answer: In 1900 the population of Ohio was nearly 4 million more than the population of Florida. Both populations grew between 1900 and 1950. At this point, the population of Ohio exceeded that of Florida by approximately 5 million, indicating a greater growth rate for Ohio than Florida during those decades. Then from 1950 to 2000, the population of Ohio grew by about 3.4 million, whereas the population of Florida grew by about 13 million, indicating a significantly greater growth rate for Florida during those decades. In fact, by 2000, the population of Florida surpassed Ohio by more than 4 million people.

Module 4

Quick Check

1, 3, 5.

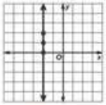


7. $y = -3x + 1$ **9.** $y = \frac{5}{2}x - 6$ **11.** $y = -10x + 6$

Lesson 4-1

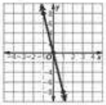
1.

x	y
−2	0
−2	1
−2	2



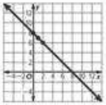
3.

x	y
−1	8
0	0
1	−8



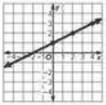
5.

x	y
0	8
1	7
2	6

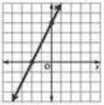


7.

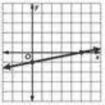
x	y
0	1
2	2
4	3



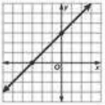
9.



11.

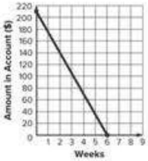


13.

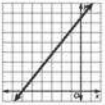


15a. The x-intercept is 6. This means that after 6 weeks, Amanda will have $0 in her school lunch account. The y-intercept is 210. This means that there was initially $210 in Amanda's school lunch account.

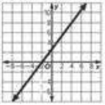
15b.



17.



19.

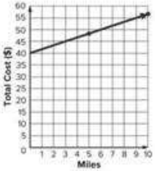


21. x-int: 7; y-int: −2

23. x-int: $1\frac{1}{3}$; y-int: 4

25. x-int: $-1\frac{1}{2}$; y-int: 1

27. $y = 1.7x + 40$; The y-intercept is 40. This means that it would cost $40 to hook up the car.



29. Sample answer: The x-intercept is −4. The x-intercept is not reasonable because the football team cannot lose −4 games. The y-intercept is 4. The y-intercept is reasonable because the y-intercept means that if the football team won 4 games, they lost 0 games. **31.** No; sample answer: A horizontal line only has a y-intercept and a vertical line only has an x-intercept. **33.** In the equation, let $y = 0$ to find the x-intercept: $2x + (0) = 4$. So the x-intercept is 2. In the equation, let $x = 0$ to find the y-intercept: $2(0) + y = 4$. So the y-intercept is 4. Robert graphed points at (2, 0) and (0, 4) and connected the points with a line. **35.** Sample answer: $y = 8$; horizontal line **37.** Sample answer: $x - y = 0$; line through (0, 0)

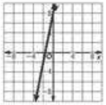
Lesson 4-2

1. $\frac{1}{5}$ **3.** increased about 1.9 people per square mile **5a.** −5; This means the temperature decreased 5°F per hour from 6 A.M. to 7 A.M. **5b.** −5; This means the temperature decreased 5°F per hour from 1 P.M. to 2 P.M. **7.** linear; $-\frac{1}{1}$ or −1 **9.** not linear **11.** $-\frac{3}{5}$ **13.** 1 **15.** 0 **17.** $\frac{1}{6}$ **19.** $\frac{4}{3}$ **21.** undefined **23.** 1 **25.** undefined **27.** 2 **29.** undefined **31.** −1 **33.** undefined **35.** $-\frac{7}{2}$ **37.** $\frac{5}{2}$ **39.** $\frac{3}{4}$ **41.** 6 **43.** 8 **45.** 11 **47.** $\frac{1}{20}$ **49.** $-\frac{1}{2}$ **51.** $\frac{1}{3}$ **53.** $\frac{1}{2}$ **55.** −1 **57.** $\frac{7}{4}$ **59.** 3 **61.** After drawing a graph, use the two points on the graph to determine the slope. This can be done by counting squares for the rise and run of the line or by using the coordinates of the points in the slope formula. **63.** The rate of change is $2\frac{1}{4}$ inches of growth per week. **65.** Step 1; she reversed the order of the x-coordinates in the formula. **67.** The difference in the x-values is always 0, and division by 0 is undefined.

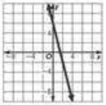
Lesson 4-3

1. $y = 5x - 3$ **3.** $y = -6x - 2$ **5.** $y = 3x + 2$ **7.** $y = x - 12$ **9.** $y = 5x + 6$ **11.** $y = \frac{1}{3}x - 2$ **13.** $y = -0.25x - 3$ **15.** $y = 25x + 100$ **17.** $y = 0.12x + 9$

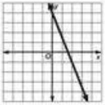
19.



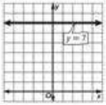
21.



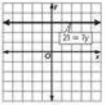
23.



25.

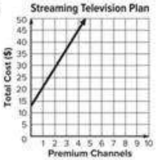


27.



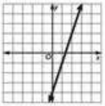
29a. $c = 13 + 8p$

29b.

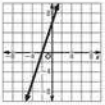


29c. \$37 **31.** $y = \frac{1}{2}x - 3$

33.



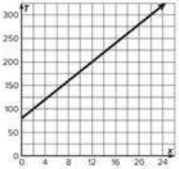
35.



37. $y = 2x - 3$ **39.** $y = -x - 1$

41a. $T = 10x + 80$

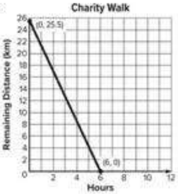
41b.



41c. 300°F **41d.** $T = -5x + 300$

43a. $y = -4.25x + 25.5$

43b.



43c. The x-intercept (6) represents the number of hours it will take Jazmin to complete the walk. The y-intercept (25.5) represents the length of the walk. **43d.** Using the graph, I can determine the value of x when y equals $-17 + 25.5$ or 8.5 km, and use the value of the x-intercept. The value of x is 4 when $y = 8.5$ and the x-intercept is 6. Therefore, Jazmin has $6 - 4$ or 2 hours more to walk. **45.** Yes; you can find the value of x on the graph when $y = 0$; $x = \frac{1}{2}$.

47. Sample answer: $y = 25x + 200$; I have \$200 in savings and will save \$25 per week until I have enough money to buy a new phone. I can predict how much money I'll have after x number of weeks.

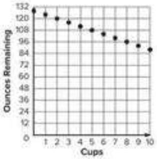
Lesson 4-4

1. $g(x)$ is a translation of the parent function 11 units up **3.** $g(x)$ is a translation of the parent function 7 units right **5.** $g(x)$ is a translation of the parent function 10 units left and 1 unit down **7.** $g(x) = 4x$; $g(x)$ is the translation of $f(x)$ 3.5 units down. **9.** $g(h) = 8h + 15$; $g(h)$ is the translation $f(h)$ of 5 units up. **11.** $g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{3}$ **13.** $g(x)$ is a horizontal compression of the parent function by a factor of $\frac{1}{3}$ **15.** $g(x)$ is a horizontal stretch of the parent function by a factor of 2.5 **17.** $g(x)$ is a vertical stretch of the parent function by a factor of 8 and a reflection across the x-axis **19.** $g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{5}{4}$ and a reflection across the y-axis **21.** $g(x)$ is a horizontal compression of the parent function by a factor of $\frac{2}{3}$ and a reflection across the y-axis **23.** $g(x)$ is a translation of the parent function 2 units right and 8 units down **25.** $g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{5}$ **27.** $g(x)$ is a horizontal compression of the parent function by a factor of 0.4 **29.** $g(x) = x - 7$ **31.** $g(x) = 1.5x$; The graph of $g(x) = 1.5x$ is the graph of $f(x) = 0.50x$ stretched vertically by a factor of 3. **33a.** $g(x) = 1.29x$ **33b.** The graph of $g(x) = 1.29x$ is the graph of $f(x) = x$ stretched vertically by a factor of 1.29. **35.** $y = \frac{1}{a}x$; The function is horizontally stretched by a factor of a.

Lesson 4-5

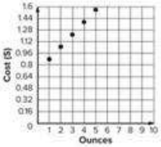
1. This sequence has a common difference of 4 between its terms. This is an arithmetic sequence. **3.** This sequence does not have a common difference between its terms. This is not an arithmetic sequence. **5.** This sequence does not have a common difference between its terms. This is not an arithmetic sequence. **7.** This sequence has a common difference of 3 between its terms. This is an arithmetic sequence. **9.** 1.06; 4.26, 5.32, 6.38 **11.** −2; 13, 11, 9 **13.** $\frac{1}{3}$; $3\frac{2}{3}$, 4, $4\frac{1}{3}$ **15.** 4; 19, 23, 27 **17.** 2; −5, −3, −1 **19.** $a_n = -5n + 2$; −33 **21.** $a_n = -4n - 7$; −35 **23a.** $f(n) = -4n + 128$

23b.



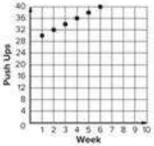
23c. 72 ounces **25a.** $f(n) = 0.17n + 0.71$

25b.



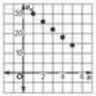
25c. 8 ounces **27a.** $f(n) = 2n + 28$

27b.



27c. 11th week **29.** This sequence does not have a common difference between its terms. This is not an arithmetic sequence. **31.** This sequence has a common difference of 2 between its terms. This is an arithmetic sequence.

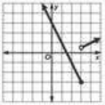
33. $a_n = -4n + 34$



35a. $a_n = 3000 + 500n$ **35b.** $15,000 **37a.** $a_n = 3n - 1$ **37b.** 59 **39.** Sample answer: 5, 3, 8, 6, 11, 9, 14, ...; The pattern is to subtract 2 from the first term to find the second term, then add 5 to the second term to find the third term. **41.** Sample answer: 2, −8, −18, −28, ... **43a.** Sample answer: $a_n = -2 - 3n$ **43b.** $a_n = -19 + 7n$ **43c.** $a_n = 12 - 2n$ **45.** On day 9, Andre has read 270 pages while Sam has 270 pages left to read. The table shows that both functions have a value of 270 when $x = 9$.

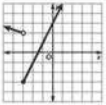
Lesson 4-6

1.



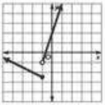
D = all real numbers,
R = $f(x) \geq -3$

3.



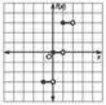
D = all real numbers,
R = $f(x) \geq -3$

5.



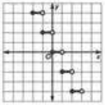
D = all real numbers,
R = $f(x) \geq -2.5$

7.



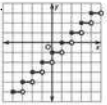
D = all real numbers,
R = all integer multiples of 3

9.



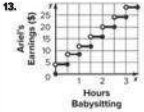
D = all real numbers,
R = all even integers

11.



D = all real number,
R = all integers

13.



15.

$$f(x) = \begin{cases} 16.20 & \text{if } 0 < x \leq 1 \\ 19.30 & \text{if } 1 < x \leq 2 \\ 22.40 & \text{if } 2 < x \leq 3 \\ 25.50 & \text{if } 3 < x \leq 4 \\ 28.60 & \text{if } 4 < x \leq 5 \end{cases}$$

D = $\{x \mid 0 < x < 5\}$;
R = {16.20, 19.30, 22.40, 25.50, 28.60}

17. $g(x) = \begin{cases} 2x + 1 & \text{if } x \leq 2 \\ x - 2 & \text{if } x > 2 \end{cases}$

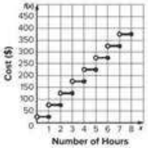
19. $133.00

21a.

x	0	2	4	6	8
f(x)	0	75	175	275	375

21b. $f(x) = 25 + 50[[x]]$

21c.



21d. $2 < x \leq 3$

23. Sample answer:

$$y = \begin{cases} -x & x < -4 \\ 2x & -4 \leq x \leq 2 \\ x & x > 2 \end{cases}$$

25. A step function has different constants over different intervals of its domain. A piecewise-defined function can have different algebraic rules over different intervals of its domain.

27. $f(x) = \begin{cases} \frac{1}{2}x - 3 & x > 6 \\ -\frac{1}{2}x + 3 & x \leq 6 \end{cases}$

29. R = $f(x) \geq 0$

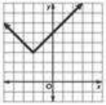
31. 2.4

Lesson 4-7

1. The graph of $g(x)$ is the parent function translated 5 units down. **3.** The graph of $g(x)$ is the parent function translated 2 units right and 7 units up. **5.** The graph of $g(x)$ is the parent function translated 1 unit up.

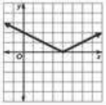
7. $f(x) = |x + 2|$ **9.** $f(x) = |x| - 3$ **11.** $f(x) = |x| + 1$ **13.** The graph of $g(x)$ is a horizontal compression of the parent function. **15.** The graph of $g(x)$ is a vertical stretch of the parent function. **17.** The graph of $g(x)$ is a horizontal stretch of a parent function. **19.** The graph of $g(x)$ is a reflection of the parent function across the x-axis and a vertical stretch. **21.** The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal stretch. **23.** The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal compression.

25.

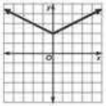


D = all real numbers,
R = $g(x) \geq 3$

27.

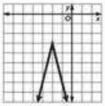


29.



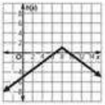
D = all real numbers,
R = $f(x) \geq 2$

31.



D = all real numbers,
R = $f(x) \leq -3$

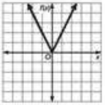
33.



D = all real numbers,
R = $h(x) \leq 1$

35. $y = 65|10 - x|$

37.



D = all real numbers,
R = $f(x) \geq 0$
The graph of $f(x)$ is the parent function horizontally compressed by a factor of $\frac{1}{2}$.

39. $f(x) = |-3x - 5|$ **41.** $f(x) = \left|\frac{1}{3}x + 2\right|$

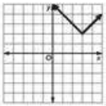
43. $x = |s - 16|$

45. $x = |t - 21.7|$; The range of times is twice the value of x, 3.2(2) = 6.4 s; The solution to the equation is 24.9 and 18.5, which has a range of 24.9 − 18.5 = 6.4 s.

47. $x = |b - 12|$

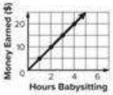
49. To get the graph of $h(x)$, the parent absolute value function is reflected in the x-axis, then translated 2 units left and 3 units down.

51. $f(x) = \begin{cases} -x + 5 & \text{if } x < 3 \\ x - 1 & \text{if } x \geq 3 \end{cases}$

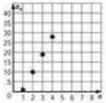


Module 4 Review

1.



3. −500 gallons/hr **5.** B **7.** A
9. dilation **11.** C
13. $f(n) = 9n - 8$



15.

Hours Worked, x	Money Earned, $f(x)$
30	270
35	315
40	360
45	427.5
50	495

17. B

19. Sample answer: It is translated 5 units up.

21. $f(x) = -|x - 4| + 3$

Module 5

Quick Check

1. $y = 5 - x$ **3.** $y = x + 5$ **5.** (4, 2) **7.** (2, −4)
9. (−3, −3)

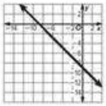
Lesson 5-1

1. $y = \frac{1}{2}x$ **3.** $-\frac{3}{4}x + \frac{17}{2}$ **5.** $y = \frac{1}{2}x + 1$
7. $d = 3t + 12$ **9.** $C = 2.54y + 62.38$
11. $y = -4$ **13.** $y = \frac{4}{3}x - \frac{1}{3}$ **15.** $y = -\frac{3}{2}x - \frac{9}{2}$
17. $y = -\frac{4}{11}x + \frac{58}{11}$ **19.** $y = -\frac{1}{2}x - \frac{9}{2}$
21. $y = \frac{1}{6}x + \frac{19}{24}$ **23.** $C = 10d + 12$
25. $T = -4.5x + 103$ **27.** $y = 3x - 1$
29. $y = -x - 4$ **31.** $y = -x + 3$ **33.** No;

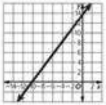
substituting 3 and −1 for x and y results in an equation that is not true. **35.** Yes; substituting 15 and −13 for x and y results in an equation that is true. **37.** Sample answer: (3, −3) **39.** Sample answer: (0, −5) **41.** Sample answer: (0, 4) **43.** C; x represents the number of plane tickets per order and y represents the total cost of an order. **45.** A; x represents the number of hours and y represents the oil level in the tank, in inches. **47a.** $y = x + 2.5$ **47b.** 10 **47c.** $y = x + 1.5$ **49a.** $y = 7.5x + 1$ **49b.** 1; Koby's puppy weighed 1 pound at birth (0 months) **49c.** 7.5; Koby's puppy gained 7.5 pounds a month for the first 6 months. **51.** Jacinta; Tess switched the x- and y-coordinates on the point that she entered in Step 3. **53.** Sample answer: Let y represent the number of quarts of water in a pitcher, and let x represent the time in seconds that water is pouring from the pitcher. As time increase by 1 second, the amount of water in the pitcher decrease by $\frac{1}{2}$ qt. An equation is $y = -\frac{1}{2}x + 4$. The slope is the rate at which the water is leaving the pitcher, $\frac{1}{2}$ quart per second. The y-intercept represents the amount of water in the pitcher when it is full, 4 qt.

Lesson 5-2

1. $y + 3 = -1(x + 6)$



3. $y - 11 = \frac{4}{3}(x + 2)$



5. Sample answer: $y + 3 = -4(x - 1)$
7. Sample answer: $y - 3 = \frac{4}{3}(x - 3)$
9. $y = -6x - 47$ **11.** $y = \frac{1}{6}x - \frac{8}{3}$
13. $y - 18 = 3.5(x - 5)$ **15.** $2x - y = 6$
17. $x + 6y = -7$ **19.** $x - y = -1$
21. Sample answer: $2x + 3y = -13$
23. Sample answer: $3x + y = -3$
25. Sample answer: $y = x - 5$; $y = -x + 1$
27. Sample answer: $y = -5x + 2$; $y = \frac{1}{5}x + 2$
29. Sample answer:
$y = -\frac{3}{4}x + \frac{3}{2}$; $y = \frac{4}{3}x + \frac{17}{3}$
31. neither
33. perpendicular **35.** neither
37. Sample answer: $5x + 4y = 20$
39. $y = 9x + 5$; $9x - y = -5$
41. $y = -6x - 45$; $6x + y = -45$
43. $y = \frac{9}{10}x - 4\frac{3}{10}$; $9x - 10y = 43$
45. Yes; sample answer: The line that represents one of the ceiling walls has a slope of $-\frac{1}{4}$ and the line that represents the other ceiling wall has a slope of 4.
47a. Sample answer: $y - 0 = 0.5(x - 0)$
47b. $y = 0.5x$ **47c.** $x - 2y = 0$
49. Sample answer: You need to know the slope of the line and the y-intercept of the line, the slope and the coordinates of another point on the line, or the coordinates of two points on the line.

51. No; the line through (7, −10) and (3, −2) has a slope of −2 and $2x - y = -5$ has a slope of 2.

53. Sample answer: $y - g = \frac{j-g}{h-f}(x - f)$

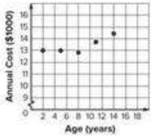
55. Sample answer: Jocari spent $18 to go to a carnival and play games. The price she paid included admission. The games cost $2 each; $y - 18 = 2(x - 5)$, $y = 2x + 8$.

Lesson 5-3

1. Positive; as time spent exercising increases, the more Calories are burned. **3.** Negative; as weight increases, the number of repetitions decreases. **5a.** $y = -328.275x + 3142.15$

5b. about 187.675 million

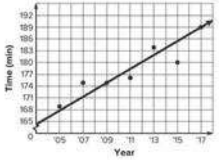
7a. There is a positive correlation between the child's age and annual cost.



7b. $y = 270x + 10{,}640$ **7c.** about $15,230

9. no correlation

11a.

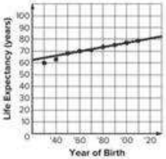


11b. Sample answer: x represents the number of years since 2005, so year 2005 is represented by $x = 0$ and year 2020 is represented by $x = 15$. Two points on the line of best fit are (4, 175) and (17, 189). Use these two points to find the slope to be 1.75 and the equation of the line of best fit to be $y = 1.75x + 168$. **11c.** Sample answer: about 196 minutes **11d.** Sample answer: Not all of the data points are close to the line of best fit, so there is not a consistent trend regarding the length of games. Therefore, the predicted game length may or may not be accurate.

13a. positive correlation; As the number of years since 2007 increases, the price of a ticket increases.

13b. $y = 2.87x + 64.24$ **13c.** about $133.12

15a.

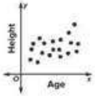


15b. Sample answer: About 81.8; The data show a positive correlation, so as the years increase, the life expectancy also increases. Therefore, the life expectancy should be higher than that of a baby born in 2010. **15c.** Sample answer: I assumed that the trend continues, so as the year increases, the life expectancy also increases.

17. Sample answer: The salary of an individual and the years of experience that he or she has could be modeled using a scatter plot. This would be a positive correlation because the more experience an individual has, the higher the salary would likely be.

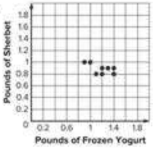
19. Neither; line g has the same number of points above the line and below the line. Line f is close to 2 of the points; but for the rest of the data, there are 3 points above and 3 points below the line.

21. Sample answer: You can visualize a line to determine whether the data has a positive or negative correlation. The graph shows the ages and heights of people. To predict a person's age given his or her height, write a linear equation for the line of fit. Then substitute the person's height and solve for the corresponding age. You can use the pattern in the scatter plot to make decisions.



Lesson 5-4

1a.

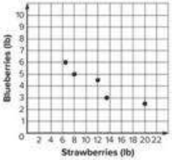


1b. Negative; as the number of pounds of frozen yogurt consumed increases, the number of pounds of sherbet consumed decreases.

1c. The relationship may be a causation. Since both are frozen desserts, eating more frozen yogurt may cause people to decrease the amount of sherbet they eat. Other things that might influence the data are an increase in frozen yogurt stores and a decrease in popularity or availability of sherbet.

3. Correlation, sample answer: Having a wider palm does not cause someone to watch less television. **5.** Causation; sample answer: An increase in the price of cereal likely causes customers to buy less cereal.

7a.



7b. Negative; as the number of pounds of strawberries produced increases, the number of pounds of blueberries produced decreases.

7c. The relationship is a correlation, but not a causation. A better yield of strawberries does not cause the blueberries to grow poorly. Other factors, such as temperature and rain, could be affecting the plants that week.

9. positive correlation and causation; Sample answer: Because pizzas are topped with cheese, an increase in the number of pizzas made cause more cheese to be used.

11. Sample answer: Two elements can have a strong correlation, but it does not mean that one causes the other. There could be an unknown factor affecting the elements.

13. Sample answer: Correlation does not mean causation. Even though there is a strong correlation that does not mean buying swimsuits causes the use of air conditioners. Another factor, like the temperature, could be affecting both swimsuit sales and use of air conditioners.

Lesson 5-5

1a. $y = -1.31x + 50.95$ **1b.** $r \approx -0.714$; The equation models the data fairly well. Its negative value means that as the years since 2010 increase, the total number of goals the soccer team scores each season decreases.

3a. $y = 8.52x + 3.18$

3b. $r \approx 0.999$; The equation models the data very well. Its positive value means that as the years since 2010 increase, sales, in millions of dollars, increase.

5a. $y = 103.77x + 108.06$ **5b.** about $3221.16 **7a.** $y = 0.59x + 1.51$ **7b.** The residuals are randomly scattered and are centered about the line $y = 0$. So, the best-fit line models that data well. **9a.** $y = 0.26x + 21.21$

9b. $r \approx 0.359$; The equation does not model the data well. Its value means that as the years since the 2011–2012 school year increase, the percentage of students in public school who met all six of California's physical fitness standards each year varies. **9c.** Because the data on the students who meet all six standards is reported as a percentage, it cannot exceed 100. **11a.** $y = 140.4x + 13.8$

11b. $r \approx 0.999$; The equation models the data very well. Its positive value means that as the number of games increases, the cumulative number of yards increases.

11c. Sample answer: Because the data have a positive correlation, the total number of yards will increase and then number of games increases. So, the running back will have run for 950 yards between games 6 and 9.

11d. during game 7

13a. $y = 9619x + 443{,}918.8$ **13b.** $r \approx 0.999$; The equation models the data very well. Its positive value means that as the number of years since the 2010-2011 school year increases, the number of student athletes participating in college athletics each year increases.

13c. The residuals are randomly scattered and are centered about the line $y = 0$. So, the best-fit line models that data well.

13d. about 684,394

15. Apply a linear regression model to the data. Use the number of each test as the independent variable. If there is no correlation, the r-value will not be close enough to 1 or −1. If this is the case, the line of fit could not be used to predict the scores of the other students.

17a. $y = 84{,}345.0x + 5{,}003{,}868.3$

17b. about 7,365,528

Lesson 5-6

1. $\{(-1, -9), (-4, -7), (-7, -5), (-10, -3), (-13, -1)\}$

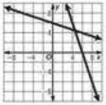
3. $\{(-2, -4), (-1, -2), (1, 0), (0, 2), (2, 4)\}$

5. $\{(-3, 5), (-9, 2), (-15, -1), (-21, -4)\}$

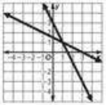
7. $\{(16, -1), (12, -2), (8, -3), (4, -4)\}$

9. $\{(-49, -4), (35, 8), (-28, -1), (7, 4)\}$

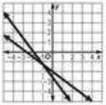
11.



13.



15.



17. $f^{-1}(x) = \frac{x}{6} - 7$

19. $f^{-1}(x) = \frac{5}{2}(x + 16)$ **21.** $f^{-1}(x) = \frac{1 - 5x}{4}$

23a. $P^{-1}(x) = \frac{x + 36}{7.6}$ **23b.** x represents Alisha's profit and represents the number of dozens of brownies sold. **23c.** 5

25a. $C^{-1}(x) = \frac{x - 125}{16}$ **25b.** 108 feet

27. $f^{-1}(x) = \frac{1}{4}x + 6$ **29.** $f^{-1}(x) = 6x - 42$

31. $f^{-1}(x) = \frac{7}{2}x - 14$ **33.** $f^{-1}(x) = \frac{1}{7}x - \frac{6}{7}$

35. $f^{-1}(x) = 2x - 22$ **37.** B **39.** A

41. $\{(-k, b), (p, -g), (-m, -w), (q, r)\}$ **43.** The slopes are reciprocals. For example, if the slope of one line is $\frac{2}{3}$, then the slope of the inverse function is $\frac{3}{2}$. **45.** Sample answer: This claim is incorrect. The −1 in the inverse function notation is not an exponent. As an example, the inverse function for $y = x + 1$ is found by switching x and y and solving for y, which gives $y = x - 1$. $y = x - 1$ is not the same as $y = \frac{1}{(x + 1)}$, which is not a line. This method does not work.

47. $a = 2$; $b = 14$

49. sometimes; Sample answer: $f(x)$ and $g(x)$ do not need to be inverse functions for $f(a) = b$ and $g(b) = a$. For example, if $f(x) = 2x + 10$, then $f(2) = 14$ and if $g(x) = x - 12$, then $g(14) = 2$, but $f(x)$ and $g(x)$ are not inverse functions. However, if $f(x)$ and $g(x)$ are inverse functions, then $f(a) = b$ and $g(b) = a$.

51. Sample answer: A situation may require substituting values for the dependent variable into a function. By finding the inverse of the function, the dependent variable becomes the independent variable. This makes the substitution an easier process.

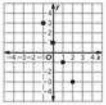
Module 5 Review

1. A **3.** $y = 1.5x + 11$ **5.** A

7. $y - 4 = 2.5(x - 2)$ **9.** A

11. a positive correlation **13.** C

15. $f^{-1}(x) = -2x + 1$;



17. A

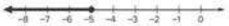
Module 6

Quick Check

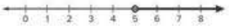
1. 4 **3.** -2 **5.** $\{-29, 7\}$ **7.** $\{-1, 15\}$

Lesson 6-1

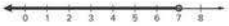
1.



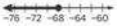
3.



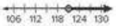
5.



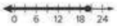
7. $t < -1$ **9.** $w < 5$ **11.** $b \geq -5$
13. $\{m \mid m < 7\}$ **15.** $\{r \mid r \leq 15\}$ **17.** $\{b \mid b \geq 2\}$
19. $\{c \mid c \leq -4\}$ **21.** $\{m \mid m \geq 4\}$
23. $\{r \mid r \geq 22\}$ **25.** $\{a \mid a \leq -4\}$
27. $\{w \mid w \geq -5\}$ **29.** $\{x \mid x \leq 5\}$
31. $\frac{3}{10}x \leq 4.50, x \leq \$15$ **33.** no more than 2.1 pounds per day **35.** at least 500 pieces
37. $\{m \mid m \leq -68\}$



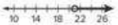
39. $\{c \mid c > 121\}$



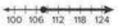
41. $\{x \mid x \leq 20\}$



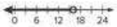
43. $\{h \mid h > 21\}$



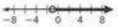
45. $\{n \mid n \geq 108\}$



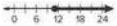
47. $\{r \mid r < 16\}$



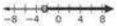
49. $\{t \mid t > -1\}$



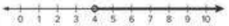
51. $\{z \mid z \geq 11\}$



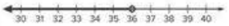
53. $\{d \mid d > -2\frac{1}{2}\}$



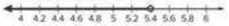
55. d **57.** a **59.** b
61. Sample answer: Let n = the number. $n + 7 \leq -18$; $\{n \mid n \leq -25\}$
63. Sample answer: Let n = the number. $n + 2 \leq 1$; $\{n \mid n \leq -1\}$
65. Sample answer: Let n = the number. $-12n \leq 84$; $\{n \mid n \geq -7\}$
67. $\{g \mid g > 4\}$



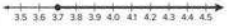
69. $\{x \mid x < 36\}$



71. $\{m \mid m < 5.4\}$



73. $\{c \mid c \geq 3.7\}$

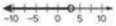


75. \$22.23.

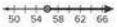
77. Sample answer: Let x represent the decibel level of the calls of a blue whale; $x - 83 \leq 105$; $x \leq 188$. The calls of a blue whale are less than or equal to 188 decibels.
79. $-\frac{x}{2} < 1$ **81a.** $x < \frac{7}{a}$ **81b.** $x \geq \frac{12}{a}$
81c. $x > 3$ **81d.** $x \geq \frac{1}{4}$

Lesson 6-2

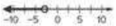
1a. $15 + 2h \leq 35$ **1b.** $h \leq 10$; 10 hours
3a. $1.50 + 0.25(5x - 1) \leq 3.75$ **3b.** $x \leq 2$; 2 mi
3c. Because the service charges per $\frac{1}{a}$ mile, multiply a by the number of miles, x, to find the number of $\frac{1}{a}$ miles. Subtract 1 from the total number of $\frac{1}{a}$ miles, ax, to find the number of additional $\frac{1}{a}$ miles. Multiply the difference by the cost per additional $\frac{1}{a}$ mile, \$0.25, and add the cost for the first $\frac{1}{a}$ mile, \$1.50. This sum is less than or equal to the total amount Demetri has in his pocket, \$3.75, so $1.50 + 0.25(ax - 1) \leq 3.75$. **5a.** $100 + 40x \leq 250$
5b. $x \leq 3.75$; 3 people
7. $21 > 15 + 2x$; $x < 3$



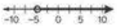
9. $\frac{x}{8} - 13 > -6$; $x > 56$



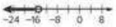
11. $37 < 7 - 10x$; $x < -3$



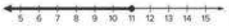
13. $-\frac{5}{4}x + 6 < 12$; $x > -\frac{24}{5}$



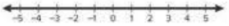
15. $15x + 30 < 10x - 45$; $x < -15$



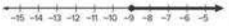
17. $\{a \mid a \leq 11\}$



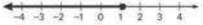
19. $\{b \mid b$ is a real number.$\}$



21. $\{a \mid a \geq -9\}$



23. $\{x \mid x \geq \frac{1}{2}\}$ **25.** $\{m \mid m \geq 18\}$
27. $\{w \mid w > -2\}$ **29.** $\{x \mid x \leq 8\}$
31. $\{x \mid x > -6\}$ **33.** $\{x \mid x \geq 1.5\}$
35. $\left\{p \mid p \leq 1\frac{1}{9}\right\}$

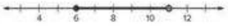


37a. $2x + 4 \leq 13$; $x \leq 4.5$
37b. 4.5 ft.
37c. 5ft.
39. Eric does not have any pencils. Based on his statement, the inequality is $6p + 15 < 20$, where p is the number of pencils. The solution of the inequality is $p < \frac{5}{6}$. However, the number of pencils must be a whole number, so $p = 0$.

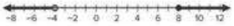
41. $10n - 7(n + 2) > 5n - 12$
(Original inequality)
$10n - 7n - 14 > 5n - 12$
(Distributive Property)
$3n - 14 > 5n - 12$
(Combine like terms.)
$3n - 14 - 5n > 5n - 12 - 5n$
(Subtract $5n$ from each side.)
$-2n - 14 > -12$
(Simplify.)
$-2n - 14 + 14 > -12 + 14$
(Add 14 to each side.)
$-2n > 2$
(Simplify.)
$\frac{-2n}{-2} < \frac{2}{-2}$
Divide each side by -2. Change $>$ to $<$.
$n < -1$
(Simplify.)
The solution set is $\{n \mid n < -1\}$.
43. $\frac{78 + 80 + 78 + x}{4} \geq 82$; $x \geq 94$; Mei needs a score of at least 94 on the next exam.
45. Sample answer: $2(2x - 1) < 10$
47. Let c = the number of baseball cards Ted has; $4c > 5c - 15$; $15 > c$; Ted has fewer than 15 cards. **49.** $\emptyset$; If the inequality is always true, the opposite inequality will always be false.
51. Sample answer: The solution set for the inequality that results in a false statement is the empty set, as in $12 \leq -15$. The solution set for an inequality in which any value of x results in a true statement is all real numbers, as in $12 \leq 12$.

Lesson 6-3

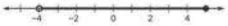
1. $\{f \mid 6 < f < 11\}$



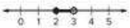
3. $\{y \mid y \geq 8$ or $y < -4\}$



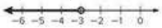
5. $\{p \mid -4 < p \leq 5\}$



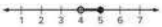
7. $\{h \mid 2 \le h < 3\}$



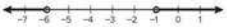
9. $\{y \mid y < -3\}$



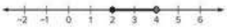
11. $\{b \mid 4 < b \le 5\}$



13. $\{m \mid m < -6 \text{ or } m > -1\}$

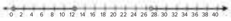


15. $\{m \mid 2 \le m < 4\}$



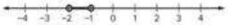
17a. $x + 8 < 20$ or $x + 8 > 35$ **17b.** $0 < x < 12$ or $x > 27$; Because the combined height of the sign and pole cannot be negative, the value of x must be greater than 0.

17c.

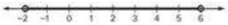


19. $-3 < x \le 3$ **21.** $x < -2$ or $x \ge 1$
23. $b > 3$ or $b \le 0$ **25.** $y < -1$ or $y \ge 1$

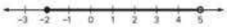
27. $f \mid -2 < f < -1\}$



29. $\{b \mid -2 < b < 6\}$



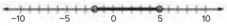
31. $\{a \mid -2 \le a < 5\}$



33. Sample answer: Let n = the number. $n - 2 \le 4$ or $n - 2 \ge 9$; $\{n \mid n \le 6 \text{ or } n \ge 11\}$ **35.** $54° \le x \le 68°$ **37.** The minimum is 67, since the solution of the inequality $2000 \le 1000 + 15x$ is $66\frac{2}{3} \le x$, and the number of students must be a whole number. The maximum is 100, since the solution of the inequality $1000 + 15x \le 3000$ is $x \le 133\frac{1}{3}$, but the bus can only hold 100 students. **39a.** The side lengths must be 5, x, and $9 - x$. Using the Triangle Inequality results in the compound inequality $x + 5 > 9 - x$ and $14 - x > x$.

39b. The solution of the compound inequality is $2 < x < 7$, so each of the lengths must be greater than 2 m but less than 7 m. The sum of the two lengths must be 9 m.

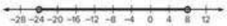
41. $\{x \mid -2 < x < 5\}$



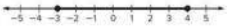
43a. $\$400 \le x \le \$800$ **43b.** $\$428 \le x \le \$856$
45. B
47. $x > -1$ or $x \ge 4$; This can be written as $x > -1$ because this is the union of two graphs.
49. The union of the two graphs is the graph on the left, so the graph on the left is the graph of the solution set for **Exercise 47.** The intersection of the two graphs is the graph on the right, so the graph on the right is the graph of the solution set for **Exercise 48.**
51a. $x > -\frac{4}{a}$ and $x \le \frac{4}{a}$
51b. $x < -\frac{6}{a}$ or $x > 5a$
53. Sometimes; The graph of $x > 2$ or $x < 5$ includes the entire number line.

Lesson 6-4

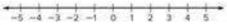
1. $\{x \mid -24 < x < 8\}$



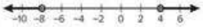
3. $\{c \mid -3 \le c \le 4\}$



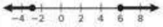
5. $\{\emptyset\}$



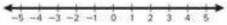
7. $\{r \mid r < -8 \text{ or } r > 4\}$



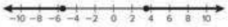
9. $\{h \mid h \le -3 \text{ or } h \ge 6\}$



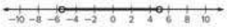
11. $\{v \mid v$ is a real number.$\}$



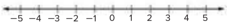
13. $\left\{n \mid n \le -5\frac{1}{4} \text{ or } n \ge 3\frac{3}{4}\right\}$



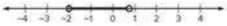
15. $\left\{h \mid -5\frac{2}{3} < h < 5\right\}$



17. $\{\emptyset\}$

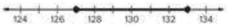


19. $\left\{r \mid -2 < r < \frac{2}{3}\right\}$

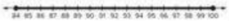


21. $\{x \mid 58.5 \le x \le 61.5\}$ **23a.** $|p - 130| \le 3.05$

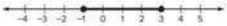
23b.



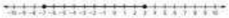
25a. $|x - 515| \le 114$ **25b.** 287 to 743
27. $|n + 2| \ge 1$ **29.** $|w - 2| < 2$ **31.** $|x| > 1$
33. d **35.** c
37. $|x - 92| \le 8$; $\{x \mid 84 \le x \le 100\}$



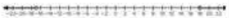
39. $\{x \mid -1 \le x \le 3\}$



41. $\{x \mid -7 \le x \le 3\}$

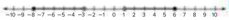


43. $\{x \mid x > 18 \text{ or } x < -17\}$



45. By definition, the absolute value is always greater than a negative number. Therefore, no matter what number is chosen, it will always be greater than −1 when evaluated in the absolute value inequality given. **47a.** Set the absolute value of an unknown variable, x, minus the recommended weight, 516, to be less than or equal to the variance of 4. So, the inequality $|x - 516| \le 4$ represents the situation.
47b. Write two inequalities, one for each case: $x - 516 \le 4$ and $-(x - 516) \le 4$. For the first case, add 516 to both sides: $x \le 520$. For the second case, distribute the negative on the left side, subtract 516 from both sides and divide by a negative 1 remembering to switch the inequality sign: $x \ge 512$. This means a box of cereal should have a minimum weight of 512 g and a maximum weight of 520 g.

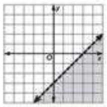
49. The solution set for $|x - 2| > 4$ is $\{x \mid x < -2 \text{ or } x > 6\}$. The solution set for $-2x < 4$ or $x > 6$ is $\{x \mid x > -2\}$. One includes numbers greater than −2, and the other includes numbers less than −2 or greater than 6. These solution sets are not the same.
51. Jordan is correct. Chloe did not distribute the negative to both x and 3.
53. $(-8 \le n < -3)$ or $(1 < n \le 6)$. To solve this compound inequality, split it into two inequalities. The first one to solve is $|n + 1| > 2$ and the second one is $|n + 1| \le 7$. The solution set of the entire problem is the overlap of the individual solutions.



55. No; Sample answer: Lucita forgot to change the direction of the inequality sign for the negative case of the absolute value.
57. Sample answer: If $t = 0$, then the absolute value is equal to 0, not greater than 0.
59. Sample answer: When an absolute value is on the left and the inequality symbol < or ≤, the compound sentence uses *and*, and if the inequality symbol is > or ≥, the compound sentence uses *or*. To solve, if $|x| < n$, then set up and solve the inequalities $x < n$ and $x > -n$, and if $|x| > n$, then set up and solve the inequalities $x > n$ or $x < -n$.

Lesson 6-5

1.



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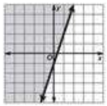

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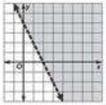

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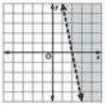

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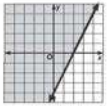

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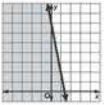
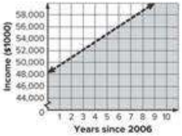
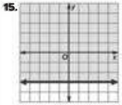


13a. $y < 1240x + 48{,}200$

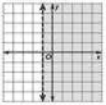


13b. no, no, yes, no

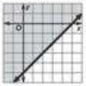
15.



17.



19.



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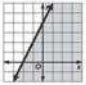
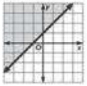
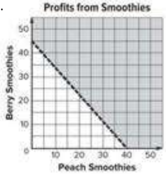


23.



25a. $2.25p + 2b > 90$; $p \geq 0$ and $b \geq 0$

25b.



25c. The café sold more than 22 berry smoothies. **25d.** 41; Sample explanation: 40 peach smoothies results in a profit of exactly \$90, so to make a profit of more than \$90, the café must have sold 41 smoothies. **27.** The value of c must be positive. Since (0, 0) is a solution of the inequality, $a(0) + b(0) < c$ must be a true statement, so $0 < c$. **29.** Sample answer: $y < -x + 1$ **31.** Sample answer: The inequality $y > 10x + 45$ represents the cost of a monthly smartphone data plan with a one-time fee of \$45, plus \$10 per GB of data used. Both the domain and range are nonnegative real numbers because the GB used, and the total cost cannot be negative.

Module 6 Review

1. A

3. 8 rows

5. C

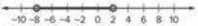
7. $\{t \mid t < -3\}$

9. A, B

11. $\{g \mid -5 \leq g\}$

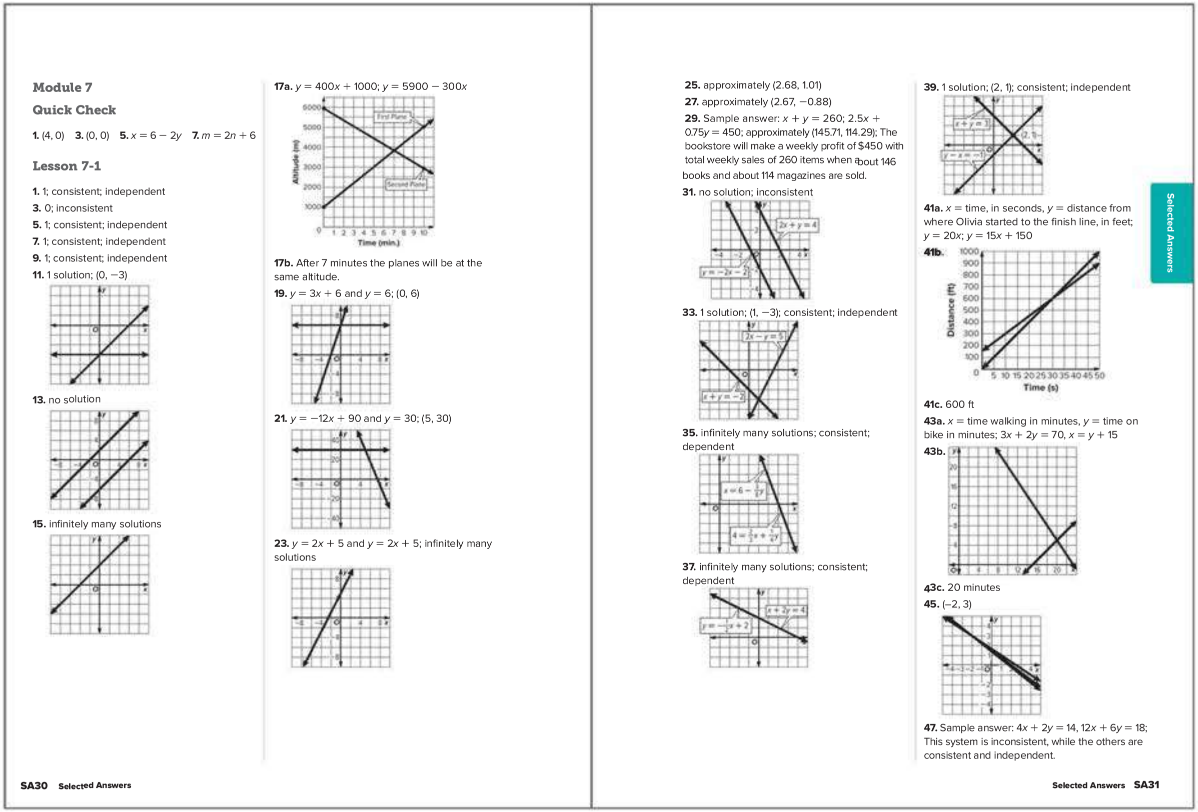
13A. $\{h \mid -8 < h < 2\}$

13B.



15. B

17. C, D, B, A



Module 7

Quick Check

1. (4, 0) **3.** (0, 0) **5.** $x = 6 - 2y$ **7.** $m = 2n + 6$

Lesson 7-1

1. 1; consistent; independent
3. 0; inconsistent
5. 1; consistent; independent
7. 1; consistent; independent
9. 1; consistent; independent
11. 1 solution; (0, −3)

13. no solution

15. infinitely many solutions

17a. $y = 400x + 1000$; $y = 5900 - 300x$

17b. After 7 minutes the planes will be at the same altitude.

19. $y = 3x + 6$ and $y = 6$; (0, 6)

21. $y = -12x + 90$ and $y = 30$; (5, 30)

23. $y = 2x + 5$ and $y = 2x + 5$; infinitely many solutions

25. approximately (2.68, 1.01)

27. approximately (2.67, −0.88)

29. Sample answer: $x + y = 260$; $2.5x + 0.75y = 450$; approximately (145.71, 114.29); The bookstore will make a weekly profit of $450 with total weekly sales of 260 items when about 146 books and about 114 magazines are sold.

31. no solution; inconsistent

33. 1 solution; (1, −3); consistent; independent

35. infinitely many solutions; consistent; dependent

37. infinitely many solutions; consistent; dependent

39. 1 solution; (2, 1); consistent; independent

41a. x = time, in seconds, y = distance from where Olivia started to the finish line, in feet; $y = 20x$; $y = 15x + 150$

41b.

41c. 600 ft

43a. x = time walking in minutes, y = time on bike in minutes; $3x + 2y = 70$, $x = y + 15$

43b.

43c. 20 minutes

45. (−2, 3)

47. Sample answer: $4x + 2y = 14$, $12x + 6y = 18$; This system is inconsistent, while the others are consistent and independent.

49. Graphing clearly shows whether a system of equations has one solution, no solution, or infinitely many solutions. However, finding the exact value of x and y from a graph can be difficult.

51. Francisca; If the item is less than $100, then $10 off is better. Of the item is more than $100, then the 10% is better.

Lesson 7-2

1. (1, 6) **3.** (29, 53) **5.** (1, 1) **7.** infinitely many

9. no solution **11.** (0, 1) **13.** (2, 5)

15. infinitely many

17a. Sample answer: $a + b = 5$; $0.7a + 0.2b = 0.65(5)$

17b. 4.5 mL from Beaker A and 0.5 mL from Beaker B

19. $\left(\frac{1}{2}, -\frac{3}{8}\right)$

21. Sample answer: In 2011, the population of Ecuador was about 15,180,000 and the population of Chile was about 17,150,000. The population of Ecuador increased by 1,210,000 and the population of Chile increased by 760,000 from 2011 to 2016. Let x = the number of 5-year periods and y = population. The system is $y = 15{,}180{,}000 + 1{,}210{,}000x$ and $y = 17{,}150{,}000 + 760{,}000x$. Solve by substitution to find that $x \approx 4.4$, or $4.4 \times 5 = 22$ years. So, the population of Ecuador and Chile will be equal in about $2011 + 22 = 2033$. (Source: World Bank)

23. Let x = tens digit and y = units digit of the original number; $10y + x = 10x + y - 45$; $x = 3y + 1$; (7, 2); The original number is 72.

25. Neither; Guillermo substituted incorrectly for b. Cara solved correctly for b, but misinterpreted the pounds of apples bought.

27. Sample answer: The solutions found by each of these methods should be the same. However, it may be necessary to estimate when using a graph. So, when a precise solution is needed, you should use substitution.

29. An equation containing a variable with a coefficient of 1 can easily be solved for the variable. That expression can then be substituted into the second equation for the variable.

Lesson 7-3

1. (−3, 4) **3.** (−3, 1) **5.** (4, −2) **7.** (8, −7)

9. (4, 7) **11.** (4, 1.5) **13.** (2, 1) **15.** (11, 0)

17. (−3, 7) **19.** (2, −1) **21.** (−3, −5)

23. (10, 4) **25.** (7, 5) **27.** (2, −3)

29. −2 and −4

31a. $r + s = 181$ and $r - s = 119$

31b. 31 state senators and 150 state representatives

33. (4, −1) **35.** $\left(-1, 3\frac{1}{3}\right)$

37. (−36, −4) **39.** 34 games

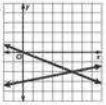
41a. Sample answer: $4p + 2n = 18.50$, $7p + 2n = 26.75$, where p is the price of a bag of popcorn and n is the price of a plate of nachos

41b. (2.75, 3.75); A bag of popcorn costs $2.75 and a plate of nachos costs $3.75.

43a. Add the equations because this will eliminate the variable y, and then you can solve for x.

43b. (5, −2)

43c.



Sample answer: The point of intersection on the graph will match the solution (5, −2).

43d. The equations would be equivalent. There would be infinitely many solutions, all real numbers x and y satisfying the equation $x - 5y = 15$.

43e. There would be no solution because the lines would be parallel and would never intersect.

45. Sample answer: $x + y = 1$ and $-x - y = 1$; This system of equations has no solutions.

47. Sample answer: $-x + y = 5$, I used the solution to create another equation with the coefficient of the x-term being opposite of its corresponding coefficient.

49. Sample answer: It would be most beneficial to use elimination to solve a system of equations when one variable has either the same coefficient in both equations or one variable has coefficients that are additive inverses in the equations.

Lesson 7-4

1. (−1, 3) **3.** (−3, 4) **5.** (−2, 3) **7.** (3, 5)

9. (1, −5) **11.** (0, 1)

13a. $2x + y = 592.30$ and $x + 2y = 691.31$, where x is the number of MLB games and y is the number of NBA games

13b. MLB: $164.43, NBA: $263.44

15. 8 and −1

17. wash: $6, vacuum: $2

19a.

	Tropical Breeze	Kona Cooler	Total
Amount of Juice (qt)	t	k	10
Amount of Pineapple Juice (qt)	$0.2t$	$0.5k$	4

19b. $\left(3\frac{1}{3}, 6\frac{2}{3}\right)$; The owner should mix $3\frac{1}{3}$ qt of Tropical Breeze and $6\frac{2}{3}$ qt of Kona Cooler.

19c. $3\frac{1}{3}$ qt + $6\frac{2}{3}$ qt = 10 qt, so the total amount is correct, and $0.2\left(3\frac{1}{3}\text{ qt}\right) + 0.5\left(6\frac{2}{3}\text{ qt}\right) = 4$ qt, so the amount of pineapple juice in the new drink is correct.

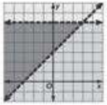
21. Jason; In order to eliminate the t-terms, you can multiply the second equation by 2 and then subtract, or multiply the equation by −2 and then add. Daniela did not subtract the equations correctly.

23. Sample answer: $2x + 3y = 6$ and $4x + 9y = 5$

25. Sample answer: It is more helpful to use substitution when one of the variables has a coefficient of 1 or if a coefficient can be reduced to 1 without turning other coefficients into fractions. Otherwise, elimination is more helpful because it will avoid the use of fractions when solving the system.

Lesson 7-5

1.



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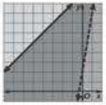
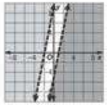
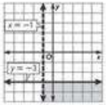


5. no solution



7.



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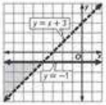
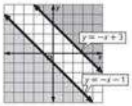
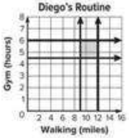


11. no solution



13a. Sample answer: Let x = hours at gym and y = miles of walking; $x \geq 9, x \leq 12, y \geq 4.5,$ $y \leq 6$

13b.



13c. Sample answers: gym 5 h, walk 9 mi; gym 6 h, walk 10 mi, gym 5.5 h, walk 11 mi

15. $y \leq x + 2, y \geq x - 3$

17. $y \geq x + 1, y < 1$

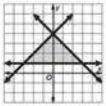
19. The solution set is the region where the graphs of the inequalities overlap. The point (2.5, 1) is not in the overlapping region, so it is not a solution. A solution must make all of the inequalities in the system true statements: $4x - 5y \geq 2 \rightarrow 4(2.5) - 5(1) \geq 2 \rightarrow 10 - 5 \geq 2 \rightarrow 5 \geq 2$; $2x + 3y > 8 \rightarrow 2(2.5) + 3(1) > 8 \rightarrow 5 + 3 > 8 \rightarrow 8 > 8$; The first inequality is true, but the second inequality is false. So, (2.5, 1) is not a solution.

21. Let x = tins of popcorn and y = tins of peanuts; $x + y \leq 200; x \geq y; 3x + 4y \leq 900;$ $x \geq 0$ and $y \geq 0$.

23. Sample answer: (3, 3)

25. Sometimes; sample answer: $y > 3, y < -3$ will have no solution, but $y < -3, y < 3$ will have solutions.

27. 9 units^2



Module 7 Review

1. B, C, D **3.** B

5. 4 wooden frames and 3 plastic frames

7. one solution; (−2, 7)

9. (10, 4)

11. A, B **13.** $r = 6, t = 5$ **15.** D **17.** C

Module 8

Quick Check

1. −196

3. 0.25

5. 32

7. −3

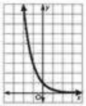
Lesson 8-1

1. No; the domain values are at regular intervals and the range values have a common difference 3.

3. Yes; the domain values are at regular intervals and the range values have a common factor 2.

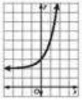
5. No; there is no common factor between the picture areas.

7.



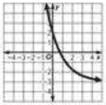
1; D = {all real numbers}, R = {y | $y > 0$}

9.



3; D = {all real numbers}, R = {y | $y > 2$}

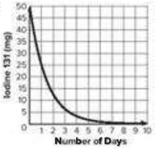
11.



2; D = {all real numbers}, R = {y | $y > -3$}

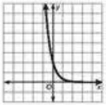
13a. y-intercept = 50; D = {all real numbers}, R = {y | $y > 0$}

13b.



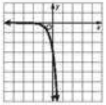
13c. Because time cannot be negative, the relevant domain is {x | $x \geq 0$}. Because the amount of Iodine 131 cannot be negative, and the amount when $x = 0$ is 50 mg, the relevant range is {y | $0 < y \leq 50$}.

9.



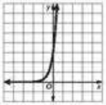
2; D = {all real numbers}, R = {y | $y > 0$}; $y = 0$

11.



−3; D = {all real numbers}, R = {y | $y < 0$}; $y = 0$

13.



3; D = {all real numbers}, R = {y | $y > 0$}; $y = 0$

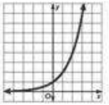
15a. 1038 millibars

15b. about 794 millibars

15c. It decreases.

17. $f(x) = 3(2^x)$

19. Sample answer: The number of teams competing in a basketball tournament can be represented by $y = 2^x$, where the number of teams competing is y and the number of rounds is x. The y-intercept of the graph is 1. The graph increases rapidly for $x > 0$. With an exponential model, each team that joins the tournament will play all of the other teams. If the scenario were modeled with a linear function, each team that joined would play a fixed number of teams.



Lesson 8-2

1. translated up 8 units **3.** compressed horizontally **5.** reflected across the x-axis; translated 1 unit right **7.** reflected across the y-axis; translated 4 units up **9.** stretched vertically **11.** translated right 3 units
13. $y = -2^x$ **15.** $y = 2^{-x} + 5$ **17.** stretched vertically by a factor of 2000 **19.** stretched vertically by a factor of 20 **21.** translated up 6 units **23.** reflected across the x-axis; compressed vertically **25.** reflected across the y-axis **27.** $g(x) = 2^x + 3$

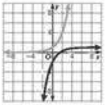
29. $g(x) = 5^{x-2}$ **31.** $g(x) = 6^x + 5$

33. $g(x) = \frac{1}{2}(4^x)$ **35.** $g(x) = 2^{3x}$

37. $g(x) = 5^x - 2$ **39.** $g(x) = 5^{x-4}$

41a. translated up 500 units **41b.** \$500

43. The graph has been reflected over the x-axis and reflected over the y-axis. It has been stretched vertically by a factor of 3 and shifted up 1 unit.



45. The graphs of these two exponential functions are the same. $f(x) = 4^{x+2} = 4^x \cdot 4^2 = 16 \cdot 4^x = g(x)$.

47. Jennifer is correct. Sample answer: As it is written, the function is multiplied by 2, which causes the graph to rise more rapidly than the parent graph, so Jennifer is correct. However, $g(x) = 2(2^x)$ is equivalent to $g(x) = 2^{x+1}$. This graph is the parent graph of $f(x) = 2^x$ shifted to the left one unit, but it still rises at the same rate.

49. The first pair; $g(x)$ is shifted right 3 units instead of left 3 units.

Lesson 8-3

1. $y = 4 \cdot 2^x$ **3.** $y = 10 \cdot 3^x$ **5.** $y = 3 \cdot 4^x$
7. $y = 3 \cdot 2^x$ **9.** $y = \left(\frac{1}{4}\right)^x$ **11.** $f(x) = 50 \cdot 2^x$, where x is the number of 30-minute time periods **13.** $f(x) = 43 \cdot (1.23)^x$, where x is the number of years since 2010.
15a. $P = 8{,}192{,}426(1.009)^t$ **15b.** about 9,370,872
17a. $Z = 60{,}000(0.90)^t$ **17b.** about \$31,886

19. \$2200 **21.** 360 million
23. $y = 2.6 \cdot 4^x$ **25.** about 77,529

27. Sample answer: The equation can be rewritten in the form $y = a(1 + r)^x$ to find the amount of original investment, a, and the rate of increase or decrease. Because $a = 2400$, he invested \$2400. Because $1 + r = 0.95$ and is less than 1, his investment is decreasing in value. A graphing calculator can be used to find that the investment will be worth \$1200 in about 13.5 years.

29a. $P(t) = 128(1.25)^t$

29b. an increase of approximately 41 deer per year **29c.** No; the amount of increase is exponential, not linear. **29d.** There is no common difference over equal intervals (differences are 32, 40 and 50). There is a common factor (factor is 1.25 in each case.)

31. about 9.2 years **33.** Sample answer: Exponential models can grow without bound, which is usually not the case for the situation that is being modeled. For instance, a population cannot grow without bound due to space and food constraints. Therefore, the situation that is being modeled should be carefully considered when used to make decisions.

35a. Sample answer: 5%; about 14.2 years

35b. Sample answer: 10%; about 6.6 years

35c. Sample answer: about 10.4 years; about \$8320

Lesson 8-4

1a. $A(t) = (1.021)^t$; $A(t) = (1.0052)^{4t}$

1b. Bank B has the better plan because the effective quarterly interest rate is 0.8%, which is greater than the quarterly interest rate of about 0.52% for Bank A.

1c. About 3.2%; sample answer: This confirms the result of part **b** because 3.2% is greater than the annual interest rate at Bank A, so Bank B has the better plan.

3. Bank A; Bank A has a quarterly interest rate of 0.95%. Bank B has a quarterly interest rate of about 0.92%. Bank A's quarterly interest rate is higher.

5. Species B; the population of Species A is decreasing at a rate of about 0.25% per quarter. The population of Species B is decreasing at a rate of about 0.4% per quarter. The population of Species B is decreasing at a faster rate.

7. Plan A

9. Account A; Account A has a semi-annual interest rate of 2.3%. Account B has a semi-annual interest rate of about 2.1%. Account A's semi-annual interest rate is greater.

11. Account A; Account A has a monthly interest rate of 0.5%. Account B has a monthly interest rate of about 0.21%. Account A's monthly interest rate is greater.

13. $T(t) = 72 + 140(0.67)^t$

15. Sample answer: Bank A offers a savings account with a 0.6% interest rate compounded quarterly. Bank B offers a savings account with a 2% interest rate compounded annually. Bank A offers the better interest rate because it has a higher effective annual interest rate of about 2.4%.

Lesson 8-5

1. The ratios are not the same, so the sequence is not geometric.

3. Since the ratio is the same for all of the terms, 5, the sequence is geometric.

5. The ratios are not the same, so the sequence is not geometric.

7. Because the ratio is the same for all of the terms, $\frac{1}{2}$, the sequence is geometric.

9. The ratios are not the same, so the sequence is not geometric.

11. The ratios are not the same, so the sequence is not geometric.

13. −250, 1250, −6250

15. 108, 324, 972

17. −2058; −14,406; −100,842

19. 54, 162, 486

21. $\frac{1}{10}, \frac{1}{20}, \frac{1}{40}$

23. $\frac{1}{3}, \frac{1}{18}, \frac{1}{108}$

25. 387,420,489

27. 177,147

29. $a_n = 4 \cdot \left(\frac{3}{2}\right)^{n-1}$

31. \$1310.72

33a. $a_n = P \cdot 1.005^n$

33b. \$538.84

35. $a_n = \frac{9}{16}\left(\frac{2}{3}\right)^{n-1}$; $\frac{4}{81}$

37. $a_n = -8\left(\frac{1}{4}\right)^{n-1}$; $-\frac{1}{2048}$

39. Sample answer: $a_n = 39{,}416(1.03)^{n-1}$; ≈ 69,116.19; This means that after 20 years of employment the average annual salary will be about $69,116.19.

41. −3, −12, −48

43a. The first method provides a starting salary of $100 and an $8 per month raise. The second method provides a starting salary of $0.01 and doubles it each month.

43b. The first situation is linear because there is a common difference of $8. The equation is $y = 8x + 92$. The second situation is exponential because it is a geometric sequence with a common ratio of 2. The equation is $y = 0.01(2)^{x-1}$.

43c. Sample answer: As long as I do not need money immediately, I would use the second method. In the last month, I would make $y = 0.01(2)^{23} = \$83{,}886.08$ due to the fact that the payment is growing exponentially. In the last month, in the first method I would make $y = 8(24) + 92 = \$284$.

45. If the values fit a geometric sequence, then $r = \sqrt{\frac{540}{180}} = \sqrt{3}$. This would mean that the interior angles of a square would have a sum of $180\sqrt{3} \approx 312°$. Since the sum of the angles in a square is 360°, this is not a geometric sequence.

47. Neither; Haro calculated the exponent incorrectly. Matthew did not calculate $(-2)^8$ correctly.

49. Sample answer: When graphed, the terms of a geometric sequence lie on a curve that can be represented by an exponential function. They are different in that the domain of a geometric sequence is the set of natural numbers, while the domain of an exponential function is all real numbers. Thus, geometric sequences are discrete, while exponential functions are continuous.

51. Sample answer: In the geometric sequence 6, 3, 1.5, ..., the value of r is 0.5 and the absolute value of a_{n+1} will be closer to zero than the value of a_n.

Lesson 8-6

1. 23, 30, 37, 44, 51

3. 8, 20, 50, 125, 312.5

5. 13, −29, 55, −113, 223

7. $a_1 = 12, a_n = a_{n-1} - 13, n \geq 2$

9. $a_1 = 2, a_n = a_{n-1} + 9, n \geq 2$

11. $a_1 = 40, a_n = -1.5a_{n-1}, n \geq 2$

13. $a_1 = 3, a_n = a_{n-1} - 1, n \geq 2$

15. $a_1 = 2, a_n = a_{n-1} + 1, n \geq 2$

17. $a_1 = \frac{5}{2}, a_n = a_{n-1} - \frac{1}{2}, n \geq 2$

19a. 875, 1050, 1225, 1400, 1575

19b. $a_1 = 175, a_n = a_{n-1} + 175, n \geq 2$

19c. $a_n = 175n$

21a. $a_1 = 6, a_n = 0.9a_{n-1}, n \geq 2$

21b. $a_n = 6(0.9)^{n-1}$

23. $a_n = -12n + 10$

25. $a_1 = 45, a_n = a_{n-1} - 7, n \geq 2$

27. $a_1 = -11, a_n = a_{n-1} + 5, n \geq 2$

29. $a_n = 16(4)^{n-1}$

31. $a_n = 500(1.05)^{n-1}$

33. Ramon has 2 parents, 4 grandparents, 8 great-grandparents, and so on. We can write a geometric sequence to count the number of ancestors in a given generation. The recursive formula is $a_1 = 2$, $a_n = 2_{n-1}$, $n >= 2$. The explicit formula is $a_n = 2^n$. Ramon's claim is about the 8th generation back: $a_8 = 2^8 = 256$. Ramon is correct.

35a. Sample answer: B3 = B2 + B1 and C2 = B2 ÷ B1

35b. The ratio approaches a constant value of 1.618034.... For larger values of n, the Fibonacci numbers behave like a geometric sequence with a common ratio of 1.618034....

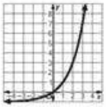
37. Both; Sample answer: The sequence can be written as the recursive formula $a_1 = 2$, $a_n = (-1)a_{n-1}, n \geq 2$. The sequence can also be written as the explicit formula $a_n = 2(-1)^{n-1}$.

39. False; sample answer: A recursive formula for the sequence 1, 2, 3, ... can be written as $a_1 = 1, a_n = a_{n-1} + 1, n \geq 2$ or as $a_1 = 1, a_2 = 2, a_n = a_{n-2} + 2, n \geq 3$.

41. Sample answer: In an explicit formula, the nth term a_n is given as a function of n. In a recursive formula, the nth term a_n is found by performing operations to one or more of the terms that precede it.

Module 8 Review

1.



3. As x increases, y increases; and, as x decreases, y approaches 0.

5. A

7. A

9. D

11. The Local Credit Union offers Joey the better savings plan; sample answer: the monthly interest rate is 0.12% higher than at First & Loan, and the annual interest rate is 1.6% higher than at First & Loan.

13. 146 people

15. $a_1 = 20, a_n = a_{n-1} + 15, n \geq 2$

17. row 2: 12; row 3: 12, 48; row 4: 48, 192

Module 9

Quick Check

1. 45.88 **3.** $3\frac{3}{20}$ **5.** 82.4% **7.** 85.6%

Lesson 9-1

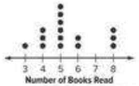
1. mean: 15.625, median: 15.5, mode: none
3. mean: 3.3, median: 2.5, mode: 2
5. mean: 5 students; median: 4 students; mode: 3 students **7.** mean: 54.75 mph; median: 54 mph; mode: 53 mph **9.** mean: about 2.8; median: 2.75; mode: 2 **11.** 25th percentile
13. Sample answer: The mean could be slightly higher because on a few of Saturday nights throughout the year, there were a very large number of people at the movies, which caused the mean to increase but did not affect the median. **15.** mean: 252; median: 245; mode: none **17.** 23 **19.** mean: 51.5, median: 51, mode: none **21.** 20 points **23.** 90th percentile **25a.** mean = 109,633; median = 66,556, no mode **25b.** Sample answer: The novels lower than the 50th percentile would be those consisting of words in the thirty-thousands and in the upper fifty-thousands. My prediction is correct because those three books are in the 47th percentile, which is just under the 50th percentile. **25c.** The median will change from 66,556 to 69,920, a difference of 3364 words. The mean will change from 109,633 to 111,065, a difference of 1432 words. **27.** Canada: 20th percentile; France: 50th percentile; Japan: 40th percentile; Russia: 60th percentile; Brazil: 10th percentile; Great Britain: 70th percentile

Olympic Medal Counts	
Country	Total Medals
Australia	29
Brazil	19
Canada	22
China	70
France	42
Great Britain	67
Japan	41
New Zealand	18
Russia	56
United States	121

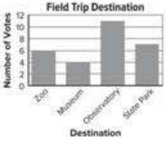
29. Sample answer: I can assume that the data is tightly clustered around 37 because all three measures of center are close. **31.** 75th percentile
33. Because the mean is an average of all the numbers in the data set, it is most affected by outliers. An outlier on the high end will cause the mean to increase. The median is the middle value in the dataset, adding one high number should not have much effect on the median unless the dataset has values, which are widely spread. The mode is the most frequent number so the outlier will have no effect on the mode unless the outlier is the same as the mode.
35. The mean, median, and mode will all be multiplied by the number. **37.** Julio should have chosen the mean because all the growth values are close together. **39.** Sample answer: To find a percentile rank, order the data set in decreasing order. Count the number of items below the item you are ranking, and divide that by the total number of items. Multiply this answer by 100 to arrive at the percentile rank.

Lesson 9-2

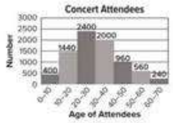
1. Summer Reading Program



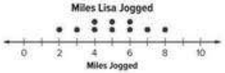
3.



5. histogram

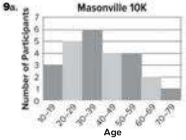


7a.



7b. 2 **7c.** 8

9a.



9b. 8 **9c.** 30-39
11. Sample answer: The scientist should break down the data into increments of two-tenths starting at 1 and going through 2.6.
13. Sample answer: 1) Because the data is clustered around ratings 7-10, it can be concluded that the product is well-liked by most customers and may have minor inconsistencies that certain people did not like. 2) Because there are only two low ratings, it can be concluded that dissatisfaction with the product could be a result of personal preference or manufacturer defect in a specific item.
15. Sample answer: If the range of the data is broad with specific, unrepeating values, then it makes the dot plot more meaningful if the range is divided up into equal intervals.
17. Sample answer: Bar graphs and histograms are similar because each displays data with bars. They are different because a bar graph is best used with data that are discrete and a histogram represents data that are continuous. For this reason, the bars in a bar graph do not touch and represent single values while the bars in a histogram touch and represent a range of values.

Lesson 9-3

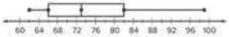
1. Sample answer: The intended population is all students. By asking only students leaving basketball practice, Awan is not getting a representative example of the entire student body. **3.** Sample answer: The first sentence states a positive outcome of music education, which may bias the respondent toward support. This bias may serve people trying to keep music education in schools. **5.** Mean: 4, median: 4, mode: 2; The mean and median are appropriate measures to use to accurately summarize the data. **7.** Sample answer: The scale for vendor 1 starts at 70, and because of the size of the bars, it looks like their sales doubled in one year, when they increased about 50%. Vendor 2 had a larger increase in sales of approximately 67%.
9. Sample answer: The required class would be better because it is more likely to contain a representative sample of students. The elective class might not be representative of the whole student body because these courses are chosen for reasons such as personal preference or future career aspirations.
11. Median; sample answer: The two lowest weights are much lower than the others, so the mean will be affected by those outliers.
13. Sample answer: The original data are very close together, so it is likely that the measures of center will all be the same or very close. Adding an outlier of 24 to the data set will cause the mean to go up, but the median and mode would likely stay unchanged or very close to the original number. So, in this case the median or mode would best represent the center of data.

15. Sample answer: To assess a sample for bias, identify the intended population and sample method; then, based on this information assess whether there is potential sample bias.   **21.** There are more extreme values in the lower end, which cause the mean to be lower than the median.

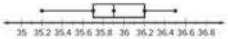
Lesson 9-4

1. 41   **3.** 62   **5.** 20

7. 62, 66, 73, 82, 99

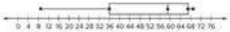


9. 35.2, 35.7, 35.9, 36.2, 36.5

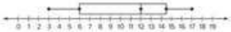


11. 25   **13.** 13   **15.** 20   **17.** 2.83   **19.** 2.97

21. 2.16; Since the standard deviation is large compared to the mean of 3, the number of goals scored each game is not relatively close to the mean.   **23a.** 9, 36, 59, 67, 69   **23b.**



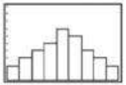
25. range: 14; minimum: 3; lower quartile: 6; median: 12; upper quartile: 14.5; maximum: 17; interquartile range: 8.5; standard deviation: 4.5



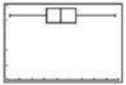
27. Both; sample answer; When an outlier is removed from a set of data, the spread and standard deviation of the data will decrease. When more values that are equal to the mean of a data set are added to the data set, the mean will be stronger and outliers will have less influence.

Lesson 9-5

1. symmetric

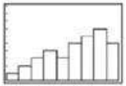


[24, 78] scl: 6 by [0, 10] scl: 1



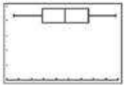
[24, 78] scl: 6 by [0, 5] scl: 1

3. Sample answer: The distribution is skewed, so use the five-number summary. The range is 53 – 12, or 41. The median is 39.5, and half of the data are between 28 and 48.



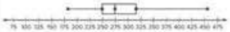
[10, 55] scl: 5 by [0, 10] scl: 1

5. Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean is about 58.7 with standard deviation of about 22.8.



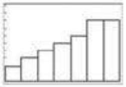
[10, 100] scl: 10 by [0, 5] scl: 1

7a.

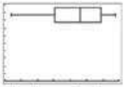


7b. min: 182, Q_1: 249, median: 274, Q_3: 315.5, max: 455   **7c.** The outlier mainly affects the mean. When the outlier is removed, the median decreases, but only $3 to $271. However, the mean changes from $289 to $266, which is more representative of the data as a whole.

9. negatively skewed

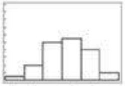


[10, 80] scl: 10 by [0, 10] scl: 1

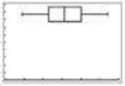


[10, 80] scl: 10 by [0, 10] scl: 1

11. symmetric

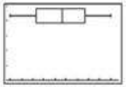


[10, 70] scl: 10 by [0, 20] scl: 2



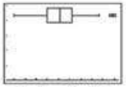
[10, 70] scl: 10 by [0, 20] scl: 2

13. symmetric



[35, 65] scl: 3 by [0, 5] scl: 1

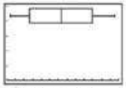
15. Sample answer: The distribution is approximately symmetric, so use the mean and standard deviation. The mean is about 54.7 years with standard deviation of about 6.2 years.



[40, 70] scl: 3 by [0, 5] scl: 1

17a. Sample answer: The distribution is skewed, so use the five-number summary. min: 62, max: 525, med: 103, Q1: 84, Q3: 290

17b. Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean is about 92.4 with standard deviation of about 18.4.



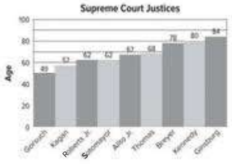
[60, 125] scl: 5 by [0, 5] scl: 1

17c. Original: mean 171.5, median 103; altered: mean about 92.4, median 92. The means differ by about 79.1, while the medians differ by 11.

19a. Sample answer: 225–230 g would be a reasonable advertised weight for either brand, so it is quite likely that they have the same advertised weight. Rafaello appears to have better control over the exact quantity in each package because its distribution is grouped more closely about the mean.   **19b.** Sample answer: Both distributions have an inverted, symmetric U-shape with “tails” on either side. Leonardo’s distribution is lower and wider.

21. Currently, Gerardo’s distribution would be positively skewed. If he lost his longest streaks, the data would represent a symmetric distribution.

23a. negatively skewed

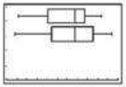


23b. The data is skewed, so use the five-number summary; min: 49, Q_1: 59.5, median: 67, Q_3: 79, max: 84. **23c.** There are no outliers in the data. **25.** Sample answer: A bimodal distribution is a distribution of data that is characterized by having data divided into two clusters, thus producing two modes and having two peaks. The distribution can be described by summarizing the center and spread of each cluster of data. **27.** Sample answer: In a symmetrical distribution, the majority of the data are located near the center of the distribution. The mean of the distribution is also located near the center of the distribution. Therefore, the mean and standard deviation should be used to describe the data. In a skewed distribution, the majority of the data lie either on the right or left side of the distribution. Because the distribution has a tail or may have outliers, the mean is pulled away from the majority of the data. The median is less affected. Therefore, the five-number summary should be used to describe the data.

Lesson 9-6

1. 60.9; 60; 60; 14; 4.7 **3.** 22.5; 21.5; no mode; 24; 7.4 **5.** 36.8; 38; 12; 56; 20.0 **7.** 26.8; 27.2; 29.6; 10.4; 3.5

9a. both negatively skewed



[45, 100] scl: 5 by [0, 5] scl: 1

9b. Sample answer: The distributions are skewed, so use the five-number summaries. The medians for both teams are 79. The upper quartile and maximum for the Marlins are 83.5 and 92. The upper quartile and maximum for the Cubs are 88 and 97. This means that the upper 50% of data for the Cubs is slightly higher than the upper 50% of data for the Marlins. Overall, we can conclude that the Cubs were slightly more successful than the Marlins during this time period. **11.** 93.5; 94.5; 97; 17; 5.1 **13.** 60; 62.5; 45; 50; 16.9 **15.** 25.9; 21.5; 17; 30; 10.3 **17.** 13.9; 14; 14.4; 1.4; 0.5 **19.** 75.8; 72; no mode; 48, 16.1 **21a.** 160.5; 166; no mode; 115; 33.9 **21b.** 216.5; 222; no mode; 115; 33.9 **23a.** 64.7, 66, 55, 46, 15.9 **23b.** 18.1, 18.9, 12.8, 25.6, 8.9

25a. Sample answer: The mean of Saeed's prices is \$11.79, which is \$0.80 more than his rival's mean price. The new prices come from subtracting \$0.80 from each price, which will reduce the mean price to be the same as his rival's.

New Prices				
14.19	3.69	9.19	17.69	12.19
6.19	7.69	21.19	12.69	13.19
9.19	10.19	11.69	3.69	12.19

25b. Current prices: $\mu = 11.79$, $\sigma = 4.60$
New prices: $\mu = 10.99$, $\sigma = 4.60$ The mean has dropped by 0.8, but the standard deviation has remained constant **27.** Sample answer: Male students, $\bar{x} = 70.0$ in., $\sigma = 2.0$ in. Female students, $\bar{x} = 66.3$ in., $\sigma = 2.7$ in. Sample answer: For male students, the mean is 70.0 in., and the standard deviation is 2.0 in. For female students, the mean is 66.3 in., and the standard deviation is 2.7 in. On average, males are taller. However, because the standard deviation of males is smaller than that of females, the heights of females are more spread out.

29. Sample answer: Histograms show the frequency of values occurring within set intervals. This makes the shape of the distribution easy to recognize. However, no specific values of the data set can be identified from looking at the histogram, and the overall spread of the data can be difficult to determine. The box plot show the data divided into four sections. This aids when comparing the spread of one set of data to another. However, the box plots are limited because they cannot display the data any more specifically than showing it divided into four sections. **31.** \$37,750
33. Sample answer: When two distributions are symmetric, determine how close the averages are and how spread out each set of data is. The mean and standard deviation are the best values to use for this comparison. When distributions are skewed, determine which direction the data is skewed and the degree to which the data is skewed. The mean and standard deviation cannot provide information in this regard, but get this information by comparing the range, quartiles, and medians found in the five-number summaries. So if one or both sets of data are skewed, it is best to compare their five-number summaries.

Lesson 9-7

1.

	Small	Large	Total
Cherry	35	20	55
Grape	25	15	40
Watermelon	15	15	30
Total	75	50	125

3. 30

5.

	Male	Female	Total
Spanish	22.5%	25%	47.5%
French	20%	15%	35%
German	7.5%	10%	17.5%
Total	50%	50%	100%

7. Sample answer: Most of the students are studying Spanish. **9.** Sample answer: Each conditional relative frequency represents the proportion of each candidate's support from each gender. **11.** 12

13.

	Male	Female	Total
Tree Swallow	5	7	12
Cardinal	5	10	15
Goldfinch	8	5	13
Total	18	22	40

15. 18

17.

	Sports or Clubs	No Sports or Clubs	Total
Freshmen	10%	12.5%	22.5%
Sophomores	12.5%	15%	27.5%
Juniors	10.6%	14.4%	25%
Seniors	11.9%	13.1%	25%
Total	45%	55%	100%

19. 55.6% **21.** 66 **23.** 100 **25.** 31 **27.** 38%

29.

Region	Apple	Sweet Potato	Pumpkin	Totals
West	77 ≈ 19.0%	4 ≈ 1.0%	13 ≈ 3.2%	94 ≈ 23.2%
Midwest	32 ≈ 7.9%	6 ≈ 1.5%	54 ≈ 13.3%	92 ≈ 22.7%
South	12 ≈ 3.0%	63 ≈ 15.6%	24 ≈ 5.9%	99 ≈ 24.4%
Northeast	92 ≈ 22.7%	2 ≈ 0.5%	26 ≈ 6.4%	120 ≈ 29.6%
Total	213 ≈ 52.6%	75 ≈ 18.5%	117 ≈ 28.9%	405 = 100%

31. Sample answer: The conditional relative frequencies based on pie preference give the probability of a person preferring a particular pie choice being from one of the U.S. regions. For example, there is an 84% probability that a person who prefers sweet potato pie is from the south.

Region	Apple	Sweet Potato	Pumpkin
West	36.2%	5.3%	11.1%
Midwest	15.0%	8.0%	46.2%
South	5.6%	84%	20.5%
Northeast	43.2%	2.7%	22.2%
Total	100%	100%	100%

33.

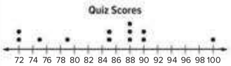
Vehicle Type	2WD	AWD	Totals
Hatchbacks	90	9	99
Sedans	60	13	73
SUVs	2	41	43
Total	152	63	215

35. Sample answer: Yes, there does appear to be an association. When the gasoline prices are higher, the distances traveled appear to be lower; when the gasoline prices are lower, the distances traveled appear to be higher.

37. Sample answer: A relative frequency is the ratio of the number in a category to the overall total of both categories. A conditional relative frequency is the ratio of the joint frequency to the marginal frequency. Therefore, it is important to understand what relationship is being analyzed because each two-way relative frequency table can provide two different conditional relative frequency tables.

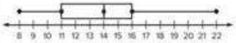
Module 9 Review

1.



3. C, D **5.** The data could be separated into intervals of 10, from 0–9, 10–19, 20–29, and so on through 70–79. **7.** scaled dot plot; histogram

9.

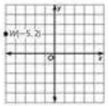


11. D **13.** C

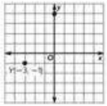
Module 10

Quick Check

1.



3.



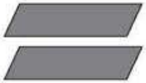
5. 20 **7.** 68

Lesson 10-1

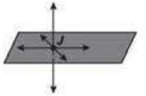
1. Sample answer: Kelsey's jersey number is greater than 5 and less than 11. Marie and Kelsey are on the same team. Kylie's team scored 26 points. **3.** Sample answer: Tom mulched the yard of all three clients this week. Ms. Martinez paid Tom $115 this week. Mr. Hansen paid Tom to mow his lawn and mulch his yard. Mrs. Johnson used all of Tom's services this week. **5.** analytic geometry **7.** synthetic geometry **9.** synthetic geometry **11.** Sample answer: Pedro and Rafael ate the same type of salad. **13.** Sample answer: Theo is likely doing analytic geometry, because he is using a graph with points. **15.** Sample answer: Because Sydney's plan is on a grid, she is likely using analytic geometry; that is, assuming the grid is used as a coordinate system. **17.** Sample answer: The three routes are not a model for the axiomatic system. Axioms 1, 2, and 4 are satisfied. Axiom 3 is not satisfied because Route 3 visits Stadium District twice and it is not the first/last stop. Axiom 5 is not satisfied because all three routes visit Stadium District. **19.** Sample answer: The rules of a game are like the axioms of an axiomatic system. They establish what can happen within the game. Plays are like theorems. They are tested against the rules or axioms to see whether they are legal in the game. In basketball, it is a rule that during playing time 5 players from each team shall be on the playing court. A play in which 6 players are on the court is a violation because the rules allow exactly 5 players.
21. Sample answer: The second figure does not satisfy all the axioms. The axioms do not specify that the line segments connecting the points need to be straight, so the first and third figures would work. **23.** Sample answer: The triangle on the coordinate grid does not belong because it illustrates analytic geometry, while the other two figures illustrate synthetic geometry.

Lesson 10-2

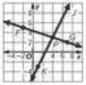
1. Sample answer: n and q **3.** plane R
5. Sample answer: point P **7.** Yes; sample answer: Line n intersects line q when the lines are extended. **9.** plane **11.** plane **13.** point on a line **15.** line **17.** plane **19.** plane
21. Sample answer:



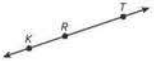
23. Sample answer:



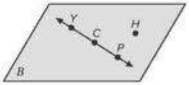
25. 5 **27.** *A*, *B*, *E*, *F* or *B*, *C*, *D*, *E* or *A*, *C*, *D*, *F*
29. $\overleftrightarrow{AB}$, $\overleftrightarrow{AG}$, $\overleftrightarrow{AH}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{BH}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{DH}$, $\overleftrightarrow{EF}$, $\overleftrightarrow{FG}$, $\overleftrightarrow{FH}$ **31a.** The lines intersect at the vanishing points. **31b.** Sample answer: The walls of the building and the ground form planes.
33. Sample answer:



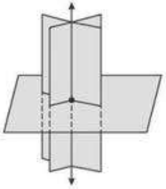
35. Sample answer: line *q* **37.** $\overleftrightarrow{CD}$ or $\overleftrightarrow{DC}$
39. Sample answer:



41. Sample answer:



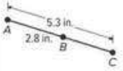
43. lines perpendicular to a plane
45. Sample answer:



47. Sample answer: Hiroshi is correct. After you draw the line from the first point to the other three, one of the lines from the second point is already drawn. **49.** Sample answer: A table is a finite plane. It is not possible to have a real-life object that is an infinite plane because all real-life objects have boundaries.

Lesson 10-3

1. 2.1 mm **3.** 1.1 cm **5.** 2.0 m **7.** $2\frac{1}{4}$ in. **9.** 5.3 mm **11.** $b = 12.5$; $YZ = 100$ **13.** $c = 1.7$; $YZ = 3.4$ **15.** $w = 4$; $YZ = 24$ **17.** $n = 4$; $YZ = 1$ **19.** $k = 6$; $YZ = 46$ **21.** $x = 6$; $YZ = 18$ **23.** $x = 10$; $YZ = 60$ **25.** 13 in. and 65 in. **27a.** 6 mi **27b.** Sample answer: I assumed the three locations were in a straight line. **29.** 4.4 mm **31.** 10.8 in. **33.** 66 units
35. 3 **37.** 4
39. Sample answer: $AP + PM = AM$
41. 5184 ft **43.** $x = 3$; 13 mi **45.** 40 ft
47. $JK = 12$, $KL = 16$ **49.** $x = 3$; $y = 4$
51. Sample answer: $2.8 + BC = 5.3$; $BC = 2.5$ in.



Lesson 10-4

1. 5 **3.** 9 **5.** 12 **7.** 3 **9.** 3 **11.** 9 **13.** 6
15. yes **17.** no **19.** no **21.** 10 units
23. $\sqrt{89}$ or about 9.4 units
25. $\sqrt{20}$ or about 4.5 units **27.** $\sqrt{20}$ or approximately 4.5 units **29.** Yes; sample answer: The distance between Mariah's house and the library is $\sqrt{74}$ or about 8.6 miles. Because $\frac{2}{3}$ of 12 miles is 8 miles, Mariah's bike ride is more than $\frac{2}{3}$ of the cycling portion of the triathlon. **31.** $\sqrt{37}$ or about 6.1 units
33. $\sqrt{29}$ or about 5.4 units **35.** $\sqrt{13}$ or about 3.6 units **37.** $\sqrt{89}$ or about 9.4 units
39. $3\sqrt{37}$ or about 18.2 units **41.** $\sqrt{52}$ or about 7.2 units **43.** no **45.** (0, −4), (0, 10)
47. 10 in. **49.** No; sample answer: We know that $QU + UR = QR = 4$ and $QU = UR$, so $QU = 2$. Further, we know that $RV + VS = RS = 2$, and $RV = VS$, so $RV = 1$. Because *QU* is not equal to *RV*, we know that $\overline{QU}$ is not congruent to $\overline{RV}$. **51.** (5, 10)

53. Sample answer: Substitute 10 for *d*, (1, 3) for (x_1, y_1), and $(9, y)$ for (x_2, y_2) in the Distance Formula: $10 = \sqrt{(9-1)^2 + (y-3)^2}$. Solve for *y*:

$$
\begin{aligned}
100 &= (9-1)^2 + (y-3)^2\\
&= 8^2 + (y-3)^2\\
&= 64 + (y-3)^2\\
36 &= (y-3)^2\\
6 &= y-3 \text{ or } -6 = y-3\\
9 &= y \text{ or } -3 = y
\end{aligned}
$$

So, the *y*-coordinate of point *B* is 9 or −3.

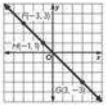
Lesson 10-5

1. 6 **3.** 9 **5.** 8.4 **7.** −1 **9.** −5.5 **11.** −4
13. −1 **15.** −2 **17.** *Y* **19.** −4 **21.** −3
23a. 4.36 mi **23b.** 10 mi **25.** 720 mi
27. $2\frac{2}{5}$ in. **29.** Sample answer: Draw $\overline{AB}$. Next, draw a construction line and place point *C* on it. From *C*, strike 6 arcs in succession of length *AB*. On the sixth segment of length *AB*, perform a segment bisector two times to create a $\frac{1}{4}AB$ length. Label the endpoint *D*. **31.** Sometimes; sample answer: If the coordinate of *X* is 0 and the coordinate of *Y* is negative, then the coordinate of *W* will be negative and less than the coordinate of *X*. If the coordinate of *X* is positive and the coordinate of *Y* is greater than the coordinate of *X*, then the coordinate of *W* will be greater than the coordinate of *X*.

Lesson 10-6

1. (−3.6, −2.2) **3.** $\left(1, 1\frac{2}{3}\right)$ **5.** $\left(\frac{14}{3}, 1\right)$
7. $\left(-\frac{7}{5}, 4\right)$ **9.** $\left(\frac{16}{7}, -3\right)$ **11.** $\left(1, \frac{5}{4}\right)$
13. $\left(\frac{20}{7}, -1\right)$ **15.** (−7, 11) and (−1, 1)
17. (−3, −2) **19.** $\frac{3}{4}$ **21a.** Julianne substituted the wrong values for (x_1, y_1) and (x_2, y_2).
21b. (1.6, 4.2) **23.** Sample answer: Because the distance from *A* to *P* is twice the distance from *P* to *D*, the distance from *A* to *P* could be 2 and the distance from *P* to *D* could be 1. Therefore, the fractional distance that *P* is from *A* to *D* is $\frac{2}{2+1}$ or $\frac{2}{3}$. The coordinates of point *P* are (5, 10).

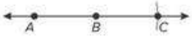
25. Sample answer:



H is $\frac{1}{3}$ of the distance from *F* to *G*.

Lesson 10-7

1. −2 **3.** 0.5 **5.** 1.5 **7.** 3 **9.** −4.5 **11.** 8.5
13. 9 **15.** 3 **17.** 9 yd **19.** (4, 6) **21.** (−3, −3)
23. (−1.5, 3.5) **25.** (5, 4) **27.** (18.5, 5.5)
29. (−6.5, −3) **31.** (4.2, 10.4) **33.** *A*(1, 6)
35. *C*(16, −4) **37.** *C*(−12, 13.25) **39.** 16
41. 2.5 **43.** 8 **45.** 7 **47.** 0.5 **49.** *Q*(8, 4)
51. *N*(−5, 1) **53.** (1, 1) **55.** (6, 7.5) **57.** 48 m
59. Sample answer: The midpoint of a segment is the average of the coordinates of the endpoints. Divide each coordinate of the endpoint that is not located at the origin by 2. For example, if the segment has coordinates (0, 0) and (−10, 6), the midpoint is located at $\left(-\frac{10}{2}, \frac{6}{2}\right)$ or (−5, 3). Using the Midpoint Formula, if the endpoints of the segment are (0, 0) and (*a*, *b*), the midpoint is $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{a}{2}, \frac{b}{2}\right)$.
61. Sample answer:



Module 10 Review

1. B, D
3. intersecting planes
5. 11
7. A
9. B
11. D
13. (3, 6)
15. B
17. C

Module 11

Quick Check

1. 63 **3.** 3 **5.** 1980 **7.** 344

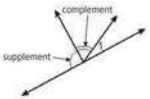
Lesson 11-1

1. *A* **3.** $\angle ADC$, $\angle CDA$ **5.** 66° **7.** 78° **9.** 61° **11.** 56° **13.** Sample answer: $\angle SRQ$ and $\angle TRP$ **15.** 65 **17.** $x = 35$, $y = 85$ **19.** *R* **21.** *P* **23.** $\overrightarrow{NM}$ and $\overrightarrow{NV}$ **25.** $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ or $\overrightarrow{RT}$ and $\overrightarrow{RQ}$ **27.** $\angle TPQ$ **29.** $\angle TPN$, $\angle NPT$, $\angle TPM$, $\angle MPT$ **31.** *S*, *Q* **33.** Sample answer: $\angle MPR$, $\angle PRQ$ **35.** $x = 48$, $y = 21$ **37.** Sample answer: $\angle HGE$, $\angle DGE$ **39.** Sample answer: $\angle BFC$, $\angle BFD$ **41.** Sample answer: $\angle STV$, $\angle VTW$, $\angle UTW$ **43a.** yes **43b.** no **43c.** no **43d.** no **45.** 45° **47a.** 45° **47b.** $\overrightarrow{EF}$ **49.** 168; sample answer: If $m\angle RMP = 21°$ and $\overrightarrow{MR}$ bisects $\angle QMP$, then $m\angle QMP = 2(21)$ or 42°. If $m\angle QMP = 42°$ and $\overrightarrow{MQ}$ bisects $\angle LMP$, then $m\angle LMP = 2(42)$ or 84°. If $m\angle LMP = 84°$ and $\overrightarrow{MP}$ bisects $\angle LMN$, then $m\angle LMN = 2(84) = 168°$.

Lesson 11-2

1. 72.5°, 107.5° **3.** 128°; 52° **5.** 45°; 135° **7.** $m\angle ABD = 47°$; $m\angle DBC = 43°$ **9.** $a = 20$ **11.** Yes; because $\angle 7$ is a right angle, $\angle 6$ and $\angle 8$ must form a right angle. **13.** Yes; the angles are nonadjacent and are formed by two intersecting lines. **15.** 40° **17.** 7 **19.** 92° **21.** No; the measures of the angles are unknown. **23.** No; the angles are not adjacent. **25.** $x = 25$ **27.** $x = 94$, $y = 79$, $z = 26$ **29.** Yes; sample answer: Angles that are right or obtuse do not have complements because their measures are greater than or equal to 90°. **31.** Sample answer: You can determine whether an angle is right if it is marked with a right angle symbol, if the angle is a vertical pair with a right angle, or if the angle forms a linear pair with a right angle.

33. Sample answer:



35. No; sample answer: Straight angles or angles that are greater than 180° do not have supplementary angles because their measures are greater than or equal to 180°.

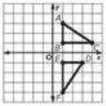
Lesson 11-3

1. 20.9 cm; 16 cm^2 **3.** 17.8 cm; 25.1 cm^2 **5.** 11.3 cm; 8 cm^2 **7.** 15.7 in.; 19.6 in^2 **9.** 42.1 ft; 92.5 ft^2 **11.** 35.1 m^2 **13a.** 398 m **13b.** In part a, I assumed that there was no space between the field and the first lane of the track. I also assumed the athlete's body was centered on the border of the track. **13c.** 402 m **13d.** 30 cm **15.** quadrilateral; 20 units; 24 units^2 **17.** quadrilateral; 20 units; 36 units^2 **19.** \$650; sample answer: The side of the play area that is adjacent to the house does not need fencing. The remaining three sides of the play area on the grid have lengths 4, 5, and 4 units. The perimeter of the play area to be fenced on the grid is $P = 4 + 5 + 4 = 13$ units. Each unit on the grid represents 5 feet, so Derek will need 13(5 ft) or 65 ft of fencing. The cost of the fencing is \$10 per foot, so the total cost will be 65(\$10) = \$650. **21.** 78.5 square miles **23a.** $s = 4$, so $A = 4^2 = 16$ units^2 **23b.** $b = 2$, $h = \sqrt{3}$, so $A = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3}$ units^2 **23c.** $16 - \sqrt{3}$ or about 14.3 units^2; sample answer: The area not covered by the triangle is equal to the area of the square minus the area of the triangle. So, $A = (16 - \sqrt{3})$ units^2 or about 14.3 units^2. **25.** 290.93 units^2 **27.** 39 units; Sample answer: An equilateral triangle has congruent side lengths. Use the Distance Formula to find $KM = \sqrt{(10 - (-2))^2 + (6 - 1)^2} = 13$. So, $P = 3(KM) = 3(13)$ or 39 units.

Lesson 11-4

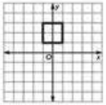
1. reflection **3.** rotation **5.** translation **7.** rotation **9.** $A'(2, 0)$, $B'(-1, -5)$, and $C'(4, -3)$ **11.** $A'(2, 2)$, $B'(-1, 7)$, and $C'(4, 5)$ **13.** $A'(-2, 0)$, $B'(1, -5)$, and $C'(-4, -3)$ **15.** $D'(4, 1)$, $E'(5, -2)$, and $F'(1, -2)$ **17.** $D'(5, -1)$, $E'(6, 2)$, and $F'(2, 2)$ **19.** $D'(-4, 1)$, $E'(-5, -2)$, and $F'(-1, -2)$ **21.** translation along vector $\langle -10, 35\rangle$ **23a.** rotation **23b.** translation **25.** reflection; translation

27.



$D(3, -1)$, $E(1, -1)$, $F(1, -4)$

29. reflection in the *x*-axis **31.** translation 3 units right and 2 units down **33.** Antwan; sample answer: When you reflect a point across the *x*-axis, the reflected point is in the same place horizontally, but not vertically. When (2, 3) is reflected across the *x*-axis, the coordinates of the reflected point are (2, −3) because it is in the same location horizontally, but the other side of the *x*-axis vertically. **35.** Sample answer:

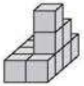


Lesson 11-5

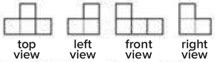
1. not a polyhedron; cone **3.** polyhedron; rectangular pyramid; base *WXYZ*; faces $\square WXYZ$, $\triangle VWX$, $\triangle VXY$, $\triangle VYZ$, $\triangle VZW$; edges $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, $\overline{ZW}$, $\overline{WV}$, $\overline{XV}$, $\overline{YV}$, $\overline{ZV}$; vertices *W*, *X*, *Y*, *Z*, *V* **5.** sphere; not a polyhedron **7.** 26.9 cm^2; 7.7 cm^3 **9.** 800 ft^2; 1280 ft^3 **11.** 90π or about 282.7 yd^2; 100π or about 314.2 yd^3 **13a.** 35 ft^2 **13b.** 13 bags **15.** 2004 in^2 **17.** 3.9 cm **19.** 3 in. **21a.** 5027 ft^3 **21b.** 402 ft^3 **21c.** 5429 ft^3 **23a.** 0.012 m^3; $V = \frac{1}{3}Bh$, so $V = \frac{1}{3}(900)(40)$ or 12,000 cm^3. One cubic meter equals 1 million cubic centimeters, so the volume of one pyramid shaped lawn ornament is 0.012 m^3. **23b.** concrete: 28.5 kg; granite: 32.2 kg; marble 32.5 kg **23c.** If the volume of the lawn ornament stays the same, then the weight of the ornament increases as the density of the material used to make it increases. **25.** Neither; sample answer: The surface area is twice the sum of the areas of the top, front, and left side of the prism or $2(5 \cdot 3 + 5 \cdot 4 + 3 \cdot 4)$, which is 94 square inches. **27.** Sample answer: The cone and pyramid have nearly the same volumes. Cone: The area of the base is approximately 154 square centimeters, so $V \approx \frac{1}{3}(154)(28)$ or about 1437 cubic centimeters. The volume of the pyramid is greater by such a small amount that we can say the volumes are approximately equal. **29.** 27 mm^3

Lesson 11-6

1.



3.



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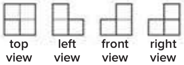
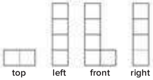
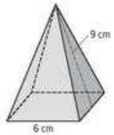


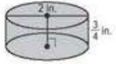
7.



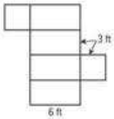
9. square pyramid, 144 cm^2



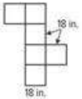
11. cylinder; 3.5π in^2 or 11.0 in^2



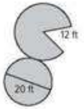
13. tetrahedron
15. octahedron
17. Sample answer:



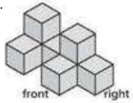
19. Sample answer:



21. Sample answer:



23.



25. Sample answer: ice cream cone

27. No; the area of this net is 48 square centimeters.

29. Julian; All of Julian's triangles are congruent. Caleb's top triangle will not match with the others when folded. **31.** Orthographic drawings and nets are both two-dimensional shapes used to describe three-dimensional figures. Orthographic drawings are views of the top, left, front, and right sides of an object, whereas nets can be folded to create a three-dimensional object.

Lesson 11-7

1. The sample is precise because there are consistently 17 or 18 rice cakes in each package. The sample indicates an innacurate claim of 20 rice cakes per package.

3. 0.05 milliamp **5.** 1.1°F **7a.** 16.5 yd **7b.** 12.5 yd **7c.** least possible area = 189.75 yd^2; greatest possible area = 218.75 yd^2 **7d.** The cost would be at least \$75.90 but less than \$87.50. **9.** 1 in. **11a.** 0.11 lb **11b.** 0.01 lb **11c.** 0.64 lb **13.** The cost would be at least \$955.94 but less than \$961.36. **15.** The cost would be at least \$107.81 but less than \$132.81. **17.** Accuracy is how well the information or data matches the true values. Precision is the repeatability of the measurement and level of measurement. Sample answer: 16.1 oz, 16.3 oz, 15.93 oz, 15.8 oz. **19a.** There are two faces that have an area of 111.4 in^2, two faces that have an area 35.82 in^2, and two faces that have an area 75.3 in^2. **19b.** 445.0 in^2 **19c.** The calculation of surface area is accurate to the nearest tenth. The true surface area falls between 444.95 in^2 and 445.05 in^2. **19d.** 440.1 in^2

Lesson 11-8

1. 5 **3.** 2 **5.** 1 **7.** Michelle **9.** 4.5 to 5.5 in. **11.** 2.5 to 3.5 cm **13.** 4.375 to 4.625 in. **15.** 2 **17.** 4 **19.** 2 **21.** 7.2 mL **23.** 15.1 cm^2 **25.** The area of the circle is between 31.2 and 33.2 cm^2. **27.** 4 **29.** 5 **31.** 5 **33.** 0.663 gram **35.** Scale 1: 4; Scale 2: 3; Scale 3: 4 **37.** 79.40 yd^2 **39.** 8.3 g **41.** 624 in^3

43. Sample answer: The 0 in each dimension, 1.40 cm and 1.60 cm, is significant. The answer should be given with 3 significant figures as 2.24 square centimeters. **45.** Yes; sample answer: The zeros before and after the decimal are not significant because a nonzero number did not come before them. Therefore, both numbers have two significant figures. **47.** Sometimes; sample answer: A zero between two nonzero significant figures is always significant, a leading zero is never significant, and a zero at the end of a number is only significant when a decimal point is given in the number.

Module 11 Review

1. A, C **3.** Sample answer: Use paddy paper or wax paper. Draw an angle on the paper. Fold the paper so that one side of the angle is directly on top of the other side. Draw the angle bisector in the crease of the fold. **5.** B **7.** Perimeter: 20.6 units; Area: 24.2 square units **9.** B **11.** C **13.** 2144.66 in^3 **15.** B

Module 12

Quick Check

1. 5 **3.** 2.3 **5.** $\angle BXD$, $\angle AXE$ **7.** 6

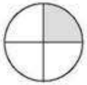
Lesson 12-1

1. Each term in the pattern is four more than the previous term; 24.

3. Each term is one half the previous term; $\frac{1}{16}$.

5. Each arrival time is 2 hours and 30 minutes prior to the previous arrival time; 7:30 A.M.

7. The shaded section in each circle has moved one section counterclockwise from its location in the previous circle.



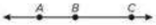
9. The product is an odd number.

11. They are equal. **13.** The lines are perpendicular.

15. $NP = PQ$ **17.** 180 cm

19. False; sample answer: Suppose $x = 2$, then $-x = -2$.

21. false;



23. False; sample answer: The length could be 4 m, and the width could be 5 m.

25. Yes; sample answer: If no team got more than 5 medals, then the total number of medals could not be more than 5 × 6 or 30 medals.

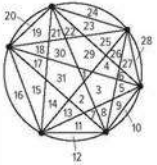
27a. 1, 3, 5 **27b.** You get all the odd numbers.

27c. Their sum equals n^2.

29. Sample answer: When $n = 4$, $2^n - 1 = 15$ and $15 = 3 \times 5$.

31a. Sample answer: The number of regions doubles when you add a point on the circle.

31b. For six points, there should be 32 regions, however only 31 regions are formed. The conjecture is false.



33. False; sample answer: If the two points create a straight angle that includes the third point, then the conjecture is true. If the two points do not create a straight angle with the third point, then the conjecture is false.

35. Sample answer: A postulate states that a line contains at least two points. These two points and all the points between them are line segments, by the definition of a line segment.

Lesson 12-2

1. $-3 - 2 = -5$, and vertical angles are congruent; p is true, and q is true, so p and q is true.

3. Vertical angles are congruent, or $2 + 8 \leq 10$; q is true, and $\sim r$ is true, so $q \vee \sim r$ is true.

5. $-3 - 2 \neq -5$, and not all vertical angles are congruent; $\sim p$ is false, and $\sim q$ is false, so $\sim p \wedge \sim q$ is false. **7.** H: there is no struggle; C: there is no progress **9.** H: you lead; C: I will follow **11.** H: two angles are vertical; C: they are congruent **13.** H: you were at the party; C: you received a gift; If you were at the party, then you received a gift. **15.** H: a figure is a circle; C: the area is πr^2; If a figure is a circle, then the area is πr^2. **17.** H: an angle is right; C: the angle measures 90°; If an angle is right, then the angle measures 90°. **19.** Converse: If you can buy five raffle tickets, then you have five dollars. The converse is true. Inverse: If you do not have five dollars, then you cannot buy five raffle tickets. The inverse is true.

Contrapositive: If you cannot buy five raffle tickets, then you do not have five dollars. The contrapositive is true.

21. Converse: If you do not take this medicine, then you can drive. The converse is true. Inverse: If you are not driving, then you can take this medicine. The inverse is true. Contrapositive: If you take this medicine, then you are not driving. The contrapositive is true.

23. Conditional: If it is raining, then the game will be cancelled. The conditional is true. Converse: If the game is cancelled, then it is raining. Counterexample: The game could be cancelled, and it is not raining. The converse is false. Because the converse is false, a biconditional statement cannot be written.

25. Conditional: If a polygon has four sides, then it is a quadrilateral. Converse: If a polygon is a quadrilateral, then it has four sides. The conditional and the converse are true, so the biconditional is true. **27.** true **29.** false

31. yes **33.** nothing **35.** If a ray does not bisect an angle, then it does not divide the angle into two congruent angles.

37. If two angles are right angles, then they are congruent.

39a. Sample answer: If you are in Houston, then you are in Texas. **39b.** Sample answer: Converse: If you are in Texas, then you are in Houston. Inverse: If you are not in Houston, then you are not in Texas. Contrapositive: If you are not in Texas, then you are not in Houston. **39c.** Converse: false; Inverse: false; Contrapositive: true

41. There exists at least one student at Hammond High school that does not have a locker.

43. For every real number x, $x^2 \neq x$. **45.** There exists a real number that does not have a real square root.

47. Truth table with the following columns:

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Because column 5 is the same as column 8, the conditional is equivalent to its contrapositive. Because column 6 is the same as column 7, the converse and the inverse are equivalent.

49. Kiri; sample answer: When the hypothesis of a conditional is false, the conditional is always true.

Lesson 12-3

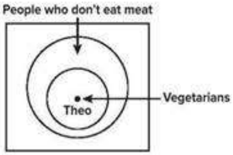
1. deductive **3.** deductive **5.** inductive

7. valid; Law of Detachment **9.** Invalid; your battery could be dead because it was old.

11. valid; Law of Detachment **13.** If Tina has a grade point average of 3.0 or greater, then she will have her name in the school paper.

15. If the measure of an angle is between 90° and 180°, then it is not acute. **17.** No valid conclusion; the conclusion of statement (1) is not the hypothesis of statement (2). **19.** The sum of the measures of $\angle 1$ and $\angle 2$ is 90°; Law of Detachment **21.** No valid conclusion; the conclusion of statement (1) is not the hypothesis of statement (2).

23. If Terryl completes a course with a grade of C, then he will have to take the course again; Law of Syllogism. **25.** Valid; Theo is inside the small and large circles, so the conclusion is valid.



27. then Mozart did not live in Vienna in the early 1800s. **29.** The child is at least 5 years old. **31.** Energy costs will be higher in Florida.

33. Law of Detachment: $[(p \to q) \wedge p] \to q$. Law of Syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$.

35. Sample answer: Jonah's statement can be restated as, "Jonah is in Group B, and Janeka is in Group B." For this compound statement to be true, both parts of the statement must be true. If Jonah was in Group A, he would not be able to say that he is in Group B, because students in Group A must always tell the truth. Therefore, the statement that Jonah is in Group B is true. For the compound statement to be false, the statement that Janeka is in Group B must be false. Therefore, Jonah is in Group B, and Janeka is in Group A.

37. Sample answer: Given: If you are at the Willis Tower, then you are in Chicago. If you are in Chicago, then you are in Illinois. Conclusion: Therefore, if you are at the Willis Tower, then you are in Illinois.

39. D; Statement D follows logically from statements (1) and (2). Statements A, B, and C do not follow logically from statements (1) and (2).

Lesson 12-4

1. The two planes meet at the edge, which lies on line t. Postulate: If two planes intersect, then their intersection is a line.

3. Postulate 3.1; Through any two points, there is exactly one line.

5. Always; Postulate 3.2 states that through any three noncollinear points, there is exactly one plane.

7. Sometimes; the points do not have to be collinear to lie in a plane.

9. Never; Postulate 3.7 states that if two planes intersect, then their intersection is a line.

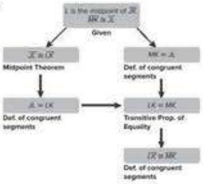
11. Statements (Reasons)

1. Y is the midpoint of $\overline{XZ}$. W is collinear with X, Y, and Z. Z is the midpoint of $\overline{YW}$. (Given)
2. $\overline{XY} \cong \overline{YZ}$ and $\overline{YZ} \cong \overline{ZW}$ (Midpoint Theorem)
3. $XY = YZ$ and $YZ = ZW$ (Definition of congruent segments)
4. $XY = ZW$ (Transitive Property of Equality)
5. $\overline{XY} \cong \overline{ZW}$ (Definition of congruent segments)

13. Statements (Reasons)

1. $SR = RT$ (Given)
2. R is the midpoint of $\overline{ST}$. (Definition of midpoint)
3. $SR = UR$ and $RT = RV$ (Given)
4. $SR = RT$, so $UR = RV$. (Substitution)
5. R is the midpoint of $\overline{UV}$. (Definition of midpoint)

15.


Given
Midpoint Theorem
Def. of congruent segments
Def. of congruent segments
Transitive Prop. of Equality
Def. of congruent segments

17. Given: B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.
Prove: $AB = CD$
Proof: Because B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$, we know by the Midpoint Theorem that $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CD}$. Because congruent segments have equal measures, $AB = BC$ and $BC = CD$. Thus, by the Transitive Property of Equality, $AB = CD$.

19. Postulate 3.3: A line contains at least two points. **21.** Iza is because a line can lie in a plane and intersect it in infinite points. **23.** 15 **25.** Ana is correct. Sample answer: The proof should begin with the given, which is that $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear. Therefore, Ana began the proof correctly. **27a.** Plane Q is perpendicular to plane P. **27b.** Line a is perpendicular to plane P. **29.** Sometimes; if the points were noncollinear, then there would be exactly one plane by Postulate 3.2 shown by Figure 1. If the points were collinear, then there would be infinitely many planes. Figure 2 shows what two planes through collinear points would look like. More planes would rotate around the three points.

Figure 1

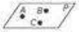
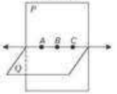


Figure 2



Lesson 12-5

1.

1. C is the midpoint of $\overline{AE}$.
   C is the midpoint of $\overline{BD}$.
   $\overline{AE} \cong \overline{BD}$
2. Definition of midpoint
3. Definition of $\cong$ segments
4. $AE = AC + CE$
   $BD = BC + CD$
5. Substitution Property
6. Substitution Property
7. $2AC = 2CD$
8. $AC = CD$
9. Definition of $\cong$ segments

3. Given: $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$
Prove: $\overline{VW} \cong \overline{VX}$
Proof
Statements (Reasons)

1. $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$ (Given)
2. $VZ = VY$ and $WY = XZ$ (Def. of $\cong$ segments)
3. $VZ = VX + XZ$ and $VY = VW + WY$ (Segment Addition Postulate)
4. $VX + XZ = VW + WY$ (Substitution Prop.)
5. $VX + WY = VW + WY$ (Substitution Prop.)
6. $VX = VW$ (Sub. Prop. of =)
7. $VW = VX$ (Symmetric Property)
8. $\overline{VW} \cong \overline{VX}$ (Def. of $\cong$ segments)

5. Clara is shorter than Chad when Maria is shorter than Luna; Clara and Chad are the same height when Maria is the same height as Luna.

7. No, it's not possible. Lola's house must be less than a mile from each house because she lives between them.

9a. Given: $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, $AC + CF + FE = GI + IL + LK$
Prove: $\overline{CF} \cong \overline{IL}$
Proof
Statements (Reasons)

1. $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, $AC + CF + FE = GI + IL + LK$ (Given)
2. $AC = GI$ and $FE = LK$ (Def. of $\cong$ segments)
3. $AC + CF + FE = AC + IL + LK$ (Substitution Property)
4. $AC - AC + CF + FE = AC - AC + IL + LK$ (Subtraction Property of Equality)
5. $CF + FE = IL + LK$ (Substitution Property)
6. $CF + FE = IL + FE$ (Substitution Property)
7. $CF + FE - FE = IL + FE - FE$ (Subtraction Property of Equality)
8. $CF = IL$ (Substitution Property)
9. $\overline{CF} \cong \overline{IL}$ (Def. of $\cong$ segments)

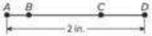
11a. Both segments are half the length of two congruent segments, so the lengths of the shorter segments must be the same.

11b.

1. $\overline{AB} \cong \overline{CD}$; M is the midpoint of $\overline{AB}$ and $\overline{CD}$.
2. Congruent segments have equal lengths.
3. Definition of midpoint
5. Segment Addition Postulate
6. Substitution Property of Equality
8. Substitution Property of Equality
9. $AM = CM$
10. $\overline{AM} \cong \overline{CM}$

13. Neither; because $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Transitive Property of Congruence.

15. Sample diagram:



17. No; The Segment Addition Postulate only applies to points that are collinear, but points P, Q, and R are not collinear.

19. Because $\overline{PQ} \cong \overline{RS}$ and congruent segments have equal lengths, $PQ = RS$. Because Q is the midpoint of $\overline{PR}$, $PQ = QR$. By the Substitution Property of Equality, $QR = RS$ so R is the midpoint of $\overline{QS}$.

Lesson 12-6

1. 113° **3.** 74° **5.** 46°

7. Statements (Reasons)

1. $\angle 1$ and $\angle 2$ form a linear pair.
   $\angle 3$ and $\angle 4$ form a linear pair. (Def. of linear pair)
2. $\angle 1$ and $\angle 2$ are supplementary.
   $\angle 3$ and $\angle 4$ are supplementary. (Supp. Thm)
3. $\angle 1 \cong \angle 3$ (Given)

4. $\angle 2 \cong \angle 4$ ($\cong$ Supp. Thm)

9. Statements (Reasons)

1. $m\angle ABC = m\angle DEF$ (Given)
2. $\angle ABC \cong \angle DEF$ (Def. of $\cong$ angles)
3. $\angle ABC$ and $\angle DEF$ are supplementary. (Given)
4. $\angle ABC$ and $\angle DEF$ are rt. angles. (If two $\angle$s are $\cong$ and supp., then each $\angle$ is a rt. $\angle$.)

11. 36°; 84°

13. $m\angle 6 = m\angle 8 = 73°$, $m\angle 7 = 107°$ ($\cong$ Supp. Thm. and Vert. $\angle$s Thm.)

15. $m\angle 3 = 62°$, $m\angle 1 = m\angle 4 = 45°$ ($\cong$ Comp. and Supp. Thm.)

17. $m\angle 9 = 156°$, $m\angle 10 = 24°$ ($\cong$ Supp. Thm.)

19. Statements (Reasons)

1. $\angle PQT$ and $\angle TQR$ form a linear pair. (Given)
2. $\angle PQR$ is a straight angle. (Given from figure)
3. $m\angle PQR = 180°$ (Def. of straight angle)
4. $m\angle PQT + m\angle TQR = m\angle PQR$ (Angle Add. Post.)
5. $m\angle PQT + m\angle TQR = 180°$ (Subs.)
6. $\angle PQT$ and $\angle TQR$ are supplementary. (Def. of supp. angles)

21. Statements (Reasons)

1. $\angle 1$ and $\angle 2$ are supplementary.
   $\angle 2$ and $\angle 3$ are supplementary. (Given)
2. $m\angle 1 + m\angle 2 = 180°$
   $m\angle 2 + m\angle 3 = 180°$ (Def. of supp. angles)
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ (Subs.)
4. $m\angle 1 = m\angle 3$ (Subtraction Prop. of =)
5. $\angle 1 \cong \angle 3$ (Def. of $\cong$ angles)

23. Statements (Reasons)

1. $\ell \perp m$ (Given)
2. $\angle 1$ is a right angle. (Def. of $\perp$)
3. $m\angle 1 = 90°$ (Def. of rt. angles)
4. $\angle 1 \cong \angle 4$ (Vert. Angles Thm)
5. $m\angle 1 = m\angle 4$ (Def. of $\cong$ angles)
6. $m\angle 4 = 90°$ (Subs.)
7. $\angle 1$ and $\angle 2$ form a linear pair.
   $\angle 3$ and $\angle 4$ form a linear pair. (Def. of linear pairs)
8. $m\angle 1 + m\angle 2 = 180°$, $m\angle 4 + m\angle 3 = 180°$ (Linear pairs are supp.)
9. $90° + m\angle 2 = 180°$, $90° + m\angle 3 = 180°$ (Subs.)
10. $m\angle 2 = 90°$, $m\angle 3 = 90°$ (Subtraction)
11. $\angle 2$, $\angle 3$, and $\angle 4$ are rt. angles. (Def. of rt. angles (Steps 6, 10))

25. Statements (Reasons)

1. $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary. (Given)
2. $m\angle 1 + m\angle 2 = 180°$ (Def. of supp. angles)
3. $m\angle 1 = m\angle 2$ (Def. of $\cong$ angles)
4. $m\angle 1 + m\angle 1 = 180°$ (Subs.)
5. $2(m\angle 1) = 180°$ (Subs.)
6. $m\angle 1 = 90°$ (Div. Prop.)
7. $m\angle 2 = 90°$ (Subs. (steps 3, 6))
8. $\angle 1$ and $\angle 2$ are rt. angles. (Def. of rt. angles)

27. By the Transitive Property, if any two angles are equal to the angle of the template, then they must be equal to each other.

29. Sample answer: $m\angle WXY = 90°$

Given: $m\angle WXZ = 45°$, $\angle WXZ \cong \angle YXZ$

Prove: $m\angle WXY = 90°$

Proof:

Statements (Reasons)

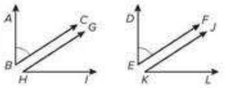
1. $m\angle WXZ = 45°$, $\angle WXZ \cong \angle YXZ$ (Given)
2. $m\angle WXZ = m\angle YXZ$ (Def. of $\cong \angle$s)
3. $m\angle YXZ = 45°$ (Substitution)
4. $m\angle WXY = m\angle WXZ + m\angle YXZ$ (Angle Add. Post.)
5. $m\angle WXY = 45° + 45°$ (Substitution)
6. $m\angle WXY = 90°$ (Substitution)

31. Each of these theorems uses the words *or to congruent angles* indicating that this case of the theorem must also be proved true. The first proof of each theorem only addressed the *to the same angle* case of the theorem.

Proof of the Congruent Complements Theorem (Case 2: Congruent Angles)

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC$, $\angle JKL$ is complementary to $\angle DEF$.

Prove: $\angle GHI \cong \angle JKL$



Proof:

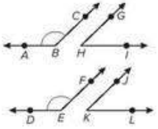
Statements (Reasons)

1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC$, $\angle JKL$ is complementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 90°$,
   $\angle DEF + \angle JKL = 90°$ (Def. of compl. angles)
3. $m\angle ABC = m\angle DEF$ (Def. of $\cong$ angles)
4. $m\angle ABC + m\angle JKL = 90°$(Subs.)
5. $90° = m\angle ABC + m\angle JKL$ (Symm. Prop.)
6. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
7. $m\angle ABC - m\angle ABC + m\angle GHI =$ $m\angle ABC - m\angle ABC + m\angle JKL$ (Subt. Prop.)
8. $m\angle GHI = m\angle JKL$ (Subs.)
9. $\angle GHI \cong \angle JKL$ (Def. of $\cong$ angles)

Proof of the Congruent Supplements Theorem (Case 2: Congruent Angles)

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is supplementary to $\angle ABC$, $\angle JKL$ is supplementary to $\angle DEF$.

Prove: $\angle GHI \cong \angle JKL$



Proof:

Statements (Reasons)

1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is suppl. to $\angle ABC$, $\angle JKL$ is suppl. to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 180°$,
   $m\angle DEF + m\angle JKL = 180°$ (Def. of suppl. angles)
3. $m\angle ABC = m\angle DEF$ (Def. of $\cong$ angles)
4. $m\angle ABC + m\angle JKL = 180°$ (Subs.)
5. $180° = m\angle ABC + m\angle JKL$ (Symm. Property)
6. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
7. $m\angle ABC - m\angle ABC + m\angle GHI =$ $m\angle ABC - m\angle ABC + m\angle JKL$ (Subt. Prop.)
8. $m\angle GHI = m\angle JKL$ (Subs.)
9. $\angle GHI \cong \angle JKL$ (Def. of $\cong$ angles)

Lesson 12-7

1. $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$ **3.** *ABCD* and *EFGH* or *ABFE* and *CDHG* **5.** $\overline{EF}$, $\overline{AB}$, and $\overline{DC}$

7. Sample answer: *ABCD* and *DCGH* could be characterized as perpendiculars, because *DCGH* contains segment $\overline{CG}$, which is perpendicular to *ABCD*. **9.** line d; alternate exterior **11.** line d; corresponding **13.** line a; consecutive interior **15.** line c; alternate interior **17.** line d; corresponding **19.** line p

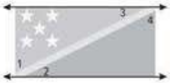
21. 100° **23.** 80° **25.** 100° **27.** 170°

29. $x = 28$, $y = 47$; Use supplementary angles to find x. Then use alternate exterior angles to find y. **31.** $x = 12$, $y = 31$ **33.** 105°

35. 105° **37.** 75° **39.** 94°

41. 64°; sample answer: Opposite sides of a rectangle are parallel. So, the top and bottom lines on the side panel are parallel and cut by a transversal, which is the dashed line. Therefore, $\angle 1$ and the 116°-angle are consecutive interior angles, so their sum is 180°. $m\angle 1 + 116° = 180°$, so $m\angle 1 = 64°$.

43. Sample answer:

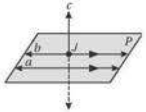


$\angle 1$ and $\angle 4$ are alternate interior angles. $\angle 2$ and $\angle 3$ are also alternate interior angles. $\angle 1$ and $\angle 2$ are complementary angles, and $\angle 3$ and $\angle 4$ are complementary angles.

45. By the Corresponding Angles Postulate, $\angle 1 \cong \angle 13$ and $\angle 13 \cong \angle 9$. By the Transitive Property, $\angle 1 \cong \angle 9$. So, $m\angle 1 = m\angle 9$. By the Corresponding Angles Postulate, $\angle 4 \cong \angle 8$ and $\angle 8 \cong \angle 12$. By the Transitive Property, $\angle 4 \cong \angle 12$. So, $m\angle 4 = m\angle 12$. It is given that $m\angle 1 - m\angle 4 = 25°$. By the Substitution Property, $m\angle 9 - m\angle 12 = 25°$.

47. By the Vertical Angles Theorem, $\angle 7 \cong \angle 5$. By the Corresponding Angles Theorem $\angle 5 \cong \angle 1$. By the Transitive Property, $\angle 1 \cong \angle 7$.

49.



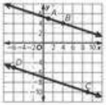
51. Sometimes; sample answer: $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{EF}$ depending on where the planes intersect.

53. $x = 171$ or $x = 155$; $y = 3$ or $y = 5$

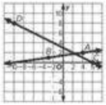
55. No; sample answer: From the definition of skew lines, the lines must not intersect and cannot be coplanar. Different planes cannot be coplanar, but they are always parallel or intersecting. Therefore, planes cannot be skew.

Lesson 12-8

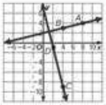
1. parallel



3. neither

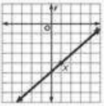


5. perpendicular

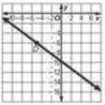


7. perpendicular **9.** parallel **11.** perpendicular
13. neither **15.** parallel

17.



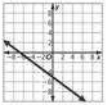
19.



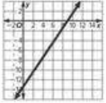
21a. $y = -\frac{2}{3}x + \frac{11}{3}$ **21b.** When the drummer crosses the x-axis, the y-coordinate will be 0, so solve $0 = -\frac{2}{3}x + \frac{11}{3}$ to find the x-coordinate; solving shows that $x = \frac{11}{2}$ or $5\frac{1}{2}$, so the x-coordinate will be greater than 5.

23. $y = -10$ **25.** $y = \frac{1}{5}x + \frac{12}{5}$

27. $y = -8$



29. $y = 0$



31. $y = -\frac{3}{2}x - 7$; $y = \frac{2}{3}x - \frac{8}{3}$

33. No; none of the slopes are equal, and no two of the slopes have a product of −1.

35a. 450 yd **35b.** −3; Ford Street and 6th Street are parallel, so they have the same slope. **35c.** Both have a slope of $\frac{1}{3}$ because both are perpendicular to Ford and 6th, and the slope of a perpendicular is given by the negative reciprocal. **35d.** 200 yd

37. $S\left(0, -5\frac{1}{2}\right)$; The slope of $\overline{QR}$ is $\frac{2-4}{3-(-2)} = -\frac{2}{5}$, so the slope of $\overline{RS}$ is $\frac{5}{2}$. Let the coordinates of S be $(0, y)$ because S must be on the y-axis. Solve $\frac{5}{2} = \frac{y-2}{0-3}$ for y. $y = -5\frac{1}{2}$, so the coordinates of S are $\left(0, -5\frac{1}{2}\right)$.

39a. $y - 1 = a(x - 5)$; the line must have slope a to be parallel to line p.

39b. $y = -\frac{1}{a}x + \frac{2}{a} + 3$; the line must have slope $-\frac{1}{a}$ to be perpendicular to line p.

41a. $B(2, 4)$ and $D(10, -4)$

41b. Sample answer: The slopes of $\overline{AB}$ and $\overline{DC}$ are undefined, so they are parallel to each other. The slopes of $\overline{AD}$ and $\overline{BC}$ are 0, so they are parallel to each other.

41c. Sample answer: Because the slope of $\overline{AB}$ is undefined and the slope of $\overline{BC}$ is zero, the lines are perpendicular to each other. Therefore, they form a right angle, which measures 90°. The same logic applies to all the sides.

43. Yes; the slope of the line through the points $(-2, 2)$ and $(2, 5)$ is $\frac{3}{4}$. The slope of the line through the points $(2, 5)$ and $(6, 8)$ is $\frac{3}{4}$. Because these lines have the same slope and have a point in common, their equations would be the same. Therefore, all the points are on the same line, and all the points are collinear.

45. Two nonvertical lines are parallel if and only if they have the same slope. Two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

Lesson 12-9

1. $a \parallel b$; Alternate Interior Angles Converse

3. $\ell \parallel m$; Alternate Exterior Angles Converse

5. $g \parallel h$; Converse of Corresponding Angles Thm.

7. 22 **9.** 6 **11.** 13 **13.** 20

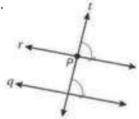
15. Parallelogram; sample answer: The top edges are perpendicular to the vertical line, so they are a single line. The bottom edge is also a single line and perpendicular to the same line as the top, so it is parallel to the top. The top edge is a transversal to the left and right slanted edges, and the angles are supplementary. So, the left and right edges are parallel.

17a. 108°; sample answer: To ensure that the horizontal part of the A is truly horizontal, it should be parallel to the dashed line. Therefore, ∠2 and the 108°-angle are alternate interior angles, and $m\angle 2 = 108°$. ∠1 and ∠2 are congruent angles, so $m\angle 1 = 108°$.

17b. Sample answer: One side of the A is longer than the other.

19. Sample answer: It is given that $\angle 1 \cong \angle 2$. Also, $\angle 1 \cong \angle 3$, because these are vertical angles. Therefore, $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence. This shows that $\ell \parallel m$ by the Converse of Corresponding Angles Theorem.

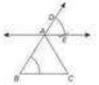
21.



23. Sample answer: Because the corners are right angles, each pair of opposite sides is perpendicular to the same line. Therefore, each pair of opposite sides is parallel.

25. Daniela is correct. ∠1 and ∠2 are alternate interior angles for $\overline{WX}$ and $\overline{YZ}$. So, if alternate interior angles are congruent, then the lines are parallel.

27a.



27b. Sample answer: Using a straightedge, the lines are equidistant. So, they are parallel.

27c. Sample answer: ∠ABC was copied to construct ∠DAE. So, $\angle ABC \cong \angle DAE$. ∠ABC and ∠DAE are corresponding angles, so by the Converse of the Corresponding Angles Theorem, $\overleftrightarrow{AE} \parallel \overleftrightarrow{BC}$.

29. Yes; sample answer: A pair of angles can be supplementary and congruent if the measure of both angles is 90°, because the sum of the angle measures would be 180°.

Lesson 12-10

1. $\sqrt{2}$ or about 1.41 units **3.** 6 units **5.** $\sqrt{10}$ or about 3.16 units

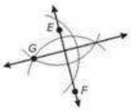
7. yes; 4.24 in.

9. 8 units **11.** $\sqrt{10}$ or about 3.16 units **13.** $3\sqrt{2}$ or about 4.24 units

15. $4\sqrt{17}$ or about 16.49 units **17.** $\sqrt{14.76}$ or about 3.84 units **19.** 0 units

21. Sample answer: Isaiah can measure the perpendicular distance between the wires in two different places. If the distances are equal, then the wires are parallel.

23.



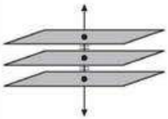
25. No; Sample answer: A path that is perpendicular to the tee box would be the shortest. The angle that the tee box makes with the path that Mark walked is less than 90°, so it is not the shortest possible path.

27. Sample answer: The point is on the line. The two lines are the same line.

29. Sample answer: First, a point on one of the parallel lines is found. Then the line that is perpendicular to the line through the point is found. Then the point of intersection is found between the perpendicular line and the other line that is not used in the first step. Last, the Distance Formula is used to determine the distance between the pair of intersection points. This value is the distance between the pair of parallel lines.

31. Sometimes; sample answer: The distance can only be found if the line is parallel to the plane.

33. If two planes are each equidistant from a third plane, then the two planes are parallel to each other.



Module 12 Review

1. 6

3. B

5. $x = 29$, $m\angle ABD = 66°$, $m\angle DBC = 24°$

7. A, D, E

9. Lines m and p

11. 102° **13.** A, B, C, D **15.** B

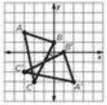
Module 13

Quick Check

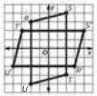
1. −14 **3.** −2 **5.** (3, 0) **7.** (1, −1)

Lesson 13-1

1. $A'(2, -3)$, $B'(1, 0)$, $C'(-3, -2)$

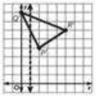


3. $R'(3, -2)$, $S'(4, 2)$, $T'(-3, 2)$, $U'(-4, -2)$



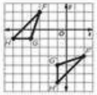
5. $S'(-7, 5)$

7.

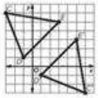


9. $W'(-7, 14)$

11.



13.



15. reflection in the line $y = -1$

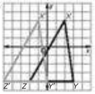
17. Sample answer: The M can be represented with the points (0, 0), (0, 3), (1, 1), (2, 0), and (2, 3). Reflecting in the line $y = x$ gives (0, 0), (3, 0), (1, 1), (0, 2), and (3, 2). **19.** $(-b, a)$

Lesson 13-2

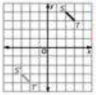
1. $\triangle J'K'L'$ is a translation of $\triangle JKL$. This translation vector can be represented as $\langle 2, 5\rangle$.

3. (4, 3), (5, 5), (6, 3)

5.



7.

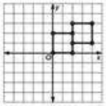


9. $Q'(2, -7)$ **11.** $B'(12, 2)$

13. No; sample answer: The size has been changed.

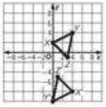
15. $(x, y) \rightarrow (x, y + 4)$

17. (0, 2), (2, 2), (0, 0), (2, 0); Sample answer: To minimize the length of the vector, I used the vertex closest to the origin, (2, 1), as the preimage for the point translated to the origin.

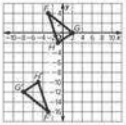


Lesson 13-3

1.



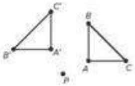
3.



5. home plate: (3, 23), first base: (3, 13), second base: (13, 13), third base: (13 , 23)

7. $K'(0, -10)$ **9.** $X'(-3, 13)$, $Y'(0, 12)$ **11.** 10 times

13a. Sample answer:



13b. Sample answer: 270° clockwise rotation about point P

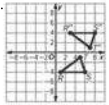
15. Sample answer: A rotation followed by another rotation is still a rotation. For example, a rotation of 30° clockwise followed by a rotation of 20° counterclockwise is the same as a rotation of 10° clockwise. A rotation of 30° counterclockwise followed by a rotation of 15° counterclockwise is the same as a rotation of 45° counterclockwise.

17. Sample answer: Distance is preserved because the lengths of segments remain the same measure. Angle measures are preserved because angle measures remain the same measure. Parallelism is preserved because parallel lines remain parallel. Collinearity is preserved because points remain on the same lines.

19. Yes; sample answer: A rotation is a transformation that maintains congruence of the original figure and its image. So, the preimage can be mapped onto the image, and corresponding segments will be congruent. Therefore, collinearity and betweenness of points are maintained in rotations.

Lesson 13-4

1.



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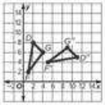

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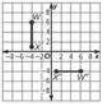

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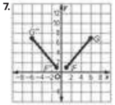
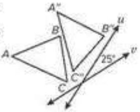
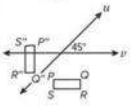


9. a 50° clockwise rotation about the point where lines u and v intersect



11. a 90° clockwise rotation about the point where lines u and v intersect

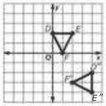


13. $\triangle JKL \cong \triangle MNP$; reflection in y-axis followed by translation along $\langle -1, 2\rangle$

15. $\triangle JKL \cong \triangle MNP$; translation along $\langle 2, 0\rangle$ followed by 180° rotation about the origin

17. Sample answer: Reflect the first two shapes vertically, and then translate to the right and repeat.

19.



21. $(3, -1)$, $(2, -5)$, $(-5, -2)$

23. $(1, 1)$, $(0, 5)$, $(-7, 2)$

25. Sometimes; sample answer: If the lines of reflection intersect, the composition is a rotation.

27. Sometimes; sample answer: This is true if the point is the origin.

29. Sample answer: Proof: It is given that a translation along $\langle a, b\rangle$ maps R to R' and S to S'. Using the definition of a translation, points R and S move the same distance in the same direction, therefore $\overline{RS} \cong \overline{R'S'}$. It is also given that a reflection in a maps R' to R'' and S' to S''. Using the definition of a reflection, points R and S are the same distance from line a, so $\overline{R'S'} \cong \overline{R''S''}$. By the Transitive Property of Congruence, $\overline{RS} \cong \overline{R''S''}$.

31. Sometimes; sample answer: The order of rotating by 180° about the origin and reflecting in the line $y = x$ does not change the location of the final image.

33. $P'''(1, -2)$, $Q'''(2, 1)$, $R'''(-1, 3)$, $S'''(-2, 0)$

Lesson 13-5

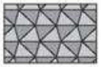
1. $x = \frac{180(5-2)}{5} = 108°$; Because 108° is not a factor of 360°, a regular pentagon will not tessellate the plane.

3. $x = \frac{180(9-2)}{9} = 140°$; Because 140° is not a factor of 360°, a regular 9-gon will not tessellate the plane.

5. yes; 2 regular hexagons, 2 equilateral triangles

7. tessellation; uniform, semi-regular

9. Yes; sample answer: reflection, rotation, translation

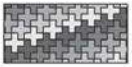


11. Never; sample answer: Each interior angle of a regular dodecagon is $\frac{180°(12-2)}{12} = 150°$. Because 150° is not a factor of 360°, a regular dodecagon will not tessellate the plane.

13. Never; sample answer: Each interior angle of a regular 15-gon is $\frac{180°(15-2)}{15} = 156°$. Because 156° is not a factor of 360°, a regular 15-gon will not tessellate the plane.

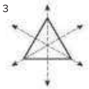
15. translation **17.** rotation **19.** Sample answer: Translations can be performed because the pieces slide. Rotations can be performed because each piece can be turned. Reflections cannot be performed because the back of a piece cannot be used to create the puzzle.

21.



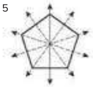
Lesson 13-6

1. yes; 3



3. no

5. yes; 5



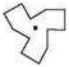
7a. Yes; the hubcap can map onto itself with a rotation that is less than 360°.

7b. Yes; the hubcap can map onto itself with a rotation that is less than 360°.

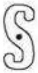
7c. Yes; the hubcap can map onto itself with a rotation that is less than 360°.

9. Yes; the symbol can map onto itself with a rotation that is less than 360°.

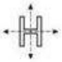
11. yes; 3; 120°



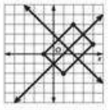
13. yes; 2; 180°



15.



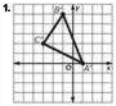
17. 2; 180° **19.** pentagon **21.** yes; order of symmetry: 3; magnitude of symmetry: 120° **23.** yes; order of symmetry: 6; magnitude of symmetry: 60°
25. Sample answer: A rectangular mirror with two lines of symmetry, one vertical and one horizontal, through the middle or a spoon with one line of symmetry down the middle.
27. 24; 360° ÷ 15° = 24, so the order of symmetry is 24. This means there are 24 sides.
29. Sample answer: (–1, 0), (2, 3), (4, 1), and (1, –2);



31. Sample answer: In both rotational and line symmetry a figure is mapped onto itself. However, in line symmetry the figure is mapped onto itself by a reflection, and in rotational symmetry a figure is mapped onto itself by a rotation. A figure can have line symmetry and rotational symmetry.

Module 13 Review

1.



3. (–9, 6)

5. Sample answer: The length of $\overline{AC}$ is not the same as the length of $\overline{EG}$.

7. C **9.** Quadrant II **11.** false **13.** A, D

15. C

17. order = 24; magnitude = 15°

Module 14

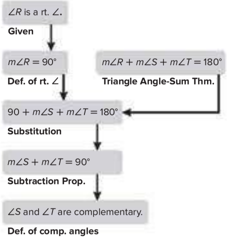
Quick Check

1. right **3.** obtuse **5.** 10.8 **7.** 18.0

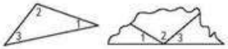
Lesson 14-1

1. $m\angle 1 = 30°$, $m\angle 2 = 60°$
3. $m\angle 1 = 109°$, $m\angle 2 = 29°$, $m\angle 3 = 71°$
5. 50° **7.** 98° **9.** 62° **11.** 26° **13.** 55°
15. $x = 20$; 40°, 60°, 80° **17.** $x = 11$; 80°, 117°
19.

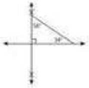
Proof:



21. $m\angle D = 27°$, $m\angle E = 81°$, $m\angle F = 72°$
23. $m\angle J = 18°$, $m\angle K = 72°$, $m\angle L = 90°$
25. $m\angle Z < 23°$; Sample answer: Because the sum of the measures of the angles of a triangle is 180° and $m\angle X = 157°$, $157° + m\angle Y + m\angle Z = 180°$, so $m\angle Y + m\angle Z = 23°$. If $m\angle Y$ was 0°, then $m\angle Z$ would equal 23°. But because an angle must have a measure greater than 0°, $m\angle Z$ must be less than 23°, so $m\angle Z < 23°$.
27. 90° **29.** $m\angle 1 = 55°$, $m\angle 2 = 75°$, $m\angle 3 = 55°$, $m\angle 4 = 15°$ **31.** Sample answer: Draw a triangle and then tear the corners off the triangle. Arrange the three corners so the angles are adjacent. The angles now form a straight angle. Because a straight angle measures 180°, the sum of the measures of the angles of a triangle is 180°.



33. Sample answer: Because an exterior angle is acute, the adjacent angle must be obtuse. Because another exterior angle is right, the adjacent angle must be right. A triangle cannot contain a right angle and an obtuse angle because the sum would be greater than 180°. Therefore, a triangle cannot have an obtuse, an acute, and a right exterior angle.
35. Sample answer: I found the measure of the second angle by subtracting the first angle from 90° because the acute angles of a right triangle are complementary.



Lesson 14-2

1. $\angle A \cong \angle D$; $\angle ABC \cong \angle DCB$; $\angle ACB \cong \angle DBC$; $\overline{AC} \cong \overline{DB}$; $\overline{AB} \cong \overline{DC}$; $\triangle ABC \cong \triangle DCB$

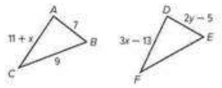
3. $\angle X \cong \angle A$; $\angle Y \cong \angle B$; $\angle Z \cong \angle C$; $\overline{XY} \cong \overline{AB}$; $\overline{XZ} \cong \overline{AC}$; $\overline{YZ} \cong \overline{BC}$; $\triangle XYZ \cong \triangle ABC$

5. $\angle R \cong \angle J$; $\angle T \cong \angle K$; $\angle S \cong \angle L$; $\overline{RT} \cong \overline{JK}$; $\overline{TS} \cong \overline{KL}$; $\overline{RS} \cong \overline{JL}$; $\triangle RTS \cong \triangle JKL$
7. 48 **9.** 5 **11.** 35 **13.** 41°
15. Proof:

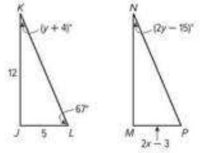
Statements (Reasons)
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$ (Given)
2. $\overline{BD} \cong \overline{BD}$ (Reflexive Prop. of Congruence)
3. $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$ (Given)
4. $\angle A \cong \angle C$ (Third Angles Theorem)
5. $\triangle ABD \cong \triangle CBD$ (Def. of congruent triangles)

17. Proof: It is given that $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$. Therefore, $\angle ABD \cong \angle CBD$ and $\angle ADB \cong \angle CDB$ by the definition of angle bisector. By the Third Angles Theorem, $\angle A \cong \angle C$. It is given that $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. By the Reflexive Property of Congruence, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ABD \cong \triangle CBD$ by Def. of congruent triangles.

19. $x = 12; y = 6$



21. $x = 4; y = 19$



23a. $\triangle ABI \cong \triangle EBF$, $\triangle CBD \cong \triangle HBG$
23b. Sample answer: $\angle A \cong \angle E$, $\angle ABI \cong \angle EBF$, $\angle I \cong \angle F$; $\overline{AB} \cong \overline{EB}$, $\overline{BI} \cong \overline{BF}$, $\overline{AI} \cong \overline{EF}$
25. Statements (Reasons)

1. $\angle P \cong \angle X$, $\angle Q \cong \angle Y$ (Given)
2. $m\angle P = m\angle X$, $m\angle Q = m\angle Y$ (Def. of congruent angles)
3. $m\angle P + m\angle Q + m\angle R = 180$
   $180 = m\angle X + m\angle Y + m\angle Z$ (Triangle Angle-Sum Thm.)
4. $m\angle P + m\angle Q + m\angle R = m\angle X + m\angle Y + m\angle Z$ (Transitive Property)
5. $m\angle X + m\angle Y + m\angle R = m\angle X + m\angle Y + m\angle Z$ (Substitution Property)
6. $m\angle R = m\angle Z$ (Subtraction Prop. of Eq.)
7. $\angle R \cong \angle Z$ (Def. of congruent angles)

27. Sample answer: Do you think that the sum of the angles of a quadrilateral is constant? If so, do you think that the final pair of corresponding angles will be congruent if three other pairs of corresponding angles are congruent for a pair of quadrilaterals?

29. Sample answer: When naming congruent triangles, it is important that the corresponding vertices be in the same location for both triangles because the location indicates congruence. For example, if $\triangle ABC$ is congruent to $\triangle DEF$, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

Lesson 14-3

1. Statements (Reasons)

1. $\overline{AB} \cong \overline{XY}$
   $\overline{AC} \cong \overline{XZ}$
   $\overline{BC} \cong \overline{YZ}$ (Given)
2. $\triangle ABC \cong \triangle XYZ$ (SSS Post.)

3. Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$. (Given)
2. $\overline{AD} \cong \overline{DC}$ (Definition of midpoint)
3. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Congruence)
4. $\triangle ABD \cong \triangle CBD$ (SSS)

5. Proof: We know that $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$. $\overline{RT} \cong \overline{RT}$ by the Reflexive Property. Because $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{RT}$, $\triangle QRT \cong \triangle SRT$ by SSS.
7. $DE = 5\sqrt{2}$, $PQ = 5\sqrt{2}$, $EF = 2\sqrt{10}$, $QR = 2\sqrt{10}$, $DF = 5\sqrt{2}$, $PR = 5\sqrt{2}$; $\triangle DEF \cong \triangle PQR$ by SSS because corresponding sides have the same measure and are congruent. **9.** $AB = 2$, $KL = 2$, $BC = 2\sqrt{2}$, $LM = 2\sqrt{2}$, $AC = 2$, $KM = 2$; The corresponding sides have the same measure and are congruent, so $\triangle ABC \cong \triangle KLM$ by SSS.
11. Proof:

Statements (Reasons)

1. $NP = PM$, $\overline{NP} \perp \overline{PL}$ (Given)
2. $\angle MPL$ and $\angle NPL$ are rt. angles. (⊥ lines form rt. angles.)
3. $\angle MPL \cong \angle NPL$ (All right angles are congruent.)
4. $\overline{PL} \cong \overline{PL}$ (Reflexive Property of ≅)
5. $\triangle NPL \cong \triangle MPL$ (SAS)

13. Proof: Because V is the midpoint of $\overline{YZ}$ and the midpoint of $\overline{WX}$, by the definition of midpoint, $\overline{YV} \cong \overline{VZ}$ and $\overline{WV} \cong \overline{XV}$. Because $\angle YVW$ and $\angle ZVX$ are vertical angles, by the Vertical Angles Theorem, the angles are congruent. Therefore, by SAS, $\triangle XVZ \cong \triangle WVY$.

15. Proof:

Statements (Reasons)

1. $\overline{BD} \perp \overline{AC}$; $\overline{BD}$ bisects $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are rt. angles. (⊥ lines form rt. angles.)

3. $\angle BDA \cong \angle BDC$ (All right angles are congruent.)
4. $\overline{AD} \cong \overline{DC}$ (Def. of segment bisector)
5. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of ≅)
6. $\triangle ABD \cong \triangle CBD$ (SAS)

17. Yes; sample answer: $\angle GLH$ and $\angle JLK$ are vertical angles, so they are congruent. Therefore, $\triangle GLH \cong \triangle JLK$ by the SAS Congruence Postulate. **19.** Yes; sample answer: The triangles share the side $\overline{AC}$, so they have two pairs of congruent sides. The given congruent angles are included angles, so $\triangle ABC \cong \triangle CDA$ by SAS. **21.** No; sample answer: The sticks do not change size, so any arrangement will yield a congruent triangle. **23.** Sample answer: She needs to measure one side of each tile because all the tiles are equilateral triangles. **25.** No; sample answer: You cannot use SAS because the angle congruence that we are given is not an included angle between two sides that are known to be congruent, and SSS cannot be used because only 2 sides of each triangle are known to be congruent. **27.** First pair; sample answer: The second pair can be shown congruent by SAS or SSS, and the third pair can be shown congruent by SSS. **29.** Case 1: You know that the hypotenuses are congruent and that one pair of legs are congruent. Then the Pythagorean Theorem says that the other pair of legs are congruent, so the triangles are congruent by SSS. Case 2: You know that the pairs of legs are congruent and that the right angles are congruent, so the triangles are congruent by SAS. **31.** Shada; to use SAS, the angle must be the included angle.

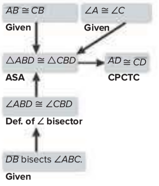
Lesson 14-4

1. Proof:

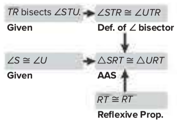
Statements (Reasons)

1. $\overline{AB} \parallel \overline{CD}$ (Given)
2. $\angle CBD \cong \angle ADB$ (Given)
3. $\angle ABD \cong \angle CDB$ (Alternate Interior Angles Theorem)
4. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Congruence)
5. $\triangle ABD \cong \triangle CDB$ (ASA)

3. Proof:


$\overline{AB} \cong \overline{CB}$
Given
$\angle A \cong \angle C$
Given
$\triangle ABD \cong \triangle CBD$
ASA
$\overline{AD} \cong \overline{CD}$
CPCTC
$\angle ABD \cong \angle CBD$
Def. of ∠ bisector
$\overline{DB}$ bisects $\angle ABC$.
Given

5. Proof: We are given that $\overline{CE}$ bisects $\angle BED$ and that $\angle BCE$ and $\angle ECD$ are right angles. Because all right angles are congruent, $\angle BCE \cong \angle ECD$. By the definition of angle bisector, $\angle BEC \cong \angle DEC$. The Reflexive Property tells us that $\overline{EC} \cong \overline{EC}$. By the Angle-Side-Angle Congruence Postulate, $\triangle ECB \cong \triangle ECD$.
7a. yes; by the ASA Congruence Postulate
7b. 10 in^2 **9a.** Sample answer: Because $\overline{AC} \parallel \overline{BK}$, $\angle CAB \cong \angle KBM$ by the Corresponding Angles Theorem. Because $\overline{CB} \parallel \overline{KM}$, $\angle ABC \cong \angle BMK$ by the Corresponding Angles Theorem. Because B is the midpoint of $\overline{AM}$, $\overline{AB} \cong \overline{BM}$ by the Midpoint Theorem. Therefore, by the ASA Congruence Postulate, $\triangle ABC \cong \triangle BMK$.
9b. 74 ft
11. Proof:


$\overline{TR}$ bisects $\angle STU$.
Given
$\angle STR \cong \angle UTR$
Def. of ∠ bisector
$\angle S \cong \angle U$
Given
$\triangle SRT \cong \triangle URT$
AAS
$\overline{RT} \cong \overline{RT}$
Reflexive Prop.

13. Proof: It is given that $\angle E \cong \angle G$ and $\overline{DE} \parallel \overline{FG}$. By the Alternate Interior Angles Theorem, $\angle DFG \cong \angle FDE$. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence. Therefore, $\triangle DFG \cong \triangle FDE$ by AAS.

15. Proof:

Statements (Reasons)
1. $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$ (Given)
2. $\angle SPM \cong \angle QPR$ (Vertical Angles Theorem)
3. $\angle SMP \cong \angle QRP$ (Alternate Interior Angles Theorem)
5. $\triangle MSP \cong \triangle RQP$ (AAS)

17. Yes; sample answer: They are congruent by AAS. **19.** Proof: Because it is given that $\overline{GE}$ bisects $\angle DEF$, $\angle DEG \cong \angle FEG$ by the definition of an angle bisector. It is given that $\angle D \cong \angle F$. By the Reflexive Property, $\overline{GE} \cong \overline{GE}$. So, $\triangle DEG \cong \triangle FEG$ by AAS. Therefore, $\overline{DG} \cong \overline{FG}$ by CPCTC. **21.** Tyrone; Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

23.

Method	Use when...
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides on one triangle must be congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.
AAS	Two angles and a non-included side of one triangle must be congruent to two angles and the corresponding non-included side of the other triangle.

Lesson 14-5

1. Statements (Reasons)
1. $\overline{XZ} \perp \overline{WY}$ (Given)
2. $\angle XZW$ and $\angle XZY$ are right angles. ($\perp$ lines form right angles.)
3. $\triangle WXZ$ and $\triangle YXZ$ are right triangles. (Definition of right triangle)
4. Z is the midpoint of $\overline{WY}$. (Given)
5. $\overline{WZ} \cong \overline{ZY}$ (Definition of midpoint)
6. $\overline{XZ} \cong \overline{XZ}$ (Reflexive Property of Congruence)
7. $\triangle WXZ \cong \triangle YXZ$ (LL Congruence Theorem)

3. Proof:

Statements (Reasons)
1. $\overline{BX} \perp \overline{AC}$ (Given)
2. $\angle AXB$ and $\angle CXB$ are rt. $\angle$s. (Definition of $\perp$ lines)
3. $\triangle AXB$ and $\triangle CXB$ are rt. $\triangle$s. (Definition of right $\triangle$s)
4. $\overline{XB} \cong \overline{XB}$ (Reflexive Property of Congruence)
5. $AB = CB$ (Given)
6. $\overline{AB} \cong \overline{CB}$ (Definition of congruent)
7. $\triangle AXB \cong \triangle CXB$ (HL Congruence Thm.)

5. Yes; LA **7.** No; not enough information
9. No; not enough information
11. Proof:

Statements (Reasons)
1. $\overline{BX} \perp \overline{XA}$, $\overline{BY} \perp \overline{YA}$ (Given)
2. $\angle BXA$ and $\angle BYA$ are rt. $\angle$s. (Definition of $\perp$ lines)
3. $\triangle BXA$ and $\triangle BYA$ are rt. $\triangle$s. (Definition of right $\triangle$s)
4. $\overline{XA} \cong \overline{YA}$ (Given)
5. $\overline{BA} \cong \overline{BA}$ (Reflexive Property of Congruence)
6. $\triangle BXA \cong \triangle BYA$ (HL Congruence Theorem)

13. Proof: By the definition of $\perp$ segments, $\angle AYB$ and $\angle AXC$ are right angles. By the definition of right triangles, $\triangle AYB$ and $\triangle AXC$ are right triangles. By the definition of congruent segments, $\overline{AX}$ is congruent to $\overline{AY}$. By the Reflexive Property of Congruence, $\angle BAY$ is congruent to $\angle CAX$. Therefore by LA, $\triangle ABY$ is congruent to $\triangle ACX$.

Lesson 14-6

1. Proof:

Statements (Reasons)
1. $\angle 1 \cong \angle 2$ (Given)
2. $\angle 2 \cong \angle 3$ (Vertical Angles Thm.)
3. $\angle 1 \cong \angle 3$ (Transitive Prop. of $\cong$)
4. $\overline{AB} \cong \overline{CB}$ (Conv. of Isos. Triangle Thm.)

3. Proof:

Statements (Reasons)
1. $\overline{DE} \parallel \overline{BC}$ (Given)
2. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$ (Corresponding angles are $\cong$.)
3. $\angle 1 \cong \angle 2$ (Given)
4. $\angle 1 \cong \angle 3$ (Transitive Property of $\cong$.)
5. $\angle 3 \cong \angle 4$ (Substitution.)
6. $\overline{AB} \cong \overline{AC}$ (Converse of Isosceles Triangle Theorem.)

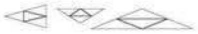
5a. The coordinates of $\triangle ABC$ are $A(0, 5)$, $B(3, 1)$, and $C(-3, 1)$.
$AC = \sqrt{[0-(-3)]^2 + (5-1)^2}$ or 5 units
$AB = \sqrt{(0-3)^2 + (5-1)^2}$ or 5 units
$BC = 6$ units

So, $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{AC}$.

5b. Because $\overline{AB} \cong \overline{AC}$, we know that $\angle C \cong \angle B$ by the Isosceles Triangle Theorem.

$m\angle A + m\angle B + m\angle C = 180°$ Triangle Angle-Sum Theorem
$m\angle A + 2m\angle C = 180°$ Definition of congruent
$m\angle A + 2(55) = 180°$ Substitute.
$m\angle A + 110 = 180°$ Multiply.
$m\angle A = 70°$ Solve.

7. $m\angle DEF = 45°$ and $m\angle EFD = 45°$ **9.** 60°; 3 m **11.** 60°; 7 in. **13.** 10 **15a.** 7 **15b.** 15
17a.

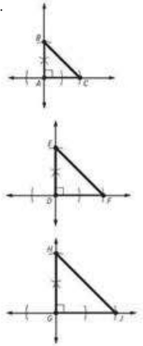


17b. Sample answer: The triangle formed by connecting the midpoints of the sides of an isosceles triangle is an isosceles triangle.

19. Proof:

Statements (Reasons)
1. $\triangle PQR$ is an equilateral triangle. (Given)
2. $\overline{PQ} \cong \overline{QR} \cong \overline{PR}$ (Def. of equilateral triangle)
3. $\angle P \cong \angle Q \cong \angle R$ (Isosceles Triangle Theorem)
4. $m\angle P = m\angle Q = m\angle R$ (Def. of congruence)
5. $m\angle P + m\angle Q + m\angle R = 180°$ (Triangle Angle-Sum Thm.)
6. $3m\angle P = 180°$ (Substitution)
7. $m\angle P = 60°$ (Division Property)
8. $m\angle P = m\angle Q = m\angle R = 60°$ (Substitution)

21. 44° **23.** 22°
25.



Sample answer: I constructed a pair of perpendicular segments and then used the same compass setting to mark points that are equidistant from their intersection. I measured both legs for each triangle. Because $AB = AC = 1.3$ cm, $DE = DF = 1.9$ cm, and

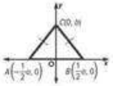
$GH = GJ = 2.3$ cm, the triangles are isosceles. I used a protractor to confirm that $\angle A$, $\angle D$, and $\angle G$ are all right angles.

27. Sometimes; sample answer: Only if the measure of the vertex angle is even.

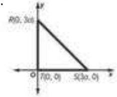
29. Sample answer: It is not possible because a triangle cannot have more than one obtuse angle.   **31.** No; $m\angle G = \frac{180-70}{2}$ or 55°.

Lesson 14-7

1.



3.



5. $C(p, q)$ **7.** $E(-2g, 0)$; $F(0, b)$ **9.** Sample answer: The midpoint P of $\overline{BC}$ is $\left(\frac{0+2a}{2}, \frac{2b+0}{2}\right) = (a, b)$. The midpoint Q of $\overline{AC}$ is $\left(\frac{0+2a}{2}, \frac{0+0}{2}\right) = (a, 0)$. The midpoint R of $\overline{AB}$ is $\left(\frac{0+0}{2}, \frac{0+2b}{2}\right) = (0, b)$. The slope of $\overline{RP}$ is $\frac{b-b}{a-0} = \frac{0}{a} = 0$, so the segment is horizontal. The slope of $\overline{PQ}$ is $\frac{b-0}{a-a} = \frac{0}{a}$, which is undefined, so the segment is vertical. $\angle RPQ$ is a right angle because any horizontal line is perpendicular to any vertical line. $\triangle PRQ$ has a right angle, so $\triangle PRQ$ is a right triangle.
11. Proof: The Midpoint Formula shows that the coordinates of M are $\left(\frac{0+2a}{2}, \frac{2a+0}{2}\right)$ or (a, a). The slope of $\overline{AC}$ is $\frac{2a-0}{0-2a} = -1$. The slope of $\overline{BM}$ is $\frac{a-0}{a-0} = 1$. The product of the slopes is –1, so $\overline{BM} \perp \overline{AC}$.   **13.** Proof: The coordinates of S are $\left(\frac{b}{2}, \frac{c}{2}\right)$, and the coordinates of T are $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

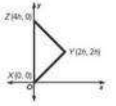
$$ST = \sqrt{\left(\frac{a+b}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2} \text{ or } \frac{a}{2}$$

$$AB = \sqrt{(a-0)^2 + (0-0)^2} \text{ or } a$$

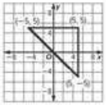
$$ST = \frac{1}{2}AB$$

15. The slope between the grandstand and the rides and games is $\frac{2}{3}$. The slope between the grandstand and the main gate is $-\frac{3}{2}$. Because $\frac{2}{3} \cdot -\frac{3}{2} = -1$, the triangle formed by these three locations is a right triangle.

17. Slope of $\overline{XY} = 1$, slope of $\overline{YZ} = -1$, slope of $\overline{ZX} = 0$; because $1(-1) = -1$, $\overline{XY} \perp \overline{YZ}$. Therefore, $\triangle XYZ$ is a right triangle.



19. $(a, 0)$ and $(0, a)$ **21.** Sample answer: Use the Distance Formula to find the length of each side of each triangle. Show that the triangles are congruent by SSS. Conclude that $\angle A \cong \angle D$ using CPCTC.   **23.** $D(a, b)$, $E(2a, 0)$, $F(3a, b)$
25. Sample answer:



27. Sample answer: $(a, 0)$
29. Sample answer: $(4a, 0)$ or $(0, 4b)$

Module 14 Review

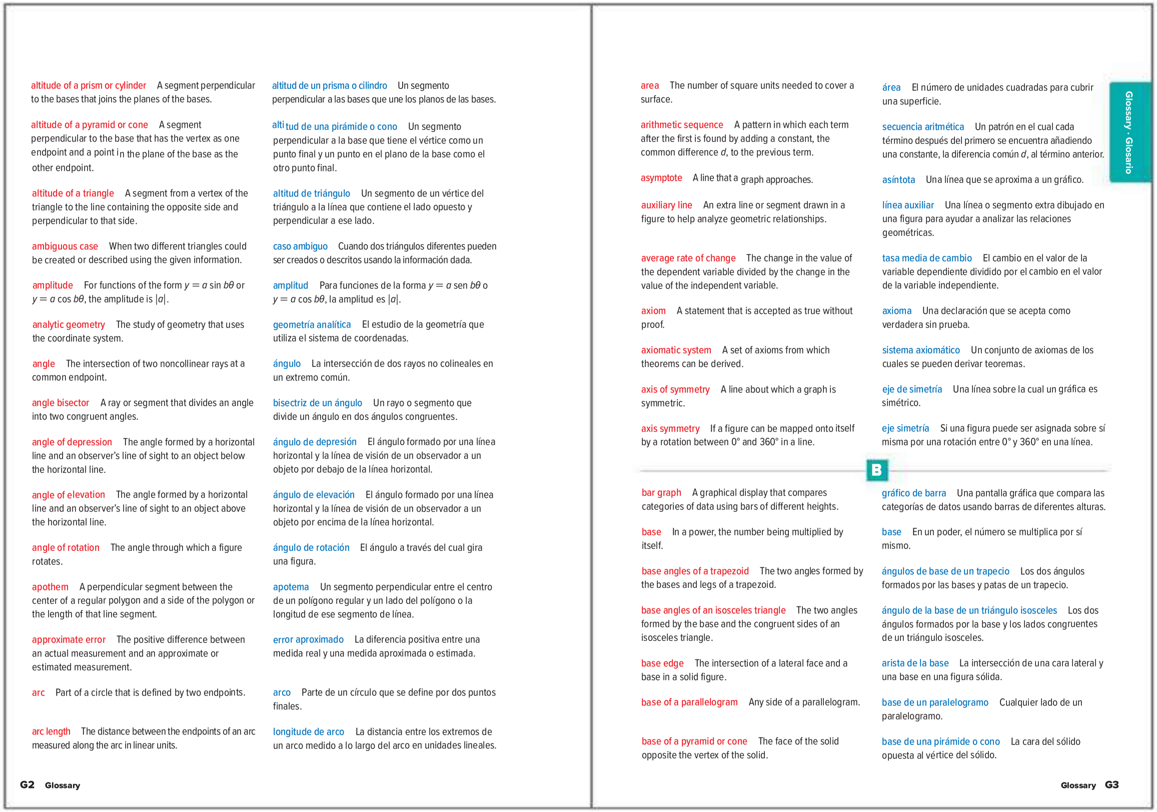
1. 106   **3.** A, D   **5.** A, C, E   **7.** A   **9.** A, B
11. A   **13.** 47   **15.** 33 units   **17.** A   **19.** B

Glossary

A

English	Español
30°-60°-90° triangle A right triangle with two acute angles that measure 30° and 60°.	triángulo 30°-60°-90° Un triángulo rectángulo con dos ángulos agudos que miden 30° y 60°.
45°-45°-90° triangle A right triangle with two acute angles that measure 45°.	triángulo 45°-45°-90° Un triángulo rectángulo con dos ángulos agudos que miden 45°.
absolute value The distance a number is from zero on the number line.	valor absoluto La distancia que un número es de cero en la línea numérica.
absolute value function A function written as $f(x) = \|x\|$, in which $f(x) \geq 0$ for all values of x.	función del valor absoluto Una función que se escribe $f(x) = \|x\|$, donde $f(x) \geq 0$, para todos los valores de x.
accuracy The nearness of a measurement to the true value of the measure.	exactitud La proximidad de una medida al valor verdadero de la medida.
additive identity Because the sum of any number a and 0 is equal to a, 0 is the additive identity.	identidad aditiva Debido a que la suma de cualquier número a y 0 es igual a, 0 es la identidad aditiva.
additive inverses Two numbers with a sum of 0.	inverso aditivos Dos números con una suma de 0.
adjacent angles Two angles that lie in the same plane and have a common vertex and a common side but have no common interior points.	ángulos adyacentes Dos ángulos que se encuentran en el mismo plano y tienen un vértice común y un lado común, pero no tienen puntos comunes en el interior.
adjacent arcs Arcs in a circle that have exactly one point in common.	arcos adyacentes Arcos en un circulo que tienen un solo punto en común.
algebraic expression A mathematical expression that contains at least one variable.	expresión algebraica Una expresión matemática que contiene al menos una variable.
algebraic notation Mathematical notation that describes a set by using algebraic expressions.	notación algebraica Notación matemática que describe un conjunto usando expresiones algebraicas.
alternate exterior angles When two lines are cut by a transversal, nonadjacent exterior angles that lie on opposite sides of the transversal.	ángulos alternos externos Cuando dos líneas son cortadas por un ángulo transversal, no adyacente exterior que se encuentran en lados opuestos de la transversal.
alternate interior angles When two lines are cut by a transversal, nonadjacent interior angles that lie on opposite sides of the transversal.	ángulos alternos internos Cuando dos líneas son cortadas por un ángulo transversal, no adyacente interior que se encuentran en lados opuestos de la transversal.
altitude of a parallelogram A perpendicular segment between any two parallel bases.	altitud de un paralelogramo Un segmento perpendicular entre dos bases paralelas.

Glossary



altitude of a prism or cylinder A segment perpendicular to the bases that joins the planes of the bases.	**altitud de un prisma o cilindro** Un segmento perpendicular a las bases que une los planos de las bases.
altitude of a pyramid or cone A segment perpendicular to the base that has the vertex as one endpoint and a point in the plane of the base as the other endpoint.	**altitud de una pirámide o cono** Un segmento perpendicular a la base que tiene el vértice como un punto final y un punto en el plano de la base como el otro punto final.
altitude of a triangle A segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side.	**altitud de triángulo** Un segmento de un vértice del triángulo a la línea que contiene el lado opuesto y perpendicular a ese lado.
ambiguous case When two different triangles could be created or described using the given information.	**caso ambiguo** Cuando dos triángulos diferentes pueden ser creados o descritos usando la información dada.
amplitude For functions of the form $y = a \sin b\theta$ or $y = a \cos b\theta$, the amplitude is $\|a\|$.	**amplitud** Para funciones de la forma $y = a$ sen $b\theta$ o $y = a \cos b\theta$, la amplitud es $\|a\|$.
analytic geometry The study of geometry that uses the coordinate system.	**geometría analítica** El estudio de la geometría que utiliza el sistema de coordenadas.
angle The intersection of two noncollinear rays at a common endpoint.	**ángulo** La intersección de dos rayos no colineales en un extremo común.
angle bisector A ray or segment that divides an angle into two congruent angles.	**bisectriz de un ángulo** Un rayo o segmento que divide un ángulo en dos ángulos congruentes.
angle of depression The angle formed by a horizontal line and an observer's line of sight to an object below the horizontal line.	**ángulo de depresión** El ángulo formado por una línea horizontal y la línea de visión de un observador a un objeto por debajo de la línea horizontal.
angle of elevation The angle formed by a horizontal line and an observer's line of sight to an object above the horizontal line.	**ángulo de elevación** El ángulo formado por una línea horizontal y la línea de visión de un observador a un objeto por encima de la línea horizontal.
angle of rotation The angle through which a figure rotates.	**ángulo de rotación** El ángulo a través del cual gira una figura.
apothem A perpendicular segment between the center of a regular polygon and a side of the polygon or the length of that line segment.	**apotema** Un segmento perpendicular entre el centro de un polígono regular y un lado del polígono o la longitud de ese segmento de línea.
approximate error The positive difference between an actual measurement and an approximate or estimated measurement.	**error aproximado** La diferencia positiva entre una medida real y una medida aproximada o estimada.
arc Part of a circle that is defined by two endpoints.	**arco** Parte de un círculo que se define por dos puntos finales.
arc length The distance between the endpoints of an arc measured along the arc in linear units.	**longitude de arco** La distancia entre los extremos de un arco medido a lo largo del arco en unidades lineales.

area The number of square units needed to cover a surface.	**área** El número de unidades cuadradas para cubrir una superficie.
arithmetic sequence A pattern in which each term after the first is found by adding a constant, the common difference d, to the previous term.	**secuencia aritmética** Un patrón en el cual cada término después del primero se encuentra añadiendo una constante, la diferencia común d, al término anterior.
asymptote A line that a graph approaches.	**asíntota** Una línea que se aproxima a un gráfico.
auxiliary line An extra line or segment drawn in a figure to help analyze geometric relationships.	**línea auxiliar** Una línea o segmento extra dibujado en una figura para ayudar a analizar las relaciones geométricas.
average rate of change The change in the value of the dependent variable divided by the change in the value of the independent variable.	**tasa media de cambio** El cambio en el valor de la variable dependiente dividido por el cambio en el valor de la variable independiente.
axiom A statement that is accepted as true without proof.	**axioma** Una declaración que se acepta como verdadera sin prueba.
axiomatic system A set of axioms from which theorems can be derived.	**sistema axiomático** Un conjunto de axiomas de los cuales se pueden derivar teoremas.
axis of symmetry A line about which a graph is symmetric.	**eje de simetría** Una línea sobre la cual un gráfica es simétrico.
axis symmetry If a figure can be mapped onto itself by a rotation between 0° and 360° in a line.	**eje simetría** Si una figura puede ser asignada sobre sí misma por una rotación entre 0° y 360° en una línea.

B

bar graph A graphical display that compares categories of data using bars of different heights.	**gráfico de barra** Una pantalla gráfica que compara las categorías de datos usando barras de diferentes alturas.
base In a power, the number being multiplied by itself.	**base** En un poder, el número se multiplica por sí mismo.
base angles of a trapezoid The two angles formed by the bases and legs of a trapezoid.	**ángulos de base de un trapecio** Los dos ángulos formados por las bases y patas de un trapecio.
base angles of an isosceles triangle The two angles formed by the base and the congruent sides of an isosceles triangle.	**ángulo de la base de un triángulo isosceles** Los dos ángulos formados por la base y los lados congruentes de un triángulo isosceles.
base edge The intersection of a lateral face and a base in a solid figure.	**arista de la base** La intersección de una cara lateral y una base en una figura sólida.
base of a parallelogram Any side of a parallelogram.	**base de un paralelogramo** Cualquier lado de un paralelogramo.
base of a pyramid or cone The face of the solid opposite the vertex of the solid.	**base de una pirámide o cono** La cara del sólido opuesta al vértice del sólido.

bases of a prism or cylinder	The two parallel congruent faces of the solid.	bases de un prisma o cilindro	Las dos caras congruentes paralelas de la figura sólida.
bases of a trapezoid	The parallel sides in a trapezoid.	bases de un trapecio	Los lados paralelos en un trapecio.
best-fit line	The line that most closely approximates the data in a scatter plot.	línea de ajuste óptimo	La línea que más se aproxima a los datos en un diagrama de dispersión.
betweenness of points	Point C is between A and B if and only if A, B, and C are collinear and $AC + CB = AB$.	intermediación de puntos	El punto C está entre A y B si y sólo si A, B, y C son colineales y $AC + CB = AB$.
bias	An error that results in a misrepresentation of a population.	sesgo	Un error que resulta en una tergiversación de una población.
biconditional statement	The conjunction of a conditional and its converse.	declaración bicondicional	La conjunción de un condicional y su inverso.
binomial	The sum of two monomials.	binomio	La suma de dos monomios.
bisect	To separate a line segment into two congruent segments.	bisecar	Separe un segmento de línea en dos segmentos congruentes.
bivariate data	Data that consists of pairs of values.	datos bivariate	Datos que constan de pares de valores.
boundary	The edge of the graph of an inequality that separates the coordinate plane into regions.	frontera	El borde de la gráfica de una desigualdad que separa el plano de coordenadas en regiones.
bounded	When the graph of a system of constraints is a polygonal region.	acotada	Cuando la gráfica de un sistema de restricciones es una región poligonal.
box plot	A graphical representation of the five-number summary of a data set.	diagram de caja	Una representación gráfica del resumen de cinco números de un conjunto de datos.

C

categorical data	Data that can be organized into different categories.	datos categóricos	Datos que pueden organizarse en diferentes categorías.
causation	When a change in one variable produces a change in another variable.	causalidad	Cuando un cambio en una variable produce un cambio en otra variable.
center of a circle	The point from which all points on a circle are the same distance.	centro de un círculo	El punto desde el cual todos los puntos de un círculo están a la misma distancia.
center of a regular polygon	The center of the circle circumscribed about a regular polygon.	centro de un polígono regular	El centro del círculo circunscrito alrededor de un polígono regular.
center of dilation	The center point from which dilations are performed.	centro de dilatación	Punto fijo en torno al cual se realizan las homotecias.
center of rotation	The fixed point about which a figure rotates.	centro de rotación	El punto fijo sobre el que gira una figura.
center of symmetry	A point in which a figure can be rotated onto itself.	centro de la simetría	Un punto en el que una figura se puede girar sobre sí misma.
central angle of a circle	An angle with a vertex at the center of a circle and sides that are radii.	ángulo central de un círculo	Un ángulo con un vértice en el centro de un círculo y los lados que son radios.
central angle of a regular polygon	An angle with its vertex at the center of a regular polygon and sides that pass through consecutive vertices of the polygon.	ángulo central de un polígono regular	Un ángulo con su vértice en el centro de un polígono regular y lados que pasan a través de vértices consecutivos del polígono.
centroid	The point of concurrency of the medians of a triangle.	baricentro	El punto de intersección de las medianas de un triángulo.
chord of a circle or sphere	A segment with endpoints on the circle or sphere.	cuerda de un círculo o esfera	Un segmento con extremos en el círculo o esfera.
circle	The set of all points in a plane that are the same distance from a given point called the center.	círculo	El conjunto de todos los puntos en un plano que están a la misma distancia de un punto dado llamado centro.
circular function	A function that describes a point on a circle as the function of an angle defined in radians.	función circular	Función que describe un punto en un círculo como la función de un ángulo definido en radianes.
circumcenter	The point of concurrency of the perpendicular bisectors of the sides of a triangle.	circuncentro	El punto de concurrencia de las bisectrices perpendiculares de los lados de un triángulo.
circumference	The distance around a circle.	circunferencia	La distancia alrededor de un círculo.
circumscribed angle	An angle with sides that are tangent to a circle.	ángulo circunscrito	Un ángulo con lados que son tangentes a un círculo.
circumscribed polygon	A polygon with vertices outside the circle and sides that are tangent to the circle.	polígono circunscrito	Un polígono con vértices fuera del círculo y lados que son tangentes al círculo.
closed	If for any members in a set, the result of an operation is also in the set.	cerrado	Si para cualquier número en el conjunto, el resultado de la operación es también en el conjunto.
closed half-plane	The solution of a linear inequality that includes the boundary line.	semi-plano cerrado	La solución de una desigualdad linear que incluye la línea de límite.
codomain	The set of all the y-values that could possibly result from the evaluation of the function.	codominar	El conjunto de todos los valores y que podrían resultar de la evaluación de la función.
coefficient	The numerical factor of a term.	coeficiente	El factor numérico de un término.
coefficient of determination	An indicator of how well a function fits a set of data.	coeficiente de determinación	Un indicador de lo bien que una función se ajusta a un conjunto de datos.

cofunction identities Identities that show the relationships between sine and cosine, tangent and cotangent, and secant and cosecant.	**identidades de cofunción** Identidades que muestran las relaciones entre seno y coseno, tangente y cotangente, y secante y cosecante.
collinear Lying on the same line.	**colineal** Acostado en la misma línea.
combination A selection of objects in which order is not important.	**combinación** Una selección de objetos en los que el orden no es importante.
combined variation When one quantity varies directly and/or inversely as two or more other quantities.	**variación combinada** Cuando una cantidad varía directamente y / o inversamente como dos o más cantidades.
common difference The difference between consecutive terms in an arithmetic sequence.	**diferencia común** La diferencia entre términos consecutivos de una secuencia aritmética.
common logarithms Logarithms of base 10.	**logaritmos comunes** Logaritmos de base 10.
common ratio The ratio of consecutive terms of a geometric sequence.	**razón común** El razón de términos consecutivos de una secuencia geométrica.
common tangent A line or segment that is tangent to two circles in the same plane.	**tangente común** Una línea o segmento que es tangente a dos círculos en el mismo plano.
complement of *A* All of the outcomes in the sample space that are not included as outcomes of event *A*.	**complemento de *A*** Todos los resultados en el espacio muestral que no se incluyen como resultados del evento *A*.
complementary angles Two angles with measures that have a sum of 90°.	**ángulo complementarios** Dos ángulos con medidas que tienen una suma de 90°.
completing the square A process used to make a quadratic expression into a perfect square trinomial.	**completar el cuadrado** Un proceso usado para hacer una expresión cuadrática en un trinomio cuadrado perfecto.
complex conjugates Two complex numbers of the form $a + bi$ and $a - bi$.	**conjugados complejos** Dos números complejos de la forma $a + bi$ y $a - bi$.
complex fraction A rational expression with a numerator and/or denominator that is also a rational expression.	**fracción compleja** Una expresión racional con un numerador y / o denominador que también es una expresión racional.
complex number Any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit.	**número complejo** Cualquier número que se puede escribir en la forma $a + bi$, donde a y b son números reales e i es la unidad imaginaria.
component form A vector written as $<x, y>$, which describes the vector in terms of its horizontal component x and vertical component y.	**forma de componente** Un vector escrito como $<x, y>$, que describe el vector en términos de su componente horizontal x y componente vertical y.

G6 Glossary

Glossary · Glosario

composite figure A figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles.	**figura compuesta** Una figura que se puede separar en regiones que son figuras básicas, tales como triángulos, rectángulos, trapezoides, y círculos.
composite solid A three-dimensional figure that is composed of simpler solids.	**solido compuesta** Una figura tridimensional que se compone de figuras más simples.
composition of functions An operation that uses the results of one function to evaluate a second function.	**composición de funciones** Operación que utiliza los resultados de una función para evaluar una segunda función.
composition of transformations When a transformation is applied to a figure and then another transformation is applied to its image.	**composición de transformaciones** Cuando una transformación se aplica a una figura y luego se aplica otra transformación a su imagen.
compound event Two or more simple events.	**evento compuesto** Dos o más eventos simples.
compound inequality Two or more inequalities that are connected by the words *and* or *or*.	**desigualdad compuesta** Dos o más desigualdades que están unidas por las palabras *y* u *o*.
compound interest Interest calculated on the principal and on the accumulated interest from previous periods.	**interés compuesto** Intereses calculados sobre el principal y sobre el interés acumulado de períodos anteriores.
compound statement Two or more statements joined by the word *and* or *or*.	**enunciado compuesto** Dos o más declaraciones unidas por la palabra *y* o *o*.
concave polygon A polygon with one or more interior angles with measures greater than 180°.	**polígono cóncavo** Un polígono con uno o más ángulos interiores con medidas superiores a 180°.
concentric circles Coplanar circles that have the same center.	**círculos concéntricos** Círculos coplanarios que tienen el mismo centro.
conclusion The statement that immediately follows the word *then* in a conditional.	**conclusión** La declaración que inmediatamente sigue la palabra *entonces* en un condicional.
concurrent lines Three or more lines that intersect at a common point.	**líneas concurrentes** Tres o más líneas que se intersecan en un punto común.
conditional probability The probability that an event will occur given that another event has already occurred.	**probabilidad condicional** La probabilidad de que un evento ocurra dado que otro evento ya ha ocurrido.
conditional relative frequency The ratio of the joint frequency to the marginal frequency.	**frecuencia relativa condicional** La relación entre la frecuencia de la articulación y la frecuencia marginal.
conditional statement A compound statement that consists of a premise, or hypothesis, and a conclusion, which is false only when its premise is true and its conclusion is false.	**enunciado condicional** Una declaración compuesta que consiste en una premisa, o hipótesis, y una conclusión, que es falsa solo cuando su premisa es verdadera y su conclusión es falsa.

Glossary G7

cone A solid figure with a circular base connected by a curved surface to a single vertex.	**cono** Una figura sólida con una base circular conectada por una superficie curvada a un solo vértice.
confidence interval An estimate of the population parameter stated as a range with a specific degree of certainty.	**intervalo de confianza** Una estimación del parámetro de población se indica como un rango con un grado específico de certeza.
congruent Having the same size and shape.	**congruente** Tener el mismo tamaño y forma.
congruent angles Two angles that have the same measure.	**ángulo congruentes** Dos ángulos que tienen la misma medida.
congruent arcs Arcs in the same or congruent circles that have the same measure.	**arcos congruentes** Arcos en los mismos círculos o congruentes que tienen la misma medida.
congruent polygons All of the parts of one polygon are congruent to the corresponding parts or matching parts of another polygon.	**polígonos congruentes** Todas las partes de un polígono son congruentes con las partes correspondientes o partes coincidentes de otro polígono.
congruent segments Line segments that are the same length.	**segmentos congruentes** Línea segmentos que son la misma longitud.
congruent solids Solid figures that have exactly the same shape, size, and a scale factor of 1 : 1.	**sólidos congruentes** Figuras sólidas que tienen exactamente la misma forma, tamaño y un factor de escala de 1 : 1.
conic sections Cross sections of a right circular cone.	**secciones cónicas** Secciones transversales de un cono circular derecho.
conjecture An educated guess based on known information and specific examples.	**conjetura** Una suposición educada basada en información conocida y ejemplos específicos.
conjugates Two expressions, each with two terms, in which the second terms are opposites.	**conjugados** Dos expresiones, cada una con dos términos, en la que los segundos términos son opuestos.
conjunction A compound statement using the word *and*.	**conjunción** Una declaración compuesta usando la palabra *y*.
consecutive interior angles When two lines are cut by a transversal, interior angles that lie on the same side of the transversal.	**ángulos internos consecutivos** Cuando dos líneas se cortan por un ángulo transversal, interior que se encuentran en el mismo lado de la transversal.
consistent A system of equations with at least one ordered pair that satisfies both equations.	**consistente** Una sistema de ecuaciones para el cual existe al menos un par ordenado que satisface ambas ecuaciones.
constant function A linear function of the form $y = b$; The function $f(x) = a$, where a is any number.	**función constante** Una función lineal de la forma $y = b$; La función $f(x) = a$, donde a es cualquier número.
constant of variation The constant in a variation function.	**constante de variación** La constante en una función de variación.
constant term A term that does not contain a variable.	**término constante** Un término que no contiene una variable.
constraint A condition that a solution must satisfy.	**restricción** Una condición que una solución debe satisfacer.
constructions Methods of creating figures without the use of measuring tools.	**construcciones** Métodos de creación de figuras sin el uso de herramientas de medición.
continuous function A function that can be graphed with a line or an unbroken curve.	**función continua** Una función que se puede representar gráficamente con una línea o una curva ininterrumpida.
continuous random variable The numerical outcome of a random event that can take on any value.	**variable aleatoria continua** El resultado numérico de un evento aleatorio que puede tomar cualquier valor.
contrapositive A statement formed by negating both the hypothesis and the conclusion of the converse of a conditional.	**antítesis** Una afirmación formada negando tanto la hipótesis como la conclusión del inverso del condicional.
convenience sample Members that are readily available or easy to reach are selected.	**muestra conveniente** Se seleccionan los miembros que están fácilmente disponibles o de fácil acceso.
converse A statement formed by exchanging the hypothesis and conclusion of a conditional statement.	**recíproco** Una declaración formada por el intercambio de la hipótesis y la conclusión de la declaración condicional.
convex polygon A polygon with all interior angles measuring less than 180°.	**polígono convexo** Un polígono con todos los ángulos interiores que miden menos de 180°.
coordinate proofs Proofs that use figures in the coordinate plane and algebra to prove geometric concepts.	**pruebas de coordenadas** Pruebas que utilizan figuras en el plano de coordenadas y álgebra para probar conceptos geométricos.
coplanar Lying in the same plane.	**coplanar** Acostado en el mismo plano.
corollary A theorem with a proof that follows as a direct result of another theorem.	**corolario** Un teorema con una prueba que sigue como un resultado directo de otro teorema.
correlation coefficient A measure that shows how well data are modeled by a regression function.	**coeficiente de correlación** Una medida que muestra cómo los datos son modelados por una función de regresión.
corresponding angles When two lines are cut by a transversal, angles that lie on the same side of a transversal and on the same side of the two lines.	**ángulos correspondientes** Cuando dos líneas se cortan transversalmente, los ángulos que se encuentran en el mismo lado de una transversal y en el mismo lado de las dos líneas.
corresponding parts Corresponding angles and corresponding sides of two polygons.	**partes correspondientes** Ángulos correspondientes y lados correspondientes.
cosecant The ratio of the length of a hypotenuse to the length of the leg opposite the angle.	**cosecante** Relación entre la longitud de la hipotenusa y la longitud de la pierna opuesta al ángulo.

cosine	The ratio of the length of the leg adjacent to an angle to the length of the hypotenuse.	coseno	Relación entre la longitud de la pierna adyacente a un ángulo y la longitud de la hipotenusa.
cotangent	The ratio of the length of the leg adjacent to an angle to the length of the leg opposite the angle.	cotangente	La relación entre la longitud de la pata adyacente a un ángulo y la longitud de la pata opuesta al ángulo.
coterminal angles	Angles in standard position that have the same terminal side.	ángulos coterminales	Ángulos en posición estándar que tienen el mismo lado terminal.
counterexample	An example that contradicts the conjecture showing that the conjecture is not always true.	contraejemplo	Un ejemplo que contradice la conjetura que muestra que la conjetura no siempre es cierta.
critical values	The *z*-values corresponding to the most common degrees of certainty.	valores críticos	Los valores z correspondientes a los grados de certeza más comunes.
cross section	The intersection of a solid and a plane.	sección transversal	Intersección de un sólido con un plano.
cube root	One of three equal factors of a number.	raíz cúbica	Uno de los tres factores iguales de un número.
cube root function	A radical function that contains the cube root of a variable expression.	función de la raíz del cubo	Función radical que contiene la raíz cúbica de una expresión variable.
curve fitting	Finding a regression equation for a set of data that is approximated by a function.	ajuste de curvas	Encontrar una ecuación de regresión para un conjunto de datos que es aproximado por una función.
cycle	One complete pattern of a periodic function.	ciclo	Un patron completo de una función periódica.
cylinder	A solid figure with two congruent and parallel circular bases connected by a curved surface.	cilindro	Una figura sólida con dos bases circulares congruentes y paralelas conectadas por una superficie curvada.

D

decay factor	The base of an exponential expression, or $1 - r$.	factor de decaimiento	La base de una expresión exponencial, o $1 - r$.
decomposition	Separating a figure into two or more nonoverlapping parts.	descomposición	Separar una figura en dos o más partes que no se solapan.
decreasing	Where the graph of a function goes down when viewed from left to right.	decreciente	Donde la gráfica de una función disminuye cuando se ve de izquierda a derecha.
deductive argument	An argument that guarantees the truth of the conclusion provided that its premises are true.	argumento deductivo	Un argumento que garantiza la verdad de la conclusión siempre que sus premisas sean verdaderas.
deductive reasoning	The process of reaching a specific valid conclusion based on general facts, rules, definitions, or properties.	razonamiento deductivo	El proceso de alcanzar una conclusión válida específica basada en hechos generales, reglas, definiciones, o propiedades.
define a variable	To choose a variable to represent an unknown value.	definir una variable	Para elegir una variable que represente un valor desconocido.
defined term	A term that has a definition and can be explained.	término definido	Un término que tiene una definición y se puede explicar.
definitions	An explanation that assigns properties to a mathematical object.	definiciones	Una explicación que asigna propiedades a un objeto matemático.
degree	The value of the exponent in a power function; $\frac{1}{360}$ of the circular rotation about a point.	grado	Valor del exponente en una función de potencia. $\frac{1}{360}$ de la rotación circular alrededor de un punto.
degree of a monomial	The sum of the exponents of all its variables.	grado de un monomio	La suma de los exponents de todas sus variables.
degree of a polynomial	The greatest degree of any term in the polynomial.	grado de un polinomio	El grado mayor de cualquier término del polinomio.
density	A measure of the quantity of some physical property per unit of length, area, or volume.	densidad	Una medida de la cantidad de alguna propiedad física por unidad de longitud, área o volumen.
dependent	A consistent system of equations with an infinite number of solutions.	dependiente	Una sistema consistente de ecuaciones con un número infinito de soluciones.
dependent events	Two or more events in which the outcome of one event affects the outcome of the other events.	eventos dependientes	Dos o más eventos en que el resultado de un evento afecta el resultado de los otros eventos.
dependent variable	The variable in a relation, usually y, with values that depend on x.	variable dependiente	La variable de una relación, generalmente y, con los valores que depende de x.
depressed polynomial	A polynomial resulting from division with a degree one less than the original polynomial.	polinomio reducido	Un polinomio resultante de la división con un grado uno menos que el polinomio original.
descriptive modeling	A way to mathematically describe real-world situations and the factors that cause them.	modelado descriptivo	Una forma de describir matemáticamente las situaciones del mundo real y los factores que las causan.
descriptive statistics	The branch of statistics that focuses on collecting, summarizing, and displaying data.	estadística descriptiva	Rama de la estadística cuyo enfoque es la recopilación, resumen y demostración de los datos.
diagonal	A segment that connects any two nonconsecutive vertices within a polygon.	diagonal	Un segmento que conecta cualquier dos vértices no consecutivos dentro de un polígono.

diameter of a circle or sphere	A chord that passes through the center of a circle or sphere.	diámetro de un círculo o esfera	Un acorde que pasa por el centro de un círculo o esfera.
difference of squares	A binomial in which the first and last terms are perfect squares.	diferencia de cuadrados	Un binomio en el que los términos primero y último son cuadrados perfectos.
difference of two squares	The square of one quantity minus the square of another quantity.	diferencia de dos cuadrados	El cuadrado de una cantidad menos el cuadrado de otra cantidad.
dilation	A nonrigid motion that enlarges or reduces a geometric figure; A transformation that stretches or compresses the graph of a function.	dilatación	Un movimiento no rígido que agranda o reduce una figura geométrica; Una transformación que estira o comprime el gráfico de una función.
dimensional analysis	The process of performing operations with units.	análisis dimensional	El proceso de realizar operaciones con unidades.
direct variation	When one quantity is equal to a constant times another quantity.	variación directa	Cuando una cantidad es igual a una constante multiplicada por otra cantidad.
directed line segment	A line segment with an initial endpoint and a terminal endpoint.	segment de línea dirigido	Un segmento de línea con un punto final inicial y un punto final terminal.
directrix	An exterior line perpendicular to the line containing the foci of a curve.	directriz	Una línea exterior perpendicular a la línea que contiene los focos de una curva.
discontinuous function	A function that is not continuous.	función discontinua	Una función que no es continua.
discrete function	A function in which the points on the graph are not connected.	función discreta	Una función en la que los puntos del gráfico no están conectados.
discrete random variable	The numerical outcome of a random event that is finite and can be counted.	variable aleatoria discreta	El resultado numérico de un evento aleatorio que es finito y puede ser contado.
discriminant	In the Quadratic Formula, the expression under the radical sign that provides information about the roots of the quadratic equation.	discriminante	En la Fórmula cuadrática, la expresión bajo el signo radical que proporciona información sobre las raíces de la ecuación cuadrática.
disjunction	A compound statement using the word *or*.	disyunción	Una declaración compuesta usando la palabra *o*.
distance	The length of the line segment between two points.	distancia	La longitud del segmento de línea entre dos puntos.
distribution	A graph or table that shows the theoretical frequency of each possible data value.	distribución	Un gráfico o una table que muestra la frecuencia teórica de cada valor de datos posible.
domain	The set of the first numbers of the ordered pairs in a relation; The set of *x*-values to be evaluated by a function.	dominio	El conjunto de los primeros números de los pares ordenados en una relación; El conjunto de valores *x* para ser evaluados por una función.

dot plot	A diagram that shows the frequency of data on a number line.	gráfica de puntos	Una diagrama que muestra la frecuencia de los datos en una línea numérica.
double root	Two roots of a quadratic equation that are the same number.	raíces dobles	Dos raíces de una función cuadrática que son el mismo número.

E

e	An irrational number that approximately equals 2.7182818....	*e*	Un número irracional que es aproximadamente igual a 2.7182818 ....
edge of a polyhedron	A line segment where the faces of the polyhedron intersect.	arista de un poliedro	Un segmento de línea donde las caras del poliedro se cruzan.
elimination	A method that involves eliminating a variable by combining the individual equations within a system of equations.	eliminación	Un método que consiste en eliminar una variable combinando las ecuaciones individuales dentro de un sistema de ecuaciones.
empty set	The set that contains no elements, symbolized by { } or ∅.	conjunto vacio	El conjunto que no contiene elementos, simbolizado por { } o ∅.
end behavior	The behavior of a graph at the positive and negative extremes in its domain.	comportamiento extremo	El comportamiento de un gráfico en los extremos positivo y negativo en su dominio.
enlargement	A dilation with a scale factor greater than 1.	ampliación	Una dilatación con un factor de escala mayor que 1.
equation	A mathematical statement that contains two expressions and an equal sign, =.	ecuación	Un enunciado matemático que contiene dos expresiones y un signo igual, =.
equiangular polygon	A polygon with all angles congruent.	polígono equiangular	Un polígono con todos los ángulos congruentes.
equidistant	A point is equidistant from other points if it is the same distance from them.	equidistante	Un punto es equidistante de otros puntos si está a la misma distancia de ellos.
equidistant lines	Two lines for which the distance between the two lines, measured along a perpendicular line or segment to the two lines, is always the same.	líneas equidistantes	Dos líneas para las cuales la distancia entre las dos líneas, medida a lo largo de una línea o segmento perpendicular a las dos líneas, es siempre la misma.
equilateral polygon	A polygon with all sides congruent.	polígono equilátero	Un polígono con todos los lados congruentes.
equivalent equations	Two equations with the same solution.	ecuaciones equivalentes	Dos ecuaciones con la misma solución.
equivalent expressions	Expressions that represent the same value.	expresiones equivalentes	Expresiones que representan el mismo valor.
evaluate	To find the value of an expression.	evaluar	Calcular el valor de una expresión.

GLOSSARY

G14-G15 Glossary

even functions Functions that are symmetric in the y-axis.	**incluso funciones** Funciones que son simétricas en el eje y.
event A subset of the sample space.	**evento** Un subconjunto del espacio de muestra.
excluded values Values for which a function is not defined.	**valores excluidos** Valores para los que no se ha definido una función.
experiment A sample is divided into two groups. The experimental group undergoes a change, while there is no change to the control group. The effects on the groups are then compared; A situation involving chance.	**experimento** Una muestra se divide en dos grupos. El grupo experimental experimenta un cambio, mientras que no hay cambio en el grupo de control. A continuación se comparan los efectos sobre los grupos; Una situación de riesgo.
experimental probability Probability calculated by using data from an actual experiment.	**probabilidad experimental** Probabilidad calculada utilizando datos de un experimento real.
exponent When n is a positive integer in the expression x^n, n indicates the number of times x is multiplied by itself.	**exponente** Cuando n es un entero positivo en la expresión x^n, n indica el número de veces que x se multiplica por sí mismo.
exponential decay Change that occurs when an initial amount decreases by the same percent over a given period of time.	**desintegración exponencial** Cambio que ocurre cuando una cantidad inicial disminuye en el mismo porcentaje durante un período de tiempo dado.
exponential decay function A function in which the independent variable is an exponent, where $a > 0$ and $0 < b < 1$.	**función exponenciales de decaimiento** Una ecuación en la que la variable independiente es un exponente, donde $a > 0$ y $0 < b < 1$.
explicit formula A formula that allows you to find any term a_n of a sequence by using a formula written in terms of n.	**fórmula explícita** Una fórmula que le permite encontrar cualquier término a_n de una secuencia usando una fórmula escrita en términos de n.
exponential equation An equation in which the independent variable is an exponent.	**ecuación exponencial** Una ecuación en la que la variable independiente es un exponente.
exponential form When an expression is in the form x^n.	**forma exponencial** Cuando una expresión está en la forma x^n.
exponential function A function in which the independent variable is an exponent.	**función exponencial** Una función en la que la variable independiente es el exponente.
exponential growth Change that occurs when an initial amount increases by the same percent over a given period of time.	**crecimiento exponencial** Cambio que ocurre cuando una cantidad inicial aumenta por el mismo porcentaje durante un período de tiempo dado.
exponential growth function A function in which the independent variable is an exponent, where $a > 0$ and $b > 1$.	**función de crecimiento exponencial** Una función en la que la variable independiente es el exponente, donde $a > 0$ y $b > 1$.

exponential inequality An inequality in which the independent variable is an exponent.	**desigualdad exponencial** Una desigualdad en la que la variable independiente es un exponente.
exterior angle of a triangle An angle formed by one side of the triangle and the extension of an adjacent side.	**ángulo exterior de un triángulo** Un ángulo formado por un lado del triángulo y la extensión de un lado adyacente.
exterior angles When two lines are cut by a transversal, any of the four angles that lie outside the region between the two intersected lines.	**ángulos externos** Cuando dos líneas son cortadas por una transversal, cualquiera de los cuatro ángulos que se encuentran fuera de la región entre las dos líneas intersectadas.
exterior of an angle The area outside of the two rays of an angle.	**exterior de un ángulo** El área fuera de los dos rayos de un ángulo.
extraneous solution A solution of a simplified form of an equation that does not satisfy the original equation.	**solución extraña** Una solución de una forma simplificada de una ecuación que no satisface la ecuación original.
extrema Points that are the locations of relatively high or low function values.	**extrema** Puntos que son las ubicaciones de valores de función relativamente alta o baja.
extreme values The least and greatest values in a set of data.	**valores extremos** Los valores mínimo y máximo en un conjunto de datos.

F

face of a polyhedron A flat surface of a polyhedron.	**cara de un poliedro** Superficie plana de un poliedro.
factored form A form of quadratic equation, $0 = a(x - p)(x - q)$, where $a \neq 0$, in which p and q are the x-intercepts of the graph of the related function.	**forma factorizada** Una forma de ecuación cuadrática, $0 = a(x - p)(x - q)$, donde $a \neq 0$, en la que p y q son las intercepciones x de la gráfica de la función relacionada.
factorial of n The product of the positive integers less than or equal to n.	**factorial de n** El producto de los enteros positivos inferiores o iguales a n.
factoring The process of expressing a polynomial as the product of monomials and polynomials.	**factorización por agrupamiento** Utilizando la Propiedad distributiva para factorizar polinomios que possen cuatro o más términos.
factoring by grouping Using the Distributive Property to factor some polynomials having four or more terms.	**factorización** El proceso de expresar un polinomio como el producto de monomios y polinomios.
family of graphs Graphs and equations of graphs that have at least one characteristic in common.	**familia de gráficas** Gráficas y ecuaciones de gráficas que tienen al menos una característica común.
feasible region The intersection of the graphs in a system of constraints.	**región factible** La intersección de los gráficos en un sistema de restricciones.
finite sample space A sample space that contains a countable number of outcomes.	**espacio de muestra finito** Un espacio de muestra que contiene un número contable de resultados.

finite sequence A sequence that contains a limited number of terms.	**secuencia finita** Una secuencia que contiene un número limitado de términos.
five-number summary The minimum, quartiles, and maximum of a data set.	**resumen de cinco números** El mínimo, cuartiles *y* máximo de un conjunto de datos.
flow proof A proof that uses boxes and arrows to show the logical progression of an argument.	**demostración de flujo** Una prueba que usa cajas y flechas para mostrar la progresión lógica de un argumento.
focus A point inside a parabola having the property that the distances from any point on the parabola to them and to a fixed line have a constant ratio for any points on the parabola.	**foco** Un punto dentro de una parábola que tiene la propiedad de que las distancias desde cualquier punto de la parábola a ellos ya una línea fija tienen una relación constante para cualquier punto de la parábola.
formula An equation that expresses a relationship between certain quantities.	**fórmula** Una ecuación que expresa una relación entre ciertas cantidades.
fractional distance An intermediary point some fraction of the length of a line segment.	**distancia fraccionaria** Un punto intermediario de alguna fracción de la longitud de un segmento de línea.
frequency The number of cycles in a given unit of time.	**frecuencia** El número de ciclos en una unidad del tiempo dada.
function A relation in which each element of the domain is paired with exactly one element of the range.	**función** Una relación en que a cada elemento del dominio de corresponde un único elemento del rango.
function notation A way of writing an equation so that $y = f(x)$.	**notación functional** Una forma de escribir una ecuación para que $y = f(x)$.

G

geometric means The terms between two nonconsecutive terms of a geometric sequence; The *n*th root, where n is the number of elements in a set of numbers, of the product of the numbers.	**medios geométricos** Los términos entre dos términos no consecutivos de una secuencia geométrica; La enésima raíz, donde n es el número de elementos de un conjunto de números, del producto de los números.
geometric model A geometric figure that represents a real-life object.	**modelo geométrico** Una figura geométrica que representa un objeto de la vida real.
geometric probability Probability that involves a geometric measure such as length or area.	**probabilidad geométrica** Probabilidad que implica una medida geométrica como longitud o área.
geometric sequence A pattern of numbers that begins with a nonzero term and each term after is found by multiplying the previous term by a nonzero constant r.	**secuencia geométrica** Un patrón de números que comienza con un término distinto de cero y cada término después se encuentra multiplicando el término anterior por una constante no nula r.
geometric series The indicated sum of the terms in a geometric sequence.	**series geométricas** La suma indicada de los términos en una secuencia geométrica.

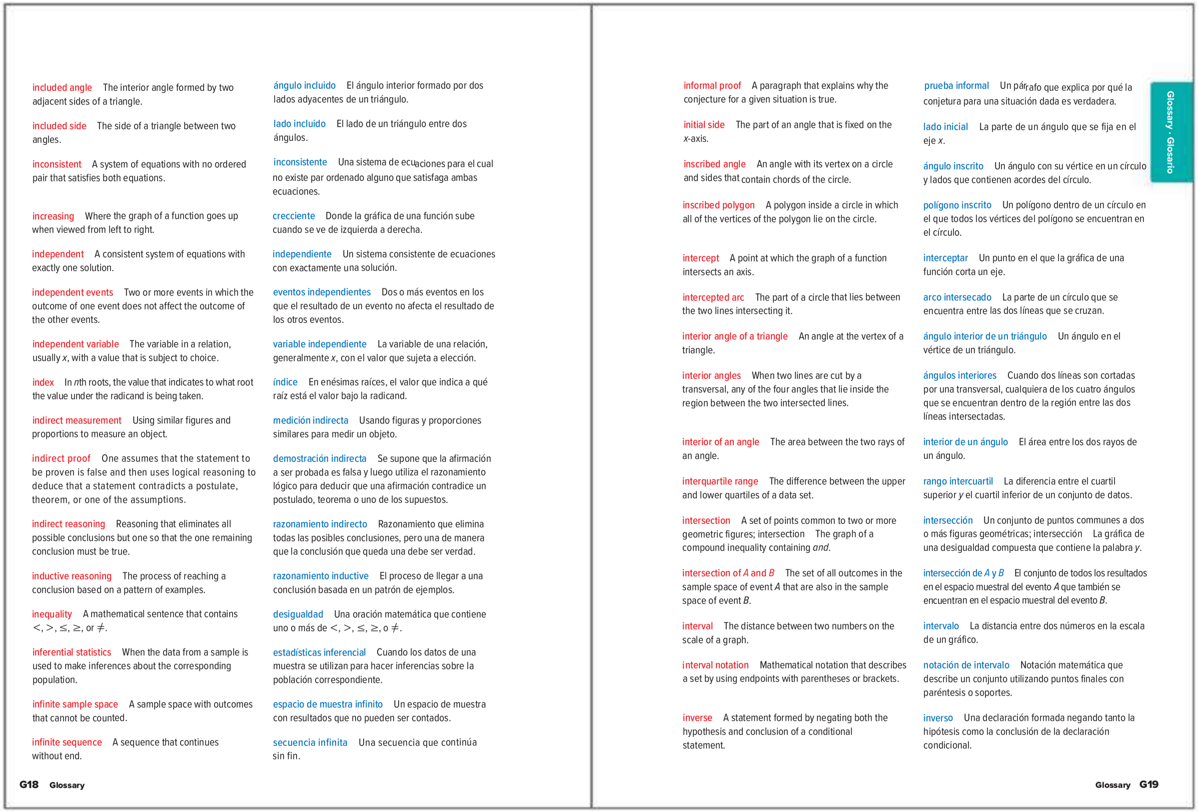
glide reflection The composition of a translation followed by a reflection in a line parallel to the translation vector.	**reflexión del deslizamiento** La composición de una traducción seguida de una reflexión en una línea paralela al vector de traslación.
greatest integer function A step function in which $f(x)$ is the greatest integer less than or equal to x.	**función entera más grande** Una función del paso en que $f(x)$ es el número más grande menos que o igual a x.
growth factor The base of an exponential expression, or $1 + r$.	**factor de crecimiento** La base de una expresión exponencial, o $1 + r$.

H

half-plane A region of the graph of an inequality on one side of a boundary.	**semi-plano** Una región de la gráfica de una desigualdad en un lado de un límite.
height of a parallelogram The length of an altitude of the parallelogram.	**altura de un paralelogramo** La longitud de la altitud del paralelogramo.
height of a solid The length of the altitude of a solid figure.	**altura de un sólido** La longitud de la altitud de una figura sólida.
height of a trapezoid The perpendicular distance between the bases of a trapezoid.	**altura de un trapecio** La distancia perpendicular entre las bases de un trapecio.
histogram A graphical display that uses bars to display numerical data that have been organized in equal intervals.	**histograma** Una exhibición gráfica que utiliza barras para exhibir los datos numéricos que se han organizado en intervalos iguales.
horizontal asymptote A horizontal line that a graph approaches.	**asíntota horizontal** Una línea horizontal que se aproxima a un gráfico.
hyperbola The graph of a reciprocal function.	**hipérbola** La gráfica de una función recíproca.
hypothesis The statement that immediately follows the word *if* in a conditional.	**hipótesis** La declaración que sigue inmediatamente a la palabra *si* en un condicional.

I

identity An equation that is true for every value of the variable.	**identidad** Una ecuación que es verdad para cada valor de la variable.
identity function The function $f(x) = x$.	**función identidad** La función $f(x) = x$.
if-then statement A compound statement of the form *if p, then q*, where p and q are statements.	**enunciado si-entonces** Enunciado compuesto de la forma *si p, entonces q*, donde p y q son enunciados.
image The new figure in a transformation.	**imagen** La nueva figura en una transformación.
imaginary unit *i* The principal square root of -1.	**unidad imaginaria *i*** La raíz cuadrada principal de -1.
incenter The point of concurrency of the angle bisectors of a triangle.	**incentro** El punto de intersección de las bisectrices interiors de un triángulo.



Glossary · Glosario

included angle The interior angle formed by two adjacent sides of a triangle.	**ángulo incluido** El ángulo interior formado por dos lados adyacentes de un triángulo.
included side The side of a triangle between two angles.	**lado incluido** El lado de un triángulo entre dos ángulos.
inconsistent A system of equations with no ordered pair that satisfies both equations.	**inconsistente** Una sistema de ecuaciones para el cual no existe par ordenado alguno que satisfaga ambas ecuaciones.
increasing Where the graph of a function goes up when viewed from left to right.	**creciente** Donde la gráfica de una función sube cuando se ve de izquierda a derecha.
independent A consistent system of equations with exactly one solution.	**independiente** Un sistema consistente de ecuaciones con exactamente una solución.
independent events Two or more events in which the outcome of one event does not affect the outcome of the other events.	**eventos independientes** Dos o más eventos en los que el resultado de un evento no afecta el resultado de los otros eventos.
independent variable The variable in a relation, usually x, with a value that is subject to choice.	**variable independiente** La variable de una relación, generalmente x, con el valor que sujeta a elección.
index In nth roots, the value that indicates to what root the value under the radicand is being taken.	**índice** En enésimas raíces, el valor que indica a qué raíz está el valor bajo la radicand.
indirect measurement Using similar figures and proportions to measure an object.	**medición indirecta** Usando figuras y proporciones similares para medir un objeto.
indirect proof One assumes that the statement to be proven is false and then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions.	**demostración indirecta** Se supone que la afirmación a ser probada es falsa y luego utiliza el razonamiento lógico para deducir que una afirmación contradice un postulado, teorema o uno de los supuestos.
indirect reasoning Reasoning that eliminates all possible conclusions but one so that the one remaining conclusion must be true.	**razonamiento indirecto** Razonamiento que elimina todas las posibles conclusiones, pero una de manera que la conclusión que queda una debe ser verdad.
inductive reasoning The process of reaching a conclusion based on a pattern of examples.	**razonamiento inductive** El proceso de llegar a una conclusión basada en un patrón de ejemplos.
inequality A mathematical sentence that contains $<, >, \leq, \geq$, or $\neq$.	**desigualdad** Una oración matemática que contiene uno o más de $<, >, \leq, \geq$, o $\neq$.
inferential statistics When the data from a sample is used to make inferences about the corresponding population.	**estadísticas inferencial** Cuando los datos de una muestra se utilizan para hacer inferencias sobre la población correspondiente.
infinite sample space A sample space with outcomes that cannot be counted.	**espacio de muestra infinito** Un espacio de muestra con resultados que no pueden ser contados.
infinite sequence A sequence that continues without end.	**secuencia infinita** Una secuencia que continúa sin fin.

informal proof A paragraph that explains why the conjecture for a given situation is true.	**prueba informal** Un párrafo que explica por qué la conjetura para una situación dada es verdadera.
initial side The part of an angle that is fixed on the x-axis.	**lado inicial** La parte de un ángulo que se fija en el eje x.
inscribed angle An angle with its vertex on a circle and sides that contain chords of the circle.	**ángulo inscrito** Un ángulo con su vértice en un círculo y lados que contienen acordes del círculo.
inscribed polygon A polygon inside a circle in which all of the vertices of the polygon lie on the circle.	**polígono inscrito** Un polígono dentro de un círculo en el que todos los vértices del polígono se encuentran en el círculo.
intercept A point at which the graph of a function intersects an axis.	**interceptar** Un punto en el que la gráfica de una función corta un eje.
intercepted arc The part of a circle that lies between the two lines intersecting it.	**arco intersecado** La parte de un círculo que se encuentra entre las dos líneas que se cruzan.
interior angle of a triangle An angle at the vertex of a triangle.	**ángulo interior de un triángulo** Un ángulo en el vértice de un triángulo.
interior angles When two lines are cut by a transversal, any of the four angles that lie inside the region between the two intersected lines.	**ángulos interiores** Cuando dos líneas son cortadas por una transversal, cualquiera de los cuatro ángulos que se encuentran dentro de la región entre las dos líneas intersectadas.
interior of an angle The area between the two rays of an angle.	**interior de un ángulo** El área entre los dos rayos de un ángulo.
interquartile range The difference between the upper and lower quartiles of a data set.	**rango intercuartil** La diferencia entre el cuartil superior y el cuartil inferior de un conjunto de datos.
intersection A set of points common to two or more geometric figures; intersection The graph of a compound inequality containing *and*.	**intersección** Un conjunto de puntos communes a dos o más figuras geométricas; intersección La gráfica de una desigualdad compuesta que contiene la palabra *y*.
intersection of A and B The set of all outcomes in the sample space of event A that are also in the sample space of event B.	**intersección de A y B** El conjunto de todos los resultados en el espacio muestral del evento A que también se encuentran en el espacio muestral del evento B.
interval The distance between two numbers on the scale of a graph.	**intervalo** La distancia entre dos números en la escala de un gráfico.
interval notation Mathematical notation that describes a set by using endpoints with parentheses or brackets.	**notación de intervalo** Notación matemática que describe un conjunto utilizando puntos finales con paréntesis o soportes.
inverse A statement formed by negating both the hypothesis and conclusion of a conditional statement.	**inverso** Una declaración formada negando tanto la hipótesis como la conclusión de la declaración condicional.

inverse cosine The ratio of the length of the hypotenuse to the length of the leg adjacent to an angle.	**inverso del coseno** Relación de la longitud de la hipotenusa con la longitud de la pierna adyacente a un ángulo.
inverse functions Two functions, one of which contains points of the form (*a*, *b*) while the other contains points of the form (*b*, *a*).	**funciones inversas** Dos funciones, una de las cuales contiene puntos de la forma (*a*, *b*) mientras que la otra contiene puntos de la forma (*b*, *a*).
inverse relations Two relations, one of which contains points of the form (*a*, *b*) while the other contains points of the form (*b*, *a*).	**relaciones inversas** Dos relaciones, una de las cuales contiene puntos de la forma (*a*, *b*) mientras que la otra contiene puntos de la forma (*b*, *a*).
inverse sine The ratio of the length of the hypotenuse to the length of the leg opposite an angle.	**inverso del seno** Relación de la longitud de la hipotenusa con la longitud de la pierna opuesta a un ángulo.
inverse tangent The ratio of the length of the leg adjacent to an angle to the length of the leg opposite the angle.	**inverso del tangente** Relación de la longitud de la pierna adyacente a un ángulo con la longitud de la pierna opuesta a un ángulo.
inverse trigonometric functions Arcsine, Arccosine, and Arctangent.	**funciones trigonométricas inversas** Arcsine, Arccosine y Arctangent.
inverse variation When the product of two quantities is equal to a constant *k*.	**variación inversa** Cuando el producto de dos cantidades es igual a una constante *k*.
isosceles trapezoid A quadrilateral in which two sides are parallel and the legs are congruent.	**trapecio isósceles** Un cuadrilátero en el que dos lados son paralelos y las patas son congruentes.
isosceles triangle A triangle with at least two sides congruent.	**triángulo isósceles** Un triángulo con al menos dos lados congruentes.

J

joint frequencies Entries in the body of a two-way frequency table. In a two-way frequency table, the frequencies in the interior of the table.	**frecuencias articulares** Entradas en el cuerpo de una tabla de frecuencias de dos vías. En una tabla de frecuencia bidireccional, las frecuencias en el interior de la tabla.
joint variation When one quantity varies directly as the product of two or more other quantities.	**variación conjunta** Cuando una cantidad varía directamente como el producto de dos o más cantidades.

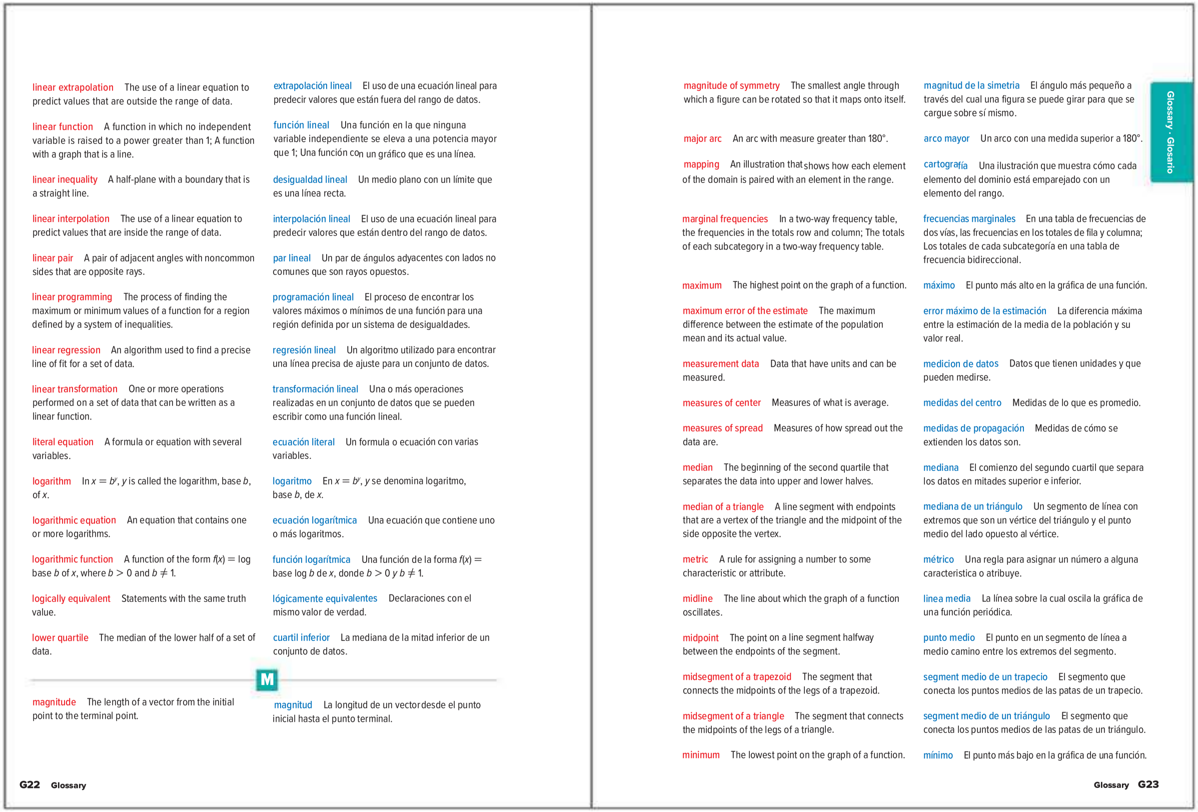
K

kite A convex quadrilateral with exactly two distinct pairs of adjacent congruent sides.	**cometa** Un cuadrilátero convexo con exactamente dos pares distintos de lados congruentes adyacentes.

L

lateral area The sum of the areas of the lateral faces of the figure.	**área lateral** La suma de las áreas de las caras laterales de la figura.

lateral edges The intersection of two lateral faces.	**aristas laterales** La intersección de dos caras laterales.
lateral faces The faces that join the bases of a solid.	**caras laterales** Las caras que unen las bases de un sólido.
lateral surface of a cone The curved surface that joins the base of a cone to the vertex.	**superficie lateral de un cono** La superficie curvada que une la base de un cono con el vértice.
lateral surface of a cylinder The curved surface that joins the bases of a cylinder.	**superficie lateral de un cilindro** La superficie curvada que une las bases de un cilindro.
leading coefficient The coefficient of the first term when a polynomial is in standard form.	**coeficiente líder** El coeficiente del primer término cuando un polinomio está en forma estándar.
legs of a trapezoid The nonparallel sides in a trapezoid.	**patas de un trapecio** Los lados no paralelos en un trapezoide.
legs of an isosceles triangle The two congruent sides of an isosceles triangle.	**patas de un triángulo isósceles** Los dos lados congruentes de un triángulo isósceles.
like radical expressions Radicals in which both the index and the radicand are the same.	**expresiones radicales semejantes** Radicales en los que tanto el índice como el radicand son iguales.
like terms Terms with the same variables, with corresponding variables having the same exponent.	**términos semejantes** Términos con las mismas variables, con las variables correspondientes que tienen el mismo exponente.
line A line is made up of points, has no thickness or width, and extends indefinitely in both directions.	**línea** Una línea está formada por puntos, no tiene espesor ni anchura, y se extiende indefinidamente en ambas direcciones.
line of fit A line used to describe the trend of the data in a scatter plot.	**línea de ajuste** Una línea usada para describir la tendencia de los datos en un diagrama de dispersión.
line of reflection A line midway between a preimage and an image; The line in which a reflection flips the graph of a function.	**línea de reflexión** Una línea a medio camino entre una preimagen y una imagen; La línea en la que una reflectión voltea la gráfica de una función.
line of symmetry An imaginary line that separates a figure into two congruent parts.	**línea de simetría** Una línea imaginaria que separa una figura en dos partes congruentes.
line segment A measurable part of a line that consists of two points, called endpoints, and all of the points between them.	**segmento de línea** Una parte medible de una línea que consta de dos puntos, llamados extremos, y todos los puntos entre ellos.
line symmetry A graph has line symmetry if it can be reflected in a vertical line so that each half of the graph maps exactly to the other half.	**simetría de línea** Un gráfico tiene simetría de línea si puede reflejarse en una línea vertical, de modo que cada mitad del gráfico se asigna exactamente a la otra mitad.
linear equation An equation that can be written in the form $Ax + By = C$ with a graph that is a straight line.	**ecuación lineal** Una ecuación que puede escribirse de la forma $Ax + By = C$ con un gráfico que es una línea recta.



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linear extrapolation The use of a linear equation to predict values that are outside the range of data.	**extrapolación lineal** El uso de una ecuación lineal para predecir valores que están fuera del rango de datos.
linear function A function in which no independent variable is raised to a power greater than 1; A function with a graph that is a line.	**función lineal** Una función en la que ninguna variable independiente se eleva a una potencia mayor que 1; Una función con un gráfico que es una línea.
linear inequality A half-plane with a boundary that is a straight line.	**desigualdad lineal** Un medio plano con un límite que es una línea recta.
linear interpolation The use of a linear equation to predict values that are inside the range of data.	**interpolación lineal** El uso de una ecuación lineal para predecir valores que están dentro del rango de datos.
linear pair A pair of adjacent angles with noncommon sides that are opposite rays.	**par lineal** Un par de ángulos adyacentes con lados no comunes que son rayos opuestos.
linear programming The process of finding the maximum or minimum values of a function for a region defined by a system of inequalities.	**programación lineal** El proceso de encontrar los valores máximos o mínimos de una función para una región definida por un sistema de desigualdades.
linear regression An algorithm used to find a precise line of fit for a set of data.	**regresión lineal** Un algoritmo utilizado para encontrar una línea precisa de ajuste para un conjunto de datos.
linear transformation One or more operations performed on a set of data that can be written as a linear function.	**transformación lineal** Una o más operaciones realizadas en un conjunto de datos que se pueden escribir como una función lineal.
literal equation A formula or equation with several variables.	**ecuación literal** Un formula o ecuación con varias variables.
logarithm In $x = b^y$, y is called the logarithm, base b, of x.	**logaritmo** En $x = b^y$, y se denomina logaritmo, base b, de x.
logarithmic equation An equation that contains one or more logarithms.	**ecuación logarítmica** Una ecuación que contiene uno o más logaritmos.
logarithmic function A function of the form $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$.	**función logarítmica** Una función de la forma $f(x) = \log_b x$, donde $b > 0$ y $b \neq 1$.
logically equivalent Statements with the same truth value.	**lógicamente equivalentes** Declaraciones con el mismo valor de verdad.
lower quartile The median of the lower half of a set of data.	**cuartil inferior** La mediana de la mitad inferior de un conjunto de datos.

M

magnitude The length of a vector from the initial point to the terminal point.	**magnitud** La longitud de un vector desde el punto inicial hasta el punto terminal.
magnitude of symmetry The smallest angle through which a figure can be rotated so that it maps onto itself.	**magnitud de la simetría** El ángulo más pequeño a través del cual una figura se puede girar para que se cargue sobre sí mismo.
major arc An arc with measure greater than 180°.	**arco mayor** Un arco con una medida superior a 180°.
mapping An illustration that shows how each element of the domain is paired with an element in the range.	**cartografía** Una ilustración que muestra cómo cada elemento del dominio está emparejado con un elemento del rango.
marginal frequencies In a two-way frequency table, the frequencies in the totals row and column; The totals of each subcategory in a two-way frequency table.	**frecuencias marginales** En una tabla de frecuencias de dos vías, las frecuencias en los totales de fila y columna; Los totales de cada subcategoría en una tabla de frecuencia bidireccional.
maximum The highest point on the graph of a function.	**máximo** El punto más alto en la gráfica de una función.
maximum error of the estimate The maximum difference between the estimate of the population mean and its actual value.	**error máximo de la estimación** La diferencia máxima entre la estimación de la media de la población y su valor real.
measurement data Data that have units and can be measured.	**medicion de datos** Datos que tienen unidades y que pueden medirse.
measures of center Measures of what is average.	**medidas del centro** Medidas de lo que es promedio.
measures of spread Measures of how spread out the data are.	**medidas de propagación** Medidas de cómo se extienden los datos son.
median The beginning of the second quartile that separates the data into upper and lower halves.	**mediana** El comienzo del segundo cuartil que separa los datos en mitades superior e inferior.
median of a triangle A line segment with endpoints that are a vertex of the triangle and the midpoint of the side opposite the vertex.	**mediana de un triángulo** Un segmento de línea con extremos que son un vértice del triángulo y el punto medio del lado opuesto al vértice.
metric A rule for assigning a number to some characteristic or attribute.	**métrico** Una regla para asignar un número a alguna caracteristica o atribuye.
midline The line about which the graph of a function oscillates.	**linea media** La línea sobre la cual oscila la gráfica de una función periódica.
midpoint The point on a line segment halfway between the endpoints of the segment.	**punto medio** El punto en un segmento de línea a medio camino entre los extremos del segmento.
midsegment of a trapezoid The segment that connects the midpoints of the legs of a trapezoid.	**segment medio de un trapecio** El segmento que conecta los puntos medios de las patas de un trapecio.
midsegment of a triangle The segment that connects the midpoints of the legs of a triangle.	**segment medio de un triángulo** El segmento que conecta los puntos medios de las patas de un triángulo.
minimum The lowest point on the graph of a function.	**mínimo** El punto más bajo en la gráfica de una función.

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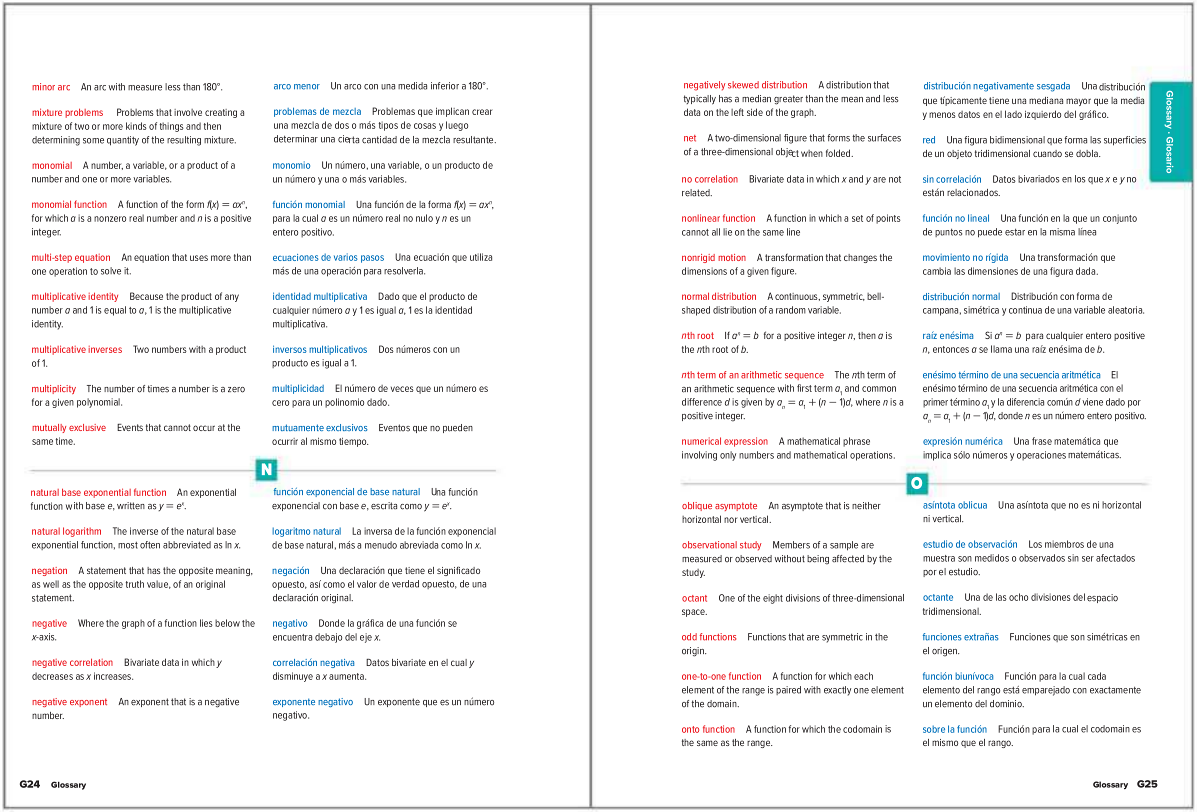
Glossary G23



minor arc An arc with measure less than 180°.	**arco menor** Un arco con una medida inferior a 180°.
mixture problems Problems that involve creating a mixture of two or more kinds of things and then determining some quantity of the resulting mixture.	**problemas de mezcla** Problemas que implican crear una mezcla de dos o más tipos de cosas y luego determinar una cierta cantidad de la mezcla resultante.
monomial A number, a variable, or a product of a number and one or more variables.	**monomio** Un número, una variable, o un producto de un número y una o más variables.
monomial function A function of the form $f(x) = ax^n$, for which a is a nonzero real number and n is a positive integer.	**función monomial** Una función de la forma $f(x) = ax^n$, para la cual a es un número real no nulo y n es un entero positivo.
multi-step equation An equation that uses more than one operation to solve it.	**ecuaciones de varios pasos** Una ecuación que utiliza más de una operación para resolverla.
multiplicative identity Because the product of any number a and 1 is equal to a, 1 is the multiplicative identity.	**identidad multiplicativa** Dado que el producto de cualquier número a y 1 es igual a, 1 es la identidad multiplicativa.
multiplicative inverses Two numbers with a product of 1.	**inversos multiplicativos** Dos números con un producto es igual a 1.
multiplicity The number of times a number is a zero for a given polynomial.	**multiplicidad** El número de veces que un número es cero para un polinomio dado.
mutually exclusive Events that cannot occur at the same time.	**mutuamente exclusivos** Eventos que no pueden ocurrir al mismo tiempo.

N

natural base exponential function An exponential function with base e, written as $y = e^x$.	**función exponencial de base natural** Una función exponencial con base e, escrita como $y = e^x$.
natural logarithm The inverse of the natural base exponential function, most often abbreviated as ln x.	**logaritmo natural** La inversa de la función exponencial de base natural, más a menudo abreviada como ln x.
negation A statement that has the opposite meaning, as well as the opposite truth value, of an original statement.	**negación** Una declaración que tiene el significado opuesto, así como el valor de verdad opuesto, de una declaración original.
negative Where the graph of a function lies below the x-axis.	**negativo** Donde la gráfica de una función se encuentra debajo del eje x.
negative correlation Bivariate data in which y decreases as x increases.	**correlación negativa** Datos bivariate en el cual y disminuye a x aumenta.
negative exponent An exponent that is a negative number.	**exponente negativo** Un exponente que es un número negativo.

negatively skewed distribution A distribution that typically has a median greater than the mean and less data on the left side of the graph.	**distribución negativamente sesgada** Una distribución que típicamente tiene una mediana mayor que la media y menos datos en el lado izquierdo del gráfico.
net A two-dimensional figure that forms the surfaces of a three-dimensional object when folded.	**red** Una figura bidimensional que forma las superficies de un objeto tridimensional cuando se dobla.
no correlation Bivariate data in which x and y are not related.	**sin correlación** Datos bivariados en los que x e y no están relacionados.
nonlinear function A function in which a set of points cannot all lie on the same line	**función no lineal** Una función en la que un conjunto de puntos no puede estar en la misma línea
nonrigid motion A transformation that changes the dimensions of a given figure.	**movimiento no rígida** Una transformación que cambia las dimensiones de una figura dada.
normal distribution A continuous, symmetric, bell-shaped distribution of a random variable.	**distribución normal** Distribución con forma de campana, simétrica y continua de una variable aleatoria.
***n*th root** If $a^n = b$ for a positive integer n, then a is the nth root of b.	**raíz enésima** Si $a^n = b$ para cualquier entero positive n, entonces a se llama una raíz enésima de b.
***n*th term of an arithmetic sequence** The nth term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.	**enésimo término de una secuencia aritmética** El enésimo término de una secuencia aritmética con el primer término a_1 y la diferencia común d viene dado por $a_n = a_1 + (n - 1)d$, donde n es un número entero positivo.
numerical expression A mathematical phrase involving only numbers and mathematical operations.	**expresión numérica** Una frase matemática que implica sólo números y operaciones matemáticas.

O

oblique asymptote An asymptote that is neither horizontal nor vertical.	**asíntota oblicua** Una asíntota que no es ni horizontal ni vertical.
observational study Members of a sample are measured or observed without being affected by the study.	**estudio de observación** Los miembros de una muestra son medidos o observados sin ser afectados por el estudio.
octant One of the eight divisions of three-dimensional space.	**octante** Una de las ocho divisiones del espacio tridimensional.
odd functions Functions that are symmetric in the origin.	**funciones extrañas** Funciones que son simétricas en el origen.
one-to-one function A function for which each element of the range is paired with exactly one element of the domain.	**función biunívoca** Función para la cual cada elemento del rango está emparejado con exactamente un elemento del dominio.
onto function A function for which the codomain is the same as the range.	**sobre la función** Función para la cual el codomain es el mismo que el rango.

open half-plane	The solution of a linear inequality that does not include the boundary line.	medio plano abierto	La solución de una desigualdad linear que no incluye la línea de límite.
opposite rays	Two collinear rays with a common endpoint.	rayos opuestos	Dos rayos colineales con un punto final común.
optimization	The process of seeking the optimal value of a function subject to given constraints.	optimización	El proceso de buscar el valor óptimo de una función sujeto a restricciones dadas.
order of symmetry	The number of times a figure maps onto itself.	orden de la simetría	El número de veces que una figura se asigna a sí misma.
ordered triple	Three numbers given in a specific order used to locate points in space.	triple ordenado	Tres números dados en un orden específico usado para localizar puntos en el espacio.
orthocenter	The point of concurrency of the altitudes of a triangle.	ortocentro	El punto de concurrencia de las altitudes de un triángulo.
orthographic drawing	The two-dimensional views of the top, left, front, and right sides of an object.	dibujo ortográfico	Las vistas bidimensionales de los lados superior, izquierdo, frontal y derecho de un objeto.
oscillation	How much the graph of a function varies between its extreme values as it approaches positive or negative infinity.	oscilación	Cuánto la gráfica de una función varía entre sus valores extremos cuando se acerca al infinito positivo o negativo.
outcome	The result of a single event; The result of a single performance or trial of an experiment.	resultado	El resultado de un solo evento; El resultado de un solo rendimiento o ensayo de un experimento.
outlier	A value that is more than 1.5 times the interquartile range above the third quartile or below the first quartile.	parte aislada	Un valor que es más de 1,5 veces el rango intercuartílico por encima del tercer cuartil o por debajo del primer cuartil.

P

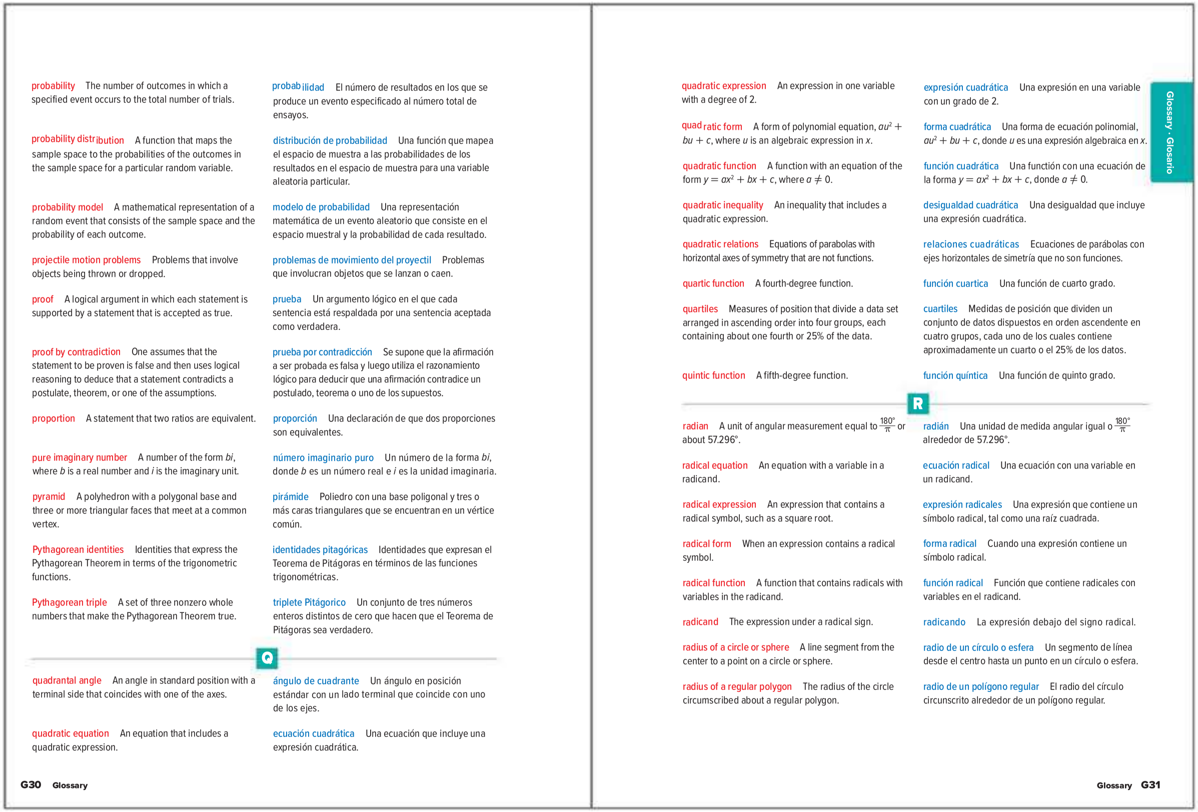
parabola	A curved shape that results when a cone is cut at an angle by a plane that intersects the base; The graph of a quadratic function.	parábola	Forma curvada que resulta cuando un cono es cortado en un ángulo por un plano que interseca la base; La gráfica de una función cuadrática.
paragraph proof	A paragraph that explains why the conjecture for a given situation is true.	prueba de párrafo	Un párrafo que explica por qué la conjetura para una situación dada es verdadera.
parallel lines	Coplanar lines that do not intersect; Nonvertical lines in the same plane that have the same slope.	líneas paralelas	Líneas coplanares que no se intersecan; Líneas no verticales en el mismo plano que tienen pendientes iguales.
parallel planes	Planes that do not intersect.	planos paralelas	Planos que no se intersecan.
parallelogram	A quadrilateral with both pairs of opposite sides parallel.	paralelogramo	Un cuadrilátero con ambos pares de lados opuestos paralelos.
parameter	A measure that describes a characteristic of a population; A value in the equation of a function that can be varied to yield a family of functions.	parámetro	Una medida que describe una característica de una población; Un valor en la ecuación de una función que se puede variar para producir una familia de funciones.
parent function	The simplest of functions in a family.	función basica	La función más fundamental de un familia de funciones.
Pascal's triangle	A triangle of numbers in which a row represents the coefficients of an expanded binomial $(a + b)^n$.	triángulo de Pascal	Un triángulo de números en el que una fila representa los coeficientes de un binomio expandido $(a + b)^n$.
percent rate of change	The percent of increase per time period.	por ciento tasa de cambio	El porcentaje de aumento por período de tiempo.
percentile	A measure that tells what percent of the total scores were below a given score.	percentil	Una medida que indica qué porcentaje de las puntuaciones totales estaban por debajo de una puntuación determinada.
perfect cube	A rational number with a cube root that is a rational number.	cubo perfecto	Un número racional con un raíz cúbica que es un número racional.
perfect square	A rational number with a square root that is a rational number.	cuadrado perfecto	Un número racional con un raíz cuadrada que es un número racional.
perfect square trinomials	Squares of binomials.	trinomio cuadrado perfecto	Cuadrados de los binomios.
perimeter	The sum of the lengths of the sides of a polygon.	perimetro	La suma de las longitudes de los lados de un polígono.
period	The horizontal length of one cycle.	periodo	La longitud horizontal de un ciclo.
periodic function	A function with y-values that repeat at regular intervals.	función periódica	Una función con y-valores aquella repetición con regularidad.
permutation	An arrangement of objects in which order is important.	permutación	Un arreglo de objetos en el que el orden es importante.
perpendicular	Intersecting at right angles.	perpendicular	Intersección en ángulo recto.
perpendicular bisector	Any line, segment, or ray that passes through the midpoint of a segment and is perpendicular to that segment.	mediatriz	Cualquier línea, segmento o rayo que pasa por el punto medio de un segmento y es perpendicular a ese segmento.
perpendicular lines	Nonvertical lines in the same plane for which the product of the slopes is -1.	líneas perpendiculares	Líneas no verticales en el mismo plano para las que el producto de las pendientes es -1.
phase shift	A horizontal translation of the graph of a trigonometric function.	cambio de fase	Una traducción horizontal de la gráfica de una función trigonométrica.
pi	The ratio $\frac{\text{circumference}}{\text{diameter}}$.	pi	Relación $\frac{\text{circunferencia}}{\text{diámetro}}$.

piecewise-defined function A function defined by at least two subfunctions, each of which is defined differently depending on the interval of the domain.	**función definida por piezas** Una función definida por al menos dos subfunciones, cada una de las cuales se define de manera diferente dependiendo del intervalo del dominio.
piecewise-linear function A function defined by at least two linear subfunctions, each of which is defined differently depending on the interval of the domain.	**función lineal por piezas** Una función definida por al menos dos subfunciones lineal, cada una de las cuales se define de manera diferente dependiendo del intervalo del dominio.
plane A flat surface made up of points that has no depth and extends indefinitely in all directions.	**plano** Una superficie plana compuesta de puntos que no tiene profundidad y se extiende indefinidamente en todas las direcciones.
plane symmetry When a plane intersects a three-dimensional figure so one half is the reflected image of the other half.	**simetría plana** Cuando un plano cruza una figura tridimensional, una mitad es la imagen reflejada de la otra mitad.
Platonic solid One of five regular polyhedra.	**sólido platónico** Uno de cinco poliedros regulares.
point A location with no size, only position.	**punto** Una ubicación sin tamaño, solo posición.
point discontinuity An area that appears to be a hole in a graph.	**discontinuidad de punto** Un área que parece ser un agujero en un gráfico.
point of concurrency The point of intersection of concurrent lines.	**punto de concurrencia** El punto de intersección de líneas concurrentes.
point of symmetry The point about which a figure is rotated.	**punto de simetría** El punto sobre el que se gira una figura.
point of tangency For a line that intersects a circle in one point, the point at which they intersect.	**punto de tangencia** Para una línea que cruza un círculo en un punto, el punto en el que se cruzan.
point symmetry A figure or graph has this when a figure is rotated 180° about a point and maps exactly onto the other part.	**simetría de punto** Una figura o gráfica tiene esto cuando una figura se gira 180° alrededor de un punto y se mapea exactamente sobre la otra parte.
polygon A closed plane figure with at least three straight sides.	**polígono** Una figura plana cerrada con al menos tres lados rectos.
polyhedron A closed three-dimensional figure made up of flat polygonal regions.	**poliedros** Una figura tridimensional cerrada formada por regiones poligonales planas.
polynomial A monomial or the sum of two or more monomials.	**polinomio** Un monomio o la suma de dos o más monomios.
polynomial function A continuous function that can be described by a polynomial equation in one variable.	**función polinómica** Función continua que puede describirse mediante una ecuación polinómica en una variable.

polynomial identity A polynomial equation that is true for any values that are substituted for the variables.	**identidad polinomial** Una ecuación polinómica que es verdadera para cualquier valor que se sustituya por las variables.
population All of the members of a group of interest about which data will be collected.	**población** Todos los miembros de un grupo de interés sobre cuáles datos serán recopilados.
population proportion The number of members in the population with a particular characteristic divided by the total number of members in the population.	**proporción de la población** El número de miembros en la población con una característica particular dividida por el número total de miembros en la población.
positive Where the graph of a function lies above the x-axis.	**positiva** Donde la gráfica de una función se encuentra por encima del eje x.
positive correlation Bivariate data in which y increases as x increases.	**correlación positiva** Datos bivariate en el cual y aumenta a x disminuye.
positively skewed distribution A distribution that typically has a mean greater than the median.	**distribución positivamente sesgada** Una distribución que típicamente tiene una media mayor que la mediana.
postulate A statement that is accepted as true without proof.	**postulado** Una declaración que se acepta como verdadera sin prueba.
power function A function of the form $f(x) = ax^n$, where a and n are nonzero real numbers.	**función de potencia** Una ecuación polinomial que es verdadera para una función de la forma $f(x) = ax^n$, donde a y n son números reales no nulos.
precision The repeatability, or reproducibility, of a measurement.	**precisión** La repetibilidad, o reproducibilidad, de una medida.
preimage The original figure in a transformation.	**preimagen** La figura original en una transformación.
prime polynomial A polynomial that cannot be written as a product of two polynomials with integer coefficients.	**polinomio primo** Un polinomio que no puede escribirse como producto de dos polinomios con coeficientes enteros.
principal root The nonnegative root of a number.	**raíz principal** La raíz no negativa de un número.
principal square root The nonnegative square root of a number.	**raíz cuadrada principal** La raíz cuadrada no negativa de un número.
principal values The values in the restricted domains of trigonometric functions.	**valores principales** Valores de los dominios restringidos de las functiones trigonométricas.
principle of superposition Two figures are congruent if and only if there is a rigid motion or series of rigid motions that maps one figure exactly onto the other.	**principio de superposición** Dos figuras son congruentes si y sólo si hay un movimiento rígido o una serie de movimientos rígidos que traza una figura exactamente sobre la otra.
prism A polyhedron with two parallel congruent bases connected by parallelogram faces.	**prisma** Un poliedro con dos bases congruentes paralelas conectadas por caras de paralelogramo.

GLOSSARY

G30-G31 Glossary



Glossary · Glosario

probability	The number of outcomes in which a specified event occurs to the total number of trials.	probabilidad	El número de resultados en los que se produce un evento especificado al número total de ensayos.
probability distribution	A function that maps the sample space to the probabilities of the outcomes in the sample space for a particular random variable.	distribución de probabilidad	Una función que mapea el espacio de muestra a las probabilidades de los resultados en el espacio de muestra para una variable aleatoria particular.
probability model	A mathematical representation of a random event that consists of the sample space and the probability of each outcome.	modelo de probabilidad	Una representación matemática de un evento aleatorio que consiste en el espacio muestral y la probabilidad de cada resultado.
projectile motion problems	Problems that involve objects being thrown or dropped.	problemas de movimiento del proyectil	Problemas que involucran objetos que se lanzan o caen.
proof	A logical argument in which each statement is supported by a statement that is accepted as true.	prueba	Un argumento lógico en el que cada sentencia está respaldada por una sentencia aceptada como verdadera.
proof by contradiction	One assumes that the statement to be proven is false and then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions.	prueba por contradicción	Se supone que la afirmación a ser probada es falsa y luego utiliza el razonamiento lógico para deducir que una afirmación contradice un postulado, teorema o uno de los supuestos.
proportion	A statement that two ratios are equivalent.	proporción	Una declaración de que dos proporciones son equivalentes.
pure imaginary number	A number of the form bi, where b is a real number and i is the imaginary unit.	número imaginario puro	Un número de la forma bi, donde b es un número real e i es la unidad imaginaria.
pyramid	A polyhedron with a polygonal base and three or more triangular faces that meet at a common vertex.	pirámide	Poliedro con una base poligonal y tres o más caras triangulares que se encuentran en un vértice común.
Pythagorean identities	Identities that express the Pythagorean Theorem in terms of the trigonometric functions.	identidades pitagóricas	Identidades que expresan el Teorema de Pitágoras en términos de las funciones trigonométricas.
Pythagorean triple	A set of three nonzero whole numbers that make the Pythagorean Theorem true.	triplete Pitágorico	Un conjunto de tres números enteros distintos de cero que hacen que el Teorema de Pitágoras sea verdadero.

Q

quadrantal angle	An angle in standard position with a terminal side that coincides with one of the axes.	ángulo de cuadrante	Un ángulo en posición estándar con un lado terminal que coincide con uno de los ejes.
quadratic equation	An equation that includes a quadratic expression.	ecuación cuadrática	Una ecuación que incluye una expresión cuadrática.

quadratic expression	An expression in one variable with a degree of 2.	expresión cuadrática	Una expresión en una variable con un grado de 2.
quadratic form	A form of polynomial equation, $au^2 + bu + c$, where u is an algebraic expression in x.	forma cuadrática	Una forma de ecuación polinomial, $au^2 + bu + c$, donde u es una expresión algebraica en x.
quadratic function	A function with an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.	función cuadrática	Una función con una ecuación de la forma $y = ax^2 + bx + c$, donde $a \neq 0$.
quadratic inequality	An inequality that includes a quadratic expression.	desigualdad cuadrática	Una desigualdad que incluye una expresión cuadrática.
quadratic relations	Equations of parabolas with horizontal axes of symmetry that are not functions.	relaciones cuadráticas	Ecuaciones de parábolas con ejes horizontales de simetría que no son funciones.
quartic function	A fourth-degree function.	función cuartica	Una función de cuarto grado.
quartiles	Measures of position that divide a data set arranged in ascending order into four groups, each containing about one fourth or 25% of the data.	cuartiles	Medidas de posición que dividen un conjunto de datos dispuestos en orden ascendente en cuatro grupos, cada uno de los cuales contiene aproximadamente un cuarto o el 25% de los datos.
quintic function	A fifth-degree function.	función quíntica	Una función de quinto grado.

R

radian	A unit of angular measurement equal to $\frac{180°}{\pi}$ or about 57.296°.	radián	Una unidad de medida angular igual o $\frac{180°}{\pi}$ alrededor de 57.296°.
radical equation	An equation with a variable in a radicand.	ecuación radical	Una ecuación con una variable en un radicand.
radical expression	An expression that contains a radical symbol, such as a square root.	expresión radicales	Una expresión que contiene un símbolo radical, tal como una raíz cuadrada.
radical form	When an expression contains a radical symbol.	forma radical	Cuando una expresión contiene un símbolo radical.
radical function	A function that contains radicals with variables in the radicand.	función radical	Función que contiene radicales con variables en el radicand.
radicand	The expression under a radical sign.	radicando	La expresión debajo del signo radical.
radius of a circle or sphere	A line segment from the center to a point on a circle or sphere.	radio de un círculo o esfera	Un segmento de línea desde el centro hasta un punto en un círculo o esfera.
radius of a regular polygon	The radius of the circle circumscribed about a regular polygon.	radio de un polígono regular	El radio del círculo circunscrito alrededor de un polígono regular.

range The difference between the greatest and least values in a set of data; The set of second numbers of the ordered pairs in a relation; The set of y-values that actually result from the evaluation of the function.	**rango** La diferencia entre los valores de datos más grande or menos en un sistema de datos; El conjunto de los segundos números de los pares ordenados de una relación; El conjunto de valores y que realmente resultan de la evaluación de la función.
rate of change How a quantity is changing with respect to a change in another quantity.	**tasa de cambio** Cómo cambia una cantidad con respecto a un cambio en otra cantidad.
rational equation An equation that contains at least one rational expression.	**ecuación racional** Una ecuación que contiene al menos una expresión racional.
rational exponent An exponent that is expressed as a fraction.	**exponente racional** Un exponente que se expresa como una fracción.
rational expression A ratio of two polynomial expressions.	**expresión racional** Una relación de dos expresiones polinomiales.
rational function An equation of the form $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial expressions and $b(x) \neq 0$.	**función racional** Una ecuación de la forma $f(x) = \frac{a(x)}{b(x)}$, donde $a(x)$ y $b(x)$ son expresiones polinomiales y $b(x) \neq 0$.
rational inequality An inequality that contains at least one rational expression.	**desigualdad racional** Una desigualdad que contiene al menos una expresión racional.
rationalizing the denominator A method used to eliminate radicals from the denominator of a fraction or fractions from a radicand.	**racionalizando el denominador** Método utilizado para eliminar radicales del denominador de una fracción o fracciones de una radicand.
ray Part of a line that starts at a point and extends to infinity.	**rayo** Parte de una línea que comienza en un punto y se extiende hasta el infinito.
reciprocal function An equation of the form $f(x) = \frac{n}{b(x)}$, where n is a real number and $b(x)$ is a linear expression that cannot equal 0.	**función recíproca** Una ecuación de la forma $f(x) = \frac{n}{b(x)}$, donde n es un número real y $b(x)$ es una expresión lineal que no puede ser igual a 0.
reciprocal trigonometric functions Trigonometric functions that are reciprocals of each other.	**funciones trigonométricas recíprocas** Funciones trigonométricas que son reciprocales entre sí.
reciprocals Two numbers with a product of 1.	**recíprocos** Dos números con un producto de 1.
rectangle A parallelogram with four right angles.	**rectángulo** Un paralelogramo con cuatro ángulos rectos.
recursive formula A formula that gives the value of the first term in the sequence and then defines the next term by using the preceding term.	**formula recursiva** Una fórmula que da el valor del primer término en la secuencia y luego define el siguiente término usando el término anterior.
reduction A dilation with a scale factor between 0 and 1.	**reducción** Una dilatación con un factor de escala entre 0 y 1.
reference angle The acute angle formed by the terminal side of an angle and the x-axis.	**ángulo de referencia** El ángulo agudo formado por el lado terminal de un ángulo en posición estándar y el eje x.

reflection A function in which the preimage is reflected in the line of reflection; A transformation in which a figure, line, or curve is flipped across a line.	**reflexión** Función en la que la preimagen se refleja en la línea de reflexión; Una transformación en la que una figura, línea o curva se voltea a través de una línea.
regression function A function generated by an algorithm to find a line or curve that fits a set of data.	**función de regresión** Función generada por un algoritmo para encontrar una línea o curva que se ajuste a un conjunto de datos.
regular polygon A convex polygon that is both equilateral and equiangular.	**polígono regular** Un polígono convexo que es a la vez equilátero y equiangular.
regular polyhedron A polyhedron in which all of its faces are regular congruent polygons and all of the edges are congruent.	**poliedro regular** Un poliedro en el que todas sus caras son polígonos congruentes regulares y todos los bordes son congruentes.
regular pyramid A pyramid with a base that is a regular polygon.	**pirámide regular** Una pirámide con una base que es un polígono regular.
regular tessellation A tessellation formed by only one type of regular polygon.	**teselado regular** Un teselado formado por un solo tipo de polígono regular.
relation A set of ordered pairs.	**relación** Un conjunto de pares ordenados.
relative frequency In a two-way frequency table, the ratios of the number of observations in a category to the total number of observations; The ratio of the number of observations in a category to the total number of observations.	**frecuencia relativa** En una tabla de frecuencia bidireccional, las relaciones entre el número de observaciones en una categoría y el número total de observaciones; La relación entre el número de observaciones en una categoría y el número total de observaciones.
relative maximum A point on the graph of a function where no other nearby points have a greater y-coordinate.	**máximo relativo** Un punto en la gráfica de una función donde ningún otro punto cercano tiene una coordenada y mayor.
relative minimum A point on the graph of a function where no other nearby points have a lesser y-coordinate.	**mínimo relativo** Un punto en la gráfica de una función donde ningún otro punto cercano tiene una coordenada y menor.
remote interior angles Interior angles of a triangle that are not adjacent to an exterior angle.	**ángulos internos no adyacentes** Ángulos interiores de un triángulo que no están adyacentes a un ángulo exterior.
residual The difference between an observed y-value and its predicted y-value on a regression line.	**residual** La diferencia entre un valor de y observado y su valor de y predicho en una línea de regresión.
rhombus A parallelogram with all four sides congruent.	**rombo** Un paralelogramo con los cuatro lados congruentes.
rigid motion A transformation that preserves distance and angle measure.	**movimiento rígido** Una transformación que preserva la distancia y la medida del ángulo.

root A solution of an equation.	**raíz** Una solución de una ecuación.
rotation A function that moves every point of a preimage through a specified angle and direction about a fixed point.	**rotación** Función que mueve cada punto de una preimagen a través de un ángulo y una dirección especificados alrededor de un punto fijo.
rotational symmetry A figure can be rotated less than 360° about a point so that the image and the preimage are indistinguishable.	**simetría rotacional** Una figura puede girar menos de 360° alrededor de un punto para que la imagen y la preimagen sean indistinguibles.

S

sample A subset of a population.	**muestra** Un subconjunto de una población.
sample space The set of all possible outcomes.	**espacio muestral** El conjunto de todos los resultados posibles.
sampling error The variation between samples taken from the same population.	**error de muestreo** La variación entre muestras tomadas de la misma población.
scale The distance between tick marks on the *x*- and *y*-axes.	**escala** La distancia entre las marcas en los ejes *x* e *y*.
scale factor of a dilation The ratio of a length on an image to a corresponding length on the preimage.	**factor de escala de una dilatación** Relación de una longitud en una imagen con una longitud correspondiente en la preimagen.
scatter plot A graph of bivariate data that consists of ordered pairs on a coordinate plane.	**gráfica de dispersión** Una gráfica de datos bivariados que consiste en pares ordenados en un plano de coordenadas.
secant Any line or ray that intersects a circle in exactly two points; The ratio of the length of the hypotenuse to the length of the leg adjacent to the angle.	**secante** Cualquier línea o rayo que cruce un círculo en exactamente dos puntos; Relación entre la longitud de la hipotenusa y la longitud de la pierna adyacente al ángulo.
sector A region of a circle bounded by a central angle and its intercepted arc.	**sector** Una región de un círculo delimitada por un ángulo central y su arco interceptado.
segment bisector Any segment, line, plane, or point that intersects a line segment at its midpoint.	**bisectriz del segmento** Cualquier segmento, línea, plano o punto que interseca un segmento de línea en su punto medio.
self-selected sample Members volunteer to be included in the sample.	**muestra auto-seleccionada** Los miembros se ofrecen como voluntarios para ser incluidos en la muestra.
semicircle An arc that measures exactly 180°.	**semicírculo** Un arco que mide exactamente 180°.
semiregular tessellation A tessellation formed by two or more regular polygons.	**teselado semiregular** Un teselado formado por dos o más polígonos regulares.
sequence A list of numbers in a specific order.	**secuencia** Una lista de números en un orden específico.

series The indicated sum of the terms in a sequence.	**serie** La suma indicada de los términos en una secuencia.
set-builder notation Mathematical notation that describes a set by stating the properties that its members must satisfy.	**notación de construción de conjuntos** Notación matemática que describe un conjunto al declarar las propiedades que sus miembros deben satisfacer.
sides of an angle The rays that form an angle.	**lados de un ángulo** Los rayos que forman un ángulo.
sigma notation A notation that uses the Greek uppercase letter S to indicate that a sum should be found.	**notación de sigma** Una notación que utiliza la letra mayúscula griega S para indicar que debe encontrarse una suma.
significant figures The digits of a number that are used to express a measure to an appropriate degree of accuracy.	**dígitos significantes** Los dígitos de un número que se utilizan para expresar una medida con un grado apropiado de precisión.
similar polygons Two figures are similar polygons if one can be obtained from the other by a dilation or a dilation with one or more rigid motions.	**polígonos similares** Dos figuras son polígonos similares si uno puede ser obtenido del otro por una dilatación o una dilatación con uno o más movimientos rígidos.
similar solids Solid figures with the same shape but not necessarily the same size.	**sólidos similares** Figuras sólidas con la misma forma pero no necesariamente del mismo tamaño.
similar triangles Triangles in which all of the corresponding angles are congruent and all of the corresponding sides are proportional.	**triángulos similares** Triángulos en los cuales todos los ángulos correspondientes son congruentes y todos los lados correspondientes son proporcionales.
similarity ratio The scale factor between two similar polygons.	**relación de similitud** El factor de escala entre dos polígonos similares.
similarity transformation A transformation composed of a dilation or a dilation and one or more rigid motions.	**transformación de similitud** Una transformación compuesto por una dilatación o una dilatación y uno o más movimientos rígidos.
simple random sample Each member of the population has an equal chance of being selected as part of the sample.	**muestra aleatoria simple** Cada miembro de la población tiene la misma posibilidad de ser seleccionado como parte de la muestra.
simplest form An expression is in simplest form when it is replaced by an equivalent expression having no like terms or parentheses.	**forma reducida** Una expresión está reducida cuando se puede sustituir por una expresión equivalente que no tiene ni términos semejantes ni paréntesis.
simulation The use of a probability model to imitate a process or situation so it can be studied.	**simulación** El uso de un modelo de probabilidad para imitar un proceso o situación para que pueda ser estudiado.
sine The ratio of the length of the leg opposite an angle to the length of the hypotenuse.	**seno** La relación entre la longitud de la pierna opuesta a un ángulo y la longitud de la hipotenusa.

sinusoidal function	A function that can be produced by translating, reflecting, or dilating the sine function.	función sinusoidal	Función que puede producirse traduciendo, reflejando o dilatando la función sinusoidal.
skew lines	Noncoplanar lines that do not intersect.	líneas alabeadas	Líneas no coplanares que no se cruzan.
slant height of a pyramid or right cone	The length of a segment with one endpoint on the base edge of the figure and the other at the vertex.	altura inclinada de una pirámide o cono derecho	La longitud de un segmento con un punto final en el borde base de la figura y el otro en el vértice.
slope	The rate of change in the y-coordinates (rise) to the corresponding change in the x-coordinates (run) for points on a line.	pendiente	La tasa de cambio en las coordenadas y (subida) al cambio correspondiente en las coordenadas x (carrera) para puntos en una línea.
slope criteria	Outlines a method for proving the relationship between lines based on a comparison of the slopes of the lines.	criterios de pendiente	Describe un método para probar la relación entre líneas basado en una comparación de las pendientes de las líneas.
solid of revolution	A solid figure obtained by rotating a shape around an axis.	sólido de revolución	Una figura sólida obtenida girando una forma alrededor de un eje.
solution	A value that makes an equation true.	solución	Un valor que hace que una ecuación sea verdadera.
solve an equation	The process of finding all values of the variable that make the equation a true statement.	resolver una ecuación	El proceso en que se hallan todos los valores de la variable que hacen verdadera la ecuación.
solving a triangle	When you are given measurements to find the unknown angle and side measures of a triangle.	resolver un triángulo	Cuando se le dan mediciones para encontrar el ángulo desconocido y las medidas laterales de un triángulo.
space	A boundless three-dimensional set of all points.	espacio	Un conjunto tridimensional ilimitado de todos los puntos.
sphere	A set of all points in space equidistant from a given point called the center of the sphere.	esfera	Un conjunto de todos los puntos del espacio equidistantes de un punto dado llamado centro de la esfera.
square	A parallelogram with all four sides and all four angles congruent.	cuadrado	Un paralelogramo con los cuatro lados y los cuatro ángulos congruentes.
square root	One of two equal factors of a number.	raíz cuadrada	Uno de dos factores iguales de un número.
square root function	A radical function that contains the square root of a variable expression.	función raíz cuadrada	Función radical que contiene la raíz cuadrada de una expresión variable.
square root inequality	An inequality that contains the square root of a variable expression.	square root inequality	Una desigualdad que contiene la raíz cuadrada de una expresión variable.
standard deviation	A measure that shows how data deviate from the mean.	desviación tipica	Una medida que muestra cómo los datos se desvían de la media.

standard error of the mean	The standard deviation of the distribution of sample means taken from a population.	error estandar de la media	La desviación estándar de la distribución de los medios de muestra se toma de una población.
standard form of a linear equation	Any linear equation can be written in this form, $Ax + By = C$, where $A \geq 0$, A and B are not both 0, and A, B, and C are integers with a greatest common factor of 1.	forma estándar de una ecuación lineal	Cualquier ecuación lineal se puede escribir de esta forma, $Ax + By = C$, donde $A \geq 0$, A y B no son ambos 0, y A, B y C son enteros con el mayor factor común de 1.
standard form of a polynomial	A polynomial that is written with the terms in order from greatest degree to least degree.	forma estándar de un polinomio	Un polinomio que se escribe con los términos en orden del grado más grande a menos grado.
standard form of a quadratic equation	A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, and c are integers.	forma estándar de una ecuación cuadrática	Una ecuación cuadrática puede escribirse en la forma $ax^2 + bx + c = 0$, donde $a \neq 0$ y a, b, y c son enteros.
standard normal distribution	A normal distribution with a mean of 0 and a standard deviation of 1.	distribución normal estándar	Distribución normal con una media de 0 y una desviación estándar de 1.
standard position	An angle positioned so that the vertex is at the origin and the initial side is on the positive x-axis.	posición estándar	Un ángulo colocado de manera que el vértice está en el origen y el lado inicial está en el eje x positivo.
statement	Any sentence that is either true or false, but not both.	enunciado	Cualquier oración que sea verdadera o falsa, pero no ambas.
statistic	A measure that describes a characteristic of a sample.	estadística	Una medida que describe una característica de una muestra.
statistics	An area of mathematics that deals with collecting, analyzing, and interpreting data.	estadísticas	El proceso de recolección, análisis e interpretación de datos.
step function	A type of piecewise-linear function with a graph that is a series of horizontal line segments.	función escalonada	Un tipo de función lineal por piezas con un gráfico que es una serie de segmentos de línea horizontal.
straight angle	An angle that measures 180°.	ángulo recto	Un ángulo que mide 180°.
stratified sample	The population is first divided into similar, nonoverlapping groups. Then members are randomly selected from each group.	muestra estratificada	La población se divide primero en grupos similares, sin superposición. A continuación, los miembros se seleccionan aleatoriamente de cada grupo.
substitution	A process of solving a system of equations in which one equation is solved for one variable in terms of the other.	sustitución	Un proceso de resolución de un sistema de ecuaciones en el que una ecuación se resuelve para una variable en términos de la otra.
supplementary angles	Two angles with measures that have a sum of 180°.	ángulos suplementarios	Dos ángulos con medidas que tienen una suma de 180°.

surface area The sum of the areas of all faces and side surfaces of a three-dimensional figure.	**área de superficie** La suma de las áreas de todas las caras y superficies laterales de una figura tridimensional.
survey Data are collected from responses given by members of a group regarding their characteristics, behaviors, or opinions.	**encuesta** Los datos se recogen de las respuestas dadas por los miembros de un grupo con respecto a sus características, comportamientos u opiniones.
symmetric distribution A distribution in which the mean and median are approximately equal.	**distribución simétrica** Un distribución en la que la media y la mediana son aproximadamente iguales.
symmetry A figure has this if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself.	**simetría** Una figura tiene esto si existe una reflexión-reflexión, una traducción, una rotación o una reflexión de deslizamiento rígida-que mapea la figura sobre sí misma.
synthetic division An alternate method used to divide a polynomial by a binomial of degree 1.	**división sintética** Un método alternativo utilizado para dividir un polinomio por un binomio de grado 1.
synthetic geometry The study of geometric figures without the use of coordinates.	**geometría sintética** El estudio de figuras geométricas sin el uso de coordenadas.
synthetic substitution The process of using synthetic division to find a value of a polynomial function.	**sustitución sintética** El proceso de utilizar la división sintética para encontrar un valor de una función polynomial.
system of equations A set of two or more equations with the same variables.	**sistema de ecuaciones** Un conjunto de dos o más ecuaciones con las mismas variables.
system of inequalities A set of two or more inequalities with the same variables.	**sistema de desigualdades** Un conjunto de dos o más desigualdades con las mismas variables.
systematic sample Members are selected according to a specified interval from a random starting point.	**muestra sistemática** Los miembros se seleccionan de acuerdo con un intervalo especificado desde un punto de partida aleatorio.

T

tangent The ratio of the length of the leg opposite an angle to the length of the leg adjacent to the angle.	**tangente** La relación entre la longitud de la pata opuesta a un ángulo y la longitud de la pata adyacente al ángulo.
tangent to a circle A line or segment in the plane of a circle that intersects the circle in exactly one point and does not contain any points in the interior of the circle.	**tangente a un círculo** Una línea o segmento en el plano de un círculo que interseca el círculo en exactamente un punto y no contiene ningún punto en el interior del círculo.
tangent to a sphere A line that intersects the sphere in exactly one point.	**tangente a una esfera** Una línea que interseca la esfera exactamente en un punto.
term A number, a variable, or a product or quotient of numbers and variables.	**término** Un número, una variable, o un producto o cociente de números y variables.

term of a sequence A number in a sequence.	**término de una sucesión** Un número en una secuencia.
terminal side The part of an angle that rotates about the center.	**lado terminal** La parte de un ángulo que gira alrededor de un centro.
tessellation A repeating pattern of one or more figures that covers a plane with no overlapping or empty spaces.	**teselado** Patrón repetitivo de una o más figuras que cubre un plano sin espacios superpuestos o vacíos.
theorem A statement that can be proven true using undefined terms, definitions, and postulates.	**teorema** Una afirmación o conjetura que se puede probar verdad utilizando términos, definiciones y postulados indefinidos.
theoretical probability Probability based on what is expected to happen.	**probabilidad teórica** Probabilidad basada en lo que se espera que suceda.
transformation A function that takes points in the plane as inputs and gives other points as outputs. The movement of a graph on the coordinate plane.	**transformación** Función que toma puntos en el plano como entradas y da otros puntos como salidas. El movimiento de un gráfico en el plano de coordenadas.
translation A function in which all of the points of a figure move the same distance in the same direction; A transformation in which a figure is slid from one position to another without being turned.	**traslación** Función en la que todos los puntos de una figura se mueven en la misma dirección; El movimiento de un gráfico en el plano de coordenadas.
translation vector A directed line segment that describes both the magnitude and direction of the slide if the magnitude is the length of the vector from its initial point to its terminal point.	**vector de traslación** Un segmento de línea dirigido que describe tanto la magnitud como la dirección de la diapositiva si la magnitud es la longitud del vector desde su punto inicial hasta su punto terminal.
transversal A line that intersects two or more lines in a plane at different points.	**transversal** Una línea que interseca dos o más líneas en un plano en diferentes puntos.
trapezoid A quadrilateral with exactly one pair of parallel sides.	**trapecio** Un cuadrilátero con exactamente un par de lados paralelos.
trend A general pattern in the data.	**tendencia** Un patrón general en los datos.
trigonometric equation An equation that includes at least one trigonometric function.	**ecuación trigonométrica** Una ecuación que incluye al menos una función trigonométrica.
trigonometric function A function that relates the measure of one nonright angle of a right triangle to the ratios of the lengths of any two sides of the triangle.	**función trigonométrica** Función que relaciona la medida de un ángulo no recto de un triángulo rectángulo con las relaciones de las longitudes de cualquiera de los dos lados del triángulo.
trigonometric identity An equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.	**identidad trigonométrica** Una ecuación que implica funciones trigonométricas que es verdadera para todos los valores para los cuales se define cada expresión en la ecuación.

trigonometric ratio A ratio of the lengths of two sides of a right triangle.	**relación trigonométrica** Una relación de las longitudes de dos lados de un triángulo rectángulo.
trigonometry The study of the relationships between the sides and angles of triangles.	**trigonometría** El estudio de las relaciones entre los lados y los ángulos de los triángulos.
trinomial The sum of three monomials.	**trinomio** La suma de tres monomios.
truth value The truth or falsity of a statement.	**valor de verdad** La verdad o la falsedad de una declaración.
two-column proof A proof that contains statements and reasons organized in a two-column format.	**prueba de dos columnas** Una prueba que contiene declaraciones y razones organizadas en un formato de dos columnas.
two-way frequency table A table used to show frequencies of data classified according to two categories, with the rows indicating one category and the columns indicating the other.	**tabla de frecuencia bidireccional** Una tabla utilizada para mostrar las frecuencias de los datos clasificados de acuerdo con dos categorías, con las filas que indican una categoría y las columnas que indican la otra.
two-way relative frequency table A table used to show frequencies of data based on a percentage of the total number of observations.	**tabla de frecuencia relativa bidireccional** Una tabla usada para mostrar las frecuencias de datos basadas en un porcentaje del número total de observaciones.

U

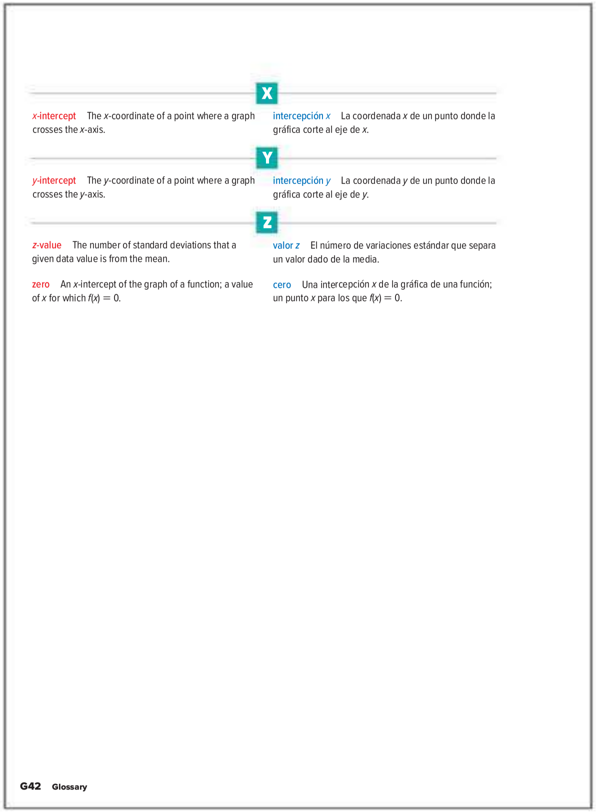
unbounded When the graph of a system of constraints is open.	**no acotado** Cuando la gráfica de un sistema de restricciones está abierta.
undefined terms Words that are not formally explained by means of more basic words and concepts.	**términos indefinidos** Palabras que no se explican formalmente mediante palabras y conceptos más básicos.
uniform motion problems Problems that use the formula $d = rt$, where d is the distance, r is the rate, and t is the time.	**problemas de movimiento uniforme** Problemas que utilizan la fórmula $d = rt$, donde d es la distancia, r es la velocidad y t es el tiempo.
uniform tessellation A tessellation that contains the same arrangement of shapes and angles at each vertex.	**teselado uniforme** Un teselado que contiene la misma disposición de formas y ángulos en cada vértice.
union The graph of a compound inequality containing *or*.	**unión** La gráfica de una desigualdad compuesta que contiene la palabra *o*.
union of *A* and *B* The set of all outcomes in the sample space of event *A* combined with all outcomes in the sample space of event *B*.	**unión de *A* y *B*** El conjunto de todos los resultados en el espacio muestral del evento *A* combinado con todos los resultados en el espacio muestral del evento *B*.
unit circle A circle with a radius of 1 unit centered at the origin on the coordinate plane.	**círculo unitario** Un círculo con un radio de 1 unidad centrado en el origen en el plano de coordenadas.
univariate data Measurement data in one variable.	**datos univariate** Datos de medición en una variable.
upper quartile The median of the upper half of a set of data.	**cuartil superior** La mediana de la mitad superior de un conjunto de datos.

V

valid argument An argument is valid if it is impossible for all of the premises, or supporting statements, of the argument to be true and its conclusion false.	**argumento válido** Un argumento es válido si es imposible que todas las premisas o argumentos de apoyo del argumento sean verdaderos y su conclusión sea falsa.
variable A letter used to represent an unspecified number or value; Any characteristic, number, or quantity that can be counted or measured.	**variable** Una letra utilizada para representar un número o valor no especificado; Cualquier característica, número, o cantidad que pueda ser contada o medida.
variable term A term that contains a variable.	**término variable** Un término que contiene una variable.
variance The square of the standard deviation.	**varianza** El cuadrado de la desviación estándar.
vertex Either the lowest point or the highest point of a function.	**vértice** El punto más bajo o el punto más alto en una función.
vertex angle of an isosceles triangle The angle between the sides that are the legs of an isosceles triangle.	**ángulo del vértice de un triángulo isósceles** El ángulo entre los lados que son las patas de un triángulo isósceles.
vertex form A quadratic function written in the form $f(x) = a(x - h)^2 + k$.	**forma de vértice** Una función cuadrática escribirse de la forma $f(x) = a(x - h)^2 + k$.
vertex of a polyhedron The intersection of three edges of a polyhedron.	**vértice de un polígono** La intersección de tres bordes de un poliedro.
vertex of an angle The common endpoint of the two rays that form an angle.	**vértice de un ángulo** El punto final común de los dos rayos que forman un ángulo.
vertical angles Two nonadjacent angles formed by two intersecting lines.	**ángulos verticales** Dos ángulos no adyacentes formados por dos líneas de intersección.
vertical asymptote A vertical line that a graph approaches.	**asíntota vertical** Una línea vertical que se aproxima a un gráfico.
vertical shift A vertical translation of the graph of a trigonometric function.	**cambio vertical** Una traducción vertical de la gráfica de una función trigonométrica.
volume The measure of the amount of space enclosed by a three-dimensional figure.	**volumen** La medida de la cantidad de espacio encerrada por una figura tridimensional.

W

work problems Problems that involve two people working at different rates who are trying to complete a single job.	**problemas de trabajo** Problemas que involucran a dos personas trabajando a diferentes ritmos que están tratando de completar un solo trabajo.



X

x-intercept The *x*-coordinate of a point where a graph crosses the *x*-axis.	intercepción *x* La coordenada *x* de un punto donde la gráfica corte al eje de *x*.

Y

y-intercept The *y*-coordinate of a point where a graph crosses the *y*-axis.	intercepción *y* La coordenada *y* de un punto donde la gráfica corte al eje de *y*.

Z

z-value The number of standard deviations that a given data value is from the mean.	valor *z* El número de variaciones estándar que separa un valor dado de la media.
zero An *x*-intercept of the graph of a function; a value of *x* for which $f(x) = 0$.	cero Una intercepción *x* de la gráfica de una función; un punto *x* para los que $f(x) = 0$.

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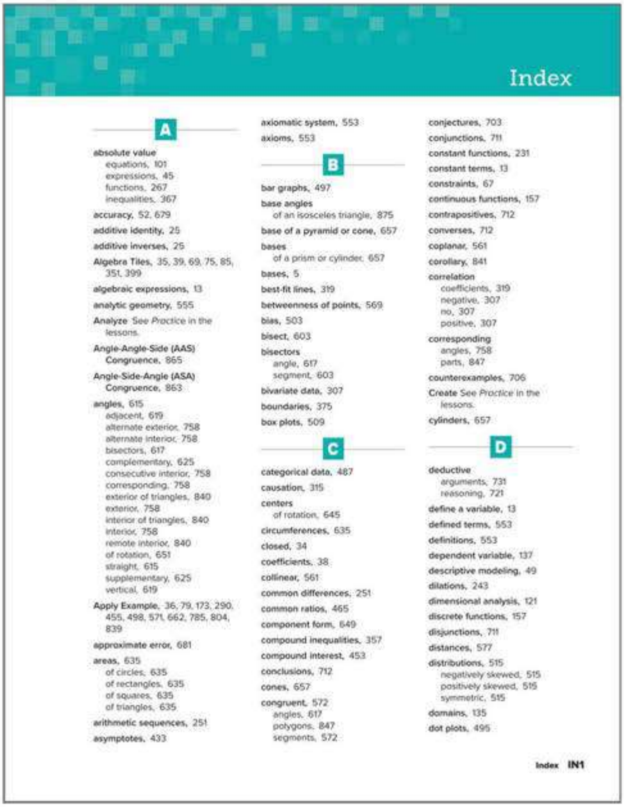

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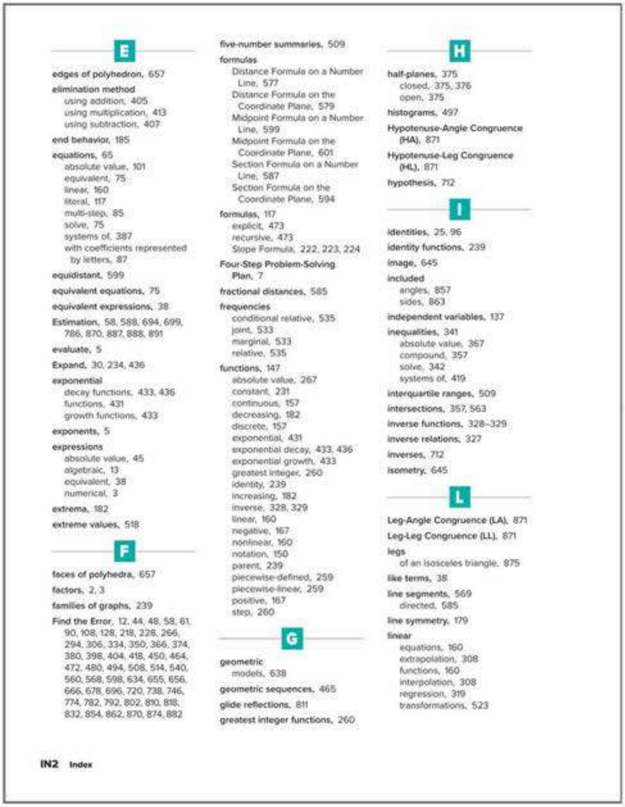

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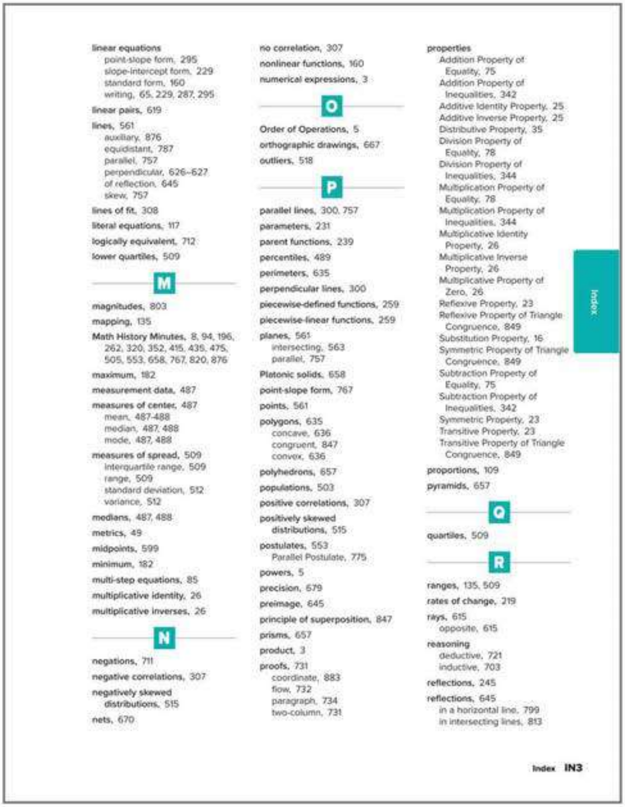

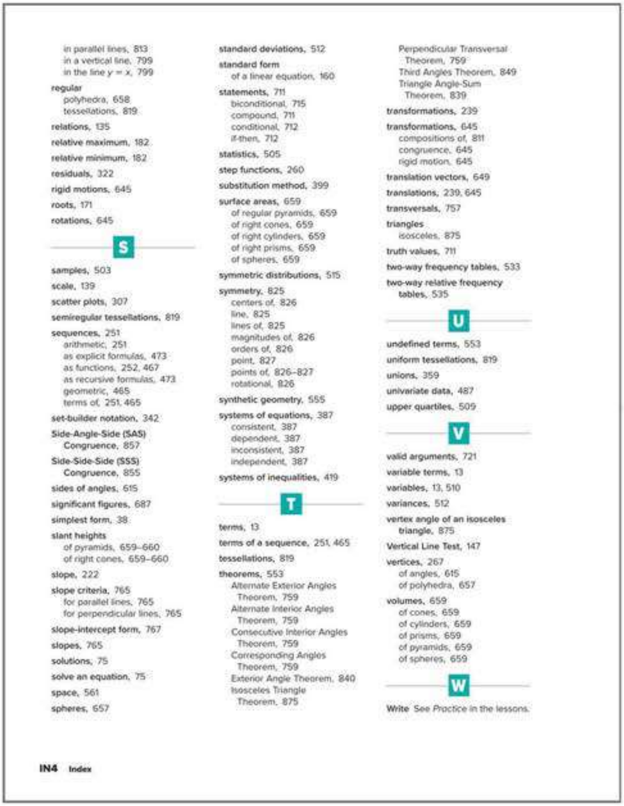

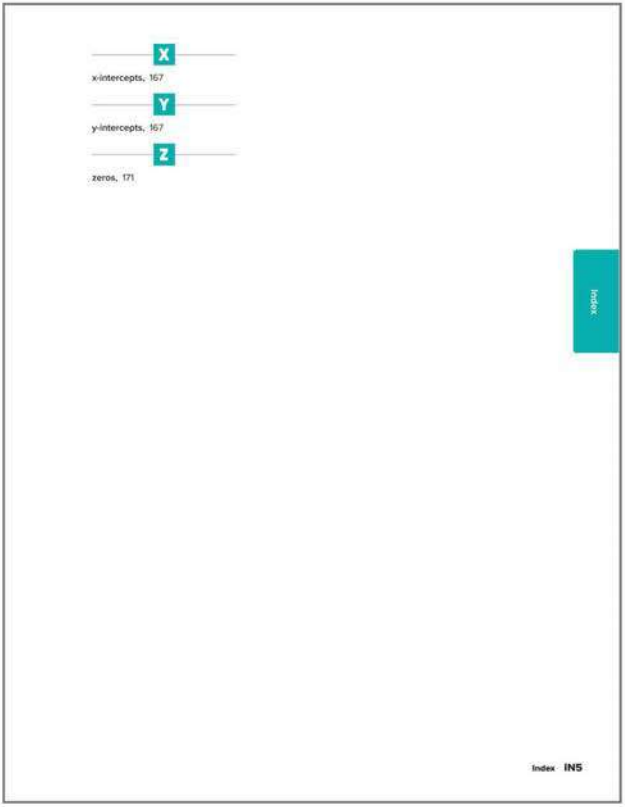

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MHID 0-07-700693-3
EAN
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