

Lesson 2: The concept of limit.

A limit is a value that a function approaches as the input approaches a specific value.

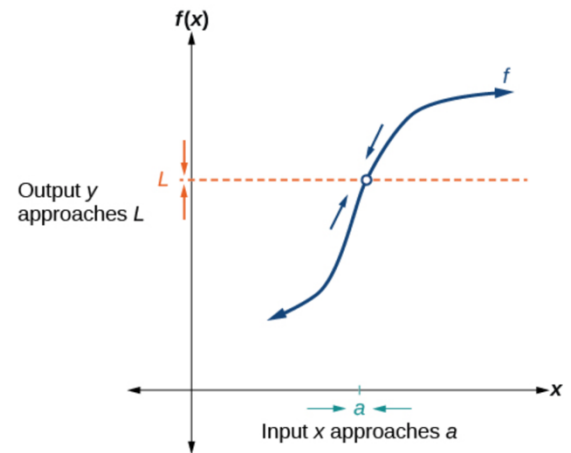
A limit exists if and only if both corresponding one-sided limits exists and are equal

That is,

$$\lim_{x \rightarrow a} f(x) = L$$

for some number L if and only if

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$$



We can find the limit of function :

Numerically

Graphically

Algebraically



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Exercises page 74: use numerical and graphical evidence to conjecture values for each limit.

Q1

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

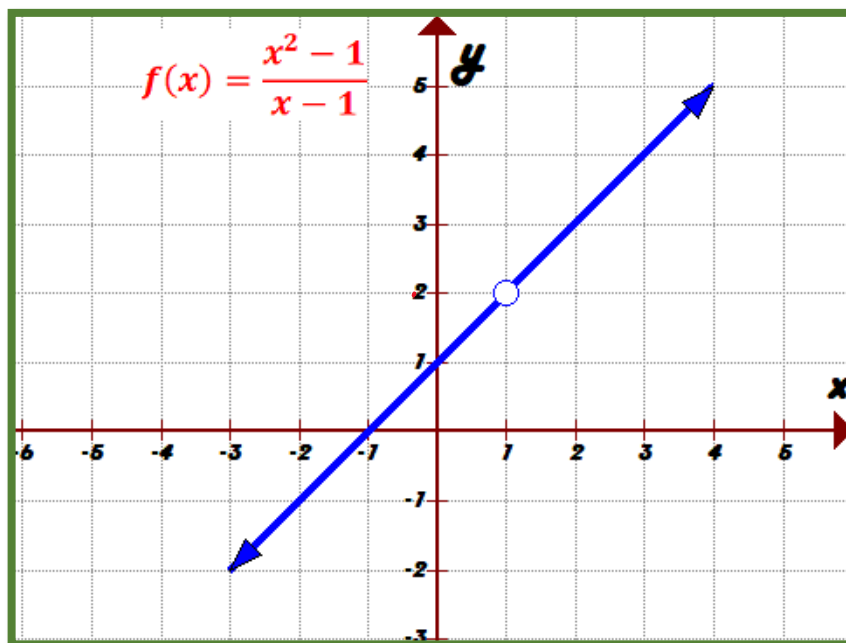
Numerical evidence:

$$x < 1$$

$$x > 1$$

x				1			
$f(x)$							

Graphical evidence:



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Q4

$$\lim_{x \rightarrow 1} \frac{(x-1)^2}{x^2 + 2x - 3}$$

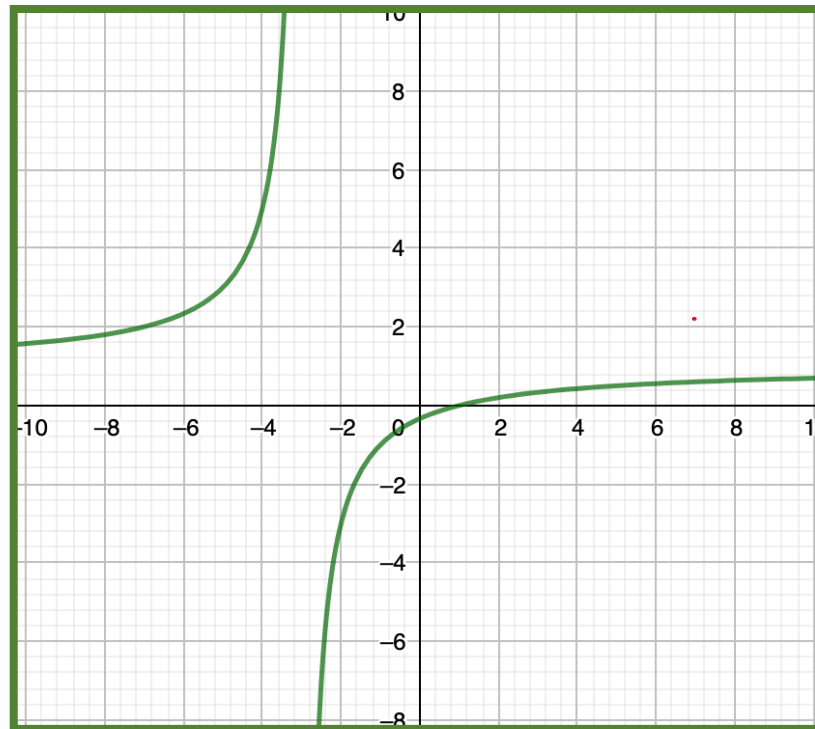
Numerical evidence:

$x < 1$

$x > 1$

x				1			
$f(x)$							

Graphical evidence:



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Q3

$$\lim_{x \rightarrow -2} \frac{x-2}{x^2-4}$$

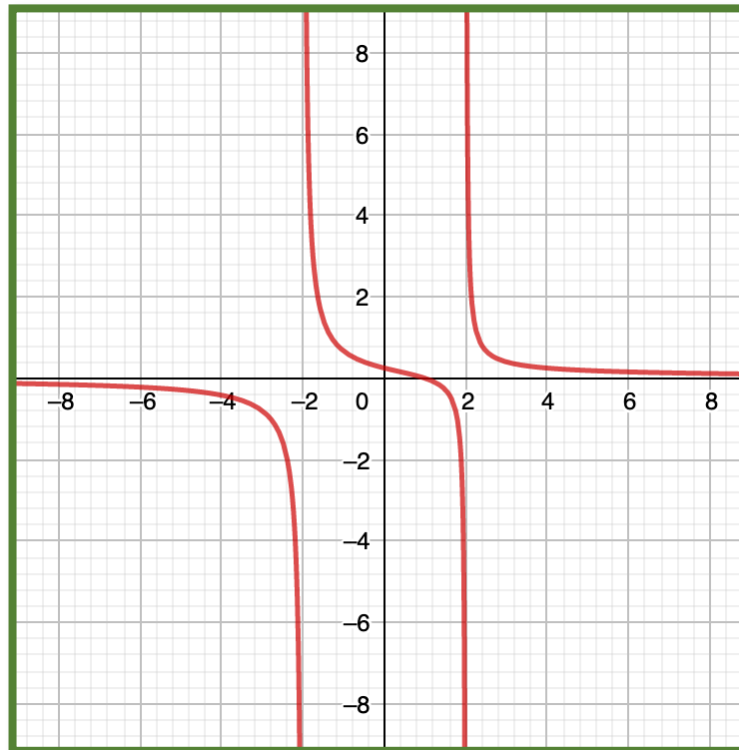
Numerical evidence:

$$x < -2$$

$$x > -2$$

x				-2			
$f(x)$							

Graphical evidence:



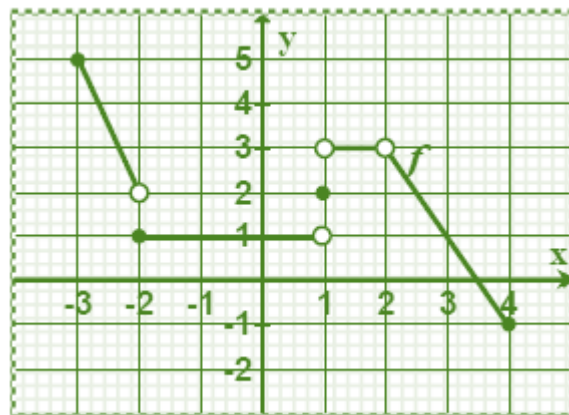
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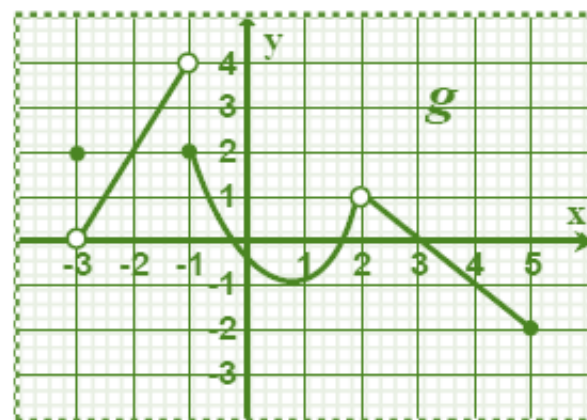
Use the graph of $f(x)$ to find the following:

$\lim_{x \rightarrow 2} f(x)$	$\lim_{x \rightarrow 1^-} f(x)$
$\lim_{x \rightarrow 1^+} f(x)$	$\lim_{x \rightarrow 1} f(x)$
$\lim_{x \rightarrow 3} f(x)$	$\lim_{x \rightarrow -2} f(x)$
$f(1)$	$f(-2)$



Use the graph of $f(x)$ to find the following:

$\lim_{x \rightarrow -2} g(x)$	$\lim_{x \rightarrow 2} g(x)$
$\lim_{x \rightarrow -1^+} g(x)$	$\lim_{x \rightarrow -1^-} g(x)$
$\lim_{x \rightarrow -1} f(x)$	
$g(-3)$	$g(2)$



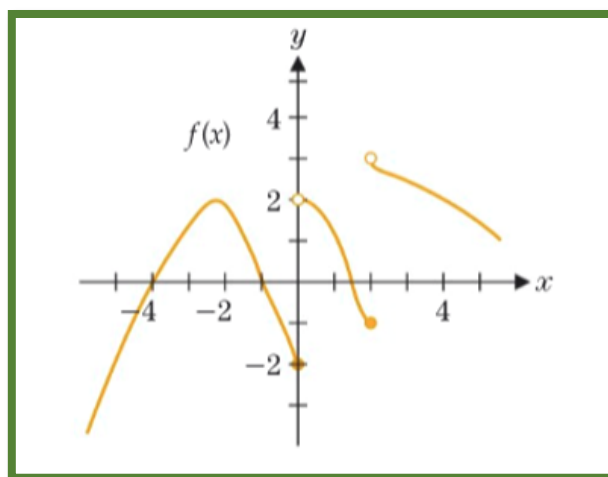
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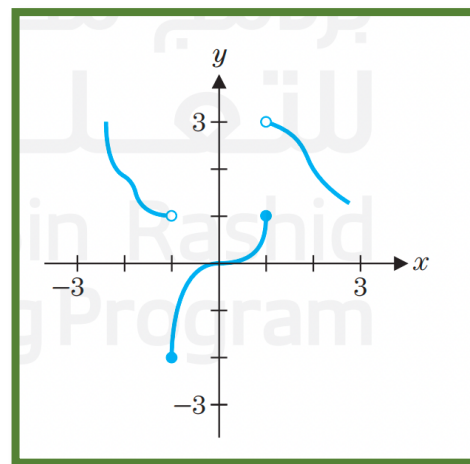
Exercise 8 page 75 :Identify each limit or state that it does not exist

$\lim_{x \rightarrow 1^-} f(x)$	$\lim_{x \rightarrow 1^+} f(x)$
$\lim_{x \rightarrow 1} f(x)$	$\lim_{x \rightarrow 2^-} f(x)$
$\lim_{x \rightarrow 2^+} f(x)$	$\lim_{x \rightarrow 2} f(x)$
$\lim_{x \rightarrow -3} f(x)$	$\lim_{x \rightarrow 3^-} f(x)$



Exercises 11 and 12 page 127 :Identify each limit or state that it does not exist

$\lim_{x \rightarrow -1^-} f(x)$	$\lim_{x \rightarrow -1^+} f(x)$
$\lim_{x \rightarrow -1} f(x)$	$\lim_{x \rightarrow 2} f(x)$
$\lim_{x \rightarrow 1^-} f(x)$	$\lim_{x \rightarrow 1^+} f(x)$
$\lim_{x \rightarrow 1} f(x)$	$\lim_{x \rightarrow 0} f(x)$

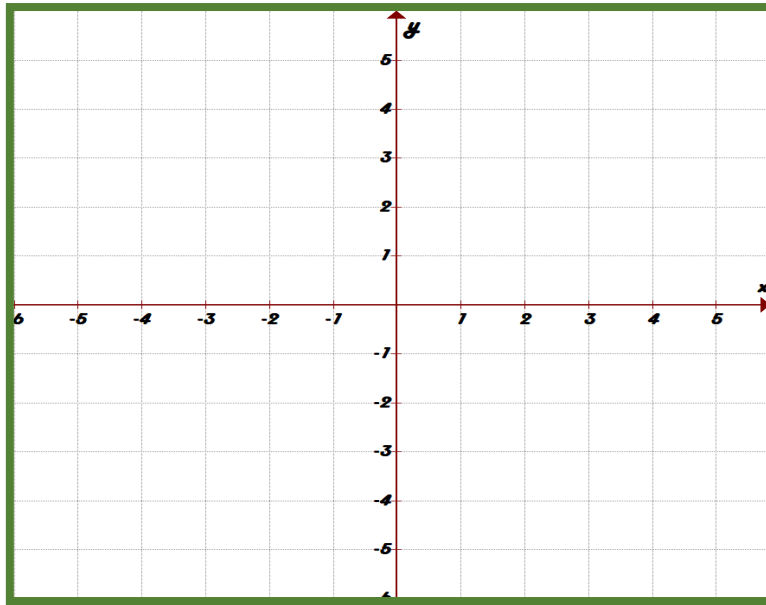


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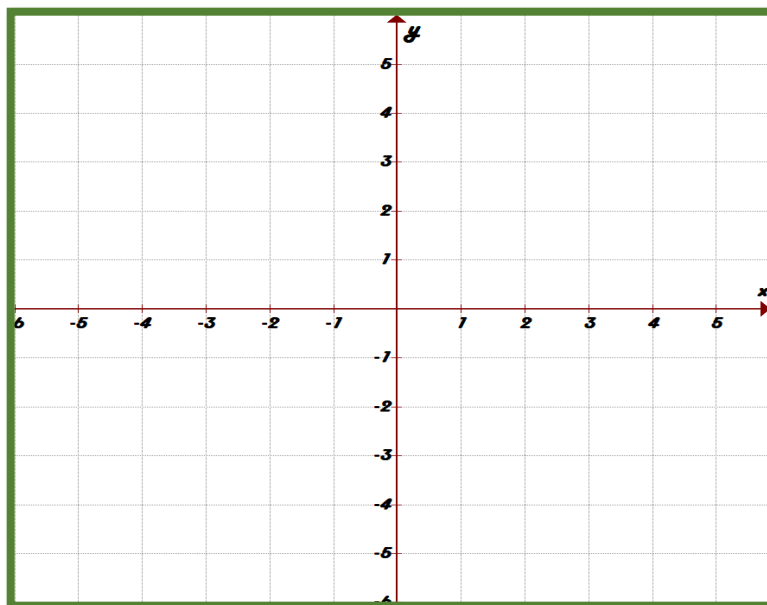
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Exercises page 75 : Sketch a graph of a function with the given properties.

24 . $f(x) = 1$ for $-2 \leq x \leq 1$, $\lim_{x \rightarrow 1^+} f(x) = 3$ and $\lim_{x \rightarrow -2} f(x) = 1$



25 . $f(0) = 1$, $\lim_{x \rightarrow 0^-} f(x) = 2$, and $\lim_{x \rightarrow 0^+} f(x) = 3$



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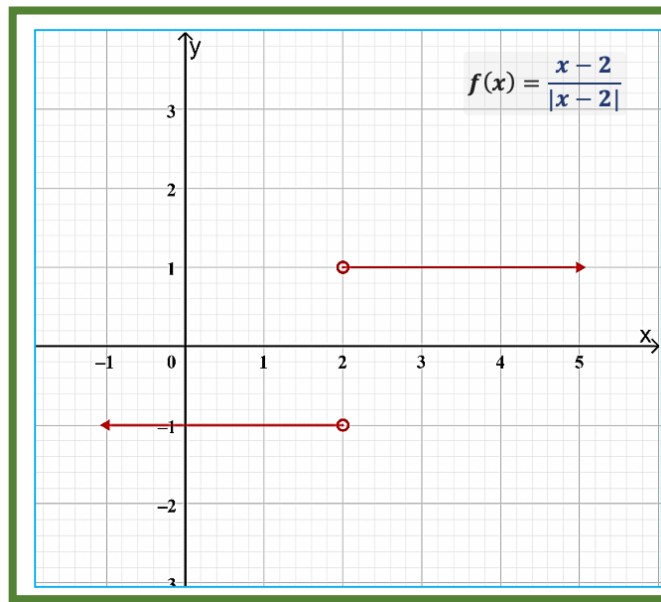
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Exercises page 75 : Evaluate the following limits

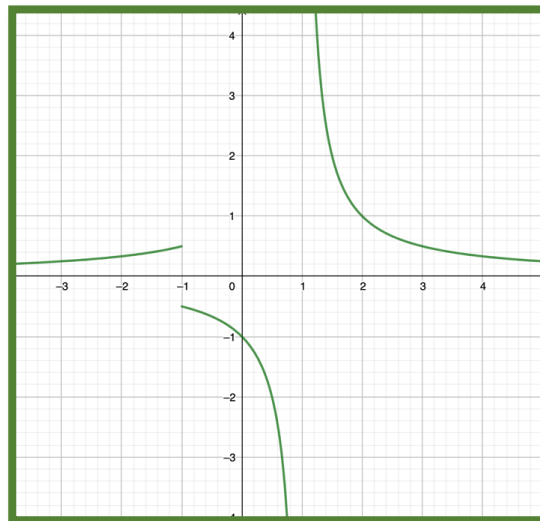
Q21

$$\lim_{x \rightarrow 2} \frac{x - 2}{|x - 2|}$$



$$\lim_{x \rightarrow -1} \frac{|x + 1|}{x^2 - 1}$$

Q22



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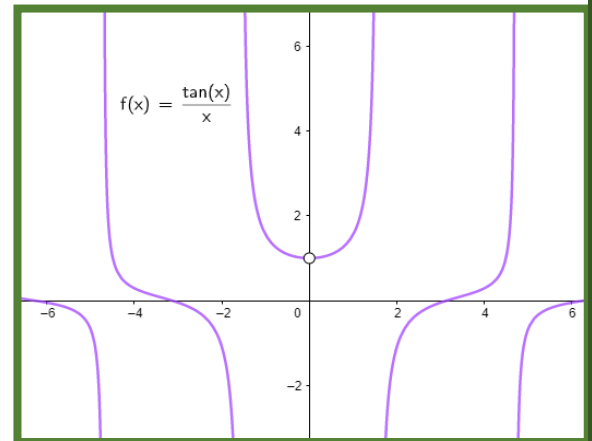
Exercise 17 page 75: Evaluate $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



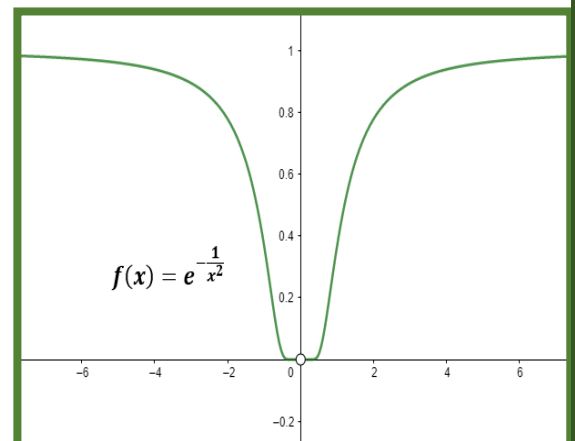
Exercise 17 page 75: Evaluate $\lim_{x \rightarrow 0} e^{-\frac{1}{x^2}}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



Homework of lesson 2

Example 1 page 70:

Evaluate (by using the numerical method)

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

Example 2 page 71:

Evaluate (by using the numerical method)

$$\lim_{x \rightarrow 2} \frac{x^2 - 5}{x - 2}$$



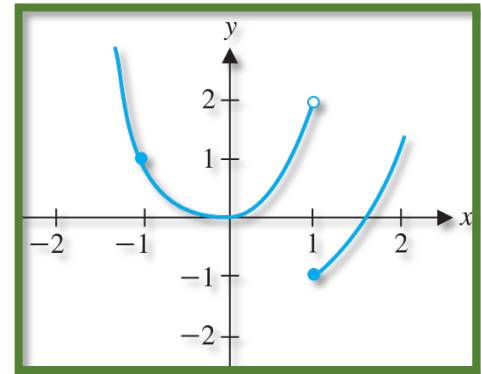
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Example 3 page 71:

Use the graph in figure 2.8 to determine

$\lim_{x \rightarrow -1} f(x)$	$\lim_{x \rightarrow 1^-} f(x)$
$\lim_{x \rightarrow 1^+} f(x)$	$\lim_{x \rightarrow 1} f(x)$



Example 4 page 72:

Evaluate (by using the numerical method)

$$\lim_{x \rightarrow -3} \frac{3x + 9}{x^2 - 9}$$



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Example 6 page 73:

Evaluate (by using the numerical method)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

Example 7 page 73:

Evaluate (by using the numerical method)

$$\lim_{x \rightarrow 0} \frac{x}{|x|}$$



The only way to Learn Mathematics is to Do Mathematics.

More Practice

<https://forms.office.com/r/S5yUqujHHN>

