(2-2) The Concept of a Limit

Learning objectives

- Find the limit of a function algebraically and graphically if it exists.
- Study the existence of a limit by checking one-sided limits.

The value a function approaches as the input approaches a specific value.

$\lim\limits_{x \to c} f(x) = L$: The limit of $f(x)$, as x approaches **c**, is **L**.

$\lim\limits_{x \to c^-} f(x) = L_1$: The limit of $f(x)$, as x approaches c from the left side is L_1.

$\lim\limits_{x \to c^+} f(x) = L_2$: The limit of $f(x)$, as x approaches c from the right side is L_2.

A limit exists if and only if both corresponding one-sided limits exist and are equal.

That is,

$\lim\limits_{x \to c} f(x) = L$, for some number **L**, if and only if $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^-} f(x) = L$

First: Evaluating a Limit Numerically

Exercise page 76

use numerical and graphical evidence to conjecture values for each limit. If possible, use factoring to verify your conjecture.

Q 1
pege76

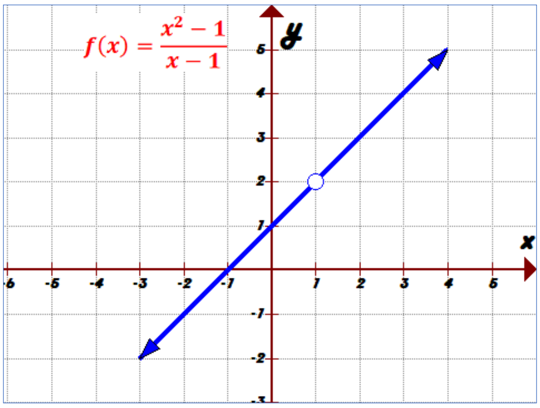
$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$$\lim_{x\to\ ^-} f(x) =$$

$$\lim_{x\to\ ^+} f(x) =$$

$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

Q 5
pege77

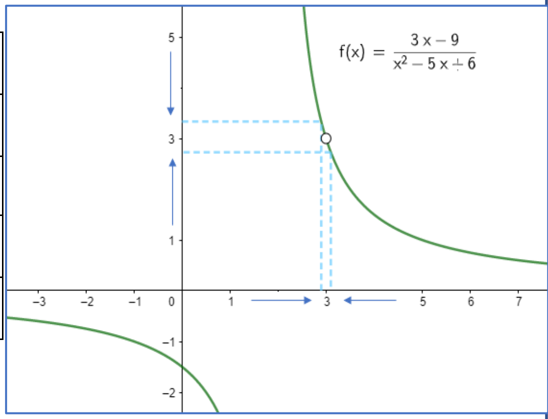
$$\lim_{x\to 3} \frac{3x-9}{x^2-5x+6}$$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$$\lim_{x\to\ ^-} f(x) =$$

$$\lim_{x\to\ ^+} f(x) =$$

$$\lim_{x\to 3} \frac{3x-9}{x^2-5x+6}$$

Q 3 pege77 $\lim\limits_{x\to 2}\dfrac{x-2}{x^2-4}$

For $x <$

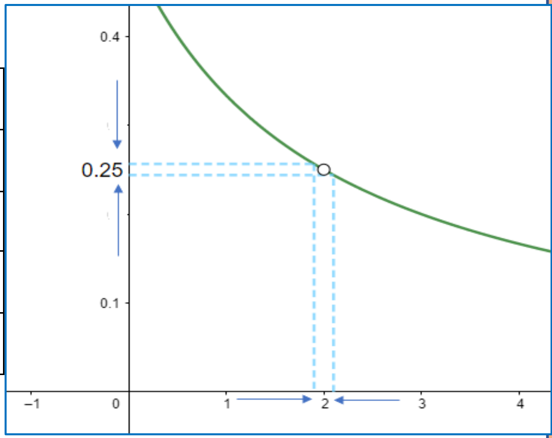
x	$f(x)$

$\lim\limits_{x\to\ ^-} f(x) =$

For $x >$

x	$f(x)$

$\lim\limits_{x\to\ ^+} f(x) =$



$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

Secondly: A Limit That Does Not Exist

3 repeated) - $\lim\limits_{x\to -2}\dfrac{x-2}{x^2-4}$

For $x <$

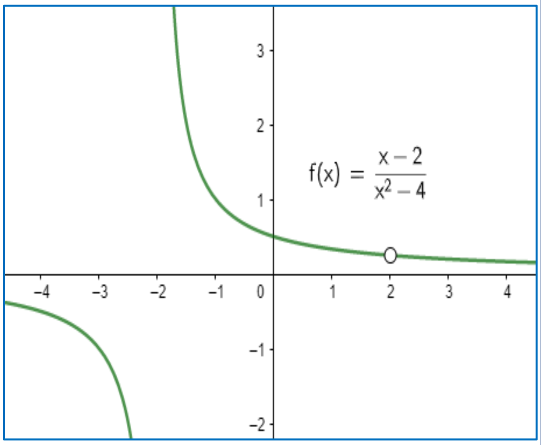
x	$f(x)$

$\lim\limits_{x\to\ ^-} f(x) =$

For $x >$

x	$f(x)$

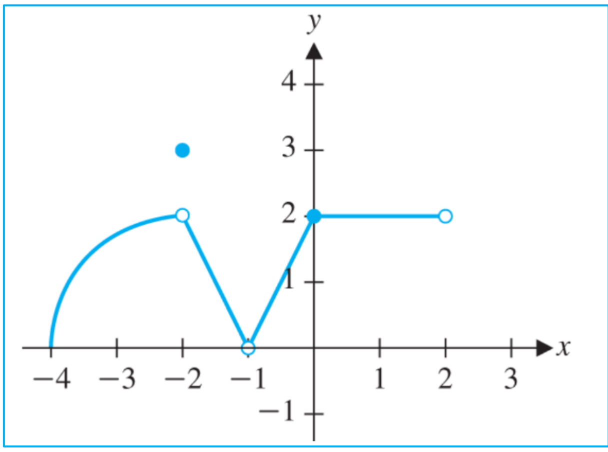
$\lim\limits_{x\to\ ^+} f(x) =$



$$\lim_{x\to -2}\frac{x-2}{x^2-4}$$

Third Determining Limits Graphically

Use the graph of $f(x)$ to answer the following questions.



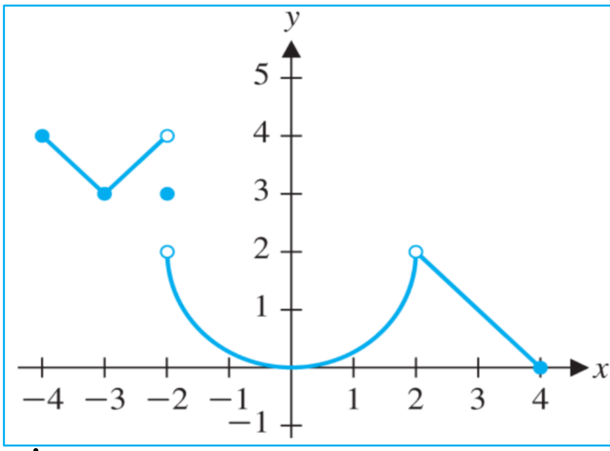
(a)- $f(-2)$

(b)- $\lim\limits_{x \to -2} f(x)$

(c)- $f(-1)$

(d)- $\lim\limits_{x \to 0^+} f(x)$

Use the graph of $f(x)$ to answer the following questions.



(a)- $f(-2)$

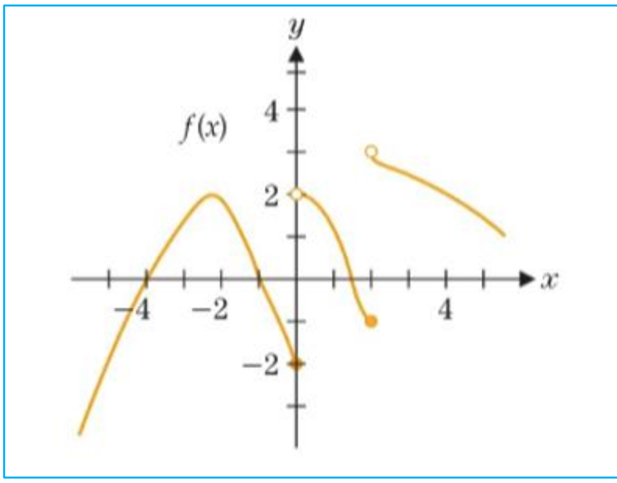
(b)- $\lim\limits_{x \to -2^+} f(x)$

(c)- $\lim\limits_{x \to -2} f(x)$

(d)- $\lim\limits_{x \to 2} f(x)$

Q 7
pege 77

Identify each limit or state that it does not exist



(a)- $\lim\limits_{x \to 0^-} f(x)$

(b)- $\lim\limits_{x \to 0^+} f(x)$

(c)- $\lim\limits_{x \to 0} f(x)$

(d)- $\lim\limits_{x \to 2^-} f(x)$

(e)- $\lim\limits_{x \to 2^+} f(x)$

(f)- $\lim\limits_{x \to 2} f(x)$

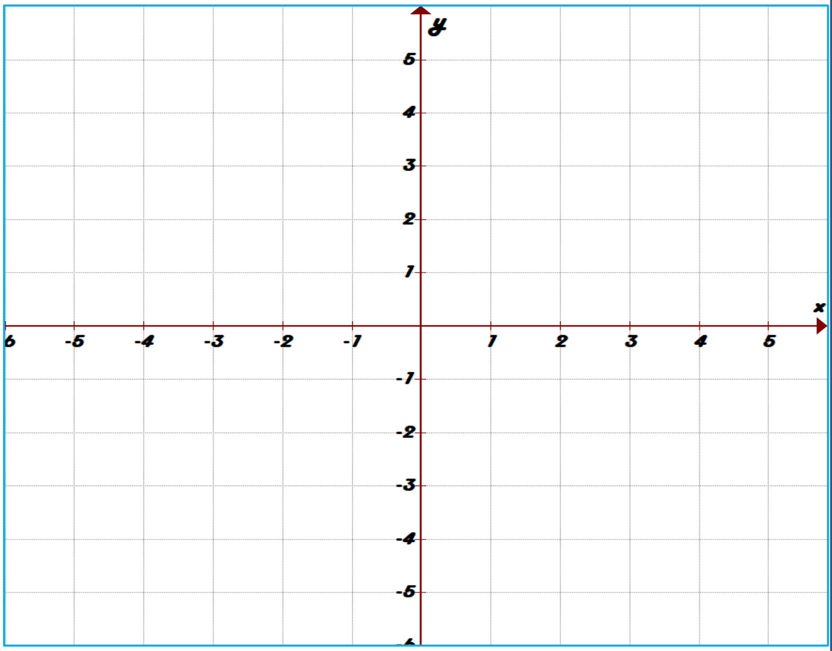
(g)- $\lim\limits_{x \to -1} f(x)$ (h)- $\lim\limits_{x \to 1^-} f(x)$

Fourthly Graph a Function

Sketch a graph of a function with the given properties.

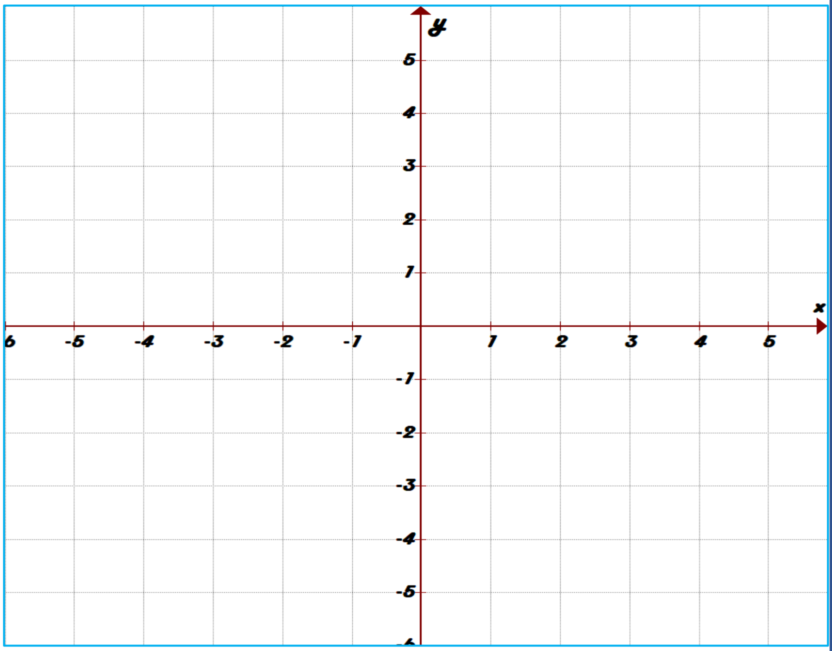
Q 23
pege 77

$f(-1) = 2,\ f(0) = -1,\ f(1) = 3 \quad and\ \lim_{x \to 1} f(x)\ does\ not\ exist$



Q 25
pege 77

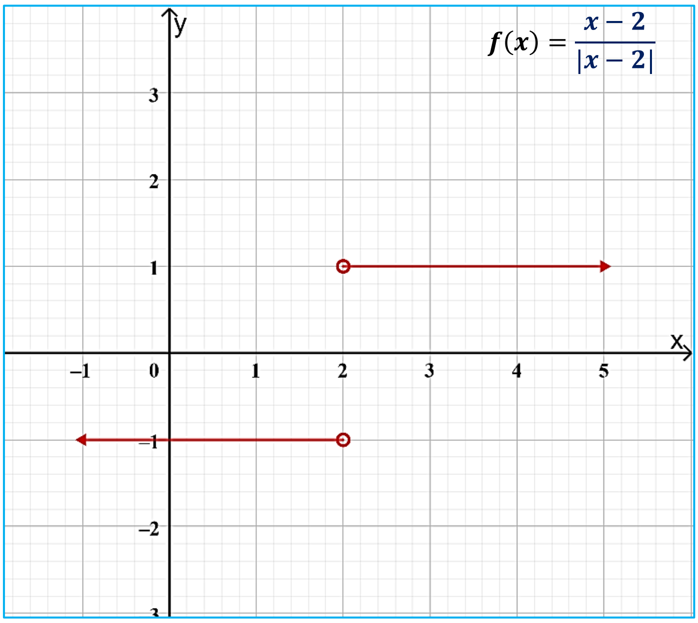
$f(0) = 1, \quad \lim_{x \to 0^-} f(x) = 2\ ,\ and\ \lim_{x \to 0^+} f(x) = 3$



fifth A Case Where One-sided Limits Disagree

Q 21
pege 77

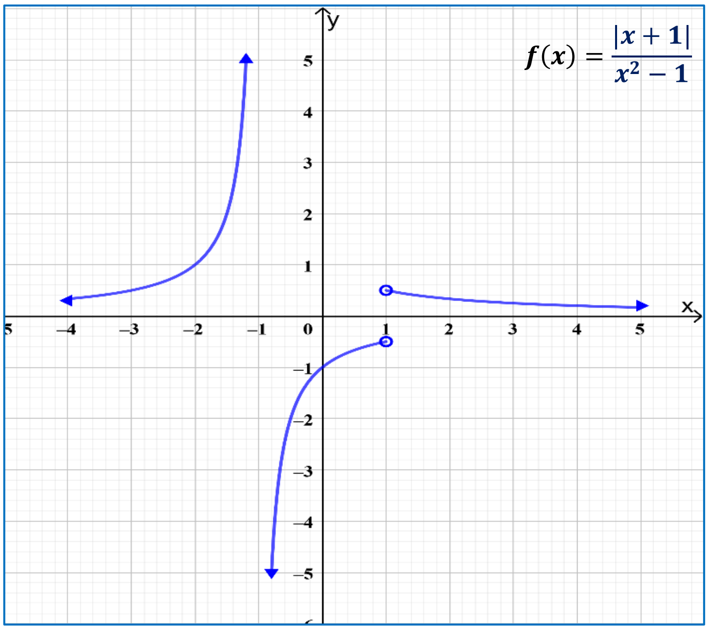
Evaluate $\lim_{x \to 2} \frac{x-2}{|x-2|}$



Q 22
pege 77

Evaluate $\lim_{x \to 1} \frac{|x+1|}{x^2-1}$

H.O.T. Problems Use Higher-Order Thinking Skills



Sixthly Approximating the Value of a Limit

Q 17
pege 77

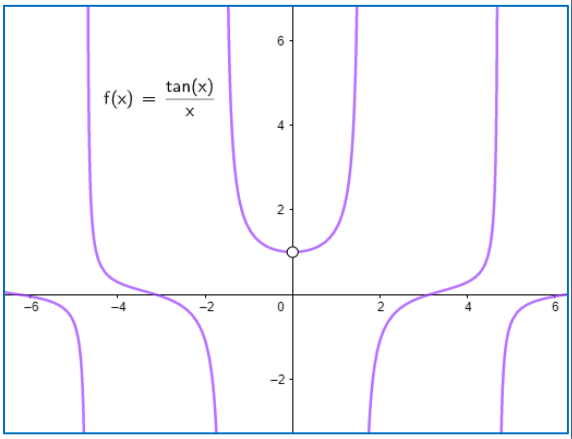
Evaluate: $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x\to\ ^-} f(x) =$

$\lim\limits_{x\to\ ^+} f(x) =$

$$\lim_{x\to 0}\frac{\tan x}{x}$$

Q 15
pege 77

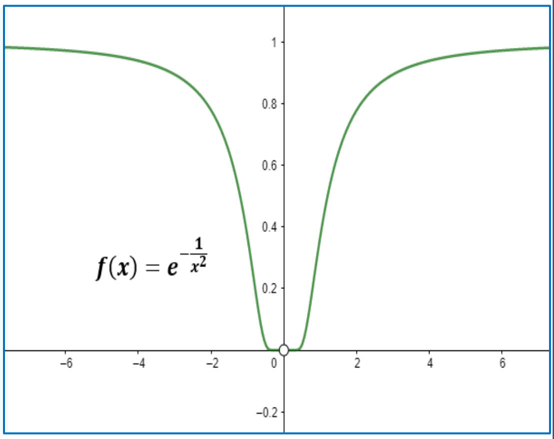
Evaluate: $\lim\limits_{x\to 0} e^{-\frac{1}{x^2}}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x\to\ ^-} f(x) =$

$\lim\limits_{x\to\ ^+} f(x) =$

$$\lim_{x\to 0} e^{-\frac{1}{x^2}}$$

Notice:

Assessment: Please go to the link and solve the assessment

https://forms.office.com/r/8y4B0ZBXjG