Chapter 2	Limits and Continuity
Lesson 2 - 1	The Concept of a Limit
Learning objectives	a) Learn the concept of limits. b) Study the existence of a limit by checking one-sided limits. c) Find the limit of a function algebraically and graphically if it exists.
Keywords	▪ One-sided limit. ▪ Numerical Evidence. ▪ Graphical Evidence.

The value a function approaches as the input approaches a specific value.

$\lim\limits_{x \to c} f(x) = L$: The limit of $f(x)$, as x approaches c, is L.

The limit of $f(x)$, as x approaches c from the right side is L_1.

$$\lim_{x \to c^+} f(x) = L_1$$

The limit of $f(x)$, as x approaches c from the left side is L_2.

$$\lim_{x \to c^-} f(x) = L_2$$

A limit exists if and only if both corresponding one-sided limits exist and are equal.

That is,

$\lim\limits_{x \to c} f(x) = L$, for some number L, if and only if $\lim\limits_{x \to c^+} f(x) = \lim\limits_{x \to c^-} f(x) = L$

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First: Evaluating a Limit Numerically

Exercise page 76

Use numerical and graphical evidence to conjecture values for each limit. If possible, use factoring to verify your conjecture.

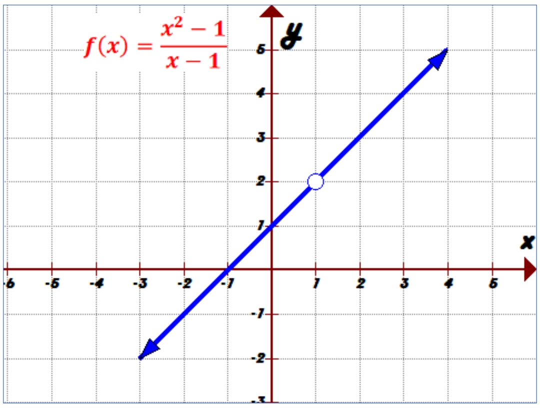
Q 1 pege76 $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x \to \;^{-}} \dfrac{x^2-1}{x-1} =$

$\lim\limits_{x \to \;^{+}} \dfrac{x^2-1}{x-1} =$

$\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1}$

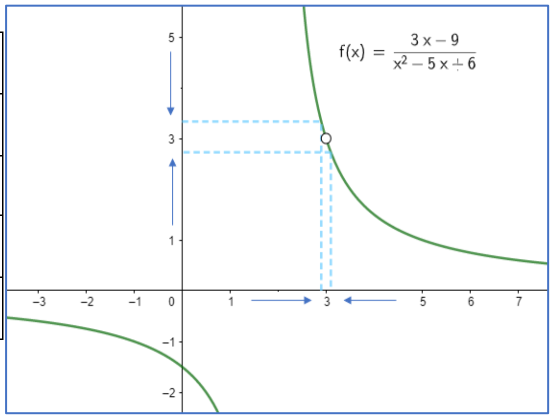
Q 5 pege77 $\lim\limits_{x \to 3} \dfrac{3x-9}{x^2-5x+6}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x \to \;^{-}} \dfrac{3x-9}{x^2-5x+6} =$

$\lim\limits_{x \to \;^{+}} \dfrac{3x-9}{x^2-5x+6} =$

$\lim\limits_{x \to 3} \dfrac{3x-9}{x^2-5x+6}$

Q 3
pege77

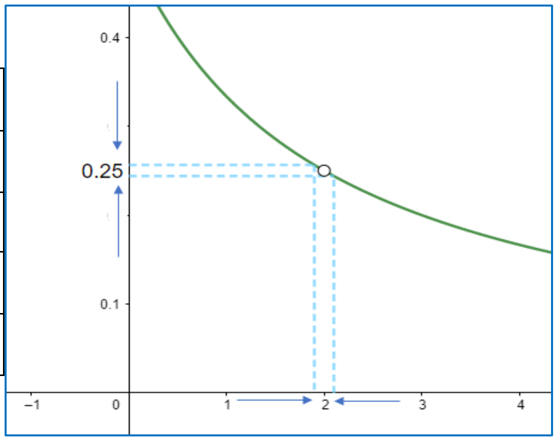
$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$$\lim_{x\to\ ^-}\frac{x-2}{x^2-4} =$$

$$\lim_{x\to\ ^+}\frac{x-2}{x^2-4} =$$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

Secondly: A Limit That Does Not Exist

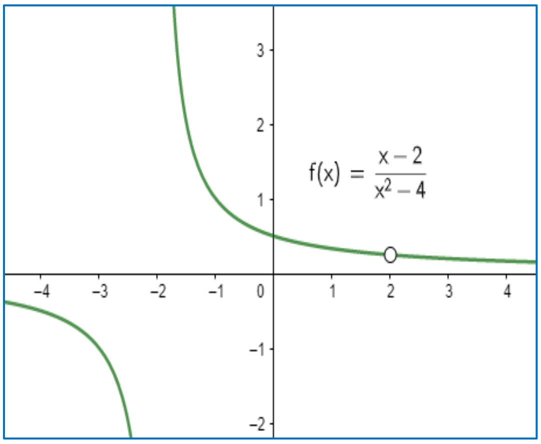
3 repeated) - $$\lim_{x\to -2}\frac{x-2}{x^2-4}$$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$$\lim_{x\to\ ^-}\frac{x-2}{x^2-4} =$$

$$\lim_{x\to\ ^+}\frac{x-2}{x^2-4} =$$

$$\lim_{x\to -2}\frac{x-2}{x^2-4}$$

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Third Determining Limits Graphically

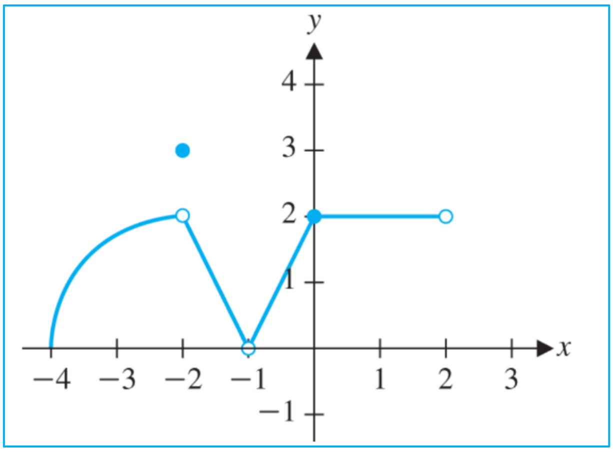
Use the graph of $f(x)$ to answer the following questions.

(a)- $f(-2)$

(b)- $\lim\limits_{x\to-2} f(x)$

(c)- $f(-1)$

(d)- $\lim\limits_{x\to0^+} f(x)$



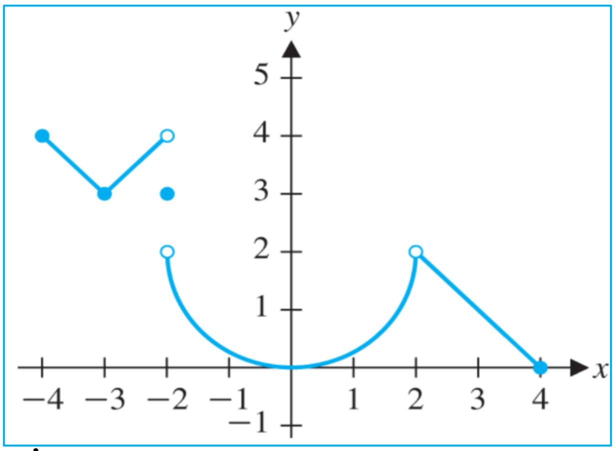
Use the graph of $f(x)$ to answer the following questions.

(a)- $f(-2)$

(b)- $\lim\limits_{x\to-2^+} f(x)$

(c)- $\lim\limits_{x\to-2} f(x)$

(d)- $\lim\limits_{x\to2} f(x)$



Q 7 pege 77 Identify each limit or state that it does not exist

(a)- $\lim\limits_{x\to0^-} f(x)$

(b)- $\lim\limits_{x\to0^+} f(x)$

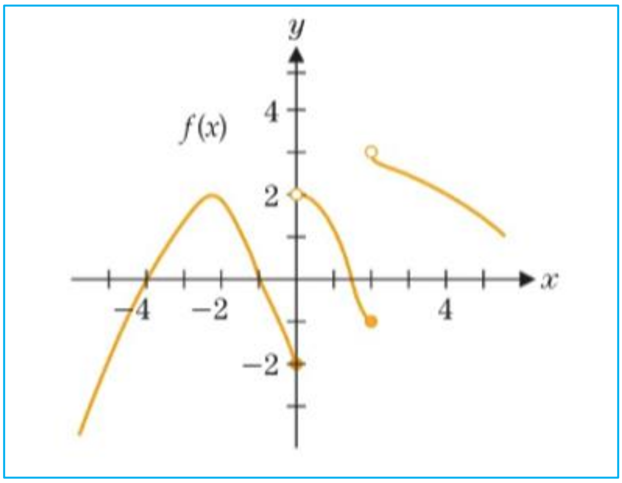
(c)- $\lim\limits_{x\to0} f(x)$

(d)- $\lim\limits_{x\to2^-} f(x)$

(e)- $\lim\limits_{x\to2^+} f(x)$

(f)- $\lim\limits_{x\to2} f(x)$

(g)- $\lim\limits_{x\to-1} f(x)$ (h)- $\lim\limits_{x\to1^-} f(x)$



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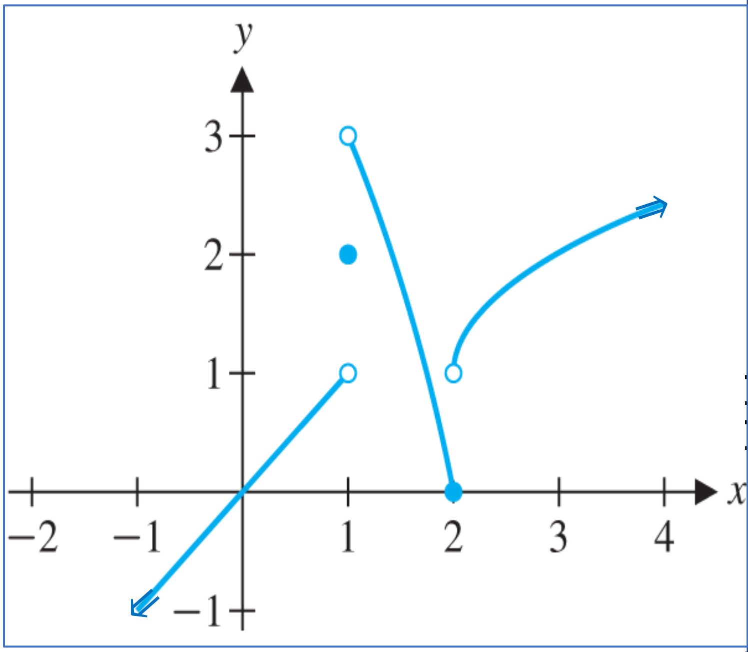
Q
pege

Given $$f(x)=\begin{cases} x & \text{if } x<1 \\ 2 & \text{if } x=1 \\ 4-x^2 & \text{if } 1<x\le 2 \\ \sqrt{x-2}+1 & \text{if } x>2 \end{cases}$$

Use the graph of $f(x)$ to answer the following



(a)- $f(1)=$

(b)- $\lim\limits_{x\to 1^-} f(x)=$

(c)- $\lim\limits_{x\to 1^+} f(x)=$

(d)- $\lim\limits_{x\to 1} f(x)=$

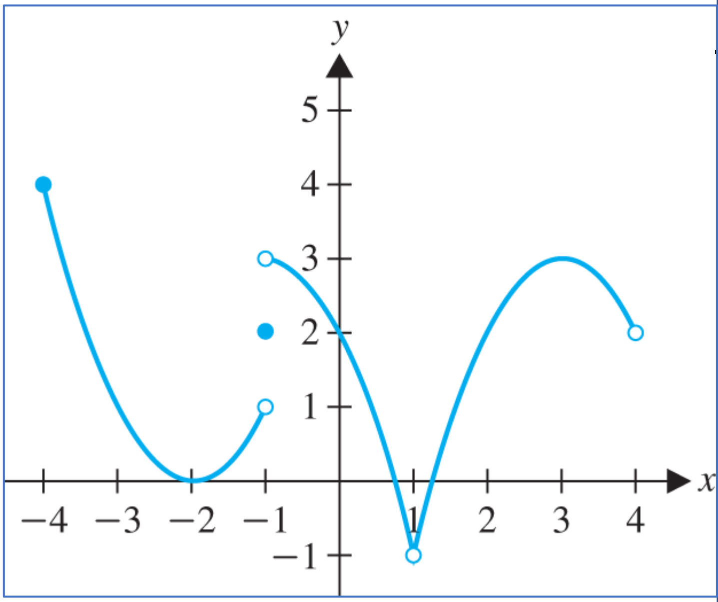
(e)- $f(2)=$

(f)- $\lim\limits_{x\to 2} f(x)=$

(g)- $\lim\limits_{x\to 0} f(x)=$

Q
pege

Indicate whether the following statements are true or false. Justify your answer.



(a)- $f(4)$ is undefined

(b)- $f(-1)=1$

(c)- $\lim\limits_{x\to -1^-} f(x)=1$

(d)- $f(x)$ is continuous at $x=-1$

(e)- $f(x)$ is continuous at $x=0$

(f)- $f(x)$ is continuous at $x=1$

(g)- $\lim\limits_{x\to 0} f(x)=$

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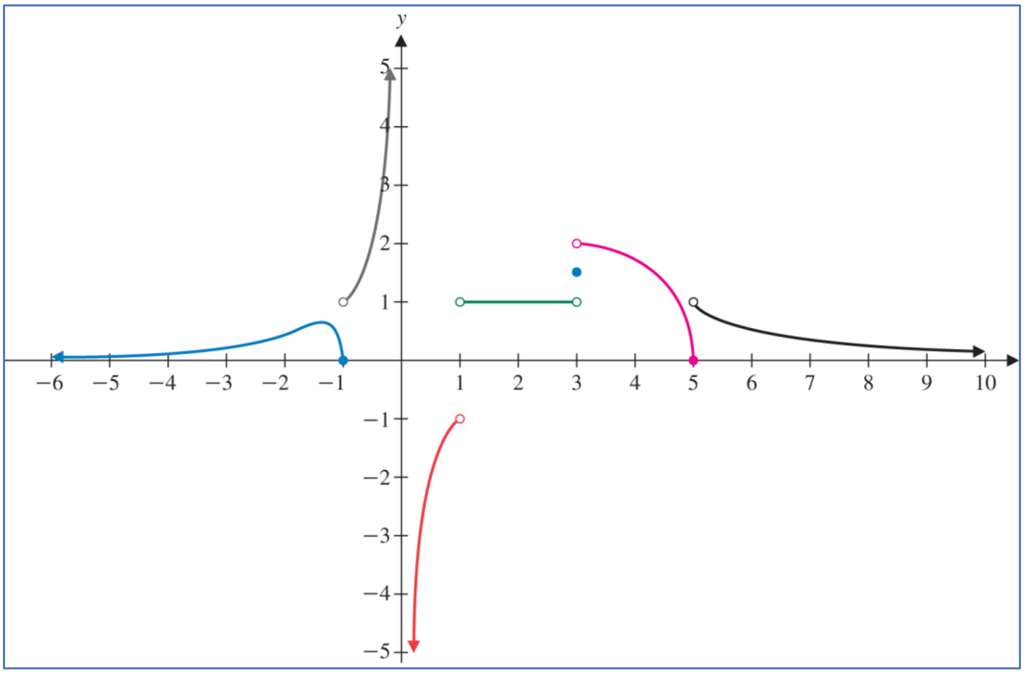
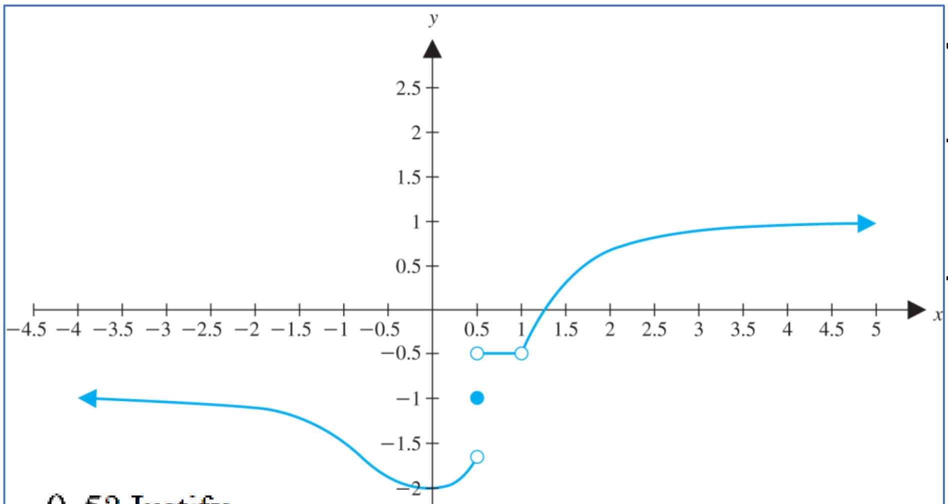
Q pege . Use the graph of $f(x)$ to answer the following questions



(a) $\lim\limits_{x\to -1^-} f(x)$	(b) $\lim\limits_{x\to -1^+} f(x)$	(c) $\lim\limits_{x\to -1} f(x)$
(d) $\lim\limits_{x\to 0^-} f(x)$	(e) $\lim\limits_{x\to 0^+} f(x)$	(f) $\lim\limits_{x\to 0} f(x)$
(g) $\lim\limits_{x\to +\infty} f(x)$	(h) $\lim\limits_{x\to -\infty} f(x)$	(i) Is $f(x)$ continuous at $x=0$?
(j) Is $f(x)$ continuous at $x=2$?	(k) $f(5)$	(l) Is $f(x)$ continuous at $x=5$?

Q pege . Use the graph of $f(x)$ to answer the following questions



(a) $f(0) =$

(b) $\lim\limits_{x\to 0.5^-} f(x) =$

(c) Is the function continuous $f(x)$ $x = 0.5$? Justify

(d) Is the function continuous $f(x)$ $x = 1$? Justify

(e) $\lim\limits_{x\to -\infty} f(x)$

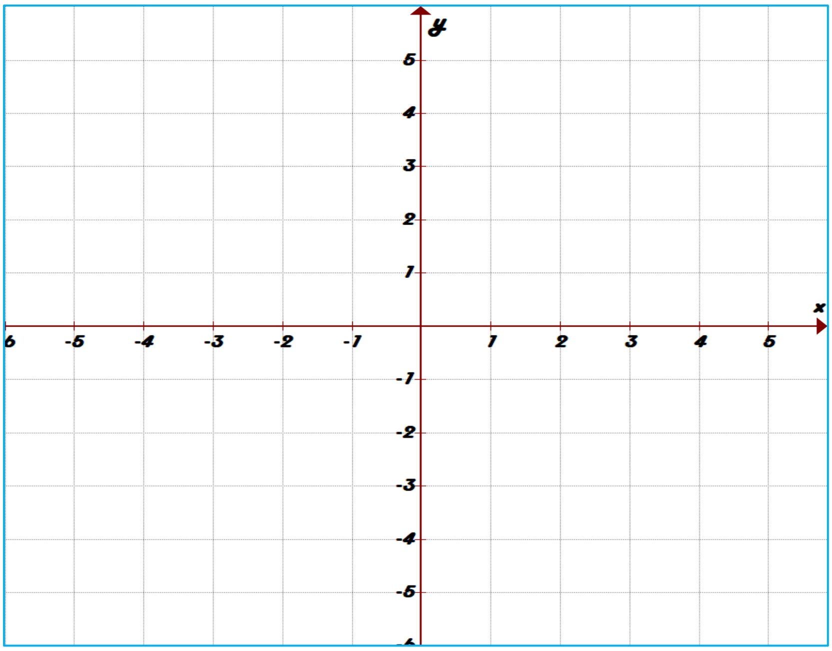
(f) $\lim\limits_{x\to +\infty} f(x)$

Fourthly Graph a Function

Sketch a graph of a function with the given properties.

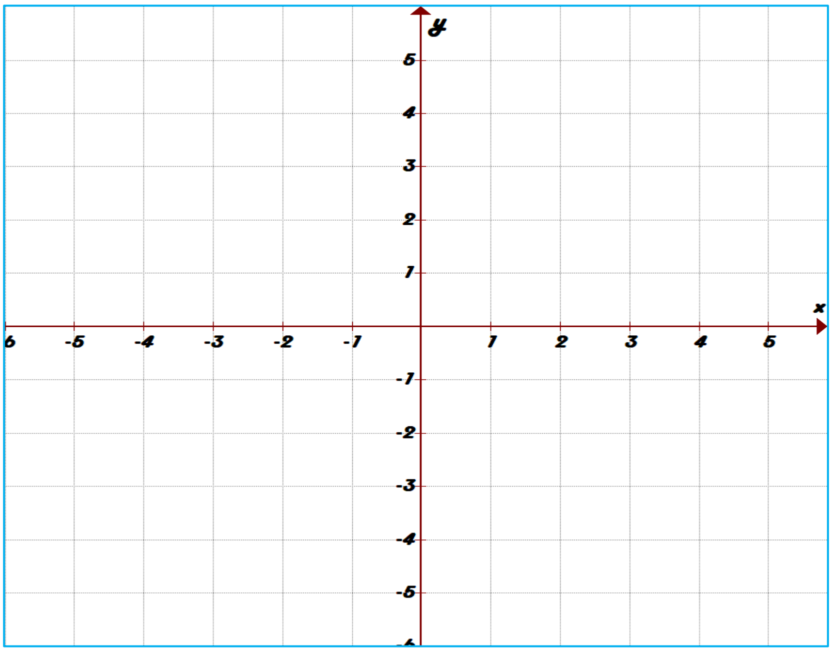
Q 23
pege 77

$$f(-1) = 2,\ f(0) = -1,\ f(1) = 3 \quad and \ \lim_{x\to 1} f(x)\ does\ not\ exist$$



Q 25
pege 77

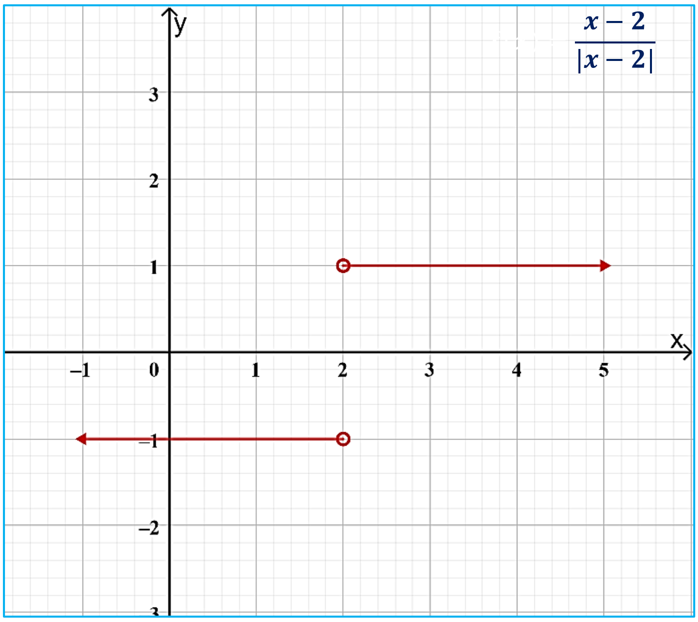
$$f(0) = 1,\quad \lim_{x\to 0^-} f(x) = 2\ ,\ and\ \lim_{x\to 0^+} f(x) = 3$$



fifth A Case Where One-sided Limits Disagree

Q 21
pege 77

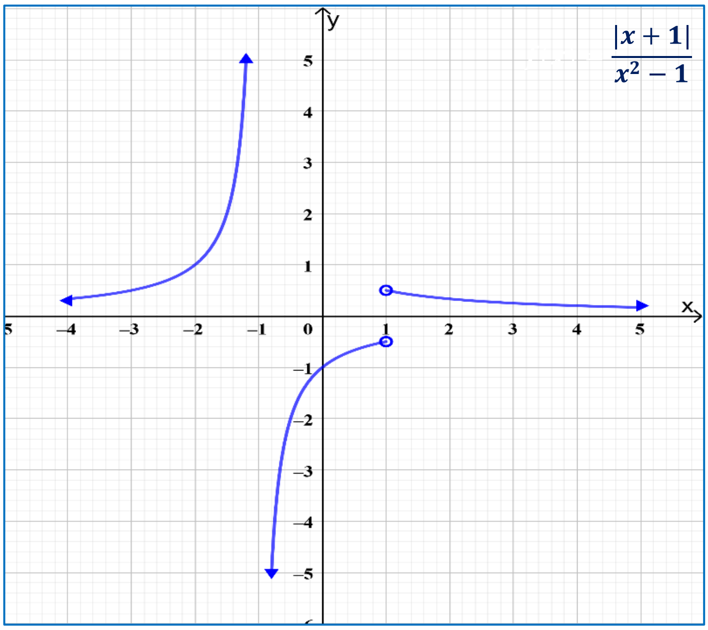
Evaluate $\lim_{x \to 2} \frac{x-2}{|x-2|}$



Q 22
pege 77

Evaluate $\lim_{x \to 1} \frac{|x+1|}{x^2-1}$

H.O.T. Problems Use Higher-Order Thinking Skills



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Sixthly Approximating the Value of a Limit

Q 17
pege 77

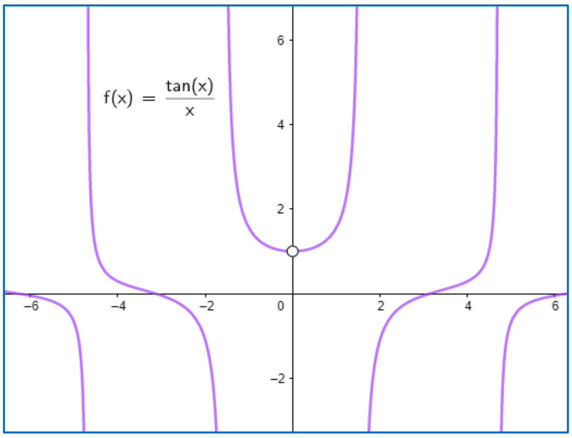
Evaluate: $\lim\limits_{x\to 0}\frac{\tan x}{x}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x\to\ ^-}\frac{\tan x}{x} =$

$\lim\limits_{x\to\ ^+}\frac{\tan x}{x} =$

$\lim\limits_{x\to 0}\frac{\tan x}{x}$

Q 15
pege 77

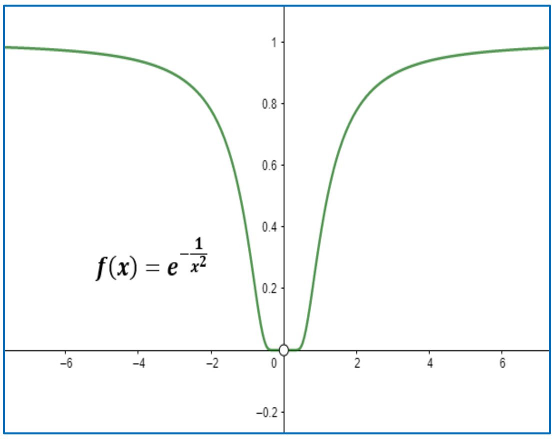
Evaluate: $\lim\limits_{x\to 0} e^{-\frac{1}{x^2}}$

For $x <$

x	$f(x)$

For $x >$

x	$f(x)$



$\lim\limits_{x\to\ ^-} e^{-\frac{1}{x^2}} =$

$\lim\limits_{x\to\ ^+} e^{-\frac{1}{x^2}} =$

$\lim\limits_{x\to 0} e^{-\frac{1}{x^2}}$

Notice:

Assessment: Please go to the link and solve the assessment

https://forms.office.com/r/vXKMMTQA55

The End

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