(6-4) Arc Length and Surface Area

Learning objectives

- Find arc length in a given interval using definite integration.
- Find surface area of a solid of revolution using definite integration.
- Solve mathematical problems involving applications on arc length or surface area.

Keywords:

1) Arc Length 2) Surface Area

1) Arc Length

Since f is continuous on all $[a, b]$ and differentiable on (a, b).

The arc length is given exactly by the definite integral

Arc length of $y = f(x)$ on the interval $[a, b]$

$$s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Arc length of $x = f(y)$ on the interval $[c, d]$

$$s = \int_c^d \sqrt{1 + (f'(y))^2}\, dy$$

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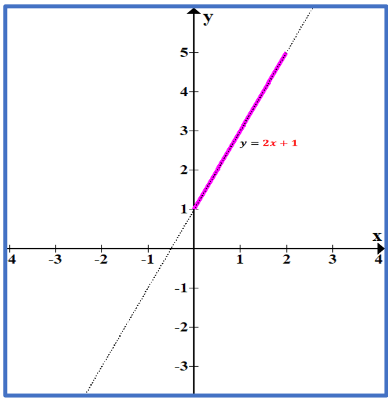
Exercises page 446, and 447

Compute the arc length exactly.



Q5- $y = 2x+1 , 0 \le x \le 2.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

Q6- $y = ln(secx) , 0 \le x \le \frac{\pi}{2}.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

Q7- $y = 4x^{\frac{3}{2}}+1 , 1 \le x \le 2.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

Q8- $y = \frac{1}{4}(e^{2x}+e^{-2x}) , 1 \le x \le 1.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

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Q11- $x = \frac{1}{8}y^4 + \frac{1}{4y^2}, -2 \le y \le -1.$

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$s = \int_c^d \sqrt{1 + (f'(y))^2}\, dy$

Q- $y = \sqrt{1 - x^2}, -1 \le x \le 1.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

Q- $f'(x) = \sqrt{64x^6 - 1} \quad ,1 \le x \le 2$

Set up an integral for arc length.

Q18- $y = \tan x, 0 \le x \le \frac{\pi}{2}.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

Q19- $y = \cos x, 0 \le x \le \pi.$

$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

Q21- $y = \int_0^x u \sin u \, du \qquad , 0 \le x \le \pi.$

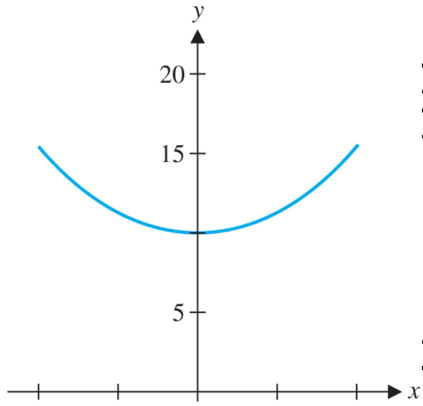
$f(x) =$

$f'(x) =$

$[f'(x)]^2 =$

$s = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$

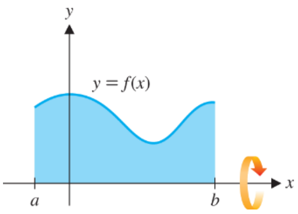
Q23-A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 10(e^{\frac{x}{20}} + e^{-\frac{x}{20}})$, , $-20 \le x \le 20$ compute the length of the rope.

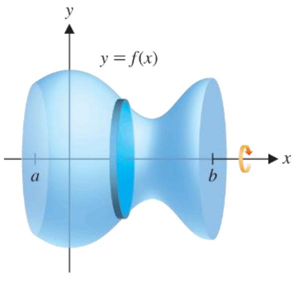


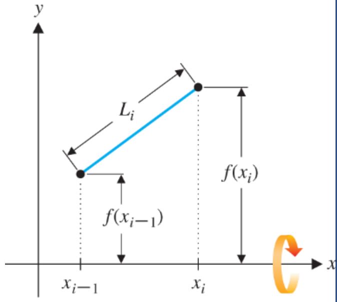
2) Surface Area

Surface Area:is the total area of the outer layer of an object

For the general problem of finding the curved surface area of a surface of revolution, consider the case where $f(x) \ge 0$ and where f is continuous on the interval $[a , b]$ and differentiable on (a , b).If we revolve the graph of $y = f(x)$ about the X-axis on the interval $[a , b]$, we get the surface of revolution s







$$S = \int_a^b 2\pi f(x). \sqrt{1 + (f'(x))^2} \, dx$$

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Set up the integral for the surface area of the surface of revolution.

Q31- $y = 2x - x^2$, $0 \leq x \leq 2$,revolved about the X–axis

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$$S = \int_a^b 2\pi f(x).\sqrt{1 + (f'(x))^2}\, dx$$

Q33-$y = e^x$, $0 \leq x \leq 1$,revolved about the X–axis

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$$S = \int_a^b 2\pi f(x).\sqrt{1 + (f'(x))^2}\, dx$$

Q34-$y = \ln x$, $1 \leq x \leq 2$,revolved about the X–axis

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$$S = \int_a^b 2\pi f(x).\sqrt{1 + (f'(x))^2}\, dx$$

Q35- $y = \cos x$, $0 \leq x \leq \frac{\pi}{2}$,revolved about the X–axis

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$$S = \int_a^b 2\pi f(x).\sqrt{1 + (f'(x))^2}\, dx$$

Q36- $y = \sqrt{x}$, $1 \leq x \leq 2$,revolved about the X–axis

$f(y) =$

$f'(y) =$

$[f'(y)]^2 =$

$$S = \int_a^b 2\pi f(x).\sqrt{1 + (f'(x))^2}\, dx$$