الرياضيات

للصف الثاني عشر متقدم

الفصل الدراسي الثالث 2021/2022

المراجعة النهائية

الوحدتين السادسة والسابعة
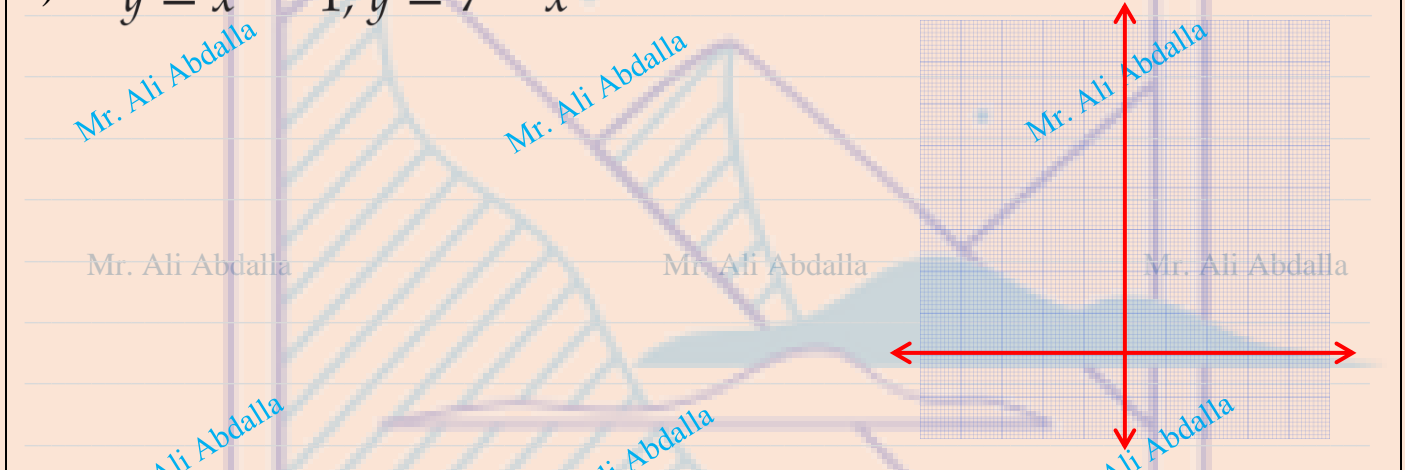
Unit 6 and Unit 7

إعداد الاستاذ

علي عبد الله

Find the area between the curves on the given interval

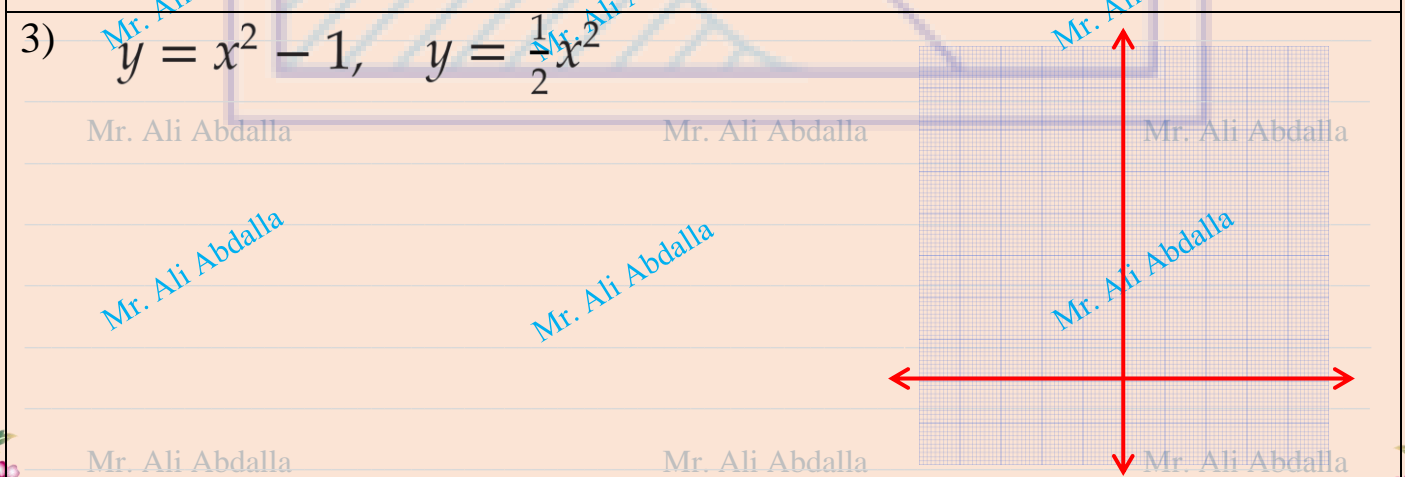1) $y = e^{-x}, y = x^2, 1 \le x \le 4$

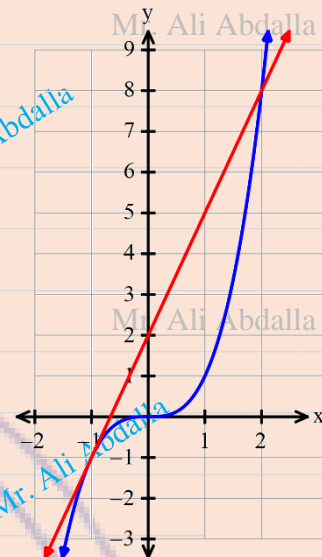Sketch and find the area of the region determined by the intersections of the curves.
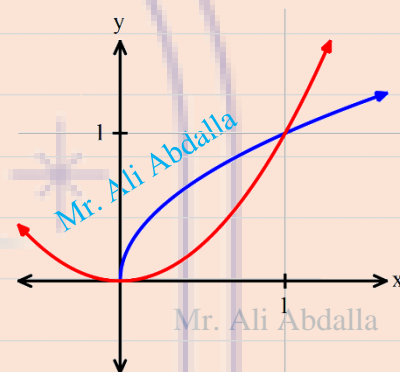
2) $y = x^2 - 1, y = 7 - x^2$

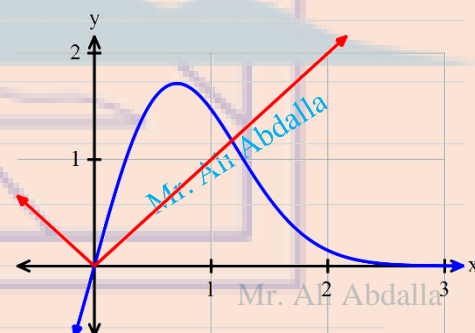3) $y = x^2 - 1, \quad y = \frac{1}{2}x^2$

4) $y = x^3$, $y = 3x + 2$
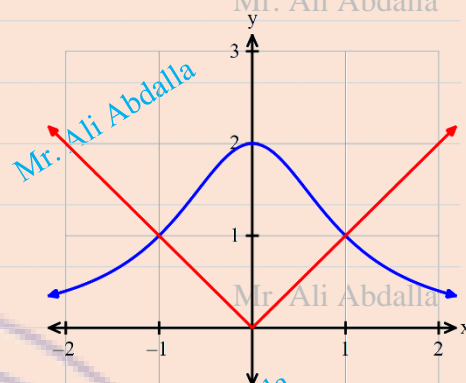

5) $y = \sqrt{x}$, $y = x^2$

6) $y = 4xe^{-x^2}$, $y = |x|$
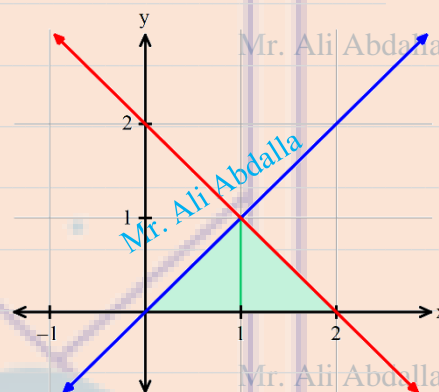
7) $y = \dfrac{2}{x^2 + 1}, \qquad y = |x|$

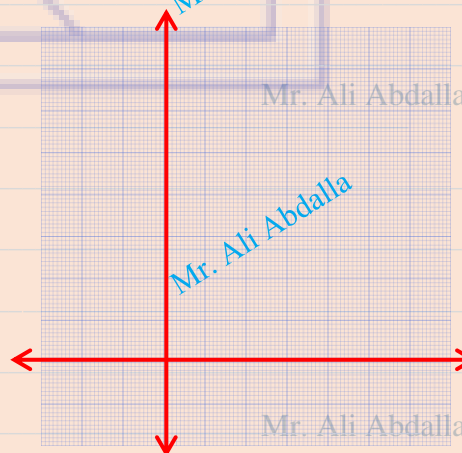| Compute the area of a region using definite integration with y as a variable	إيجاد مساحة منطقة كتكامل محدود بمعلومية y عوضاً عن x

Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.
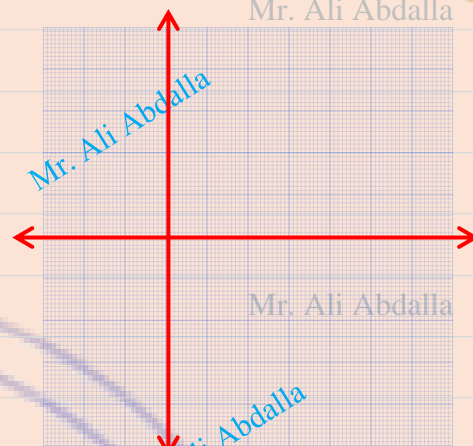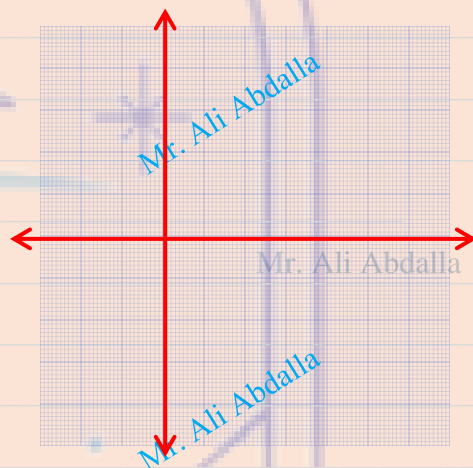
1) $y = x, y = 2 - x, y = 0$

2) $y = x, y = 2, y = 6 - x, y = 0$

3) $x = 3y, \qquad x = 2 + y^2$

4) $x = y^2, \qquad x = 4$

Compute volume by means of definite integration using areas of cross sections	حساب الحجم بالتكامل المحدود مع استخدام مساحات المقاطع العرضية

Find the volume of the solid with cross sectional area $A(x)$

1) $A(x) = x + 2, \ -1 \leq x \leq 3$

2)
$$A(x) = 10e^{0.01x}, 0 \le x \le 10$$

3)
$$A(x) = \pi(4 - x)^2, 0 \le x \le 2$$

4)
$$A(x) = \pi(3 + x)^2 \text{ for } 0 \le x \le 2.$$

5)
$$A(x) = 2(x + 1)^2, 1 \le x \le 4$$

Compute the **volume** of the solid formed by revolving the given region about the given line.

1)   Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about the x-axis

2)   Region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about the y-axis

3) Let R be the region bounded by $y = 4 - 2x$, the x-axis and the y-axis. Compute the volume of the solid formed by revolving R about the given line (a) the y-axis
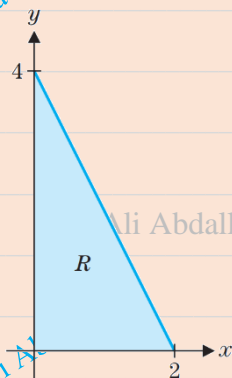
(b) the x-axis
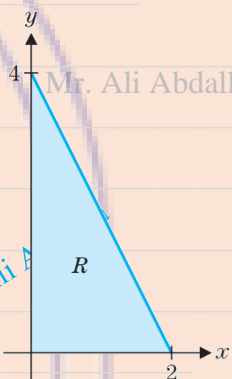
4) Let R be the region bounded by $y = x^2$, $y = 0$ and $x = 2$. Compute the volume of the solid formed by revolving R about the given line.
   (a) the x-axis

   (b) $x = 2$

5) Let R be the region bounded by $y = x$, $y = -x$ and $x = 1$.
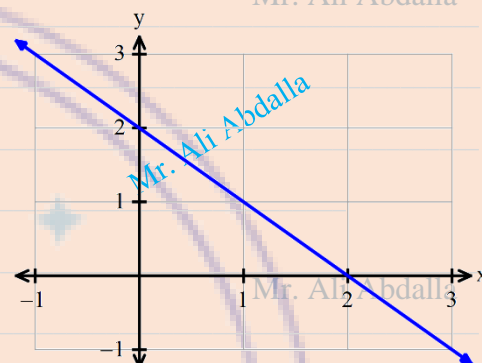Compute the volume of the solid formed by revolving R about the x-axis

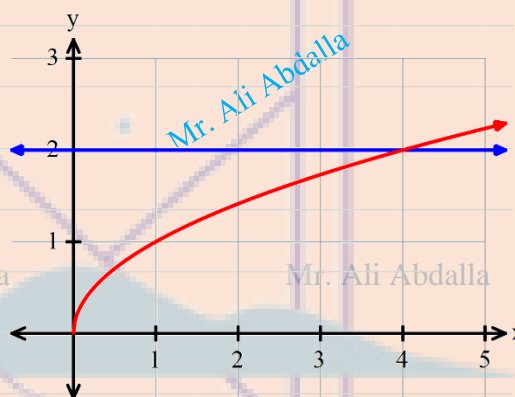| Find the volume of a solid of revolution by using the method of washers	إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات

1) Compute the volume of the solid formed by revolving the region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about $y = 3$

2) Compute the volume of the solid formed by revolving the region bounded
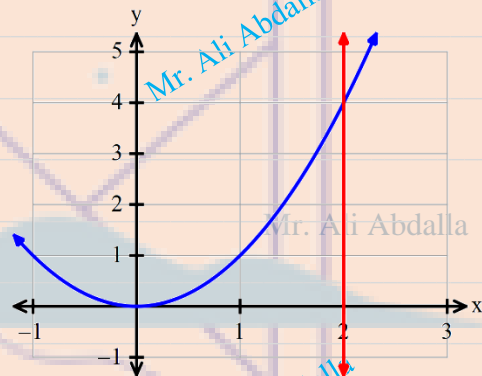by $y = x^2$, $y = 4 - x^2$ about
(a) the x-axis

(b) $y = 4$

3) Compute the volume of the solid formed by revolving the region bounded
by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about $x = 4$

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4) Compute the volume of the solid formed by revolving the region bounded by $y = x^2$ and $x = y^2$ about

(a) the y-axis
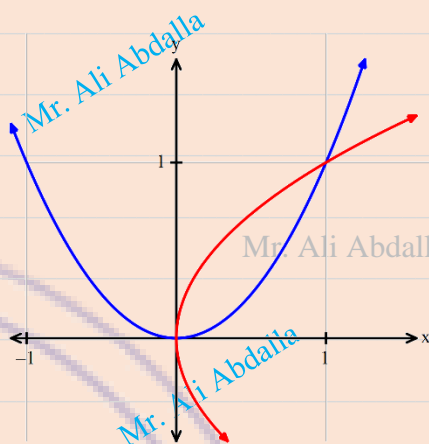
(b) $x = 1$
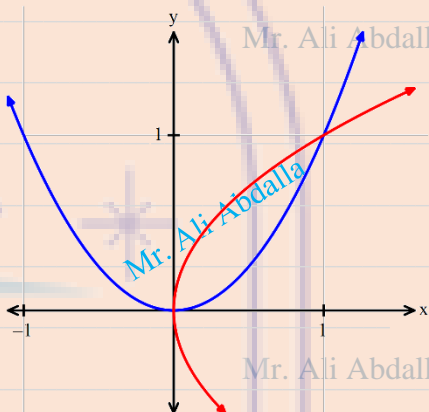
5) Let R be the region bounded by $y = 4 - 2x$, the x-axis and the y-axis. Compute the volume of the solid formed by revolving R about the given line
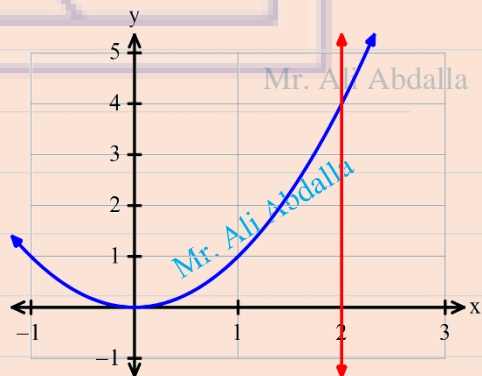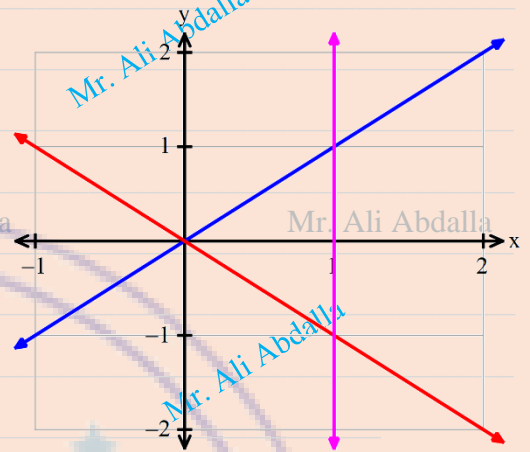
(a) $y = 4$

(b) $y = -4$

(c) $x = 2$
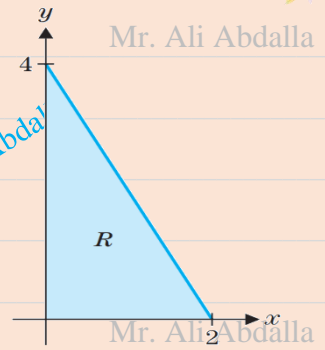
(d) $x = -2$
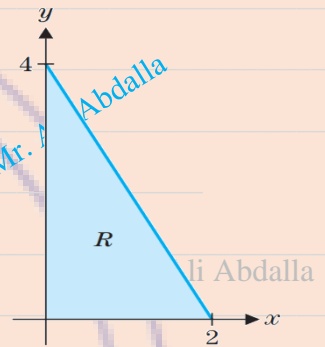
6) Let R be the region bounded by $y = x^2$, $y = 0$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.
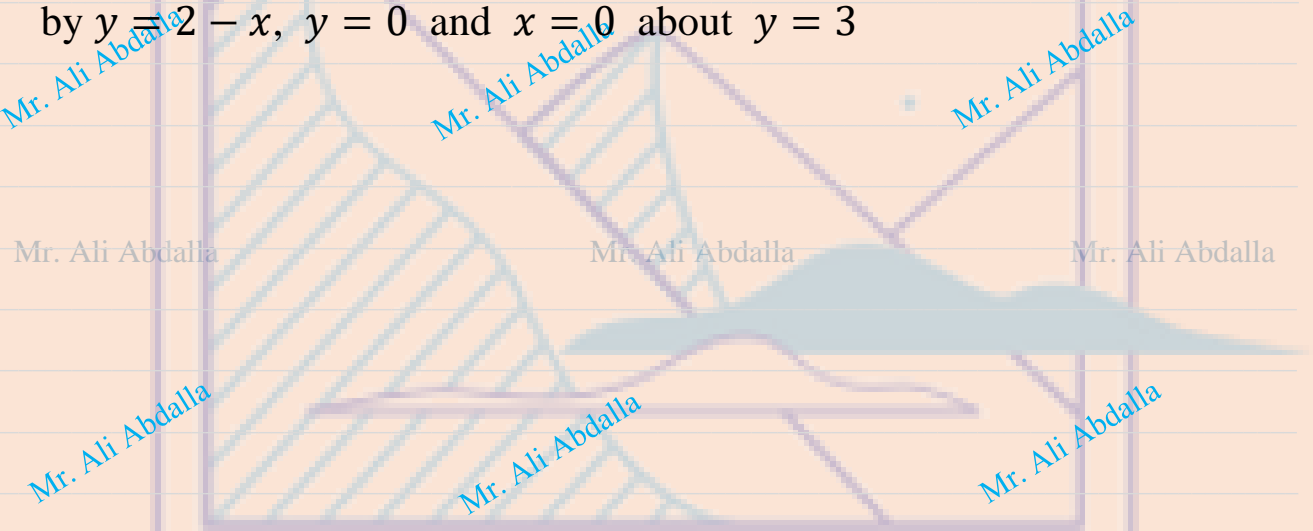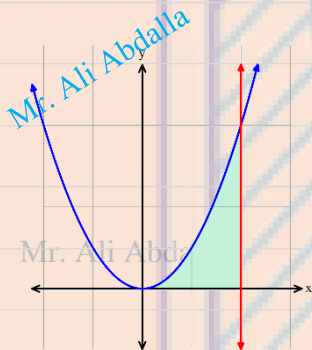
(a) the y-axis

(b) $y = 1$
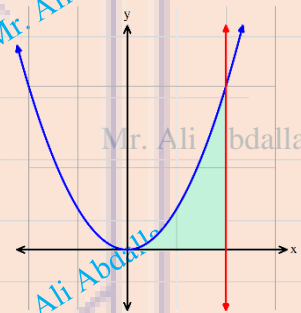
(c) $x = -1$
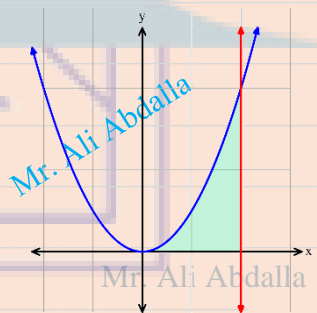
(d) $y = -1$
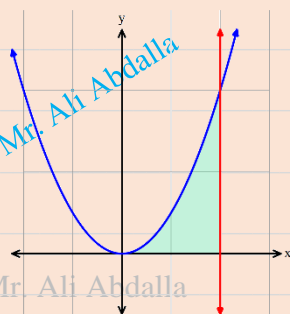
7) Let R be the region bounded by $y = x$, $y = -x$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.
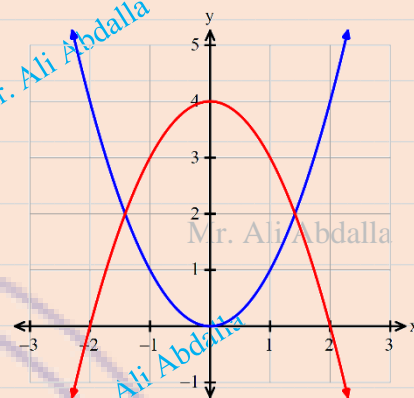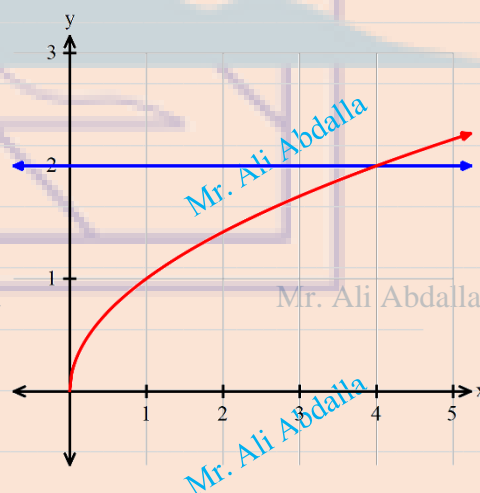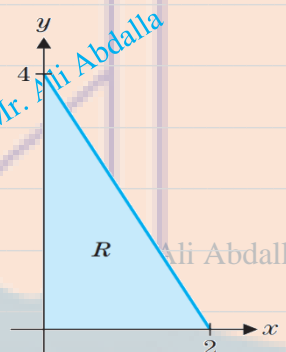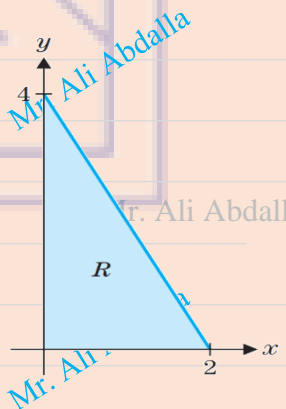
(a) the y-axis
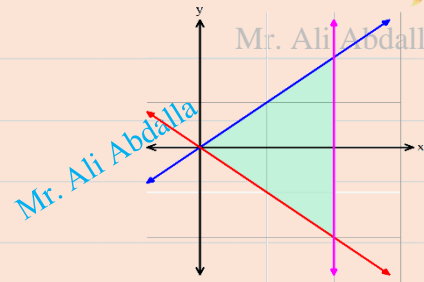
(b) $y = 1$
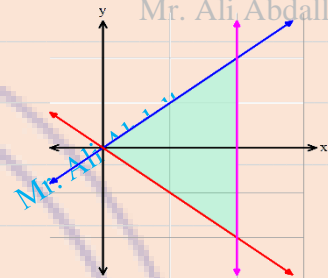
(c) $y = -1$
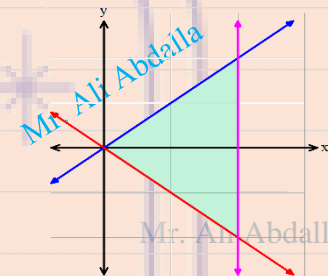
Find the volume of a solid of revolution by using the method of cylindrical shells	إيجاد حجم مجسم ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الأصداف الأسطوانية

Sketch the region, draw in a typical shell, identify the radius and height of each shell and compute the volume.

1) The region bounded by $y = x^2$ and the x-axis, $-1 \le x \le 1$, revolved about $x = 2$
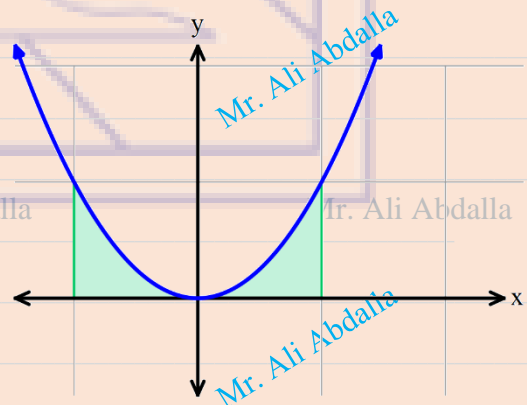
2) The region bounded by $y = x^2$ and the x-axis, $-1 \leq x \leq 1$, revolved about $x = -2$
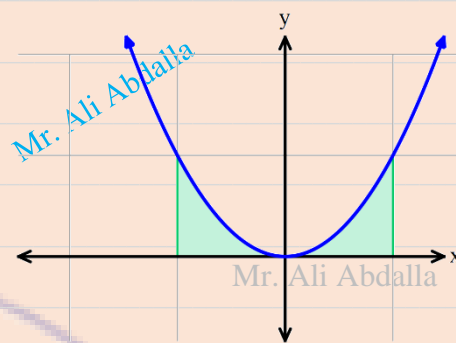
3) The region bounded by $y = x$, $y = -x$ and $x = 1$ revolved about the y-axis
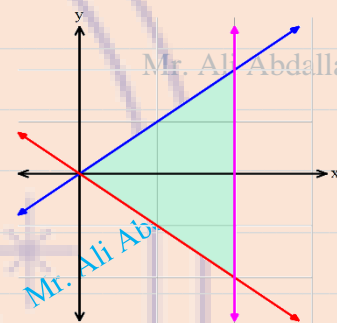
4) The region bounded by $y = x$, $y = -x$ and $x = 1$ revolved about the line $x = 1$
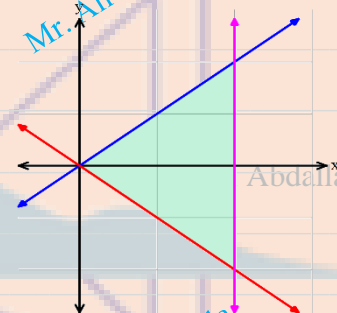
5) The region bounded by $y = \sqrt{x^2 + 1}$ and $y = 0$, $0 \leq x \leq 4$ revolved about $x = 0$
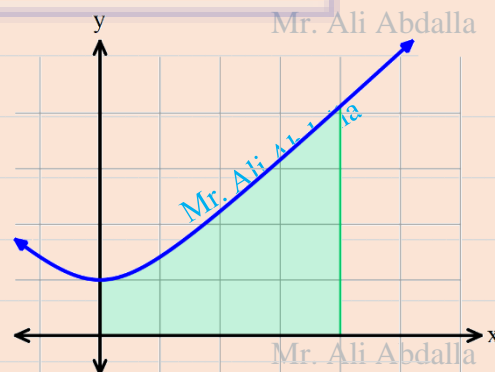
6) The region bounded by $y = x^2$ and $y = 0, -1 \leq x \leq 1$, revolved about the line $x = 2$

(repeated return to #1)
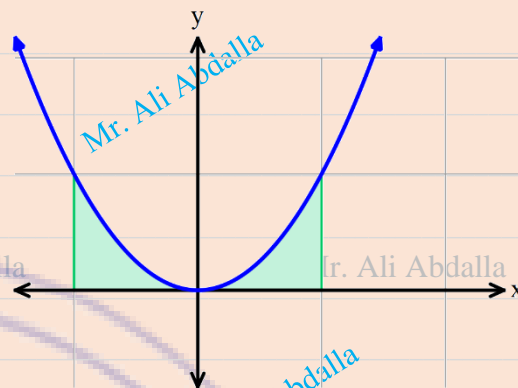
7) The region bounded by $x^2 + y^2 = 1$ revolved about $y = 2$

Radius of a shell: $r = 2 - y$.
Height of a shell: $h = f(y) = 2\sqrt{1 - y^2}$.

$$V = \int_{-1}^{1} 2\pi \,(2 - y)\, 2\sqrt{1 - y^2}\, dy$$

$$= 4\pi \int_{-1}^{1} (2 - y)\sqrt{1 - y^2}\, dy \qquad = 8\pi \int_{-1}^{1} \sqrt{1 - y^2}\, dy - 4\pi \int_{-1}^{1} y\sqrt{1 - y^2}\, dy$$

$$= 16\pi \left(\frac{\pi}{4}\right) - 0 = 4\pi^2$$
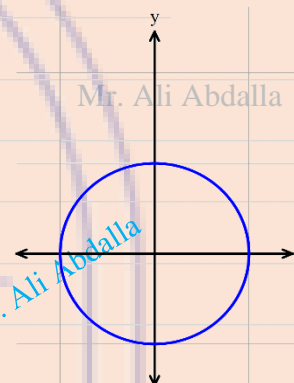
8) The region bounded by $x^2 + y^2 = 2y$ revolved about $y = 4$

9) The region bounded by $x^2 + y^2 = 4$ revolved about $y = 4$
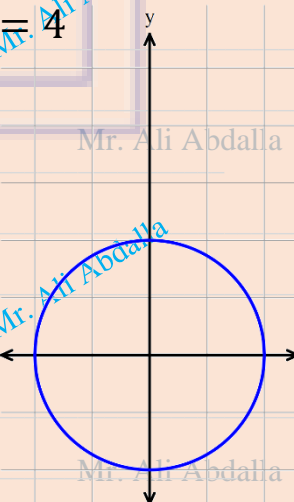
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Compute the arc length exactly:

1) $y = 2x + 1, 0 \leq x \leq 2$

2) $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$

3) $y = 4x^{3/2} + 1, 1 \leq x \leq 2$

4) $y = \frac{1}{4}(e^{2x} + e^{-2x}), 0 \leq x \leq 1$

5) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, 1 \le x \le 2$

6) $y = \frac{1}{6}x^3 + \frac{1}{2x}, 1 \le x \le 3$

Find surface area of a solid of revolution using definite integration	حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معطاة باستخدام التكامل المحدود

Set up the integral for the surface area of the surface of revolution and approximate the integral with a numerical method.

1) $y = x^2, 0 \le x \le 1$, revolved about the x-axis

2) $y = \sin x, 0 \le x \le \pi$, revolved about the x-axis

3) $y = 2x - x^2, 0 \le x \le 2$, revolved about the x-axis

4) $y = x^3 - 4x, -2 \le x \le 0$, revolved about the x-axis

5) $y = e^x, 0 \le x \le 1$, revolved about the x-axis

6) $y = \ln x, 1 \le x \le 2$, revolved about the x-axis

7) $y = \cos x, 0 \le x \le \pi/2$, revolved about the x-axis

8) $y = \sqrt{x}, 1 \le x \le 2$, revolved about the x-axis

1) A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 10\left(e^{x/20} + e^{-x/20}\right)$, $-20 \leq x \leq 20$, compute the length of the rope.
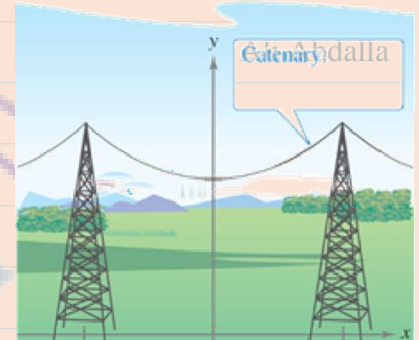
2) A rope is to be hung between two poles 60 meters apart. If the rope assumes the shape of the catenary $y = 15\left(e^{x/30} + e^{-x/30}\right)$, $-30 \leq x \leq 30$, compute the length of the rope.

Identify the initial conditions $y(0)$ and $y'(0)$.

6) a) An object is dropped from a height of 80 feet

Units in feet

b) An object is dropped from a height of 100 feet.

c) An object is released from a height of 60 feet with an upward velocity of 10 ft/s.

d) An object is released from a height of 20 feet with a downward velocity of 4 ft/s

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Units in meters

e) An object is dropped from a height of 24 meters

f) An object is dropped from a height of 30 meters.

g) An object is released from a height of 18 meters with an upward velocity of 3 m/s.

h) An object is released from a height of 6 meters with a downward velocity of 1.2 m/s

1) An object is launched at angle $\theta = \frac{\pi}{3}$ radians from the horizontal with an initial speed of 98 m/s. Determine the time of flight and the horizontal range.

98 m/s

$\frac{\pi}{3}$
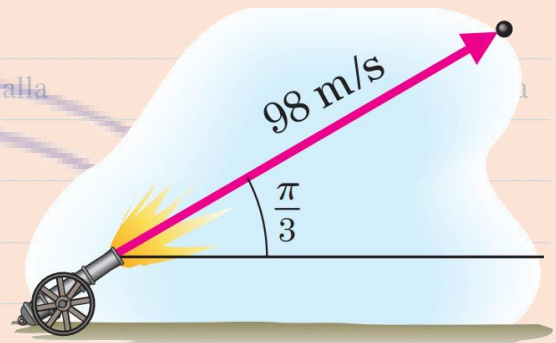
2) Find the time of flight and horizontal range of an object launched at angle 30° with initial speed 40 m/s. Repeat with an angle of 60°.

3) Venus Williams has one of the fastest serves in women's tennis. Suppose that she hits a serve from a height of 10 feet at an initial speed of 120 mph and at an angle of 6° below th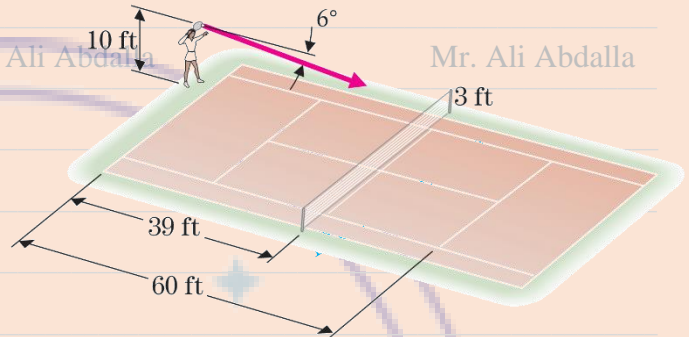e horizontal. The serve is "in" if the ball clears a 3′-high net that is 39′ away and hits the ground in front of the service line 60′ away. Determine whether the serve is in or out. (1 mile = 5280 ft)

Repeat with an initial speed of 170 ft/s

4) A baseball pitcher releases the ball horizontally from a height of 6 ft with an initial speed of 130 ft/s. Find the height of the ball when it reaches home plate 60 ft away

130 ft/s

6 ft

$x = 0$ $x = 60$
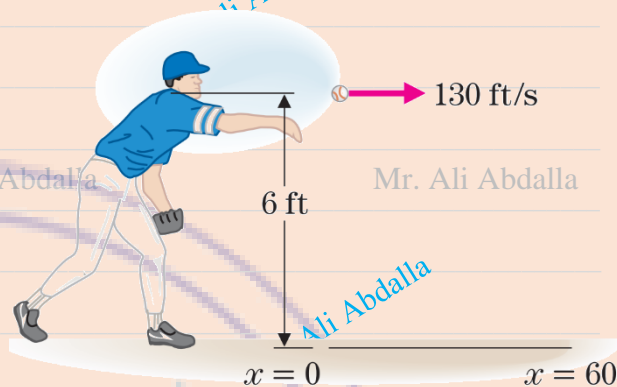
5) Repeat with an initial speed of 80 ft/s.

6) A baseball player throws a ball toward first base 120 ft away. The ball is released from a height of 5 ft with an initial speed of 120 ft/s at an angle of 5° above the horizontal. Find the height of the ball when it reaches first base.

Let $(x(t), y(t))$ be the trajectory. In this case 5° is converted to $\pi/36$ radians.

$y(0) = 5, \ x(0) = 0$

$y'(0) = 120\sin\frac{\pi}{36} \approx 10.46$

$x'(0) = 120\cos\frac{\pi}{36} \approx 119.54$

$x''(0) \equiv 0$

$x'(t) \equiv 119.54$

$x(t) = 119.54t$

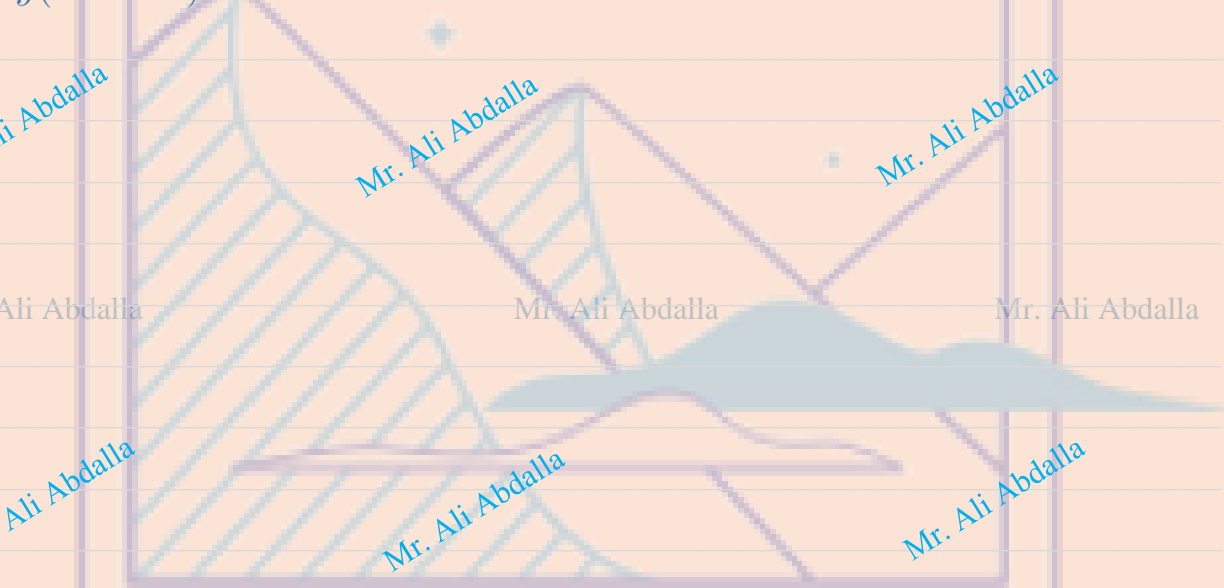This is 120 when

$t = 120/119.54 = 1.00385\ldots$

Meanwhile,

$y''(t) = -32 \qquad y'(t) = -32t + 10.46$

$y(t) = -16t^2 + 10.46t + 5$

$y(1.00385) = -16(1.00385)^2 + 10.46(1.00385) + 5$

$y(1.00385) \approx -.62$ ft

I M P O R T A N A T   R U L E S

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad for \quad n \neq -1 \ (power \ rule)$$

$$\int \sin x \, dx = -\cos x + c \qquad\qquad \int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c \qquad\qquad \int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c \qquad \int \csc x \cot x \, dx = -\csc x + c$$

$$\int e^x dx = e^x + c \qquad\qquad\qquad \int e^{-x} dx = -e^{-x} + c$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \qquad \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

$$\int \frac{1}{|x|\sqrt{1-x^2}} dx = \sec^{-1} x + c \qquad \int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c \qquad \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax + c \qquad\qquad \text{بالمثل مع باقي النسب المثلثية}$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c \qquad \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c \qquad \int f'(x) a^{f(x)} dx = \frac{1}{\ln a} a^{f(x)} + c$$

$$\int \frac{b}{a^2+x^2} dx = \frac{b}{a}\tan^{-1}\left(\frac{x}{a}\right) + c \qquad \int \frac{b}{\sqrt{a^2-x^2}} dx = b\sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{b}{|x|\sqrt{a^2-x^2}} dx = \frac{b}{|a|}\sec^{-1}\left(\frac{x}{a}\right) + c$$

Evaluate the integral.

1) $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\, dx,\ a > 0$

2) $\displaystyle\int \frac{1}{|x|\sqrt{x^2 - a^2}}\, dx,\ a > 0$

3) $\displaystyle\int \sin 6t\, dt$

4) $\displaystyle\int \sec 2t \tan 2t\, dt$

5) $\displaystyle\int (x^2 + 4)^2\, dx$

6) $\displaystyle\int x(x^2 + 4)^2\, dx$

7) $\displaystyle\int \frac{3}{16 + x^2}\, dx$

8) $\displaystyle\int \frac{2}{4 + 4x^2}\, dx$

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1) $\displaystyle\int \frac{4x + 4}{5 + 2x + x^2}\,dx$

2) $\displaystyle\int \frac{t + 1}{t^2 + 2t + 4}\,dt$

3) $\displaystyle\int \frac{4}{x^{1/3}(1 + x^{2/3})}\,dx$

4) $\displaystyle\int \frac{2}{x^{1/4} + x}\,dx$

5) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

6) $\displaystyle\int \frac{\cos(1/x)}{x^2}\, dx$

7) $\displaystyle\int_0^\pi \cos x\ e^{\sin x}\, dx$

8) $\displaystyle\int_0^{\pi/4} \sec^2 x\ e^{\tan x}\, dx$

Final Review Term 3 , 2021-2022   0503050917

Compute integrals using completing a square before integrating	إيجاد تكاملات دوال متنوعة عبر الاستعانة بإكمال المربع واستخدام التعويض

Evaluate the integral.

1)

$$\int \frac{x+1}{\sqrt{3-2x-x^2}} \, dx$$

$$\int \frac{b}{a^2 + (x \pm k)^2} dx = \frac{b}{a} \tan^{-1}\left(\frac{x \pm k}{a}\right) + c$$

$$\int \frac{b}{\sqrt{a^2 - (x \pm k)^2}} dx = b \sin^{-1}\left(\frac{x \pm k}{a}\right) + c$$
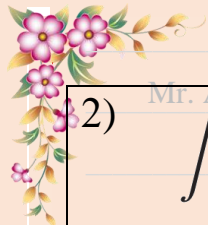
2) $\int \dfrac{1}{\sqrt{3 - 2x - x^2}}\, dx$

3) $\int \dfrac{4}{5 + 2x + x^2}\, dx$

1)

$$\int x \cos x \, dx$$

2)

$$\int x \sin 4x \, dx$$

3)

$$\int x e^{2x} \, dx$$
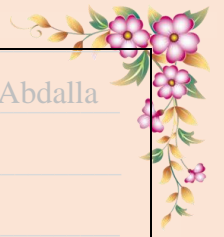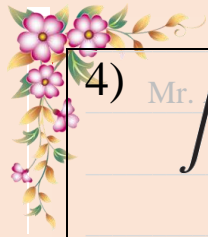
4) $\displaystyle\int x\ln x\,dx$ $\qquad\qquad$ $\displaystyle\int_1^2 x\ln x\,dx$

5) $\displaystyle\int x^2\ln x\,dx$

6) $\displaystyle\int \frac{\ln x}{x}\,dx$

7) $\displaystyle\int_1^{10} \ln 2x\,dx$

Evaluate the integrals

1) $\int_0^1 x \sin 2x \, dx$

2) $\int_0^\pi 2x \cos x \, dx$

3) $\int_0^1 x^2 \cos \pi x \, dx$

4) $\int_0^1 x^2 e^{3x} \, dx$

Evaluate the integrals

1) $\displaystyle \int \cos x \sin^4 x \, dx$

2) $\displaystyle \int \cos^3 x \sin^4 x \, dx$

3) $\displaystyle \int_0^{\pi/4} \cos 2x \, \sin^3 2x \, dx$

4) $\int_{\pi/4}^{\pi/3} \cos^3 3x \, \sin^3 3x \, dx$

5) $\int_0^{\pi/2} \cos^2 x \, \sin x \, dx$

6) $\int_{-\pi/2}^{0} \cos^3 x \, \sin x \, dx$

7) $\int \cos^2(x+1)\,dx$

8) $\int \sin^4(x-3)\,dx$

Integrate functions of the form $\tan^n(x)\sec^m(x)$	إيجاد تكاملات دوال بصيغة $\tan^n(x)\sec^m(x)$

Evaluate the integral

1) $\int \tan x \sec^3 x\,dx$

2)

$$\int \cot x \csc^4 x \, dx$$

3)

$$\int x \tan^3 (x^2 + 1) \, \sec (x^2 + 1) \, dx$$

4)

$$\int \tan (2x + 1) \, \sec^3 (2x + 1) \, dx$$

5)

$$\int \cot^2 x \csc^4 x \, dx$$

6)

$$\int \cot^2 x \csc^2 x \, dx$$

7)

$$\int_0^{\pi/4} \tan^4 x \, \sec^4 x \, dx$$

8)
$$\int_{-\pi/4}^{\pi/4} \tan^4 x \sec^2 x \, dx$$

Integrate trigonometric functions using the substitution $x = a\sin\theta$	إيجاد تكاملات دوال مثلثية باستخدام التبديل $x = a\sin\theta$

Compute the average value of the function on the given interval.

1)
$$\int \frac{1}{x^2\sqrt{9-x^2}} \, dx$$

2) $\displaystyle\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

3) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

4) $\int \dfrac{x^3}{\sqrt{9-x^2}}\, dx$

5) $\int_0^2 \sqrt{4-x^2}\, dx$

6) $\int_0^1 \dfrac{x}{\sqrt{4-x^2}}\, dx$

Evaluate the integral.

1) $\displaystyle \int \frac{x^2}{\sqrt{9 + x^2}} \, dx$

2) $\displaystyle \int x^3 \sqrt{8 + x^2} \, dx$

3) $\displaystyle \int \sqrt{16 + x^2} \, dx$

4) $\displaystyle\int \frac{1}{\sqrt{4+x^2}}\, dx$

5) $\displaystyle\int_0^1 x\sqrt{x^2+8}\, dx$

6) $\displaystyle\int_0^2 x^2\sqrt{x^2+9}\, dx$

7) $\int \dfrac{x^3}{\sqrt{1+x^2}}\, dx$

8) $\int \dfrac{x+1}{\sqrt{4+x^2}}\, dx$

9) $\int \dfrac{x}{\sqrt{x^2+4x}}\, dx$

Evaluate the integral.

1)

$$\int \frac{x^2}{\sqrt{x^2-9}}\, dx$$

2)

$$\int x^3 \sqrt{x^2-1}\, dx$$

3) $\int \dfrac{2}{\sqrt{x^2 - 4}}\, dx$

4) $\int \dfrac{x}{\sqrt{x^2 - 4}}\, dx$

5) $\int \dfrac{\sqrt{4x^2 - 9}}{x}\, dx$

6) $$\int \frac{\sqrt{x^2 - 4}}{x^2} \, dx$$

Find the partial fractions decomposition and an antiderivative

1) $\dfrac{x - 5}{x^2 - 1}$	2) $\dfrac{5x - 2}{x^2 - 4}$
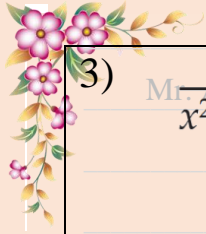
3) $\dfrac{6x}{x^2 - x - 2}$

4) $\dfrac{3x}{x^2 - 3x - 4}$

5) $\dfrac{-x + 5}{x^3 - x^2 - 2x}$

6) $\dfrac{3x + 8}{x^3 + 5x^2 + 6x}$

7) $\dfrac{5x - 23}{6x^2 - 11x - 7}$

8) $\dfrac{3x + 5}{5x^2 - 4x - 1}$

9) $\dfrac{x - 1}{x^3 + 4x^2 + 4x}$

10) $\dfrac{4x - 5}{x^3 - 3x^2}$

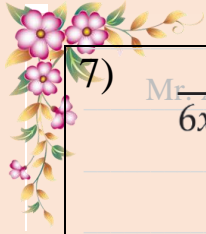| 11) $\dfrac{x+2}{x^3+x}$	12) $\dfrac{1}{x^3+4x}$

Learn differential equations of the form $y'=ky$ and their general solution	التعرف على مفهوم المعادلة التفاضلية $y'=ky$ وحلها العام

Find the solution of the given differential equation satisfying the indicated initial condition.

1) $y'=4y,\quad y(0)=2$	2) $y'=3y,\quad y(0)=-2$
3) $y'=-3y,\quad y(0)=5$	4) $y'=-2y,\quad y(0)=-6$

5) $y' = 2y, \qquad y(1) = 2$

6) $y' = -y, \qquad y(1) = 2$

7) $y' = y - 50, \qquad y(0) = 70$

8) $y' = -0.1y - 10, \qquad y(0) = 80$

1)	If you invest AED 1000 at an annual morabaha rate of 8%, compare the value of the investment after 1 year under the following forms of compounding
annual	
monthly	
daily	
continuous	

2)	If you invest AED 1000 at an annual morabaha rate of 8%, compare the value of the investment after 5 year under the following forms of compounding
annual	
monthly	
daily	
continuous	

3) Person A invests AED 10,000 in 1990 and Person B invests AED 20,000 in 2000.

(a) If both receive 12% morabaha (compounded continuously), what are the values of the investments in 2010?

Person A	Person B

(b) Repeat for an morabaha rate of 4%.

Person A	Person B

(c) Determine the morabaha rate such that person A ends up exactly even with person B. (Hint: You want person A to have AED 20,000 in 2000.)

3) One of the authors bought a set of basketball trading cards in1985 for $34. In 1995, the "book price" for this set was $9800.

(a) Assuming a constant percentage return on this investment, find an equation for the worth of the set at time t years (where $t = 0$ corresponds to 1985).

(b) At this rate of return, what would the set have been worth in 2005?

(c) The author also bought a set of baseball cards in 1985, costing $22. In 1995, this set was worth $32. At this rate of return, what would the set have been worth in 2005?

4) Suppose that the value of a $40,000 asset decreases at a constant percentage rate of 10%. Find its worth after

(a) 10 years and   (b) 20 years.

Compare these values to a $40,000 asset that is depreciated to no value in 20 years using linear depreciation

Find the general solution of separable differential equations of first order	إيجاد الحل العام لمعادلات تفاضلية من الدرجة الأولى قابلة للفصل

The differential equation is separable. Find the general solution, in an explicit form if possible.

1) $y' = (x^2 + 1)y$

2) $y' = 2x(y - 1)$

3) $y' = 2x^2 y^2$

4) $y' = 2(y^2 + 1)$

5) $y' = \dfrac{6x^2}{y(1 + x^3)}$

6) $y' = \dfrac{3x}{y + 1}$

7) $y' = \dfrac{2x}{y} e^{y-x}$

8) $y' = \dfrac{\sqrt{1-y^2}}{x \ln x}$

9) $y' = \dfrac{\cos x}{\sin y}$

10) $y' = x \cos^2 y$

11) $y' = \dfrac{xy}{1 + x^2}$

12) $y' = \dfrac{2}{xy + y}$

تم بحمد الله